Heavy quark thermodynamics with anisotropic lattices

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FASTSUM collaboration

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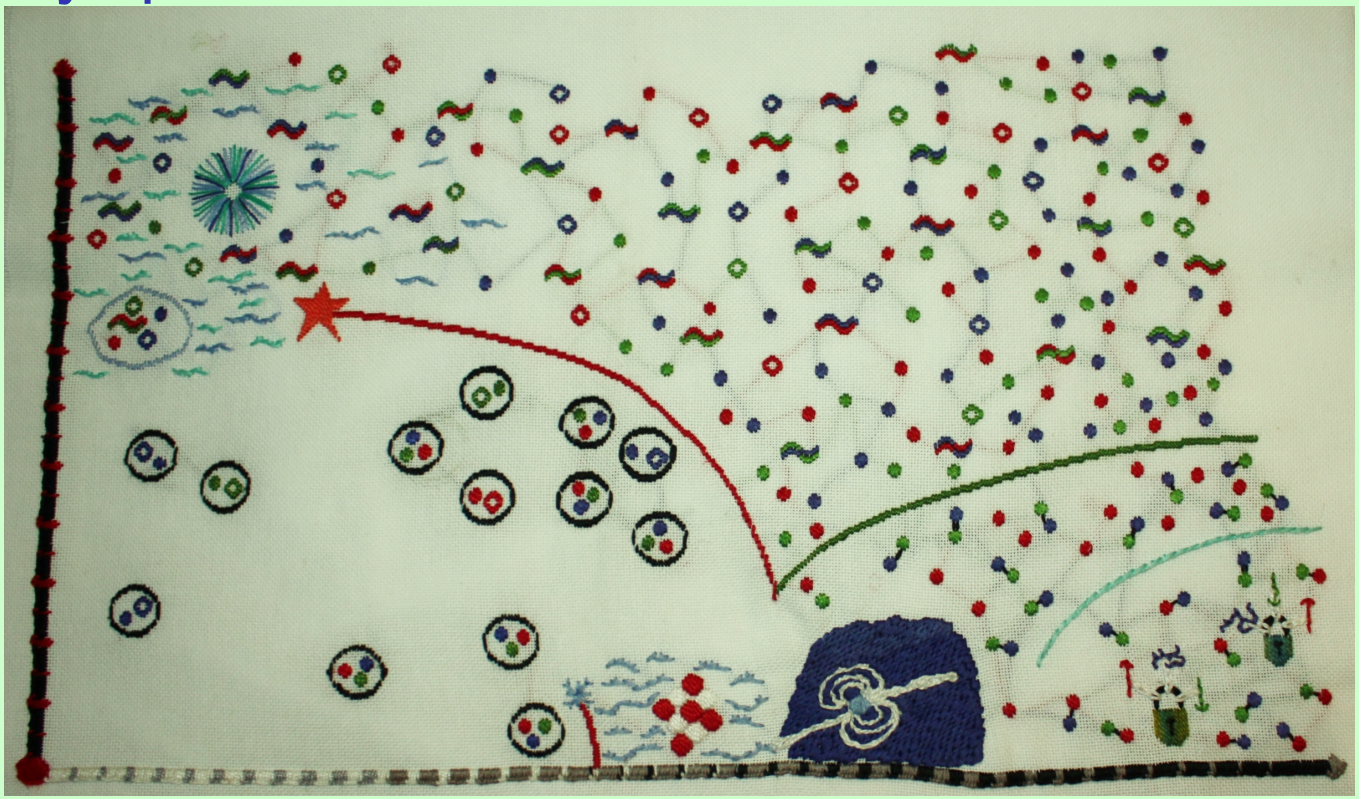
Outline

