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Chiral-scale effective field theory for dense and thermal systems

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Excited QCD 2026 Workshop, Granada, Spain, 2026.01.08-01.14

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Outline

I. Introduction

II. Basics of Chiral-scale EFT

III. Nuclear matter properties (conformality and peak of sound velocity)

IV. Chiral-scale density counting (CSDC) rule for dense and thermal systems

V. Summary and conclusion

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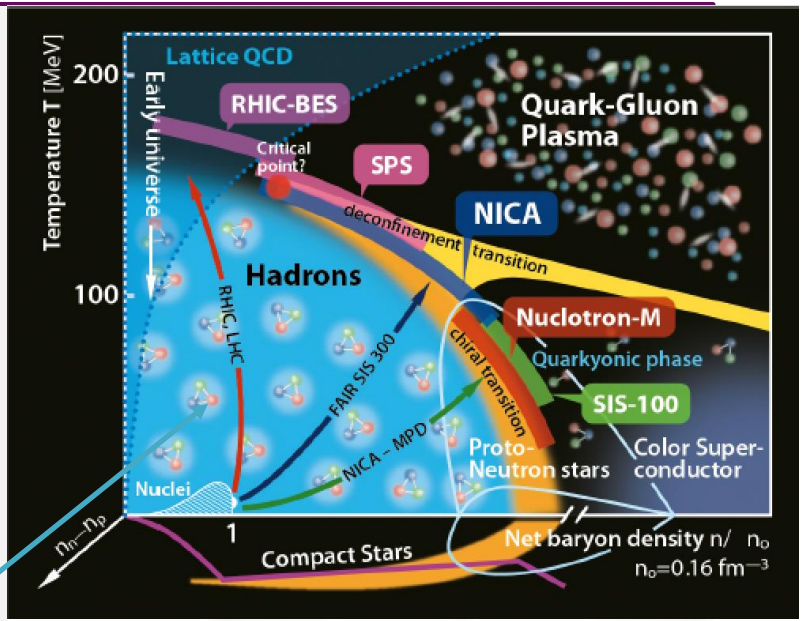
I. Introduction

- The phase structure of strong interaction matter, significant but still under debate;
- Experiments and observations (more and precise constraints):

  heavy-ion collisions (RHIC, LHC), massive neutron stars, GW detection, etc.
- More realistic theory in stead of crude model estimation is indispensable.



- **Universe 4 (2018) 3, 52, NA61/SHINE Collaboration**

In this talk, I will concerning the effective theory approach to the hadronic matter, with QCD symmetries, power counting.

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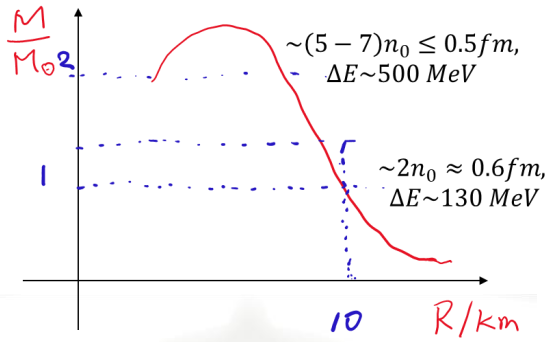
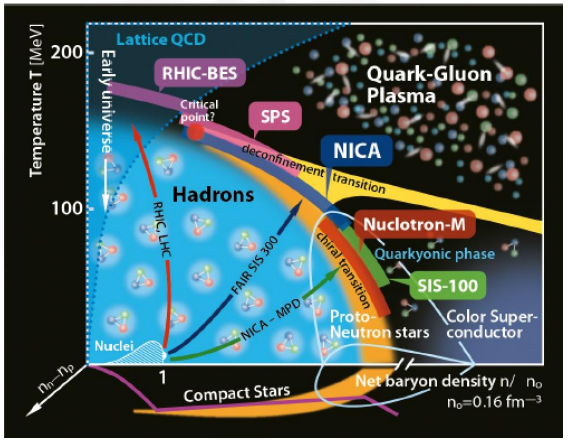
I. Introduction

- Construction of an effective field theory works in extreme environment:
  - Compact star matter density;

    $n\sim(5\sim10)n_0$
  - Temperature below deconfinement phase transition

    $T\sim200$ MeV





➢ Universe 4 (2018) 3, 52, NA61/SHINE Collaboration

I. Introduction

- Key factors to be considered:

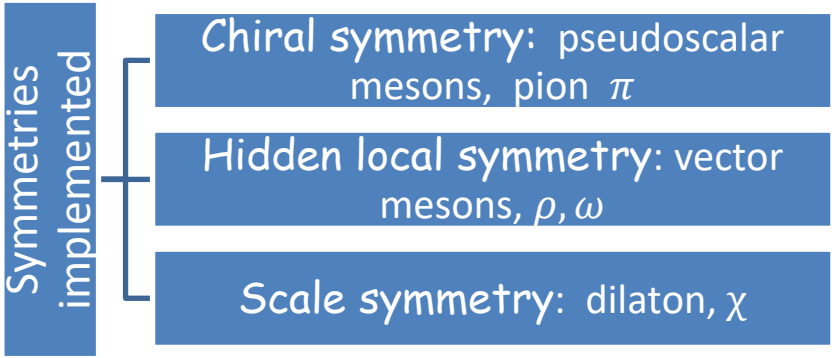
1. Indispensable degrees of freedom:

   $\sigma, \pi, \rho, \omega, N, \cdots$

2. Self-consistent power counting

   (errors can be estimated).

3. Symmetries of QCD



EFT of QCD, real QCD in the sense of symmetry and power counting.

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I. Introduction

- ChPT of pion and rho mesons, HLS

$$SU(N_f)_L \times SU(N_f)_R$$

Absence of parity partner

$$SU(N_f)_V + ({N_f}^2 - 1)\ \text{NGBs}$$
$$U(x) = e^{i\pi/f_\pi}$$

Chiral perturbation theory: power counting, derivative expansion

Weinberg, 79; Gasser and Leutwyler, 84

Redundancy in the decomposition

$$U(x) = \xi_L \; h(x)\, h(x)^\dagger \; {\xi_R}^\dagger$$

$K = 1$: π, ρ

Effective theory of vector mesons, Hidden local symmetry

Bando, *et al* 89; Harada & Yamawaki, 03

Idea of HLS: A NLσM based on the manifold G/H is gauge equivalent to a model having the symmetry $G_{globla} \times H_{local}$. After SB of H_{local}, the gauge boson of H_{local} is identified as the massive vector mesons. Integrating out the vector mesons from this model one can get the ChPT for pion only.

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I. Introduction

- Parameterization:

$$\xi_{L,R} = e^{i\sigma/f_\sigma} e^{\pm i\pi/f_\pi} \to H(x)\, \xi_{L,R}\, g^\dagger_{L,R}\,, \quad \hat{\alpha}_{\perp\mu} = \frac{1}{2i}\left(\partial_\mu \xi \cdot \xi^\dagger - \partial_\mu \xi^\dagger \cdot \xi\right)$$

$$\hat{\alpha}_{\parallel\mu} = \frac{1}{2i}\left(\partial_\mu \xi \cdot \xi^\dagger + \partial_\mu \xi^\dagger \cdot \xi\right) - g_\rho \rho_\mu - g_\omega \omega_\mu$$

$$\xi = e^{-i\pi/f_\pi}$$

$$\hat{\alpha}^\mu_{\perp,\parallel} \to h\, \hat{\alpha}^\mu_{\perp,\parallel}\, h^\dagger$$

- Particles:

$V_\mu = V^a_\mu T^a$ ⋯ HLS gauge boson

$\pi = \pi^a T^a$ ⋯ NGB of CSB

$\sigma = \sigma^a T^a$ ⋯ NGB of $[SU(N_f)_V]_{local}$ SB

$$\mathcal{L}_{\text{HLS}} = F_\pi^2\, \text{tr}\left[\hat{\alpha}^\mu_\perp \hat{\alpha}_{\perp\mu}\right] + a F_\pi^2\, \text{tr}\left[\hat{\alpha}^\mu_\parallel \hat{\alpha}_{\parallel\mu}\right] - \frac{1}{2g^2}\text{tr}\left[V_{\mu\nu} V^{\mu\nu}\right]$$

Three parameters:

- F_π ⋯ pion decay constant
- g ⋯ gauge coupling of HLS
- a ⋯ validity of the vector dominance

Power counting: $O(p) \sim O(m_\pi) \sim O(g_{\rho,\omega}) \sim O(m_{\rho,\omega})$

I. Introduction

- Both chiral symmetry breaking and HLS are based on the nonlinear realization of chiral symmetry, the 4th component of the chiral four-vector is integrated out.
- How to include the sigma meson (0^{++}) field in ChPT/HLS?

- Trace anomaly as the source：

anomaly match

Trace anomaly in QCD ⟷ Trace anomaly EFT

$$\theta_\mu^\mu = \frac{\beta(g)}{2g} G_{\mu\nu} G^{\mu\nu} \propto \chi^4 \quad \longleftrightarrow \quad \delta_D V(\chi) \propto \chi^4$$

Dilaton compensator;
A source of scalar meson in EFT

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I. Introduction

PHYSICAL REVIEW D VOLUME 2, NUMBER 4 15 AUGUST 1970

Broken Chiral and Conformal Symmetry in an Effective-Lagrangian Formalism

C. J. ISHAM, ABDUS SALAM,* AND J. STRATHDEE
International Atomic Energy Agency
and
UNESCO, International Centre for Theoretical Physics, Miramare, Trieste, Italy
(Received 13 April 1970)

The simultaneous breaking of conformal and chiral symmetry is investigated within the framework of nonlinear realizations and effective Lagrangians. The explicit introduction of a massless dilaton field, χ enables conformal invariance to be preserved in Lagrangians for massive matter fields. It is shown that the

7.A.2 Nuclear Physics B22 (1970) 478–492. North-Holland Publishing Company

ASPECTS OF CONFORMAL SYMMETRY AND CHIRALITY

John ELLIS
Department of Applied Mathematics and Theoretical Physics,
University of Cambridge, England

Received 15 June 1970

Abstract: Two simple Lagrangian models combining conformal symmetry and current algebra are exhibited. Either both symmetries are realized linearly, or both symmetries are realized in the Goldstone manner. The scalar isoscalar to pseudoscalar meson couplings differ from those obtained by naive low-energy theor-

PHYSICAL REVIEW D VOLUME 21, NUMBER 12 15 JUNE 1980

Effective Lagrangian with two color-singlet gluon fields

Joseph Schechter
Physics Department, Syracuse University, Syracuse, New York 13210
(Received 10 December 1979)

An effective Lagrangian with scalar and pseudoscalar "matter" fields as well as scalar and pseudoscalar (gauge-invariant) color-singlet gluon fields is constructed. It is a representation of quantum chromodynamics in that it has the same symmetry structure as that theory. The present model is a generalization of a one-(pseudoscalar) gluon-field effective Lagrangian which compactly summarizes the results of the $1/N_c$ approximation, gives a simple picture for the "U(1) problem," and essentially reduces to a type of generalized σ model. The new feature here is that, in addition to the chiral-anomaly equation, the trace-anomaly equation is automatically fulfilled. This gives a framework for discussion of the properties of a 0^+ color-singlet gluon field. The model may be formulated either so that this field is eliminated by a constraint

- Trace anomaly in QCD

$$\partial_\mu D^\mu = \frac{\beta(\alpha_s)}{4\alpha_s} Tr\,(G_{\mu v} G^{\mu v})$$

A possible way for a scalar meson.

