Electromagnetic form factors of Heavy-Light mesons

Based on: **Phys.Lett.B 863 (2025) 139384** and **Eur.Phys.J.C 85 (2025) 12, 1390**

Excited QCD 2026, Granada.

Ángel Miramontes
In collaboration with Joannis Papavassiliou, Jan M. Pawlowski and Fei Gao

January 12, 2026.

Introduction

Established progress in light mesons (π, K):

- **Experiment:** Precise data available for the pion.
- **Theory:** Extensive studies using BSE, lattice QCD, quark models, Dispersion relations, [1], effective field theories have achieved high precision.

Heavy–light mesons

- The heavy-light sector, i.e. D, B, D_s, B_s, remains much less tractable [2]. The asymmetry between the heavy and light valence quark masses introduces strong flavour dependence into the effective interaction, which simple approximations do not capture.

[1] Enrique Ruiz Arriola, Pablo Sanchez-Puertas Phys.Rev.D 110 (2024) 5, 054003

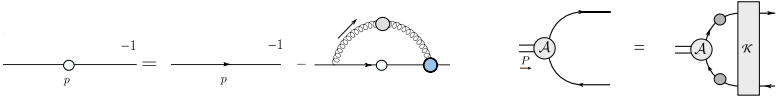
[2] Sofia Leitão, Alfred Stadler, M.T. Peña, Elmar P. Biernat Phys.Rev.D 96 (2017) 7, 074007
M. Gomez-Rocha, T. Hilger, A. Krassnigg, Phys. Rev. D 92 (5) (2015) 054030,
Yin-Zhen Xu JHEP 07 (2024) 118

Dyson-Schwinger, Bethe-Salpeter and Symmetries

- **Nonperturbative Framework**:
  - **SDE**: Quark/gluon propagators and vertices for meson properties.
  - **BSE**: Mesons as quark-antiquark bound states.
  - Can be extended to multiquark states [a]
- **QCD Symmetries**:
  - **Chiral**: Massless pions in chiral limit (Goldstone's theorem).

- **QCD Symmetries**:
  - $U(1)$ **Vector**: Ensures charge conservation for form factors.
- **Truncated Kernels**:
  - Chiral symmetry needs axial-vector Ward-Takahashi identity.
  - Charge conservation requires vector Ward-Takahashi identity.

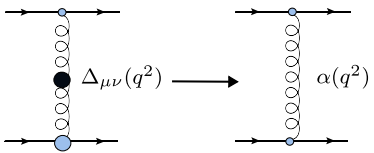
[a] Gernot Eichmann et al. Few Body Syst. 61 (2020) 4, 38



Truncations in Bethe–Salpeter Equations

Rainbow–Ladder (RL) Truncation:

- Simplest truncation[a].
- Dressing effects are collected into an effective interaction.
- Accurate for light ground-state mesons,[b] providing a good description of space-like hadronic form factors.



[a] P. Maris and P. C. Tandy, Phys. Rev. C 61, 045202 (2000)

[b] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer, Prog. Part. Nucl. Phys. 91 (2016) 1–100

Limitations

- Radial excitations,
- Time-like hadronic form factors,
- Heavy–light systems.
- States with $J \geq 2$ [a]

Beyond the Rainbow:

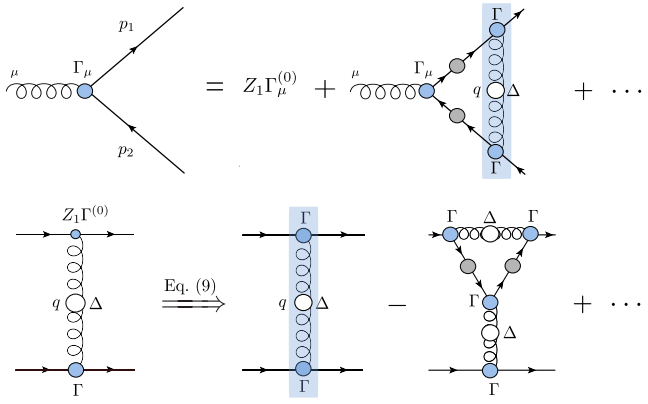
- Truncations of increasing sophistication have been constructed over the years.[b]
- Continuously developed to align with experimental and lattice QCD results.
- Different approaches improve accuracy for heavy–light mesons.[c]

[a] Stephan Hagel, Christian S. Fischer, Markus Q. Huber, Jonathan Y. Yigzaw 2510.27423 [hep-ph]

[b] M. Ferreira, A. S. M, J. M. Morgado Chavez, J. Papavassiliou, and J. M. Pawlowski, : 2512.04853 [hep-ph].
A. S. Miramontes, H. Sanchis Alepuz, R. Alkofer, Phys. Rev. D 103 (11) (2021) 116006.

[c] M. Gomez-Rocha, T. Hilger, A. Krassnigg, Phys. Rev. D 92 (5) (2015) 054030.

Skeleton Expansion of the Quark–Gluon Vertex



The Standard Taylor Charge α_T

Taylor effective charge:

$$\alpha_T(q^2) \;=\; \alpha_s(\mu^2)\, F^2(q^2)\, Z(q^2),$$

where:

- $F(q^2)$: ghost dressing function,
- $Z(q^2)$: gluon dressing function.