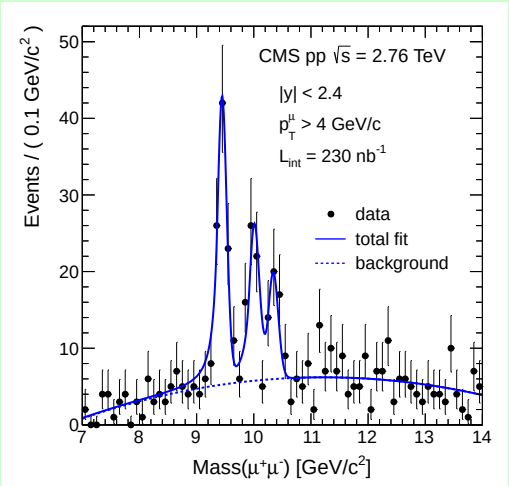
Heavy quarks in the QGP

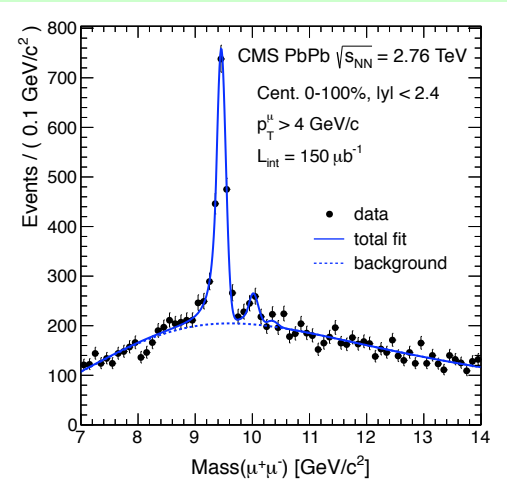


- Heavy quarks are important probes of medium
- Long history of $c\overline{c}$ studies: experiment, pheno, lattice
- Sequential $b\overline{b}$ suppression observed, numerous studies

Beauty

- Bound states expected to survive up to $T_d^{\Upsilon} \sim 3 - 5\,T_c$
- $\chi_b, \Upsilon(2S)$ melt at $T_d' \lesssim 1.2\,T_c$?
- Sequential suppression observed at CMS, ATLAS, STAR
- Detailed information on mass shifts and widths required for dynamical modelling
- Potential models may be applicable $\rightarrow$ real-time potential
- Open beauty experimental results becoming available





Spectral functions

- contain information about the fate of hadrons in the medium
  - stable states $\rho(\omega) \sim \delta(\omega - m)$
  - resonances or thermal width $\rho(\omega) \sim$ lorentzian
  - continuum above threshold

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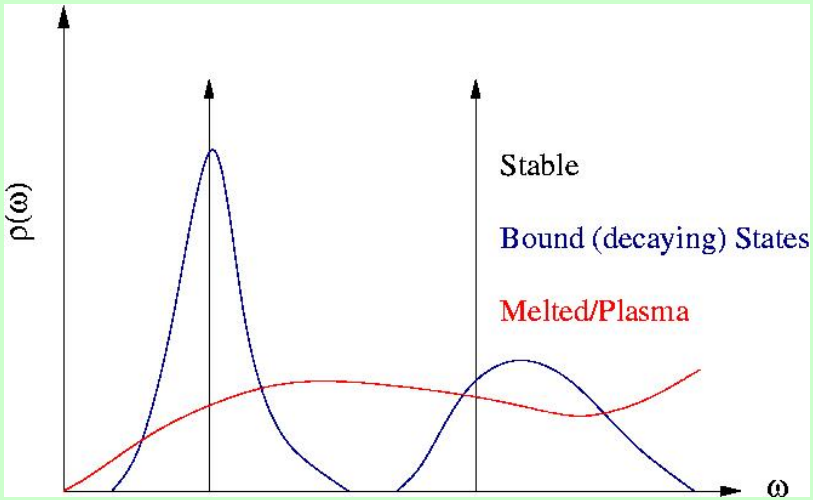

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  - stable states $\rho(\omega) \sim \delta(\omega - m)$
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- $\rho_\Gamma(\omega, \vec{p})$ related to euclidean correlator $G_\Gamma(\tau, \vec{p})$ according to

$$G_\Gamma(\tau, \vec{p}) = \int \rho_\Gamma(\omega, \vec{p}) K(\tau, \omega) d\omega \,,$$

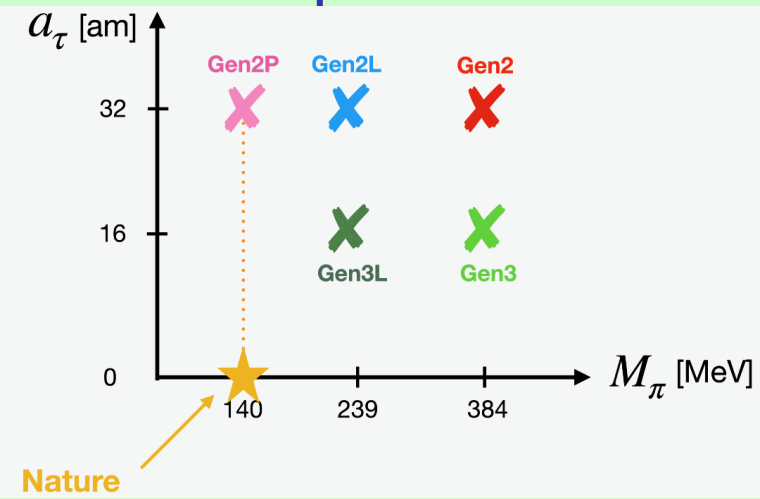
- an ill-posed problem
  - Direct correlator analysis (model driven)
  - Smeared spectral functions
  - Bayesian methods
  - Other methods are available: ML, Cuniberti, Schlessinger, ...