Anomaly matching: $\theta_\mu^\mu = -\frac{m_\sigma^2}{4 f_\sigma^2}\chi^4$

How to construct an EFT with power counting mechanism like, e.g., NGB in ChPT?

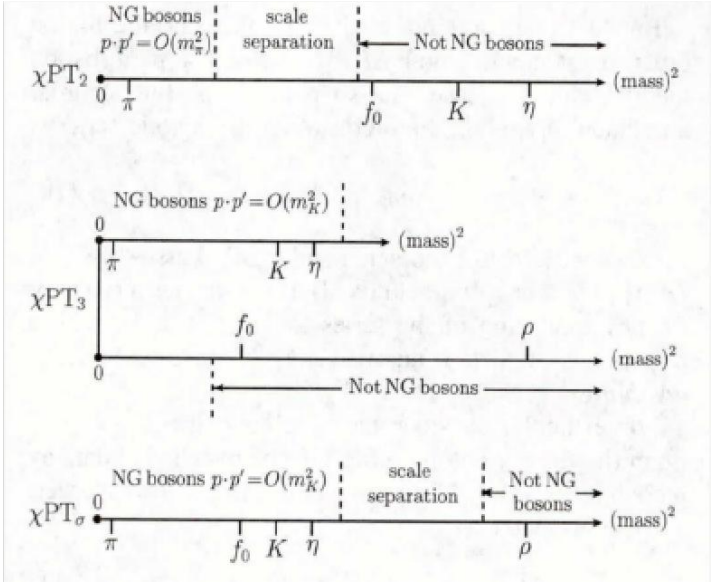
II. Basics of Chiral-scale EFT

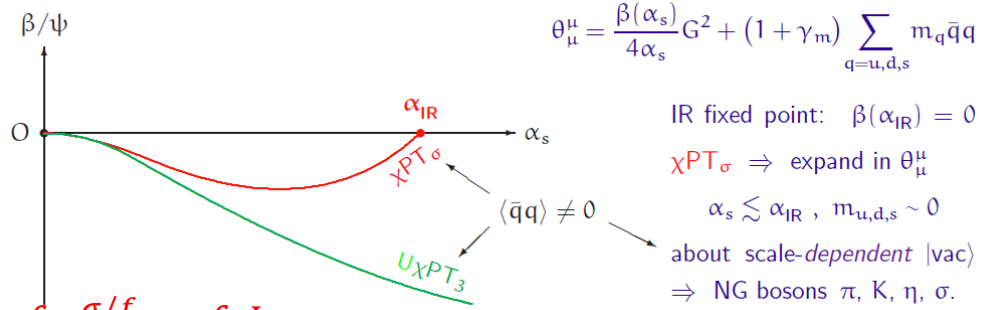
- Phenomenological consideration

$f_0(500)$ is a pNGB arising from (noted $m_{f_0} \cong m_K$). The SB of SS associated + an explicit breaking of SI.

Assumption: There is an Nonperturbative IR fixed point in the running QCD coupling constant α_s.

EB of SI: Departure of α_s from IRFP + current quark mass.





$$\chi = f_\chi e^{\sigma/f_\chi} \equiv f_\chi \Phi$$

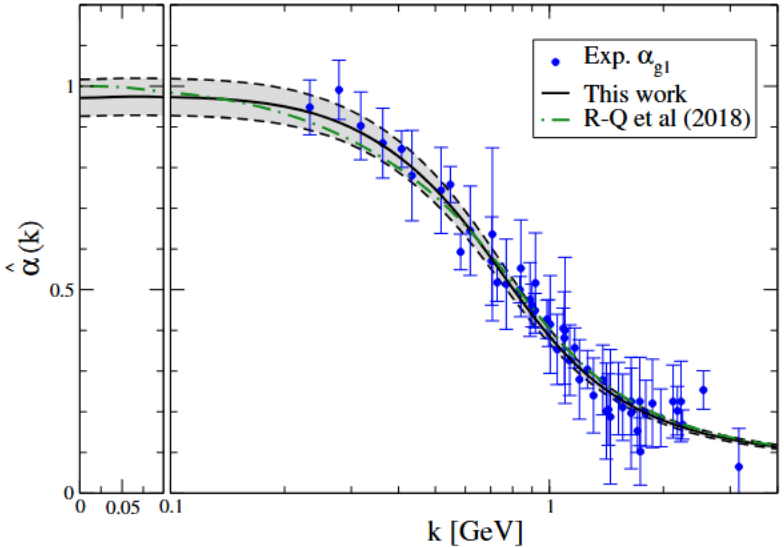
To keep chiral expansion consistent: $f_\pi \sim f_\chi$

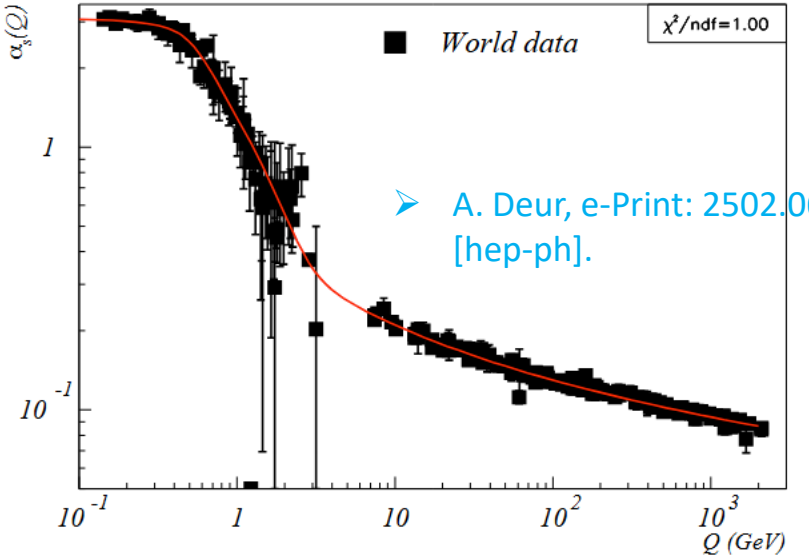
$$m_\sigma^2 \sim \Delta\alpha_s = \alpha_s - \alpha_{IR}, m_q$$

- **Crewther and Tunstall , PRD91(2015), 034016, e-Print: 1312.3319**

II. Basics of Chiral-scale EFT

- Existence of IRFP, theory and data.





- A. Deur, e-Print: 2502.06535 [hep-ph].

- Z. F. Cui, J. L. Zhang, D. Binosi, et al, e-Print: 1912.08232 [hep-ph].
  - The absence of Q^2-dependence of α_s in the IR, i.e., the vanishing QCD β-function, is variously named the freezing of α_s, the conformal window of QCD, or the $Q^2 = 0$ fixed point.

- A. Deur, e-Print: 2502.06535 [hep-ph].

II. Basics of Chiral-scale EFT

$$\mathcal{L} = \mathcal{L}^{M}_{\chi PT_\sigma}(\pi,\chi,V_\mu) + \mathcal{L}^{B}_{\chi PT_\sigma}(\psi,\pi,\chi,V_\mu) - V(\chi)$$

$$\mathcal{L}^{M}_{\chi PT_\sigma}(\pi,\chi,V_\mu) = f_\pi^2\left(\frac{\chi}{f_\sigma}\right)^2 \mathrm{Tr}[\hat{a}_{\perp\mu}\hat{a}^\mu_\perp] + a f_\pi^2\left(\frac{\chi}{f_\sigma}\right)^2 \mathrm{Tr}[\hat{a}_{\parallel\mu}\hat{a}^\mu_\parallel] + \frac{1}{2g^2}\mathrm{Tr}[V_{\mu\nu}V^{\mu\nu}] + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi$$

$$\mathcal{L}^{B}_{\chi PT_\sigma}(\psi,\pi,\chi,V_\mu) = \mathrm{Tr}(\bar{B}i\gamma_\mu D^\mu B) - \frac{\chi}{f_\sigma}\mathrm{Tr}(\bar{B}B) + \cdots$$

Trace anomaly potential:

$$V(\chi) \approx \frac{m_\sigma^2 f_\sigma^2}{4}\left(\frac{\chi}{f_\sigma}\right)^4\left[\ln\left(\frac{\chi}{f_\sigma}\right) - \frac{1}{4}\right].$$

$$\chi = f_\chi e^{\sigma/f_\chi} \equiv f_\chi \Phi$$

$$\hat{\alpha}_{\perp\mu} = \frac{1}{2i}\left(\partial_\mu\xi\cdot\xi^\dagger - \partial_\mu\xi^\dagger\cdot\xi\right)$$

$$\hat{\alpha}_{\parallel\mu} = \frac{1}{2i}\left(\partial_\mu\xi\cdot\xi^\dagger + \partial_\mu\xi^\dagger\cdot\xi\right) - g_\rho\rho_\mu - g_\omega\omega_\mu$$

$$\xi = e^{-i\pi/f_\pi}$$

$$V_\mu = \frac{g}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}(\rho^0_\mu+\omega_\mu) & \rho^+_\mu & K^{*,+}_\mu \\ \rho^-_\mu & -\frac{1}{\sqrt{2}}(\rho^0_\mu-\omega_\mu) & K^{*,0}_\mu \\ K^{*,-}_\mu & \bar{K}^{*,0}_\mu & \phi_\mu \end{pmatrix}$$

$$B(x) = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

- **W. G. Paeng, T. T. S. Kuo, H. K. Lee, *YLM*, M. Rho, arXiv: 1704.02775;**
- ***YLM*, M. Rho, e-Print: 1909.05889 [nucl-th];**
- **H. K. Lee, *YLM*, W. G. Paeng, M. Rho, e-Print: 2107.01879 [nucl-th]**

II. Basics of Chiral-scale EFT

Note:

Anomaly match in vacuum is a loose constraint, general form is complicated. To make the model feasible, some simplifications are needed.

III. Nuclear matter properties

III.1. Pseudoconformal structure

- Scale symmetry in medium is drawing more and more attentions.

  - Trace of energy momentum tensor

$$\theta^{\mu}_{\mu}(n) = \varepsilon(n) - 3p(n)$$

  - Energy density: $\varepsilon(n)$
  - Pressure: $p(n)$

$$\frac{\partial\langle\theta^{\mu}_{\mu}\rangle}{\partial n} = \frac{\partial\varepsilon(n)}{\partial n}(1 - 3v_s^2)$$

$$v_s^2 = \frac{\partial p(n)/\partial n}{\partial\varepsilon(n)/\partial n} = \frac{\partial p(n)}{\partial\varepsilon(n)}$$

➢ Stiffness of EOS, behaviour of SV, interaction among quasiparticles!

III. Nuclear matter properties

- An example: Cold non-interaction fermion matter:

$$\theta_\mu^\mu(n) = \varepsilon(n) - 3p(n)$$
$$= \frac{1}{\pi^2}\left(m^2 E_F k_F - m^4 \ln\left(\frac{E_F + k_F}{m}\right)\right)$$

$m = 0$ →

Exact scale invar. $\theta_\mu^\mu = 0$

$$E_F^2 = k_F^2 + m^2,$$
$$k_F = \left(\frac{3\pi^2}{2}n\right)^{1/3}$$

- Extreme high density regime, $k_F \gg m$

$$\epsilon \to \frac{1}{2\pi^2}k_F^4\left(1 + \frac{m^2}{k_F^2}\right) \to \frac{1}{2\pi^2}k_F^4$$

$$P \to \frac{1}{6\pi^2}k_F^4\left(1 - \frac{m^2}{k_F^2}\right) \to \frac{1}{6\pi^2}k_F^4$$

→

$$\theta_\mu^\mu \to \frac{m^2}{\pi^2}k_F^2$$

- Scale symmetry is broken by fermion mass

- Energy density and pressure → independent of m, massless fermion
- TEMT → non-zero (even divergent $\propto k_F^2$!)