Role:

- Used in lattice QCD as a practical way to extract α_s [a].

[a] S. Zafeiropoulos, P. Boucaud, F. De Soto, J. Rodríguez-Quintero, J. Segovia, Phys. Rev. Lett. 122 (16) (2019) 162002

Limitation:

- Flavor-blind: it depends only on gluon and ghost dressings.
- No explicit sensitivity to the quark sector $\Rightarrow$ **all quarks see the same coupling**.
- This is fine for light, but it fails for **heavy–light mesons**, where large mass asymmetries make quark dynamics essential.
- In such cases, the standard α_T cannot reproduce a flavor dependence of the interaction.

Modified Taylor Effective Charge

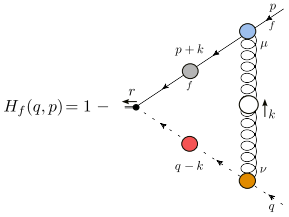
We keep only the classical form factor of the quark-gluon vertex,

$$\Gamma^{\mu}_{f}(q,p) \approx \lambda_f(q^2,p^2,r^2)\,\gamma^{\mu}$$

Determine the form factor from Slavnov-Taylor identity,

$$q^{\mu}\Gamma^{\mu}_{f} = F(q^2)\Big[S_f^{-1}(r)\,H_f(q,p) - \bar{H}_f(-q,r)\,S_f^{-1}(p)\Big]$$

$$\lambda_f = \frac{1}{2}F(q^2)\left[A_f(r^2)(X_0^f - s^2 X_3^f) + B_f(r^2)(X_2^f - X_1^f)\right] + \ldots$$



Ghost-quark kernel

$$H_f(q,p) = X_0^f 1 + X_1^f \not{r} + X_2^f \not{p} + X_3^f \sigma_{\mu\nu} r^{\mu} p^{\nu}\,,$$

Steps:

1. Keep only leading term in kernel: $H_f \simeq X_{0f}$ [a].
2. In symmetric kinematics ($q^2 = p^2 = r^2$):
$$\lambda_f(q^2) = F(q^2)\,A_f(q^2)\,X_{0f}(q^2).$$
3. Take flavor-independent part of $\tilde{X}_0$:
$$\lambda_f(q^2) \approx F(q^2)\,\tilde{X}_0(q^2)\,A_f(q^2).$$

[a] A.C.Aguilar, M.N.Ferreira, C.T.Figueiredo and J.P., Phys. Rev. D 99, no.3, 034026 (2019)

Modified Taylor Effective Charge

Effective charges:

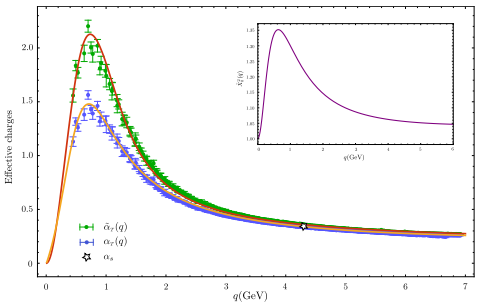
$$\alpha_T(q^2) = \alpha_s Z(q^2) F^2(q^2)$$

$$\tilde{\alpha}_T(q^2) = \alpha_T(q^2)\, \tilde{X}_0^2(q^2)$$

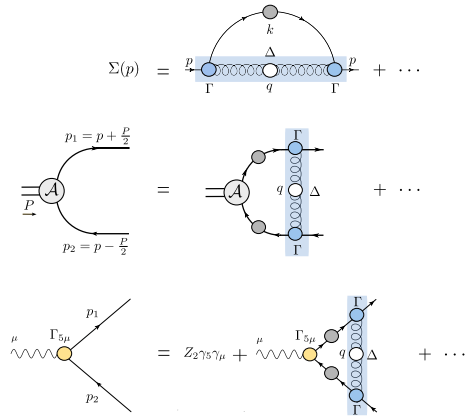
Interaction kernel:

$$I_{ff'}(q^2) = 4\pi\, \tilde{\alpha}_T(q^2)\, A_f(q^2)\, A_{f'}(q^2)$$

- Flavor-dependent via A_f, $A_{f'}$.
- Direct connection with the ingredients of the theory.



Interaction Kernel in DSE and BSE



General Considerations

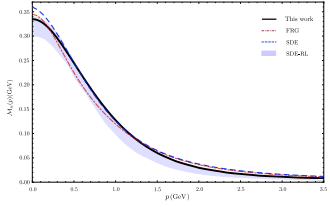
Quark propagator:

$$S_f^{-1}(p) = i\not{p}\, A_f(p^2) + B_f(p^2)$$

Renormalization (MOM, $\mu = 4.3$ GeV):

$$A_f(\mu^2) = 1, \quad B_f(\mu^2) = m_f$$

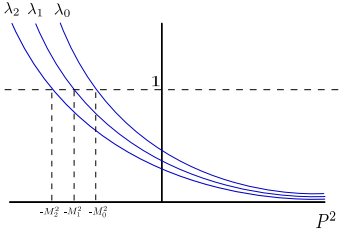
- Fixes Z_2 (wave-function) and Z_m (mass).
- Input current quark mass m_f set at μ.
- **Mass function** $M(p^2) = B(p^2)/A(p^2)$[a]



[a] F. Gao, J. Papavassiliou, J.M. Pawlowski, Phys. Rev. D 103 (9) (2021) 094013.