Dynamical anisotropic lattices

- A large number of points in time direction required to extract spectral information
- For $T = 2T_c$, $\mathcal{O}(10)$ points $\Longrightarrow$ $a_t \sim 0.025$ fm
- Far too expensive with isotropic lattices $a_s = a_t$!
- Fixed-scale approach
  - vary T by varying N_τ (not a)
  - need only 1 $T = 0$ calculation for renormalisation
  - independent handle on temperature

- Introduces 2 additional parameters
- Non-trivial tuning problem
  [PRD **74** 014505 (2006); HadSpec Collab, PRD **79** 034502 (2009)]

Simulation parameters



Gauge action:
Symanzik improved (Lüscher–Weisz)

Fermion action:
Wilson–clover, tree-level mean-field coeffs, stout links

Strange quark tuned to $\approx$ physical

Gen	N_f	ξ	a_s (fm)	a_τ^{-1} (GeV)	m_π (MeV)	N_s	L_s (fm)
1	2	6.0	0.162	7.35	490	12	1.94
2	2+1	3.45	0.123	5.63	390	24	2.95
						32	3.94
2L	2+1	3.45	0.112	6.08	240	32	3.58
2P	2+1	3.45	*0.100	*6.80	140	48	4.80
3	2+1	7.0	0.109	12.82	380	32	3.49
3L	2+1	7.0	*0.100	*13.80	240	32	3.20

Simulation parameters: temperatures

Gen 2			Gen 2L			Gen 3		
N_τ	T (MeV)	T/T_c	N_τ	T (MeV)	T/T_c	N_τ	T (MeV)	T/T_c
128	44	0.24	128	47	0.42	256	50	0.28
			64	95	0.59	128	100	0.55
			56	109	0.67	112	114	0.63
48	117	0.63	48	127	0.78	96	133	0.73
40	141	0.76	40	152	0.94	80	160	0.88
						76	169	0.93
36	156	0.84	36	169	1.04	72	178	0.98
						68	188	1.04
32	176	0.95	32	190	1.17	64	200	1.10
28	201	1.09	28	217	1.34	56	229	1.26
24	235	1.27	24	253	1.56	48	267	1.47
20	281	1.52	20	304	1.87	40	320	1.76
						36	356	1.96
16	352	1.90	16	380	2.34	32	400	2.20
			12	507	3.12	24	534	2.93

NRQCD

Scale separation $M_Q \gg T, M_Q v$

Integrate out hard scales $\longrightarrow$ Effective theory

Expand in orders of heavy quark velocity $\mathbf{v}$; we use $\mathcal{O}(\mathbf{v}^4)$ action

Advantages

- Simple (T-independent) kernel, $G(\tau) = \int \rho(\omega) e^{-\omega\tau} \frac{d\omega}{2\pi}$
- No zero-modes
- Longer euclidean time range, $\tau_{\mathrm{max}} \approx 1/T$
- High-precision correlators feasible

Disadvantages

- Not renormalisable, requires $Ma_s \gtrsim 1$
- Does not incorporate transport properties
- Energy shift: only energy differences are physical

Time-derivative moments

Basic idea

For a peaked spectral function $\rho(\omega)$ describing a (quasi)particle with mass M and width ω,

$$M = \langle\omega\rangle = \int \omega\rho(\omega)d\omega\,, \quad \Gamma = \mathrm{Var}\omega = \langle\omega^2\rangle - \langle\omega\rangle^2$$