Can we talk about conformality?

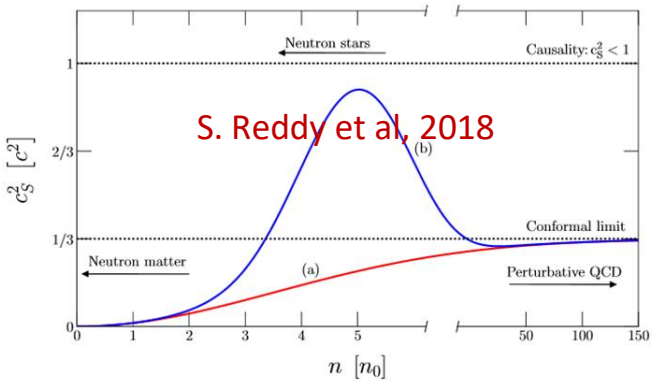
III. Nuclear matter properties

- An example: Cold non-interaction fermion matter:
  - Sound velocity: $v_s^2 = \frac{\partial p(n)/\partial n}{\partial \varepsilon(n)/\partial n} = \frac{\partial p(n)}{\partial \varepsilon(n)}$
    - $m = 0,\ \ \varepsilon(n) = 3p(n) \rightarrow$ scale invariant, conformal SV: $v_s^2 = 1/3$
    - Extreme high density regime, $k_F \gg m \rightarrow v_s^2 = \frac{1}{3}\left(1 - \frac{m^2}{k_F^2}\right) \rightarrow \frac{1}{3}$,

conformal SV, BUT scale non-invariant

Sound velocity → conformal velocity for $k_F \gg m \approx 10\, n_0$

$$\theta_\mu^\mu \rightarrow \frac{m^2}{\pi^2} k_F^2$$



- The proper criteria for conformal window $k_F \gg m$ is the SV not the TEMT.

III. Nuclear matter properties

- Compact star matter (strongly correlated matter):

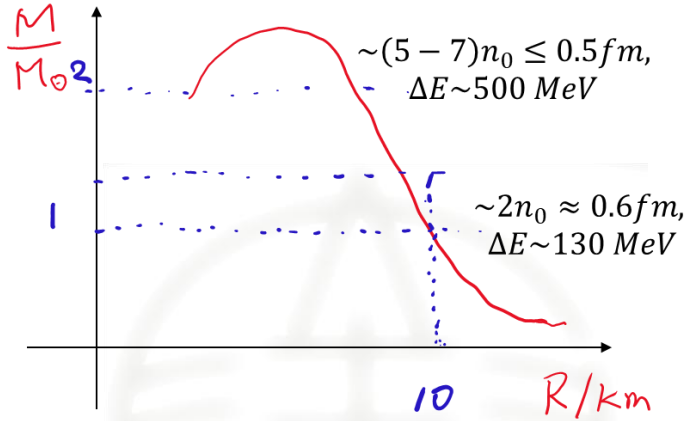
  - For compact star, the core densities are about $(5\sim10)n_0$, depending on the models.

$$k_F = \left(\frac{3\pi^2}{2}n\right)^{1/3} \sim 0.5\, m_N$$

  - ➢ Not high enough to be the conformal window,

$$k_F \gg m_N$$

Cannot talk about conformality?



✓ It is a criterion relevant to the non-interacting massive fermions, cannot be simply imposed on the strongly interacting fermionic matter at the core of compact star. A conformal window may appear in a different context.

➢ H. K. Lee, symmetry 16 (2024), 1598

III. Nuclear matter properties

$$\mathcal{L} = \mathcal{L}^{M}_{\chi PT_\sigma}(\pi, \chi, V_\mu) + \mathcal{L}^{B}_{\chi PT_\sigma}(\psi, \pi, \chi, V_\mu) - V(\chi)$$

$$\mathcal{L}^{M}_{\chi PT_\sigma}(\pi, \chi, V_\mu) = f_\pi^2 \left(\frac{\chi}{f_\sigma}\right)^2 \mathrm{Tr}[\hat{a}_{\perp\mu}\hat{a}_\perp^\mu] + a f_\pi^2 \left(\frac{\chi}{f_\sigma}\right)^2 \mathrm{Tr}[\hat{a}_{\parallel\mu}\hat{a}_\parallel^\mu] + \frac{1}{2g^2}\mathrm{Tr}[V_{\mu\nu}V^{\mu\nu}] + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi$$

$$\mathcal{L}^{B}_{\chi PT_\sigma}(\psi, \pi, \chi, V_\mu) = \mathrm{Tr}(\bar{B} i\gamma_\mu D^\mu B) - \frac{\chi}{f_\sigma}\mathrm{Tr}(\bar{B}B) + \cdots$$

$$V(\chi) \approx \frac{m_\sigma^2 f_\sigma^2}{4}\left(\frac{\chi}{f_\sigma}\right)^4 \left[\ln\left(\frac{\chi}{f_\sigma}\right) - \frac{1}{4}\right].$$

$$\chi = f_\chi e^{\sigma/f_\chi} \equiv f_\chi \Phi$$

$$m_\omega^* = \Phi^* m_\omega$$

$$m_N^* = \Phi^* m_N$$

$$\hat{\alpha}_{\perp\mu} = \frac{1}{2i}\left(\partial_\mu\xi\cdot\xi^\dagger - \partial_\mu\xi^\dagger\cdot\xi\right)$$

$$\hat{\alpha}_{\parallel\mu} = \frac{1}{2i}\left(\partial_\mu\xi\cdot\xi^\dagger + \partial_\mu\xi^\dagger\cdot\xi\right) - g_\rho\rho_\mu - g_\omega\omega_\mu$$

$$\xi = e^{-i\pi/f_\pi}$$

Trace anomaly

- W. G. Paeng, T. T. S. Kuo, H. K. Lee, *YLM*, M. Rho, arXiv: 1704.02775;
- *YLM*, M. Rho, e-Print: 1909.05889 [nucl-th];
- H. K. Lee, *YLM*, W. G. Paeng, M. Rho, e-Print: 2107.01879 [nucl-th]

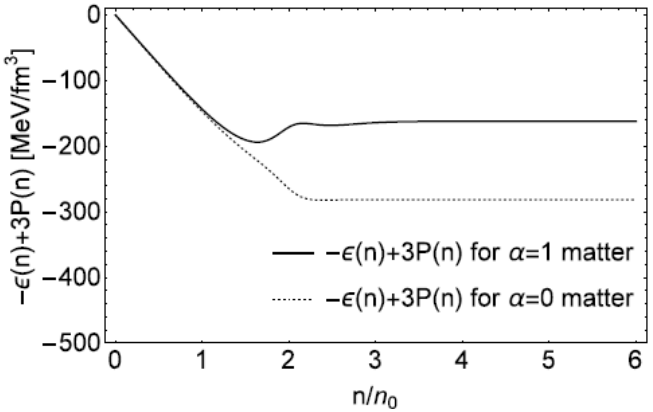
Dialing the parameters, accommodate NM properties around n_0.

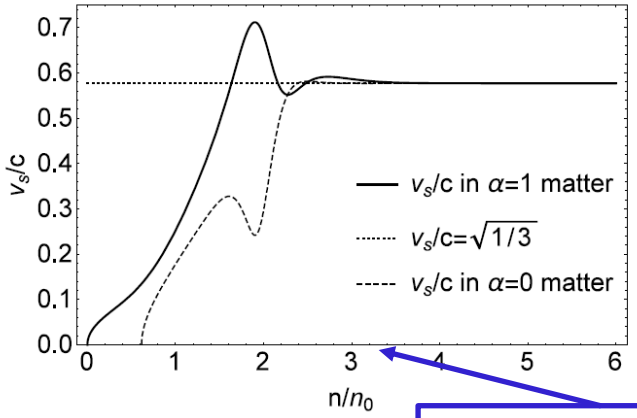
III. Nuclear matter properties

W. G. Paeng, T. T. S. Kuo, H. K. Lee, *YLM*, M. Rho, arXiv: 1704.02775.

$$\frac{\partial}{\partial n}\langle\theta^{\mu}_{\mu}\rangle = \frac{\partial\epsilon(n)}{\partial n}(1-3v_s^2) = 0$$

$$v_s^2/c^2 = \frac{\partial P(n)}{\partial n}\Big/\frac{\partial\epsilon(n)}{\partial n}$$





Relevant to the core of NS

- For $\geq 2n_0$, TEMT is a density independent constant;
- For $\geq 2n_0$, SV $\rightarrow 1/\sqrt{3}$—conformal limit.

Pseudoconformal structure

$(v_s^2 = 1/3, \theta^{\mu}_{\mu} \neq 0)$

A feature not observed before.

III. Nuclear matter properties

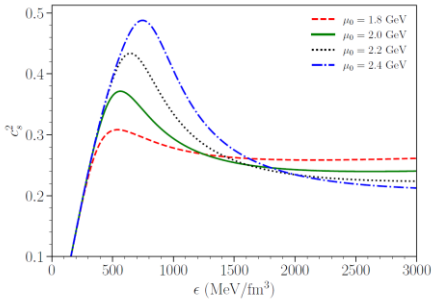
The PCM changed the previous understanding of the EoS:

- The model where the sound velocity saturating the conformal limit at asymptotic density, cannot predicted the maximum value of the NS mass satisfying the observation of the massive NSs.