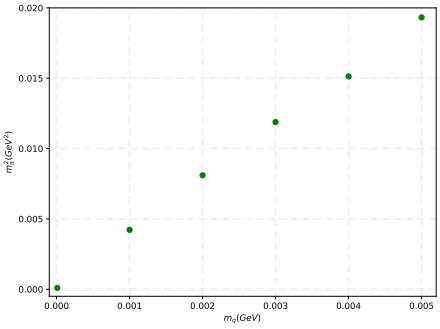
Bethe–Salpeter framework:

- Equations solved in Euclidean space.
- BSE $\rightarrow$ eigenvalue problem.
- Mass-shell condition: $P_n^2 = -M_n^2$.
- M_0: ground state, M_n $(n \geq 1)$: radial excitations.



Chiral limit

- In the chiral limit, the pion mass approaches zero, consistent with Goldstone's theorem and dynamical chiral symmetry breaking in QCD.



Numerical results: masses

mass (GeV)	π	K	ρ	K^*	ϕ	D	D^*	D_s	D_s^*	B	B^*	B_s	B_s^*	B_c	B_c^*	η_c	J/Ψ	η_b	Υ
This work	0.139†	0.495†	0.735	0.916	1.06	1.93	2.06	2.03	2.17	5.33	5.37	5.45	5.48	6.36	6.40	2.97†	3.13	9.50†	9.57
Experiment	0.139	0.494	0.775	0.890	1.02	1.87	2.01	1.97	2.11	5.28	5.33	5.37	5.42	6.28		2.98	3.10	9.40	9.46
% deviation			5.1	2.9	3.9	3.2	2.5	3.0	2.8	0.9	0.7	1.5	1.1	1.3			1.0		1.1
Lattice						1.86	2.01	1.97	2.11	5.28	5.32	5.37	5.42	6.26	6.33		3.12		
Weighted-RL	0.138	0.495	0.749	0.872	1.09	1.77	1.99	1.98	2.20	5.32	5.50	5.48	5.63	6.39	6.54	2.98	3.12	9.40	9.50

Numerical results: decay constants

f(GeV)	π	K	ρ	K^*	ϕ	D	D^*	D_s	D_s^*	B	B^*	B_s	B_s^*	B_c	B_c^*	η_c	J/Ψ	η_b	Υ
This work	0.130[†]	0.158	0.205	0.220	0.244	0.201	0.218	0.249	0.270	0.201	0.197	0.235	0.231	0.505	0.492	0.355	0.433	0.762	0.738
Experiment	0.130	0.155	0.212	0.225	0.236	0.196		0.236		0.211						0.340	0.415		0.715
% deviation		1.9	3.3	2.2	3.3	2.6		5.5		4.7						4.4	4.0		3.2
ETM						0.208	0.223	0.247	0.268	0.193	0.186	0.229	0.223						
HPQCD	0.132	0.157				0.207		0.241		0.189		0.231		0.427	0.422	0.395	0.404		
Weighted-RL	0.132	0.160	0.211	0.253	0.269	0.239	0.281	0.300	0.362	0.300	0.348	0.350	0.400	0.606	0.683	0.391	0.419	0.790	0.744

Heavy-light electromagnetic form factors

Impulse Approximation

Electromagnetic form factors are extracted from the meson current in the impulse approximation, where the photon couples directly to a quark.

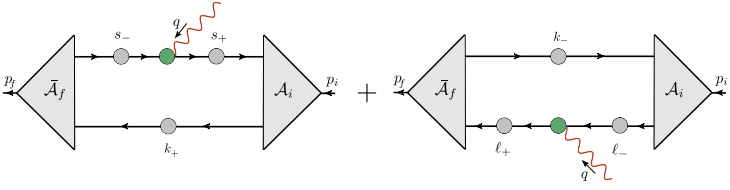
Current:

$$J^{\mu}(\bar{p}, q) = 2\,\bar{p}^{\mu}\,F(q^2)$$

Extraction of the form factor:

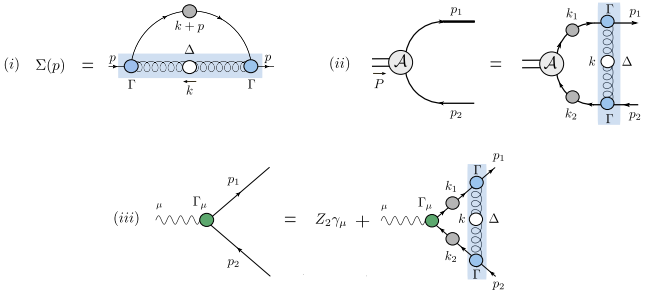
$$F(q^2) = \frac{J \cdot \bar{p}}{2\,\bar{p}^2}$$

- q = momentum transfer, $\bar{p}$ = average meson momentum.
- Form factor $F(q^2)$ fully determined by evaluating J^{μ}.