In NRQCD this is equivalent to taking time derivatives of the correlator,

$$G(\tau) = \int \rho(\omega)e^{-\omega\tau}\frac{d\omega}{2\pi}$$

$$G'(\tau) = -\int \omega\rho(\omega)e^{-\omega\tau}\frac{d\omega}{2\pi}$$

$$G''(\tau) = \int \omega^2\rho(\omega)e^{-\omega\tau}\frac{d\omega}{2\pi}$$

Gaussian moments

Assume $\rho(\omega)$ can be approximated by a sum of Gaussians,

$$\rho(\omega; T) = \sum_{i=0}^{\infty} A_i e^{-\frac{(\omega - m_i)^2}{2\Gamma_i^2}},$$

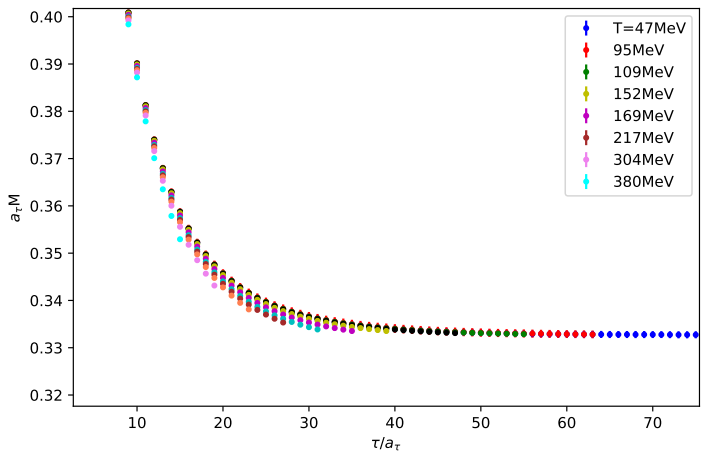
$$\implies \quad G(\tau) = \sum_{i=0}^{\infty} A_i e^{-m_i\tau + \Gamma_i^2\tau^2/2}$$

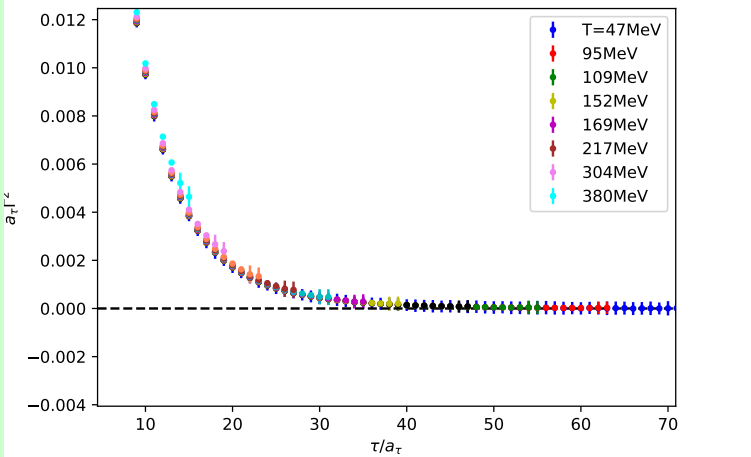
$$= A_0 e^{-m_0\tau + \Gamma_0^2\tau^2/2} \left(1 + \sum_{i=1}^{\infty} \frac{A_i}{A_0} e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}\right)$$

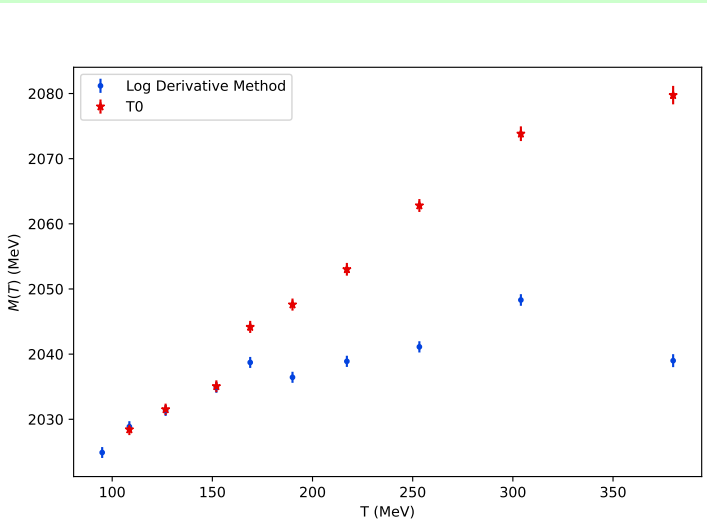
$$\frac{G'(\tau)}{G(\tau)} = \frac{d\log(G(\tau))}{d\tau} = (-m_0 + \Gamma_0^2\tau) + \sum_{i=1}^{\infty} \frac{A_i}{A_0} e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}$$