↓ Predicted in more models

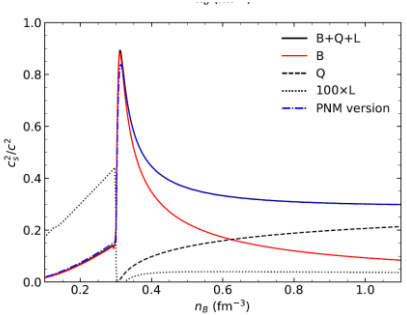
Quark-hadron crossover

$$P(\mu) = S(\mu)P_q(\mu) + (1 - S(\mu))P_h(\mu)$$



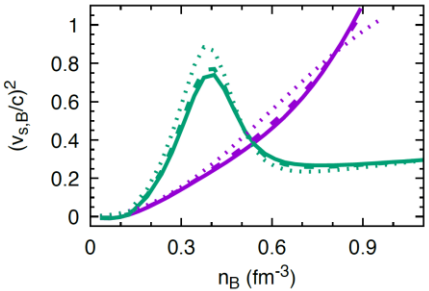
Kapusta & Welle, 2103.16633

Quarkyonic matter



Zhao & Lattimer, 2004.08293

Quarkyonic matter



Margueron et al, 2103.10209

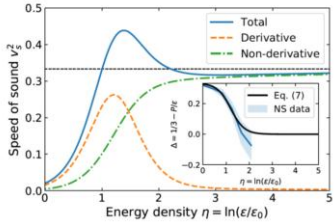


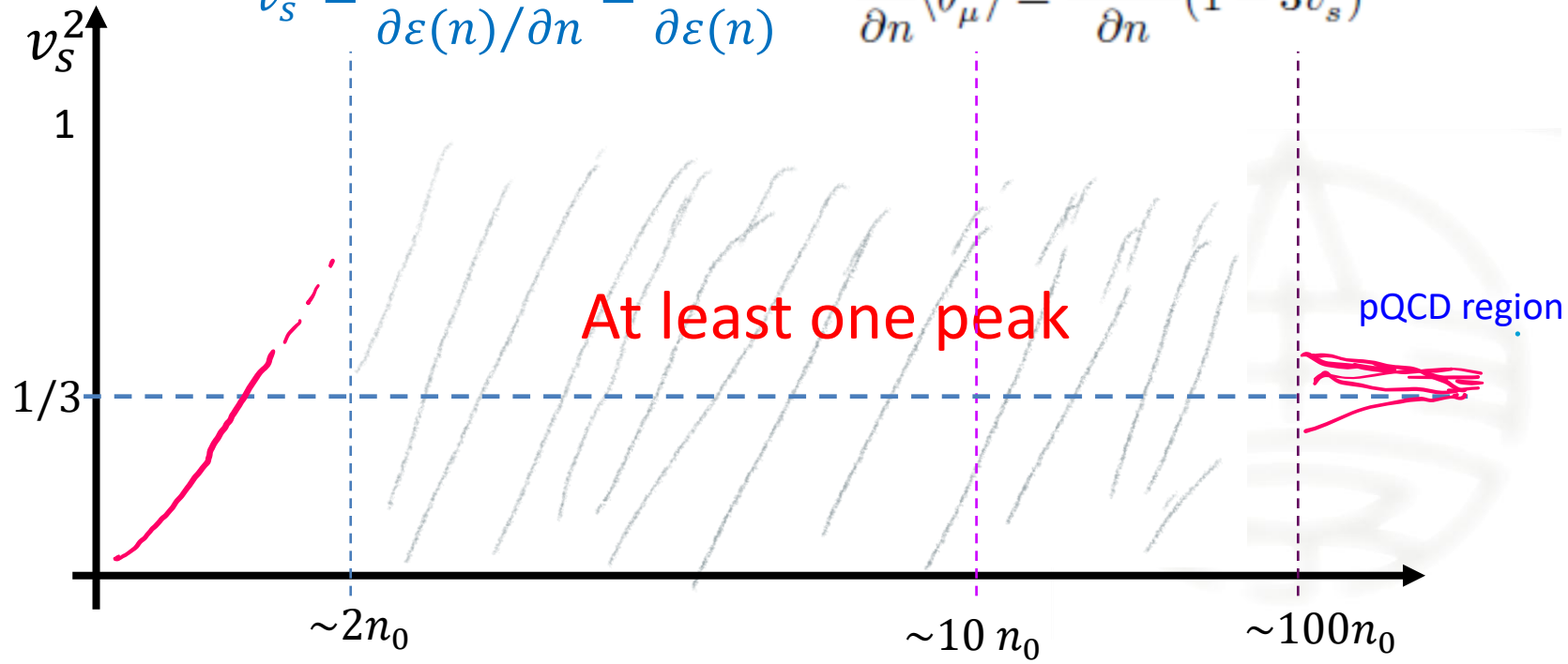
FIG. 2. The speed of sound and its decomposition (6) calculated from (7) as shown in the inset plot. The horizontal axis is the logarithmic energy η normalized to the value at the saturation point, $\varepsilon_0 = 150\,\mathrm{MeV/fm^3}$.

Fujimoto, et al, 2207.06753

III. Nuclear matter properties

III. 2. Peak of sound velocity

$$v_s^2 = \frac{\partial p(n)/\partial n}{\partial \varepsilon(n)/\partial n} = \frac{\partial p(n)}{\partial \varepsilon(n)} \qquad \frac{\partial}{\partial n}\langle \theta^\mu_\mu \rangle = \frac{\partial \epsilon(n)}{\partial n}(1 - 3v_s^2)$$



III. Nuclear matter properties

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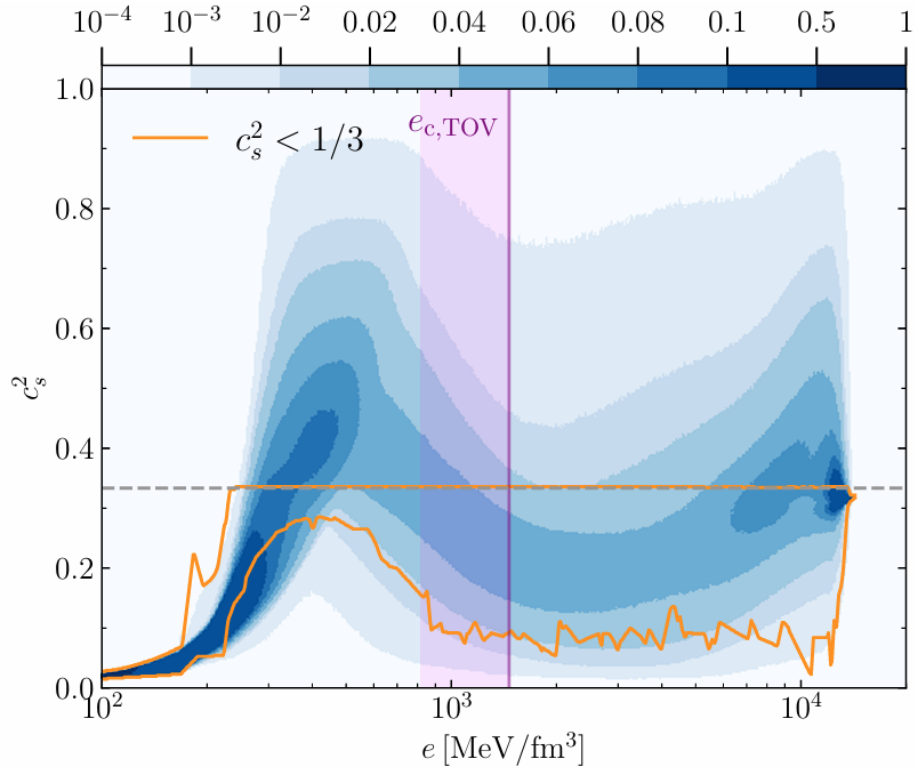

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erturbative QCD, but also with
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0.03% of our sample. Hence, it is
ge sample, we obtain estimates at

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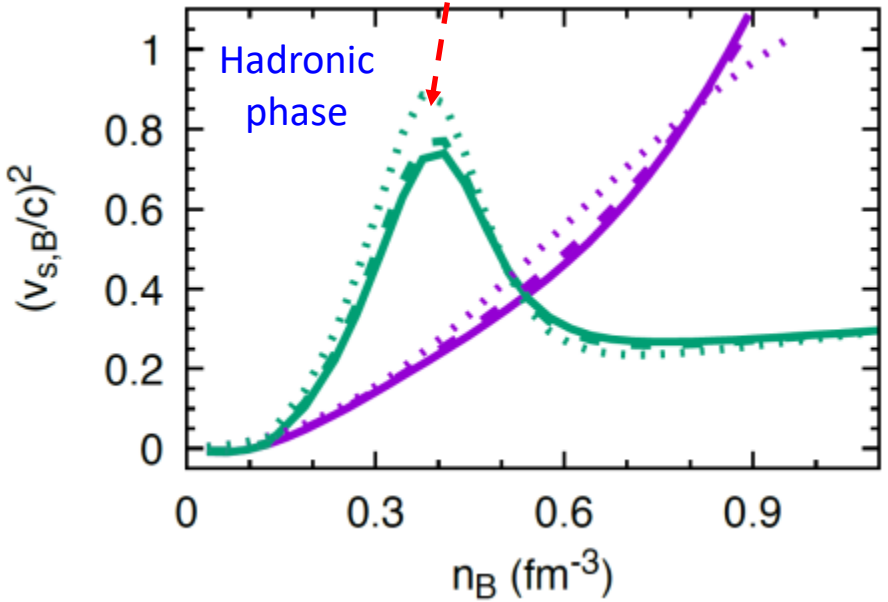
III. Nuclear matter properties

Hippert, *et al.* , 2105.04535

Emerges in the transition from a phase with broken chiral symmetry to one with gapped Fermi surface with the condensation of diquarks and dibaryons.



- Transition from hadronic phase to quarkyonic phase;
- At $(2\sim4)n_0$ due to topology change;
- Hadron-quark crossover;
- Formation of diquark gap.

- L. McLerran and S. Reddy, PRL122 (2019), 122701.
- T. Zhao and J. M. Lattimer, PRD102 (2020), 023021.
- YLM and M. Rho, PRD99 (2019), 014034; PPNP113 (2020), 103791;
- G. Baym, S. Furusawa, et al., APJ 885 (2019), 42;
- M. Leonhardt, M. Pospiech, et al., PRL125 (2020), 142502 (2020).

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III. Nuclear matter properties

$$\mathcal{L}_M = f_\pi^2\Phi^2\,\mathrm{Tr}\left(\hat{\alpha}_\perp^\mu\hat{\alpha}_{\mu\perp}\right) + f_{\sigma\rho}^2\Phi^2\,\mathrm{Tr}\left(\hat{\alpha}_\parallel^\mu\hat{\alpha}_{\mu\parallel}\right) + f_0^2\Phi^2\,\mathrm{Tr}\left(\hat{\alpha}_\parallel^\mu\right)\mathrm{Tr}\left(\hat{\alpha}_{\mu\parallel}\right)$$
$$- \frac{1}{2g_\rho^2}\,\mathrm{Tr}\left(V_{\mu v}V^{\mu v}\right) - \frac{1}{2g_0^2}\,\mathrm{Tr}\left(V_{\mu\nu}\right)\mathrm{Tr}\left(V^{\mu v}\right)$$
$$+ \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{f_\pi^2}{4}\Phi^{3-\gamma_m}\,\mathrm{Tr}\left(\mathcal{M}U^\dagger + U\mathcal{M}^\dagger\right) + h_5\Phi^4 + h_6\Phi^{4+\beta'},$$

$$\mathcal{L}_B = \bar{N}i\gamma_\mu D^\mu N - m_N\Phi\bar{N}N$$
$$- g_{\omega NN}\omega^\mu\bar{N}\gamma_\mu N - g_{\rho NN}\rho^{a\mu}\bar{N}\tau^a\gamma_\mu N$$
$$- g_{\omega NN}^{SSB}\left(\Phi^{\beta'} - 1\right)\omega^\mu\bar{N}\gamma_\mu N$$
$$- g_{\rho NN}^{SSB}\left(\Phi^{\beta'} - 1\right)\rho^{a\mu}\bar{N}\tau^a\gamma_\mu N.$$

$$\hat{\alpha}_{\perp\mu} = \frac{1}{2i}\left(\partial_\mu\xi\cdot\xi^\dagger - \partial_\mu\xi^\dagger\cdot\xi\right)$$
$$\hat{\alpha}_{\parallel\mu} = \frac{1}{2i}\left(\partial_\mu\xi\cdot\xi^\dagger + \partial_\mu\xi^\dagger\cdot\xi\right) - g_\rho\rho_\mu - g_\omega\omega_\mu$$
$$\xi = e^{-i\pi/f_\pi}$$

- L. Q. Zhang, Y. Ma and *YLM*, 2410.04142; 2412.19023.