Quark–photon vertex Γ^μ: Key ingredient for computing the meson electromagnetic current and form factors.

- Satisfies an inhomogeneous Bethe–Salpeter equation.
- Ensures electromagnetic gauge invariance via the Ward–Takahashi identity.



- For π and K mesons, our results are in very good agreement with experimental data. This provides a nice benchmark for the reliability of the method before extending it to heavier systems.

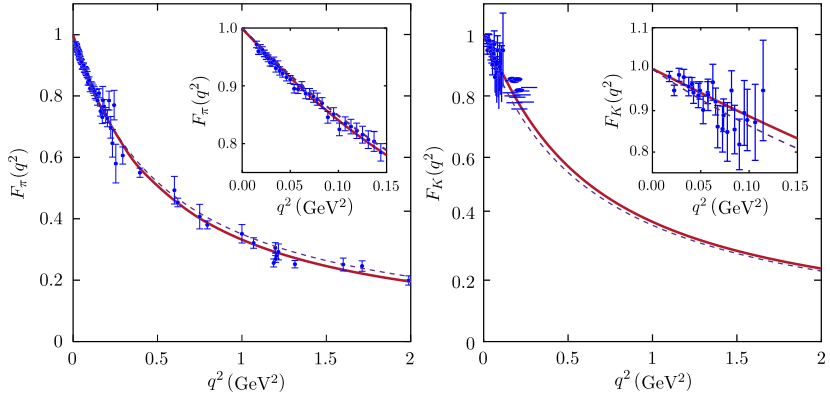
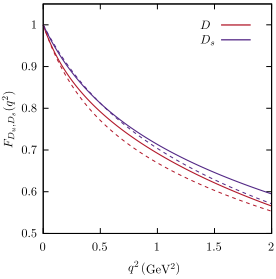
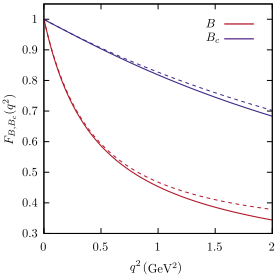


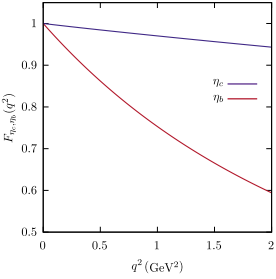
Figure 1: Computed EMFF (solid lines) compared to RL results (Dashed lines)[3]

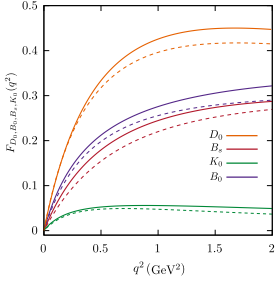
[3] Y.-Z. Xu, JHEP 07, 118 (2024)

Heavy-Light mesons









Charge Radius Predictions

Radius (fm)	π	K	K^0	D	D_0	D_s
This work	0.656(5)	0.568(4)	$0.270i$	0.428	$0.542i$	0.368
Experiment	0.659(4)	0.560(30)	$0.277(18)i$	–	–	–
LQCD [4]	0.656(11)	–	–	0.450(24)	–	0.465(57)
Data driven [5]	0.655(18)	0.599(4)	$0.245(8)i$	–	–	–
Weighted-RL [6]	0.646	0.608	$0.253i$	0.435	$0.556i$	0.352
Light front [7]	0.668(35)	0.610(20)	$0.302(22)i$	0.411(15)	$0.534(26)i$	0.301(11)
AM [8]	–	–	–	0.680		0.372
CI [9]	0.45	0.42	–	–	–	0.26

[4] G. Wang et al, Phys. Rev D 104, 074502 (2021).
[5] D. Stamen et al, Eur. Phys. J. C 82, 432 (2022)
[6] Y.-Z. Xu, JHEP 07, 118 (2024).
[7] S. Xu, et al. arXiv:2507.07523 [hep-ph]
[8] B. Almeida-Zamora. Phys. Rev. D 109, 014016 (2024).
[9] R. J. Hernandez-Pinto. Phys. Rev. D 107, 054002 (2023)

Charge radii

Radius (fm)	B	B_0	B_s	B_c	η_c	η_b
This work	0.631	$0.442i$	$0.330i$	0.213	0.267	0.082
Weighted-RL [10]	0.619	$0.435i$	$0.337i$	0.219	–	–
Lattice [11]	0.682(21)	–	–	–	–	–
Light front [12]	0.564(22)	$0.396(16)i$	$0.281(13)i$	0.189(10)	–	–
AM [13]	0.926	–	$0.345i$	0.217	–	–
CI [14]	0.34	$0.36i$	$0.24i$	0.17	0.20	0.07

[10] Y.-Z. Xu, JHEP 07, 118 (2024).
[11] Ivan Vujmilovic 2510.17549 [hep-lat]
[12] S. Xu, et al. arXiv:2507.07523 [hep-ph]
[13] B. Almeida-Zamora. Phys. Rev. D 109, 014016 (2024).
[14] R. J. Hernandez-Pinto. Phys. Rev. D 107, 054002 (2023)

Conclusions

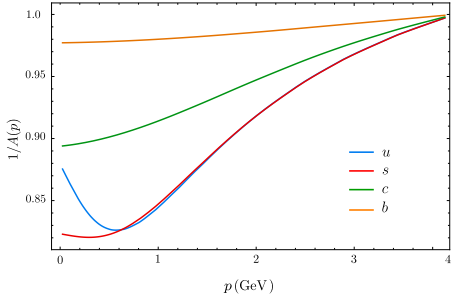
- Developed a flavour-sensitive effective charge for QCD interactions.
- Achieved very good agreement with experiment for meson spectra, decay constants, and charge radii.
- Extended framework to heavy–light mesons, improving upon rainbow–ladder truncation results.