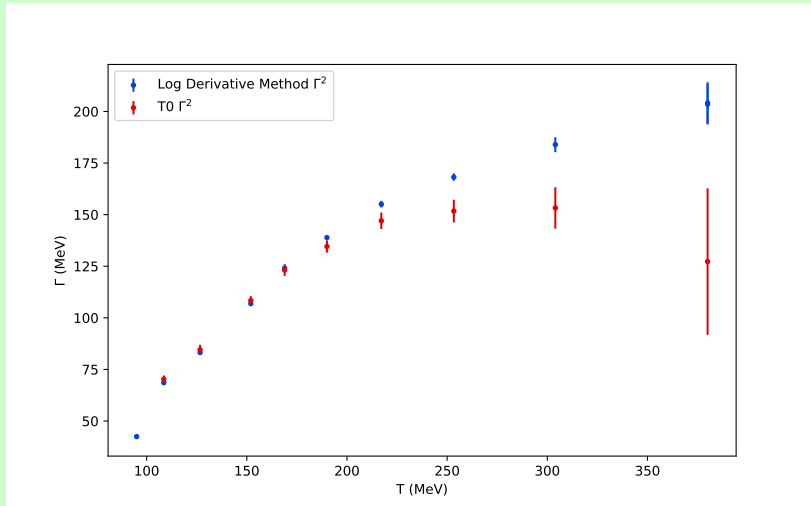
$$\frac{d^2\log(G(\tau))}{d\tau} = \Gamma_0^2 + \sum_{i=1}^{\infty} B_i e^{-\Delta m_i\tau + \Delta\Gamma_i^2\tau^2/2}$$