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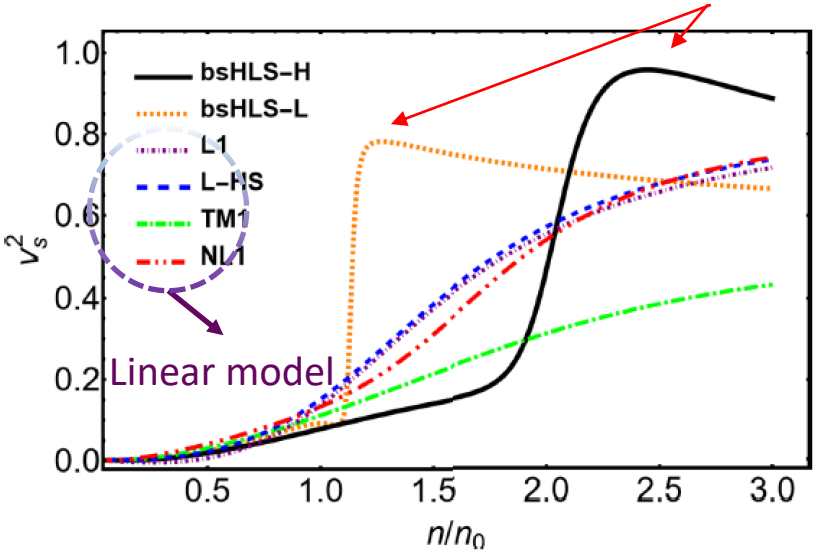
III. Nuclear matter properties

- Self-consistent mean field calculation

	Empirical	bsHLS-L	bsHLS-H
n_0	0.155 ± 0.050 [46]	0.159	0.159
e_0	-15.0 ± 1.0 [46]	-16.0	-16.0
$K(n_0)$	230 ± 30 [47]	232	284
$E_{\rm sym}(n_0)$	30.9 ± 1.9 [48]	30.5	29.2
$E_{\rm sym}(2n_0)$	46.9 ± 10.1 [49]	51.5	50.2
$L(n_0)$	52.5 ± 17.5 [48]	85.9	68.3
$J(n_0)$	-700 ± 500 [50]	-767	-599

	M_σ	β'	r	$g_{\omega NN}$	$g_{\rho NN}$	$g^{SSB}_{\omega NN}$	$g^{SSB}_{\rho NN}$
bsHLS-L	1.05	0.395	0.161	11.5	3.78	16.3	9.45
bsHLS-H	2.30	1.15	0.191	11.0	4.17	8.85	4.85

Peak of SV, not far from n_0

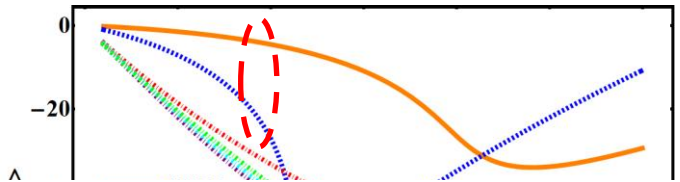


- The peak in the SV is due to the nonlinear realization of scale symmetry, NOT observed in Walecka-typy models!

➢ L. Q. Zhang, Y. Ma and *YLM*, 2410.04142; 2412.19023.

III. Nuclear matter properties

- Why in Chiral-scale EFT?



- Scale symmetry is first partially restored with density, but then stops after a critical denity.

- The nonlinear realization of scale symmetry gives a natural explanation of the peak in the SV, WITHOUT retrospect to phase transition/configuration change!

$\overline{n_0}$

$$\left(\frac{E}{B}\right)_{\mathrm{WZ}} = \frac{1}{4}\left(\frac{3g}{2}\right)^2 \int_{\mathrm{Box}} d^3x \int d^3x' \, B_0(\vec{x}) \frac{\exp(-m_\omega^*|\vec{x}-\vec{x}'|)}{4\pi|\vec{x}-\vec{x}'|} B_0(\vec{x}')$$

$$\chi = f_\chi e^{\sigma/f_\chi} \equiv f_\chi \Phi$$

$$m_\omega^* = \Phi^* m_\omega$$

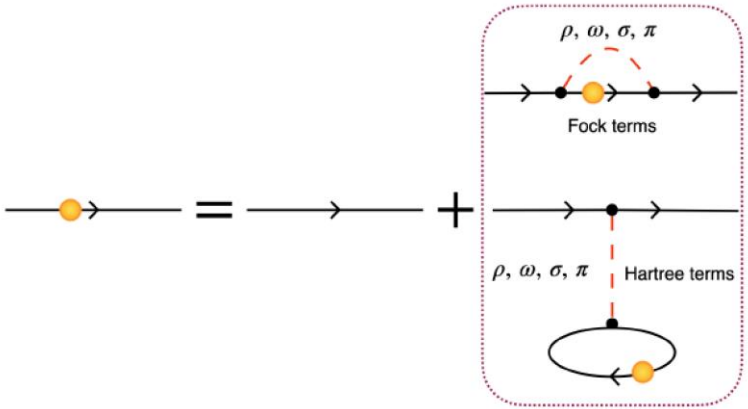
$$m_N^* = \Phi^* m_N$$

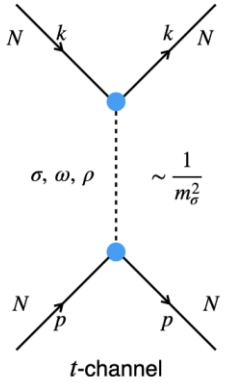
Diverge unless it is screened by m_ω^*. If the ω mass were to go down, the nucleon-nucleon interaction would become strongly repulsive with increasing density. This forces the ω mass m_ω^*, and hence $\chi^*$to increase.

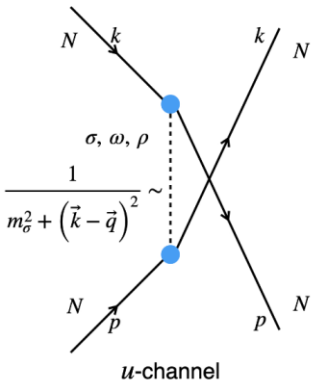
B. Y. Park, M. Rho and V. Vento, NPA807(2008), 28.

IV. CSDC rule for dense and thermal system

- So far, only the RMF approach was applied.
- Can (How can) we go beyond MFA (meson fluctuations) by using this theory?







- J. Y. Xiong, Y. Ma, B. K. Sheng, *YLM*, e-Print: 2511.04353 [nucl-th];
- Y. Ma, *YLM*, L. Q. Zhang, e-Print: 2512.23477 [nucl-th]

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IV. CSDC rule for dense and thermal systems

- At zero temperature, construct CSDC rules in relativistic Hartree-Fock method (RHF)

To illustrate the procedure, consider

$$\mathcal{L}_{\mathrm{OBE}} = \bar{N}\left[i\partial\!\!\!/ - m_N - g^{\mathrm{OBE}}_{\omega NN}\omega\!\!\!/ - g^{\mathrm{OBE}}_{\rho NN}\rho\!\!\!/^{i}\tau^{i} - g^{\mathrm{OBE}}_{\sigma NN}\sigma - g^{\mathrm{OBE}}_{\pi NN}\partial\!\!\!/\pi^{i}\tau^{i}\gamma_5\right]N + \mathcal{L}_{\mathrm{meson}}$$

Dirac equation in dense medium: $(i\gamma^{\mu}p_{\mu} - m_{\mathrm{B}} - \Sigma(p))\psi_B(x) = 0$

$\Sigma(p) = \gamma^{\mu}\Sigma_{\mathrm{V},\mu} + \Sigma_{\mathrm{S}}$ baryon's self-energy

momentum and mass in dense medium: $p^{*}_{\mu} = p_{\mu} - \Sigma_{\mathrm{V},\mu}$ $m^{*}_{\mathrm{B}} = m_{\mathrm{B}} + \Sigma_{\mathrm{S}}$

- **J. Y. Xiong, Y. Ma, B. K. Sheng, *YLM*, e-Print: 2511.04353 [nucl-th]**

IV. CSDC rule for dense and thermal systems

The solutions of baryonic and mesonic fields:

Baryon field

Fermi sphere with MFA

No-sea approximation

$$\psi_B(x) = \sum_s \int_0^{k_F} \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{\sqrt{2E^*}} \left[u(\mathbf{p}, s) e^{-ip\cdot x} a(\mathbf{p}, s) \right]$$

Effective energy at densities

Meson field

Induced background field

Nucleon source $\sim \mathcal{O}(k_F^3)$

$$\sigma(x) \simeq \int_V d^4x' D_{\sigma,R}(x - x') \left(-g_{\sigma NN}^{OBE} \bar{N}(x') N(x') \right)$$

➢ J. Y. Xiong, Y. Ma, B. K. Sheng, *YLM*, e-Print: 2511.04353 [nucl-th]

IV. CSDC rule for dense and thermal systems

Characteristic momentum expansion

- $O(k_F) \sim O(p) \sim 500$ MeV, for the typical density regions of a neutron star;
- With assigning the consistent power counting of coupling constants and label the expansion factor as characteristic momentum $k_c \sim$ hundreds of MeV;
- The induced background fields and coupling constants:

$$\sigma,\ \pi,\ \omega_\mu,\ \rho_\mu^i \ \sim \mathcal{O}(k_c^2)$$

$$g_{\sigma NN}^{OBE}, g_{\rho NN}^{OBE}, g_{\omega NN}^{OBE} \sim O(k_c), g_{\pi NN}^{OBE} \sim O(k_c^0)$$

Leading order: free Fermi gas, $\mathscr{H}_0 = \underbrace{\bar{N}\left(-i\gamma\cdot\nabla + m_N\right)N}_{\mathcal{O}(k_c^4)}$

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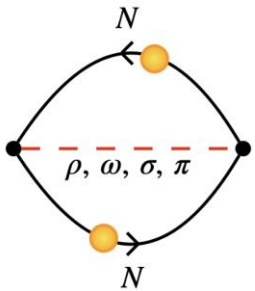
IV. CSDC rule for dense and thermal systems

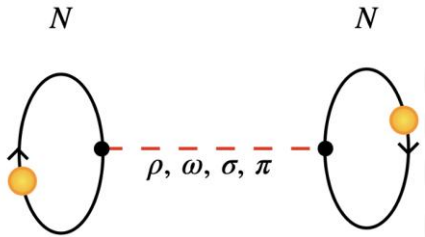
The energy density of dense nuclear matter:

$$\left\langle \Omega \left| : \hat{\mathcal{H}} : \right| \Omega \right\rangle = \left\langle 0 \left| T \left\{ : \hat{\mathcal{H}} \exp \left[-i \int_{-\infty}^{+\infty} \mathrm{d}t' \hat{H}_I(t') \right] : \right\} \right| 0 \right\rangle_c$$

$$\hat{\mathcal{H}}(x) = \bar{N}(x) \left(-i\gamma \cdot \nabla + m_N \right) N(x) + \bar{N}(x) \left(g_{\sigma NN}^{\mathrm{OBE}} \int_V \mathrm{d}^4 x' D_\sigma(x - x') \left(-g_{\sigma NN}^{\mathrm{OBE}} \bar{N}(x') N(x') \right) \right) N(x) + \cdots$$



Fock terms



Hartree terms

$\sim O(k_c^6)$

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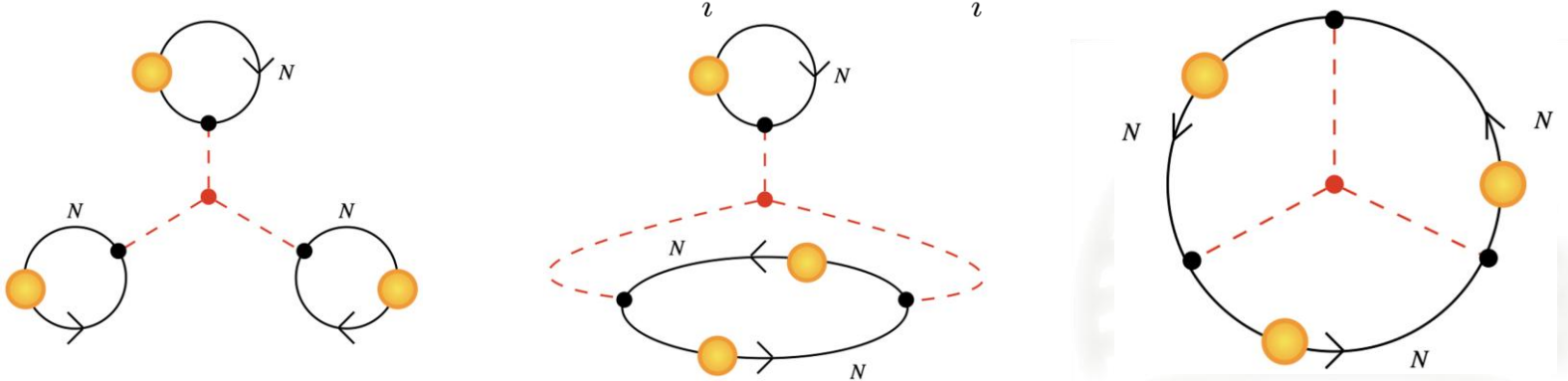
IV. CSDC rule for dense and thermal systems