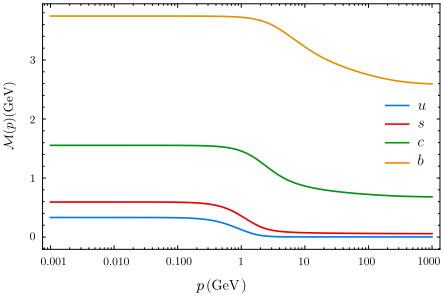
Results: Quark Propagator

- $A_f^{-1}(p^2)$ is shown for $f = u, s, c, b$.
- Distinct behavior is observed across quark flavors:

- $M_f(p^2) = B_f(p^2)/A_f(p^2)$ for $f = u, s, c, b$.
- The mass hierarchy at $p^2 = 0$:

$$M_u(0) < M_s(0) < M_c(0) < M_b(0).$$





Modified Taylor Effective Charge

The modified effective charge $\tilde{\alpha}_T(q^2)$ is constructed from lattice QCD ghost and gluon propagator data. To facilitate numerical calculations, it is parametrized as:

Parametrization

$$\tilde{\alpha}_T(q^2) = \frac{a_0 q^2 + a_1 q^4 \ln\left(1 + \frac{\Lambda_0^2}{q^2}\right) + a_2 q^4}{1 + a_3 q^2 + a_4 q^4 + a_5 q^6} + \frac{4\pi q^6}{\beta_0 \left(\Lambda_0^6 + q^6 \ln \frac{q^2}{\Lambda_T^2}\right)}$$

Parameter values:

- $a_0 = 10.35\ \text{GeV}^{-2}$, $a_1 = 23.69\ \text{GeV}^{-4}$, $a_2 = 27.94\ \text{GeV}^{-4}$
- $a_3 = 10.72\ \text{GeV}^{-2}$, $a_4 = -2.5\ \text{GeV}^{-4}$, $a_5 = 29.02\ \text{GeV}^{-6}$
- $\Lambda_0 = 1\ \text{GeV}$, $\Lambda_T = 0.5\ \text{GeV}$

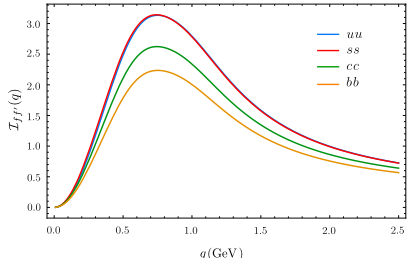
Results: Flavour dependent interaction

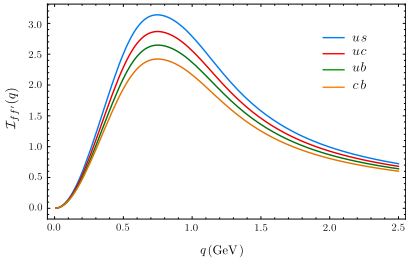
- **Diagonal Elements ($f = f'$):** The interaction strength decreases with increasing quark mass,

$$\mathcal{I}_{ff'}(q^2) = \tilde{\alpha}_T(q^2) A_f(q^2) A_{f'}(q^2)$$

$$I_{uu}(q^2) \approx I_{ss}(q^2) > I_{cc}(q^2) > I_{bb}(q^2)$$

- **Off-Diagonal Elements:** Mixed interactions, such as $I_{cu}(q^2)$, lie between $I_{uu}(q^2)$ and $I_{cc}(q^2)$, reflecting the hybrid nature of heavy-light systems.





Tensor Basis for Mesonic BSA

The Bethe-Salpeter amplitude for mesons can be expressed in terms of a Dirac tensor basis, tailored to the quantum numbers of the system.

For pseudoscalar mesons, such as pions, the tensor basis includes four elements and can be written as:

$$\tau_i^{\mathrm{PS}} = \gamma_5 \left\{ \mathbb{I}, -i\not{P}, -i(p \cdot P)\not{p}, [\not{p}, \not{P}] \right\}$$

where:

- P is the total momentum of the meson.
- p is the relative momentum between the quark and antiquark.

For vector mesons, like the ρ, the Dirac basis is more extensive, consisting of eight elements, reflecting the higher spin complexity.