Moments results

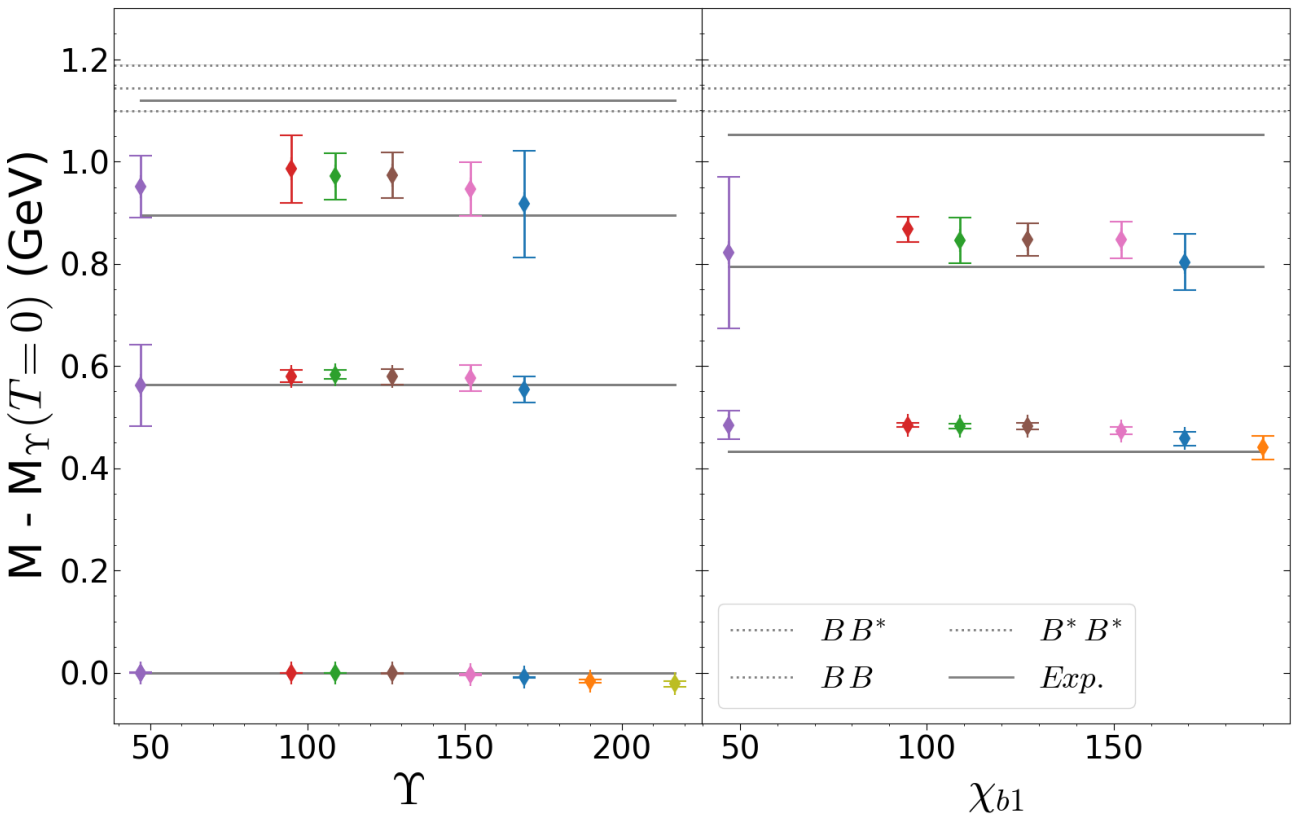






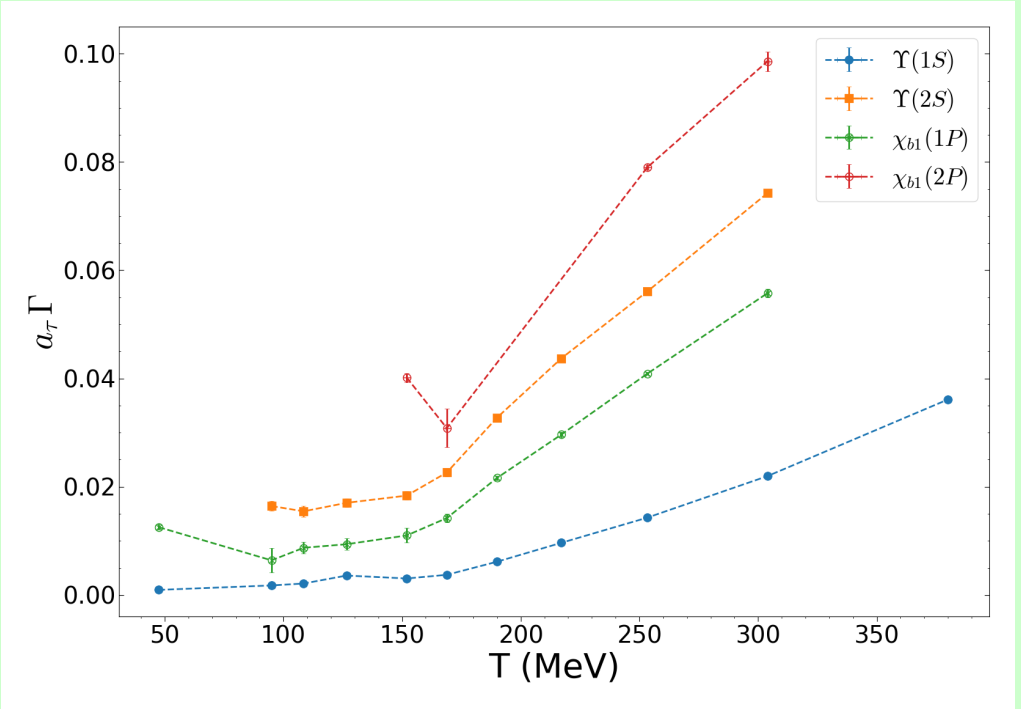


GEVP results: masses



GEVP results: widths

- Apply moments method to projected correlators
- Asymptotic term in fits stable even with noisy data



Linear methods: smeared spectral functions

Introduce smearing function

$$\overline{\Delta}(\omega,\omega_n)=\sum_{\tau=0}^{\tau_{\max}} g_\tau(\omega_n)e^{-\omega\tau} \xrightarrow{N_\tau\to\infty} \delta(\omega-\omega_n)$$

Smeared spectral function is reconstructed as

$$\hat{\rho}(\omega_n)=\sum_{\tau=0}^{\tau_{\max}} g_\tau(\omega_n)G(\tau)=\int_{\omega_{\min}}^{\infty} d\omega\overline{\Delta}(\omega,\omega_n)\rho(\omega)$$