Chiral-scale EFT brings difficultities of practical calculations due to the dilaton compensator approach with the non-linear realization of sigma. $\chi = f_\chi e^{\sigma/f_\chi} \equiv f_\chi \Phi$

$$\left(\partial_\mu \partial^\mu + m_\sigma^2\right)\sigma = -g^{\mathrm{OBE}}_{\sigma NN}\bar{N}N + \sum_i g_{\sigma^{i+1}}\sigma^i + \sum_i g_{\sigma^{i+1}NN}\sigma^i \bar{N}N$$



- The Feynman diagrams for calculating the effect of meson fluctuations are arranged in terms of physical fields;
- Expand the exponential in terms of physical field sigma. A counting mechanism is need to truncate the order. ➢ **J. Y. Xiong, Y. Ma, B. K. Sheng, *YLM*, e-Print: 2511.04353 [nucl-th]**

IV. CSDC rule for dense and thermal systems

$$\sigma(x) = \int_V \mathrm{d}^4x' D_\sigma(x-x') \left(-g^{\mathrm{OBE}}_{\sigma NN}\bar{N}(x')N(x')\right)$$

Used as the basis to solve the expansion iteratively.

- The induced background fields and coupling constants at LO:

$$\sigma,\ \pi,\ \omega_\mu,\ \rho^i_\mu \ \sim \mathcal{O}(k_c^2)$$

$$g^{OBE}_{\sigma NN}, g^{OBE}_{\rho NN}, g^{OBE}_{\omega NN} \sim O(k_c), g^{OBE}_{\pi NN} \sim O(k_c^0)$$

- Generally: $g_{\sigma^2 NN}, g_{\sigma^3 NN} \sim O(k_c), \cdots$

$$\sigma \sim \mathcal{O}(k_c^2) + \mathcal{O}(k_c^4) + \mathcal{O}(k_c^6) + \cdots$$

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IV. CSDC rule for dense and thermal systems

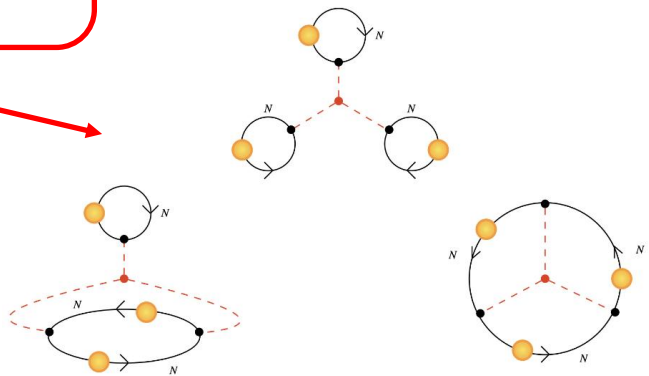
Fermion Loop expansion

$$\mathcal{H}_0 = \underbrace{\bar{N}\left(-i\gamma\cdot\nabla + m_N\right)N}_{\mathcal{O}(k_c^4)},$$

$$\mathcal{H}_I = \underbrace{g^{\rm OBE}_{\sigma NN}\bar{N}\sigma N + g^{\rm OBE}_{\omega NN}\bar{N}\gamma_\mu\omega^\mu N + g^{\rm OBE}_{\sigma NN}\bar{N}\not{\phi}^i\tau^i N + g^{\rm OBE}_{\pi NN}\bar{N}\gamma_5\not{\partial}\pi^i\tau^i N}_{\mathcal{O}(k_c^6)} - \underbrace{\text{Free Lagrangian of } \rho,\ \omega,\ \sigma \text{ mesons}}_{\mathcal{O}(k_c^6)}$$

$$+ \underbrace{g_{\sigma^3}\sigma^3 + g_{\sigma\rho^2}\sigma\rho^i_\mu\rho^{i,\mu} + g_{\sigma\omega^2}\sigma\omega_\mu\omega^\mu + g_{\sigma\pi^2}\sigma\pi^i\pi^i + g_{\rho\pi^2}\rho^{\mu,a}\partial_\mu\pi^b\pi^c\epsilon_{abc}}_{\mathcal{O}(k_c^8)}$$

$$+ \underbrace{g_{\sigma^2 NN}\bar{N}\sigma^2 N + g_{\sigma\omega NN}\bar{N}\sigma\omega^\mu\gamma_\mu N + g_{\sigma\rho NN}\bar{N}\sigma\rho^{\mu a}\tau^a\gamma_\mu N}_{\mathcal{O}(k_c^8)} + \text{higher order terms}. \quad (1$$



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IV. CSDC rule for dense and thermal systems

- Extend to finite temperatures (relativistic mean-field approximation, RMF)

Laudau potential: $\Phi_{\mathrm{G}} = -T \ln Z$

$$Z = \int \left[\mathrm{d}\bar{N}_p\right] \left[\mathrm{d}N_p\right] \left[\mathrm{d}\bar{N}_n\right] \left[\mathrm{d}N_n\right] \exp\left(\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \left(\mathscr{L}_{\mathrm{RMF}} + \mu_p N_p^\dagger N_p + \mu_n N_n^\dagger N_n \right) \right)$$

Thermodynamic potential density: $\Omega = \Phi_{\mathrm{G}}/V$

$$\begin{aligned} P &= -\Omega, \\ n_i &= -\left.\frac{\partial \Omega}{\partial \mu_i}\right|_T, \\ s &= -\left.\frac{\partial \Omega}{\partial T}\right|_{\mu_p,\mu_n}, \\ \epsilon &= \Omega + \sum_i \mu_i n_i + Ts, \end{aligned}$$

Equation of motions:

$$\frac{\partial \Omega}{\partial \sigma} = 0 \qquad \frac{\partial \Omega}{\partial \omega} = 0 \qquad \frac{\partial \Omega}{\partial \rho} = 0$$

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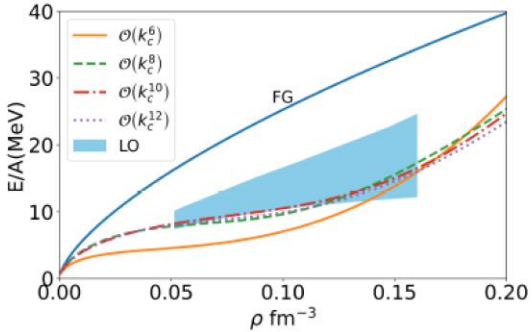
IV. CSDC rule for dense and thermal systems

Nuclear matter properties

	Empirical	$\mathcal{O}(k_c^6)$	$\mathcal{O}(k_c^8)$	$\mathcal{O}(k_c^{10})$	$\mathcal{O}(k_c^{12})$
n_0 (fm^{-3})	0.155 ± 0.050	0.160	0.160	0.160	0.158
$E(n_0)$ (MeV)	-15.0 ± 1.0	−16.3	−15.6	−16.1	−15.4
$E(1.5n_0)$ (MeV)	-13.3 ± 0.5	−6.92	−12.0	−8.99	−11.2
$K(n_0)$ (MeV)	230 ± 30	566	366	572	391
$E_{\rm sym}(n_0)$ (MeV)	30.9 ± 1.9	31.6	32.1	31.8	32.2
$L(n_0)$ (MeV)	52.5 ± 17.5	104	83.2	75.7	77.8
T_c (MeV)	20.0 ± 3.0	19.0	26.5	24.0	22.5

A. Sedrakian, J.-J. Li, and F. Weber, Prog. Part. Nucl. Phys. 131, 104041 (2023).
Y. Leifels, A. LeFevre, W. Reisdorf, J. Aichelin, and C. Hartnack, EPJ Web Conf. 88, 00018 (2015).
M. Dutra, O. Lourenco, J. S. Sa Martins, A. Delfino, J. R. Stone, and P. D. Stevenson, Phys. Rev. C 85, 035201 (2012).
J. M. Lattimer and Y. Lim, The Astrophysical Journal 771, 51 (2013).
V. A. Karnaukhov et al., Phys. Rev. C 67, 011601 (2003).



Comparison with the leading order chiral nuclear force results of PNM

C. Drischler, J. W. Holt, and C. Wellenhofer, Ann. Rev. Nucl. Part. Sci. 71, 403 (2021).

TABLE II. The values of free parameters and couplings. The couplings of relevant couplings at each order are also listed.

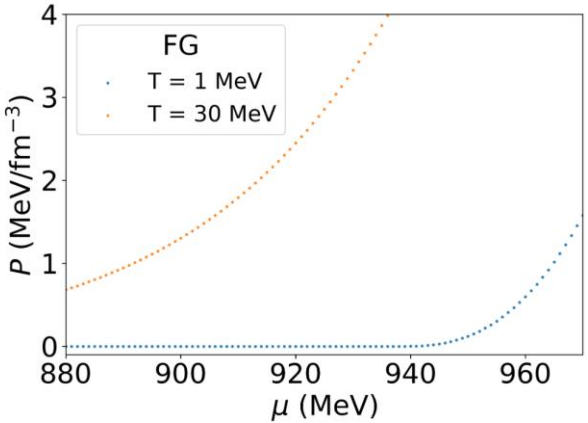
CSDC order	$\mathcal{O}(k_c^6)$	$\mathcal{O}(k_c^8)$	$\mathcal{O}(k_c^{10})$	$\mathcal{O}(k_c^{12})$	TM1
m_σ (MeV)	588	421	745	379	511
f_χ (MeV)	−82.2	−264	−122	−277	–
β'	–	1.92	0.744	0.910	–
$g_{\sigma NN}^{\rm OBE}$	−11.4	−3.54	−7.68	−3.39	10.0
$g_{\omega NN}^{\rm OBE}$	13.2	−1.50	4.14	0.311	12.6
$g_{\rho NN}^{\rm OBE}$	3.27	−4.24	−4.54	−4.86	4.63
$g_{\sigma^2 NN}$ (10^{-3}) MeV^{-1}	–	6.69	31.4	6.11	–
$g_{\sigma\omega NN}$ (10^{-1}) MeV^{-1}	–	2.00	0.681	1.23	–
$g_{\sigma\rho NN}$ (10^{-2}) MeV^{-1}	–	1.44	5.46	4.16	–
g_{σ^3} (10^3 MeV)	–	−1.09	−6.46	−0.754	−0.473
$g_{\sigma\rho^2}$ (10^3 MeV)	–	2.20	4.76	2.10	0
$g_{\sigma\omega^2}$ (10^3 MeV)	–	2.31	5.00	2.21	0
$g_{\sigma^3 NN}$ (10^{-5} MeV^{-2})	–	–	−8.55	−0.734	0
$g_{\sigma^2\omega NN}$ (10^{-5} MeV^{-2})	–	–	−20.7	−20.2	0
$g_{\sigma^2\rho NN}$ (10^{-5} MeV^{-2})	–	–	−16.6	−6.82	0
g_{σ^4}	–	–	85.5	4.51	0.152
$g_{\sigma^2\omega^2}$	–	–	−40.9	−7.96	–
$g_{\sigma^2\rho^2}$	–	–	−38.9	−7.57	–
$g_{\sigma^3\sigma NN}$(10^{-9}MeV^{-3})	–	–	–	6.62	–
$g_{\sigma^3\omega NN}$(10^{-7}MeV^{-3})	–	–	–	2.21	–
$g_{\sigma^3\rho NN}$(10^{-8}MeV^{-3})	–	–	–	7.47	–
g_{σ^5}(10^{-2}MeV^{-1})	–	–	–	−1.93	–
$g_{\sigma^3\omega^2}$(10^{-2}MeV^{-1})	–	–	–	1.91	–
$g_{\sigma^3\rho^2}$(10^{-2}MeV^{-1})	–	–	–	1.82	–
g_{ω^4}	–	–	–	–	−17.8