Numerical Considerations: Cauchy Interpolation Method

The Cauchy interpolation method is a robust approach for solving the quark DSE in the complex plane. It relies on the Cauchy integral formula for evaluating functions at any point inside a closed contour γ:

Cauchy Integral Formula:

$$f(z_0) = \frac{1}{2\pi i} \oint_\gamma \frac{f(z)}{z - z_0}\, dz$$

To enhance numerical stability, the formula can be reformulated as:

$$f(z_0) = \oint_\gamma \frac{f(z)}{z - z_0}\, dz \Bigg/ \oint_\gamma \frac{dz}{z - z_0}$$

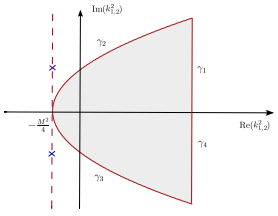
Decomposing the Contour: The closed contour γ is divided into multiple segments γ_j (with $j = 1, 2, 3, \ldots$), allowing parametrization of the integration. The final expression becomes:

$$f(z_0) = \sum_{\gamma_j} \int \frac{f(z_j(t))}{z_j(t) - z_0} z_j'(t)\, dt \Bigg/ \sum_{\gamma_j} \int \frac{z_j'(t)}{z_j(t) - z_0}\, dt$$

Cauchy Method: Integration Paths

The Cauchy method employs a closed parabola contour in the complex plane, divided into four segments for numerical integration:

- γ_1 and γ_2: Define the upper part of the parabola.
- γ_3 and γ_4: Define the lower part of the parabola.



This segmentation allows efficient parametrization of the integration paths, ensuring numerical stability and precision when solving the quark DSE.

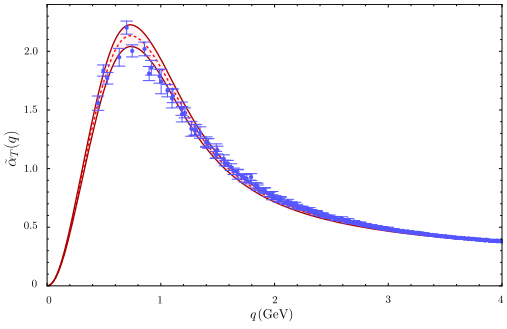
Singularities in the Quark Propagator

The Cauchy interpolation method's applicability is limited by the presence of complex conjugate poles in the quark propagator. These singularities influence the range of bound state masses that can be extracted.

- The position of the poles and the maximum bound state mass depend on the current quark masses, m_f.
- For light mesons:
  - The bound state mass lies within the range of the Cauchy parabola.
  - Accurate computation is feasible.
- For heavy-light mesons:
  - The bound state mass often exceeds the method's range of applicability.
  - Direct computation becomes unreliable.

Error analysis

The resulting masses for the pion and kaon are $m_\pi = 0.139(2)$ GeV and $m_K = 0.495(2)$ GeV. The final propagation of all these errors at the level of the EFF for the pion and kaon is shown in the bands of Figure 9. The extracted charge radii, $r_\pi = 0.656(5)$ fm and $r_K = 0.568(4)$ fm.



Decay Constants

The leptonic decay constants for pseudoscalar and vector mesons can be computed using the following expressions:

Pseudoscalar Mesons:

$$f_{\text{ps}} m_{\text{ps}}^2 = \sqrt{N_c} Z_2 \text{Tr} \int_k \gamma^5 \not{P} S(k_1) \mathcal{A}(k, P) S(k_2)$$

Vector Mesons:

$$f_{\text{v}} m_{\text{v}} = \frac{\sqrt{N_c}}{3} Z_2 \text{Tr} \int_k \gamma_\mu S(k_1) \mathcal{A}_\mu(k, P) S(k_2)$$

- $k_1 = q + P/2$: Momentum of the quark.
- $k_2 = q - P/2$: Momentum of the antiquark.

These expressions involve the quark propagator $S(k)$, the Bethe-Salpeter amplitude $\mathcal{A}(k, P)$, and the total meson momentum P. The decay constants f_{ps} and f_{v} are crucial for connecting theoretical predictions with experimental data.

Fit of the quark masses

- Since the pion mass lies within the computable region of the propagator, the quark mass u/d can be easily adjusted to reproduce the experimental value.
- In the case of the η meson, its mass lies in a region where the quark propagator cannot be directly computed. Therefore, for that region, the Ansatz containing a pair of complex conjugate poles is used.
- The current quark masses obtained through this procedure are $m_{u/d} = 0.005$ GeV, $m_s = 0.094$ GeV, $m_c = 1.1$ GeV, and $m_b = 3.5$ GeV, at $\mu = 4.3$ GeV.

Determination of $\tilde{X}_0(q^2)$

- The determination of $X_0^f(q^2)$ requires knowledge of the quark propagator, implying a coupling to the quark gap equation.
- However, $X_0^f(q^2)$ contains a **flavour-independent part**, denoted $\tilde{X}_0(q^2)$.
- This contribution emerges when the longitudinal part $k_\mu k_\nu$ of the gluon propagator $\Delta_{\mu\nu}(k)$ contracts with the quark–gluon vertex (blue circle), triggering the STI of Eq. (refeq:STI).
- After setting $H = 1$, the STI yields a term $F(k)\, S_f^{-1}(k+p)$, which cancels the internal quark propagator $S_f(k+p)$, leaving the f-independent $\tilde{X}_0(q^2)$.