Determine coefficients g_τ by minimising functional

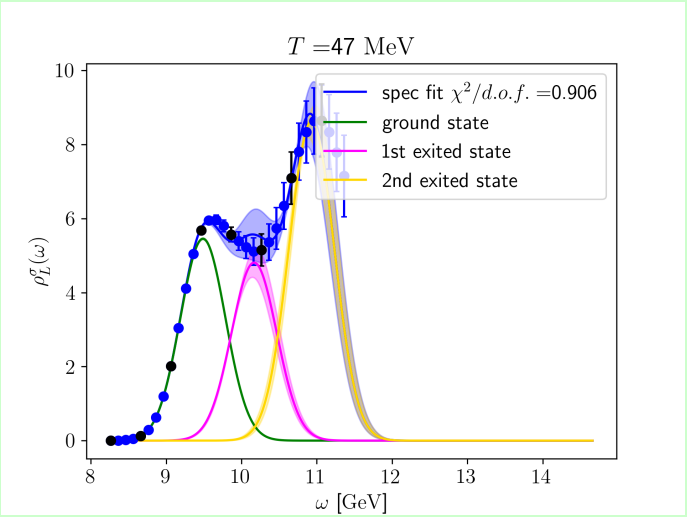
$$A[g_\tau]=\int_{\omega_{\min}}^{\infty} d\omega F[\omega,\omega_n;\overline{\Delta}]$$

Different prescriptions:

Tikhonov, Backus–Gilbert, Hansen–Lupo–Tantalo

HLT fits

- Smearing width σ is input in HLT
  $\longrightarrow$ upper bound on Γ
- Fit spectral function to sum of gaussians, each with width σ
- Number of peaks decreases with increasing T
- σ must be increased



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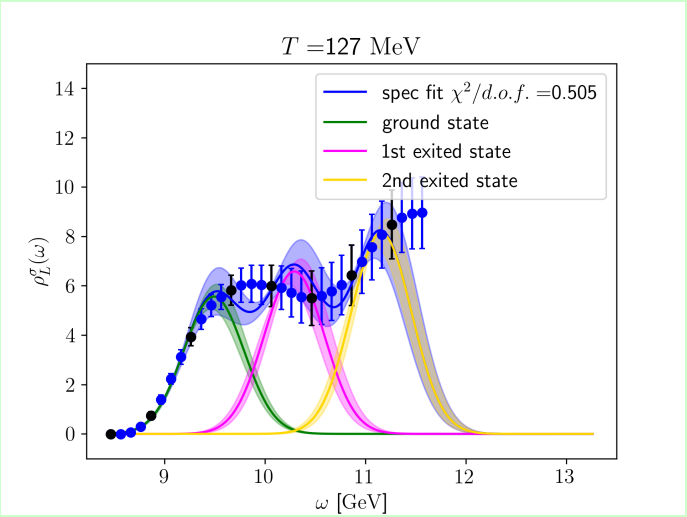

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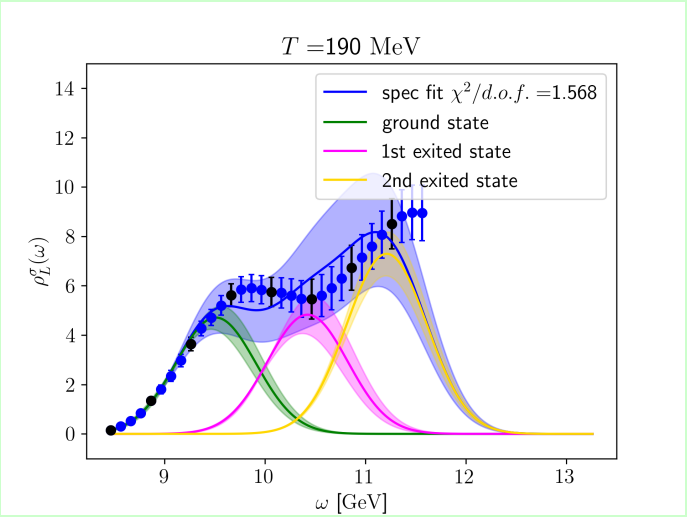