➢ **J. Y. Xiong, Y. Ma, B. K. Sheng, *YLM*, e-Print: 2511.04353 [nucl-th]**

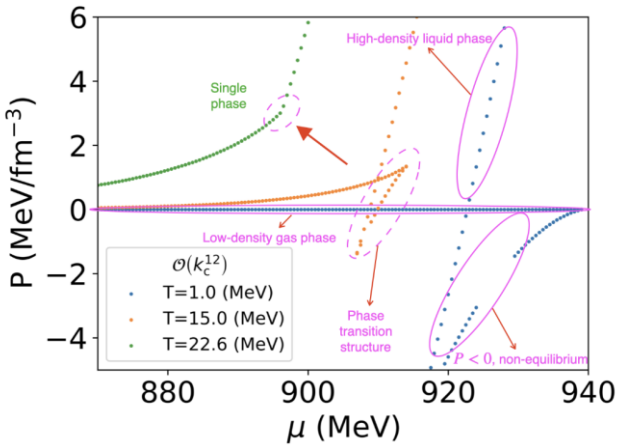
IV. CSDC rule for dense and thermal systems

■ Gas-liquid phase transition:

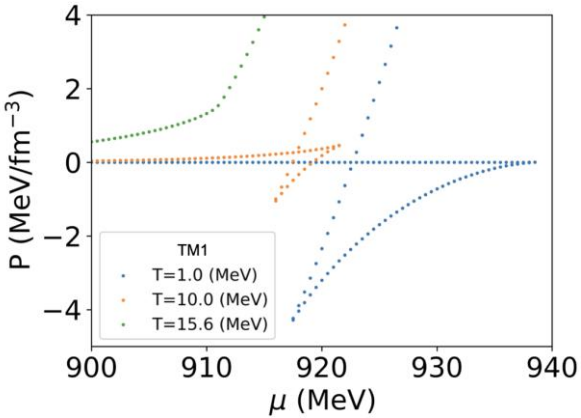
➢ **J. Y. Xiong, Y. Ma, B. K. Sheng, *YLM*, e-Print: 2511.04353 [nucl-th]**



Free Fermi gas (LO of CSDC)



$O\left(k_c^{12}\right)$ CSDC



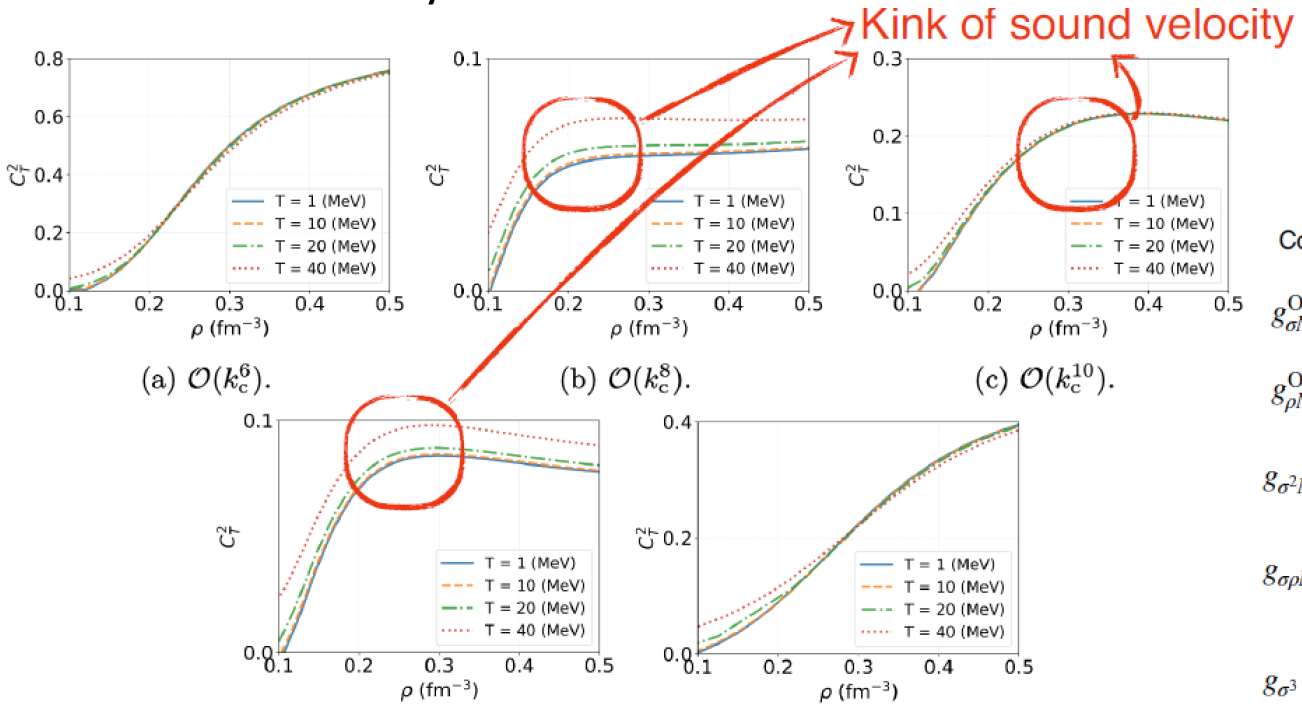
TM1 model

It depends on the model used and how the data are extracted and ranges from several MeV to20 $\pm$ 3 MeV.

- V. A. Karnaukhov et al., Phys. Rev. C 67, 011601(2003), arXiv:nucl-ex/0302006;
- V. A. Karnaukhov et al., Phys. Atom. Nucl. 71, 2067(2008), arXiv:0801.4485 [nucl-ex];
- J. B. Elliott, P. T. Lake, L. G. Moretto, and L. Phair, Phys. Rev. C 87, 054622 (2013)

IV. CSDC rule for dense and thermal systems

- Sound velocity:



(a) $\mathcal{O}(k_c^6)$. (b) $\mathcal{O}(k_c^8)$. (c) $\mathcal{O}(k_c^{10})$.

(d) $\mathcal{O}(k_c^{12})$. (e) TM1.

$$C_T^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_T$$

Coupling relations from scale compensator:

$$g_{\sigma NN}^{\mathrm{OBE}} = \frac{m_N}{f_\chi}, g_{\omega NN}^{\mathrm{OBE}} = g_{\omega N}$$

$$g_{\rho NN}^{\mathrm{OBE}} = g_{\rho NN} + g_{\rho NN}^{SSB}, g_{\pi NN}^{\mathrm{OBE}} = g_{\pi NN} + g_{\pi N}^{SSB}$$

$$g_{\sigma^2 NN} = \frac{m_N}{2f_\chi^2}, \quad g_{\sigma\omega NN} = g_{\omega NN}^{SSB}\frac{\beta'}{f_\chi},$$

$$g_{\sigma\rho NN} = g_{\rho NN}^{SSB}\frac{\beta'}{f_\chi},$$

$$g_{\sigma^3} = -\frac{32h_5}{3f_\chi^3} - \frac{(4+\beta')^3 h_6}{6f_\chi^3} - m_\pi^2 f_\pi^2 \frac{4}{3f_\chi^3},$$

Y. Sugahara and H. Toki, Nucl. Phys. A 579, 557 (1994).

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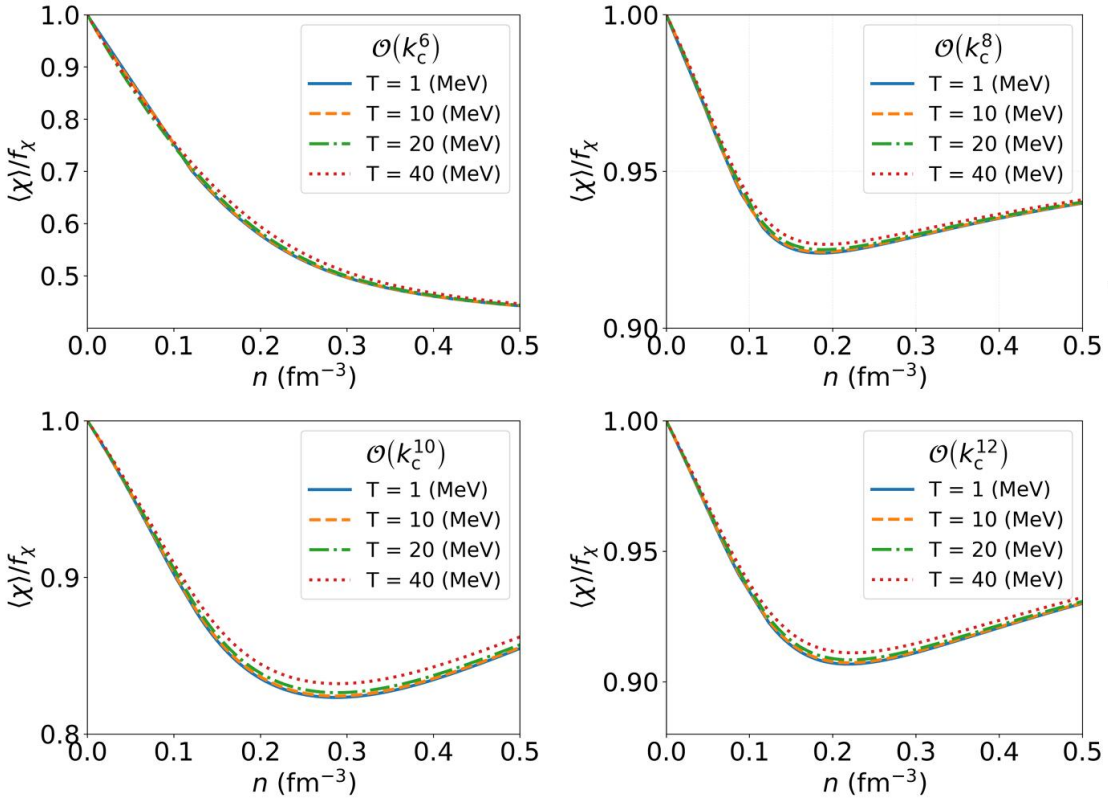
IV. CSDC rule for dense and thermal systems

- Scale symmetry:
  - At the lowest order, the scale symmetry is restored monotonously with higher densities.
  - High-order corrections bring a non-monotonic behavior of the scale restoration.
  - In low-temperature region, the scale symmetry is hardly affected by heat bath and is broken by higher temperatures.



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V. Summary and discussion

- The nonlinear realization of scale symmetry provides a source to include the lightest scalar mesons in Chiral EFT---Chiral-scale EFT;
- A novel pseudoconformal structure was proposed and a mechanism of the peak of SV was given. All phenomenological results agree with data and observstion (no shown here);
- CSDC has been established in the framework of RMF and the physics can be yielded with appropriate choice of the order. This scheme allows us to complete calculations order by order and may help us understand the phase structures of low energy QCD at finite temperatures and densities.