Flavour-independent contribution

$$\tilde{X}_0(q^2) \;=\; 1 - \frac{1}{2} g^2 N_c \int_k (k\cdot q)\, \Delta(k^2)\, D(k^2)\, B_1(k^2)\, D(t^2)$$

with $t = k + q$, $N_c = 3$, and $B_1(k^2)$ the tree-level form factor of the ghost–gluon vertex, renormalized in the Taylor scheme ($Z_{cg} = 1$) and evaluated in the symmetric configuration.

Axial Ward–Takahashi Identity (WTI) & Effective Kernel

- **WTI for axial-vector vertex:**

$$P^{\mu}\Gamma_{5\mu}^{ff'}(p_1,p_2) = S_f^{-1}(p_1)i\gamma_5 + i\gamma_5 S_{f'}^{-1}(p_2) - i(m_f + m_{f'})\Gamma_5^{ff'}(p_1,p_2)$$

- For equal flavours ($f = f'$): – Kernel $I_{ff}(q^2)$ preserves WTI. – Ensures dynamical chiral symmetry breaking.
- For unequal flavours ($f \neq f'$): – Flavour-dependent kernel $I_{ff'}(q^2)$ violates WTI. – Leads to mismatches in self-energy terms $\Sigma_f, \tilde{\Sigma}_f$. – Interpretation as genuine self-energies lost.
- **Quantification of violation:** – Compare quark propagators from diagonal vs. off-diagonal kernels. – Max errors: $\sim 3\%$ in $A(p^2)$, $\sim 9\%$ in $M(p^2)$ (for u–b system).
- **Impact:** – Example: D-meson mass shifts from 1.93 GeV (diagonal) to 1.99 GeV (off-diagonal). – Shows modest but controlled violation effect.

Excited States: Radial Excitations of Pion and Kaon

The first radial excitations of the pion and kaon were computed using the solutions of the Bethe-Salpeter equation.

Results:

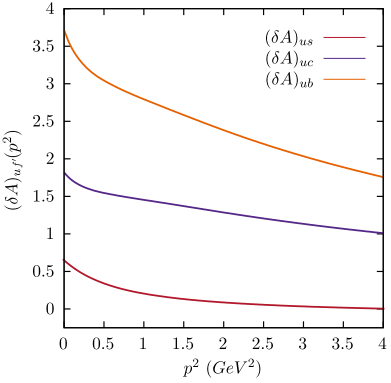
- Pion excitation mass: $m_{\pi_1} = 1.24$ GeV (compared to the experimental value $m_{\pi_1}^{\text{exp}} = 1.30$ GeV).
- Kaon excitation mass: $m_{K_1} = 1.30$ GeV (compared to the experimental value $m_{K_1}^{\text{exp}} = 1.46$ GeV).

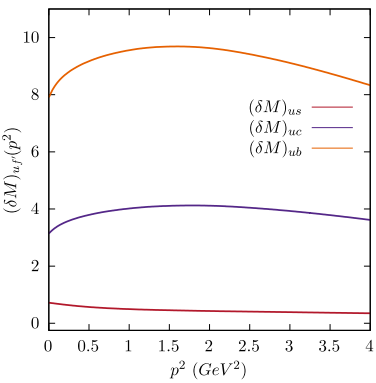
Observations:

- The computed masses show good agreement with experimental data.
- In conventional RL truncation, the mass hierarchy between pion and kaon excitations is often inverted. Usually corrected when ACM is included [15]
- This study restores the correct hierarchy.

[15] Z.-N. Xu, Z.-Q. Yao, S.-X. Qin, Z.-F. Cui, C. D. Roberts, Eur. Phys. J. A 59 (3) (2023) 39. S.-X. Qin, C. D. Roberts, Chin. Phys. Lett. 38 (7) (2021) 071201.

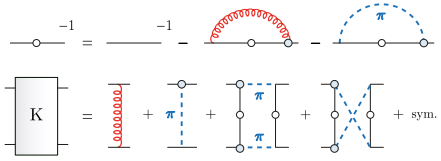
Beyond chiral limit



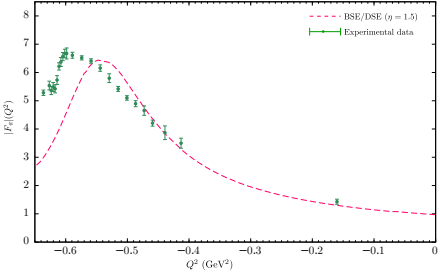


Progress on Time-Like Electromagnetic Form Factors

- Focus: **time-like pion form factor** $F_\pi(q^2)$ in continuum QCD.
- Method: Bethe–Salpeter + Dyson–Schwinger equations with **resonant contributions** in the kernel.
- Emergence of the ρ-meson pole mass and decay width.
- Connects **resonant structure** with underlying quark–gluon dynamics.

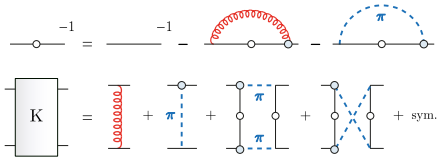


Truncation scheme for time-like dynamics.