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Thank you for your attention!




Backup

LO chiral-scale Lagrangian

$$\mathscr{L}_M = f_\pi^2 \Phi^2 \,\mathrm{Tr}\left(\hat{\alpha}_\perp^\mu \hat{\alpha}_{\mu\perp}\right) + f_{\sigma\rho}^2 \Phi^2 \,\mathrm{Tr}\left(\hat{\alpha}_\parallel^\mu \hat{\alpha}_{\mu\parallel}\right) + f_0^2 \Phi^2 \,\mathrm{Tr}\left(\hat{\alpha}_\parallel^\mu\right) \mathrm{Tr}\left(\hat{\alpha}_{\mu\parallel}\right) -$$
$$\frac{1}{2g_\rho^2} \mathrm{Tr}\left(V_{\mu\nu} V^{\mu\nu}\right) - \frac{1}{2g_0^2} \mathrm{Tr}\left(V_{\mu\nu}\right) \mathrm{Tr}\left(V^{\mu\nu}\right) +$$
$$\frac{1}{2}\partial_\mu \chi \partial^\mu \chi + \frac{f_\pi^2}{4} \Phi^{3-\gamma_m} \mathrm{Tr}\left(\mathscr{M} U^\dagger + U \mathscr{M}^\dagger\right) + h_5 \Phi^4 + h_6 \Phi^{4+\beta'}$$

$$\mathscr{L}_B = \bar{N} i \gamma_\mu D^\mu N - m_N \Phi \bar{N} N$$
$$+ \left[g_A C_A + g_A \left(1 - C_A\right) \Phi^{\beta'}\right] \bar{N} \hat{\alpha}_\perp^\mu \gamma_\mu \gamma_5 N + \left[g_{V_\rho} C_{V_\rho} + g_{V_\rho} \left(1 - C_{V_\rho}\right) \Phi^{\beta'}\right] \bar{N} \hat{\alpha}_\parallel^\mu \gamma_\mu N$$
$$+ \frac{1}{2} \left[g_{V_0} C_{V_0} + g_{V_0} \left(1 - C_{V_0}\right) \Phi^{\beta'}\right] \mathrm{Tr}\left[\hat{\alpha}_\parallel^\mu\right] \bar{N} \gamma_\mu N$$

H. K. Lee, W.-G. Paeng, and M. Rho, Phys. Rev. D 92, 125033 (2015)
Y. L. Li, Y. L. Ma, and M. Rho, Phys. Rev. D 95, 114011 (2017)

CSDC expansion

Lagrangians under CSDC

$$\mathscr{L}_{\mathrm{LO}} = \bar{N}\left(i\gamma_\mu\partial^\mu - m_N\right)N,$$

$$\mathscr{L}_{\mathrm{NLO}} = \mathscr{L}_{\mathrm{LO}} - \bar{N}\left(g^{\mathrm{OBE}}_{\omega NN}\gamma_0\omega + g^{\mathrm{OBE}}_{\rho NN}\gamma_0\rho\tau_3 + g^{\mathrm{OBE}}_{\sigma NN}\sigma\right)N + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\rho^2\rho^2 - \frac{1}{2}m_\sigma^2\sigma^2,$$

$$\mathscr{L}_{\mathrm{N^2LO}} = \mathscr{L}_{\mathrm{NLO}} - \bar{N}\left(g_{\sigma^2 NN}\sigma^2 - g_{\sigma\omega NN}\sigma\omega\gamma^0 - g_{\sigma\rho NN}\sigma\rho\gamma^0\tau_3\right)N - g_{\sigma_3}\sigma^3 - g_{\sigma\omega^2}\sigma\omega^2 - g_{\sigma\rho^2}\sigma\rho^2,$$

$$\mathscr{L}_{\mathrm{N^3LO}} = \mathscr{L}_{\mathrm{N^2LO}} - \bar{N}\left(g_{\sigma^3 NN}\sigma^3 - g_{\sigma^2\omega NN}\sigma^2\omega\gamma^0 - g_{\sigma^2\rho NN}\sigma^2\rho\gamma^0\tau_3\right)N - g_{\sigma^4}\sigma^4 - g_{\sigma^2\omega^2}\sigma^2\omega^2 - g_{\sigma^2\rho^2}\sigma^2\rho^2,$$

$$\mathscr{L}_{\mathrm{N^4LO}} = \mathscr{L}_{\mathrm{N^3LO}} - \bar{N}\left(g_{\sigma^4 NN}\sigma^4 - g_{\sigma^3\omega NN}\sigma^3\omega\gamma^0 - g_{\sigma^3\rho NN}\sigma^3\rho\gamma^0\tau_3\right)N - g_{\sigma^5}\sigma^5 - g_{\sigma^3\omega^2}\sigma^3\omega^2 - g_{\sigma^3\rho^2}\sigma^3\rho^2.$$

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The relations of parameters

Parameter definition

$$4h_5 + (4+\beta')\, h_6 + 2m_\pi^2 f_\pi^2 = 0$$

$$12h_5 + (4+\beta')\,(3+\beta')\, h_6 + 2m_\pi^2 f_\pi^2 = -\, m_\sigma^2 f_\chi^2$$

$$g_{\omega NN} = \frac{1}{2}\left(g_{V_p} C_{V_p} + g_{V_0} C_{V_0} - 1\right) g_\omega,$$

$$g_{\rho NN} = \frac{1}{2}\left(g_{V_p} C_{V_p} - 1\right) g_\rho,$$

$$g_{\pi NN} = -\frac{(g_A C_A)}{2f_\pi},$$

$$g^{SSB}_{\omega NN} = \frac{1}{2}\left[g_{V_\rho}\left(1 - C_{V_\rho}\right) + g_{V_0}\left(1 - C_{V_0}\right)\right] g_\omega,$$

$$g^{SSB}_{\rho NN} = \frac{1}{2}\left[g_{V_\rho}\left(1 - C_{V_\rho}\right)\right] g_\rho,$$

$$g^{SSB}_{\pi NN} = -\frac{\left[g_A\left(1 - C_A\right)\right]}{2f_\pi};$$

$$g^{\mathrm{OBE}}_{\sigma NN} = \frac{m_N}{f_\chi}, \quad g^{\mathrm{OBE}}_{\omega NN} = g_{\omega NN} + g^{SSB}_{\omega NN}, \quad g^{\mathrm{OBE}}_{\rho NN} = g_{\rho NN} + g^{SSB}_{\rho NN}, \quad g^{\mathrm{OBE}}_{\pi NN} = g_{\pi NN} + g^{SSB}_{\pi NN}$$

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The relations of parameters

$$g_{\sigma^2 NN} = \frac{m_N}{2f_\chi^2}, \quad g_{\sigma\omega NN} = g_{\omega NN}^{SSB}\frac{\beta'}{f_\chi}, \quad g_{\sigma\rho NN} = g_{\rho NN}^{SSB}\frac{\beta'}{f_\chi}$$

$$g_{\sigma^3} = -\frac{32h_5}{3f_\chi^3} - \frac{\left(4+\beta'\right)^3 h_6}{6f_\chi^3} - m_\pi^2 f_\pi^2 \frac{4}{3f_\chi^3}, \quad g_{\sigma\rho^2} = -\frac{m_\rho^2}{f_\chi^2}, \quad g_{\sigma\omega^2} = -\frac{m_\omega^2}{f_\chi^2}$$

$$g_{\sigma^3 NN} = \frac{m_N}{6f_\chi^3}, \quad g_{\sigma^4 NN} = \frac{m_N}{24f_\chi^4}, \quad g_{\sigma^3\omega NN} = \frac{\beta'^2 g_{\sigma\omega NN}}{6f_\chi^2}, \quad g_{\sigma^3\rho NN} = \frac{\beta'^2 g_{\sigma\rho NN}}{6f_\chi^2},$$

$$g_{\sigma^4} = -\frac{4^4 h_5 + \left(4+\beta'\right)^4 h_6 + 2^4 m_\pi^2 f_\pi^2}{24f_\chi^4}, \quad g_{\sigma^2\omega\omega} = -\frac{m_\omega^2}{f_\chi^2}, \quad g_{\sigma^2\rho\rho} = -\frac{m_\rho^2}{f_\chi^2},$$

$$g_{\sigma^5} = -\frac{4^5 h_5 + \left(4+\beta'\right)^5 h_6 + 2^5 m_\pi^2 f_\pi^2}{120f_\chi^5}, \quad g_{\sigma^3\omega\omega} = -\frac{2m_\omega^2}{3f_\chi^3}, \quad g_{\sigma^3\rho\rho} = -\frac{2m_\rho^2}{3f_\chi^3}.$$

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Effective masses and chemical potentials

Effective mass and chemical potential at each order under RMF

$$\mathcal{O}(k_c^6)$$

$$m_N^* = m_N + g_{\sigma NN}^{\mathrm{OBE}}\sigma$$

$$\mu_p^* = \mu_p - g_{\omega NN}^{\mathrm{OBE}}\omega - g_{\rho NN}^{\mathrm{OBE}}\rho$$

$$\mu_n^* = \mu_n - g_{\omega NN}^{\mathrm{OBE}}\omega + g_{\rho NN}^{\mathrm{OBE}}\rho$$

$$\mathcal{O}(k_c^8)$$

$$m_N^* = m_N + g_{\sigma NN}^{\mathrm{OBE}}\sigma + g_{\sigma^2 NN}\sigma^2$$

$$\mu_p^* = \mu_p - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma\right)\omega - \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma\right)\rho$$

$$\mu_n^* = \mu_n - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma\right)\omega + \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma\right)\rho$$

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Effective masses and chemical potentials

$\mathcal{O}(k_c^{10})$

$$m_N^* = m_N + g_{\sigma NN}^{\mathrm{OBE}}\sigma + g_{\sigma^2 NN}\sigma^2 + g_{\sigma^2\sigma NN}\sigma^3$$

$$\mu_p^* = \mu_p - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma + g_{\sigma^2\omega NN}\sigma^2\right)\omega - \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma + g_{\sigma^2\rho NN}\sigma^2\right)\rho$$

$$\mu_n^* = \mu_n - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma + g_{\sigma^2\omega NN}\sigma^2\right)\omega + \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma + g_{\sigma^2\rho NN}\sigma^2\right)\rho$$

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Effective masses and chemical potentials

$\mathcal{O}(k_c^{12})$

$$m_N^* = m_N + g_{\sigma NN}^{\mathrm{OBE}}\sigma + g_{\sigma^2 NN}\sigma^2 + g_{\sigma^3 NN}\sigma^3 + g_{\sigma^4 NN}\sigma^4$$

$$\mu_p^* = \mu_p - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma + g_{\sigma^2\omega NN}\sigma^2 + g_{\sigma^3\omega NN}\sigma^3\right)\omega - \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma + g_{\sigma^2\rho NN}\sigma^2 + g_{\sigma^3\rho NN}\sigma^3\right)\rho$$

$$\mu_n^* = \mu_n - \left(g_{\omega NN}^{\mathrm{OBE}} + g_{\sigma\omega NN}\sigma + g_{\sigma^2\omega NN}\sigma^2 + g_{\sigma^3\omega NN}\sigma^3\right)\omega + \left(g_{\rho NN}^{\mathrm{OBE}} + g_{\sigma\rho NN}\sigma + g_{\sigma^2\rho NN}\sigma^2 + g_{\sigma^3\rho NN}\sigma^3\right)\rho$$