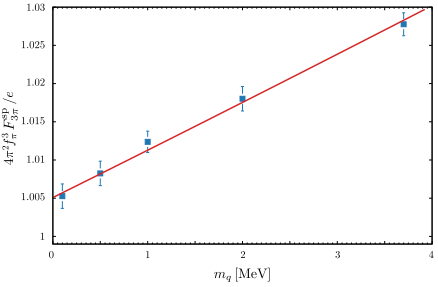


Timelike $\gamma^*\pi \to \pi\pi$ Form Factor

- Dyson–Schwinger + Bethe–Salpeter equations, beyond rainbow–ladder with **pion back-coupling**.
- Dynamical generation of the ρ resonance in the quark–photon vertex.
- Prediction: **nearly angle-independent** $F_{3\pi}(s, z)$ at low s, rising strongly toward m_ρ.
- Relevant for COMPASS Primakoff measurements and $(g-2)_\mu$ hadronic contributions.



Truncation scheme for time-like dynamics.

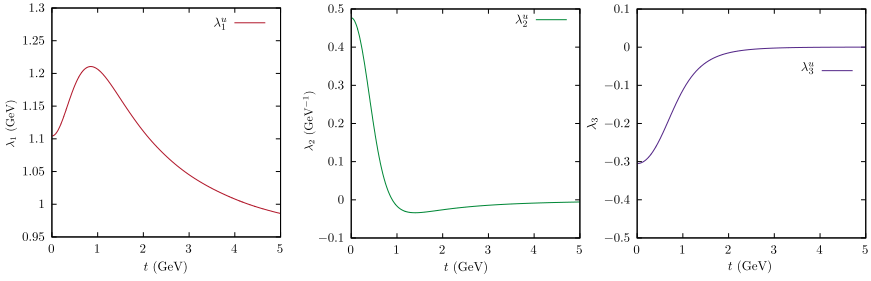


Structure of the Quark–Photon Vertex

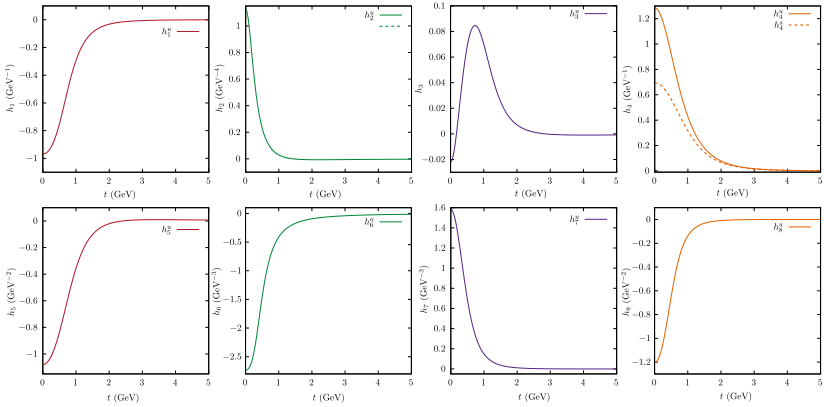
Decomposition:

$$\Gamma^\mu(p_1, p_2) = \underbrace{\Gamma^\mu_{\mathrm{BC}}(p_1, p_2)}_{\text{Ball–Chiu (longitudinal)}} + \underbrace{\Gamma^\mu_{\mathrm{T}}(p_1, p_2)}_{\text{Transverse}}$$

- **Ball–Chiu part**: Fully determined by the quark dressing functions $A(p^2)$, $B(p^2)$. Encodes dynamical chiral symmetry breaking.
- **Transverse part**: Contains 8 form factors in a covariant tensor basis. Hosts **vector meson poles** (ρ, ϕ, ...) in the timelike region. To reproduce the correct behavior in this regime, decay mechanism must be incorporated!


λ_1^u
λ_1 (GeV)
t (GeV)
λ_2^u
λ_2 (GeV^{-1})
t (GeV)
λ_3^u
λ_3
t (GeV)

QPhV II: Transverse components



Charge Radii: Light Mesons

Comparison with experiment and models:

- Our results for π^+, K^+, and K^0 agree very well with experiment: deviations of 0.45%, 1.42%, and 2.5%, respectively.
- **CI model:** underestimates radii (π: 0.450 fm, K: 0.420 fm), significantly below both experiment and our predictions.
- **Algebraic model:** close for kaons, but tends to overestimate other states.
- **LQCD and Data-driven:** generally consistent with our values, though uncertainties remain.

Charge Radii: Heavy–Light Mesons

Comparison with theoretical approaches:

- D and D_s:
  - Our results close to LQCD values.
  - AM predicts much larger D radius (0.680 fm vs. 0.428 fm ours, 0.450 fm LQCD).
  - Unlike our framework, AM shows strong D–D_s splitting; we find them similar.
- B and B_s:
  - Our results (0.631 fm, 0.330 fm) agree well with weighted-RL (0.619 fm, 0.337 fm).
- General trends:
  - CI systematically underestimates heavy–light radii (e.g. D_s: 0.260 fm vs. 0.368 fm ours).
  - LQCD is the only approach giving $r_{D_s} > r_D$, but with large errors.