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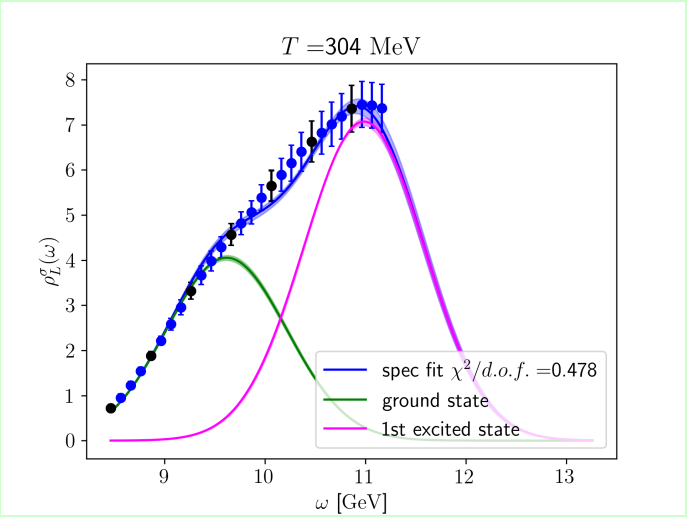

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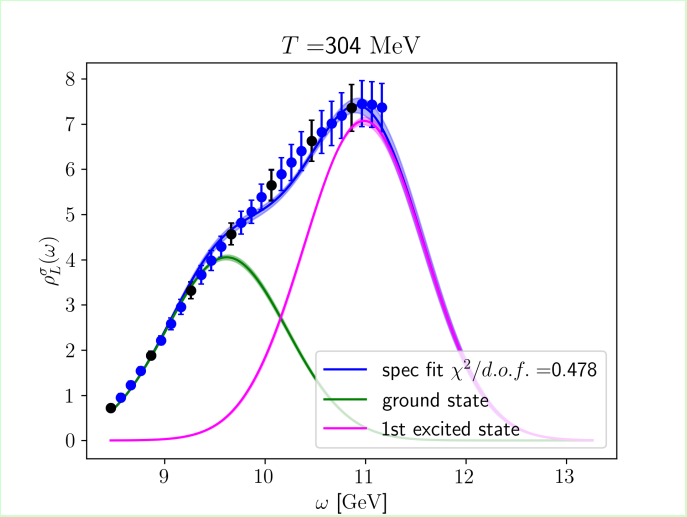
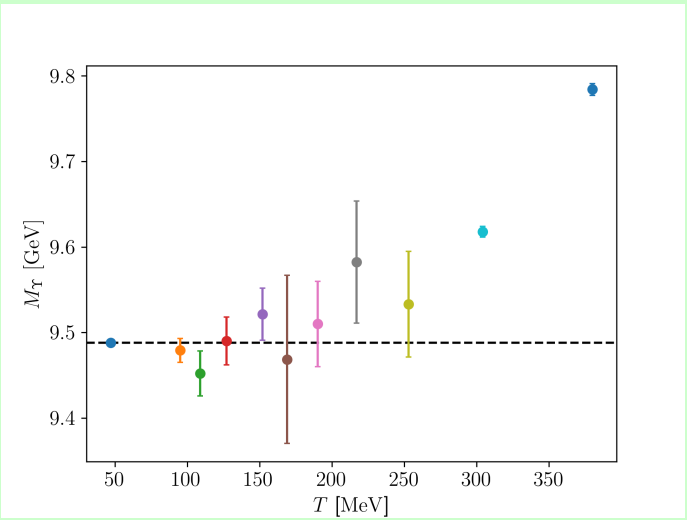




Bayesian spectral function reconstruction

Bayes theorem

$$P(\rho|DI) = \frac{P(D|\rho I)P(\rho|I)}{P(D|I)}$$

Parametrise prior probability by

$$P(\rho|I) \propto e^{\alpha S[\rho]} \quad \Longrightarrow \quad P(\rho|DI) \propto e^{-L[D,\rho]+\alpha S[\rho]}$$

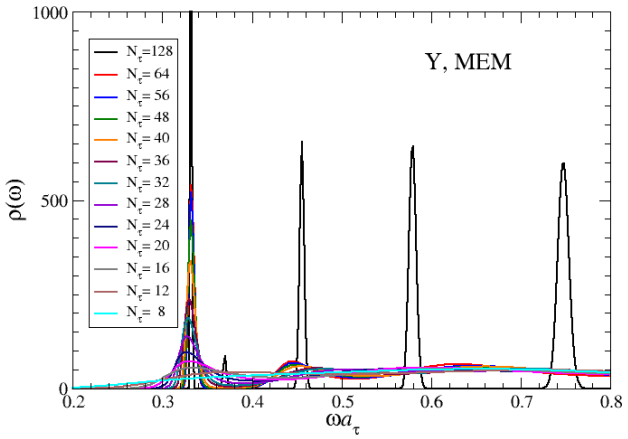
where L is the standard likelihood (χ^2)

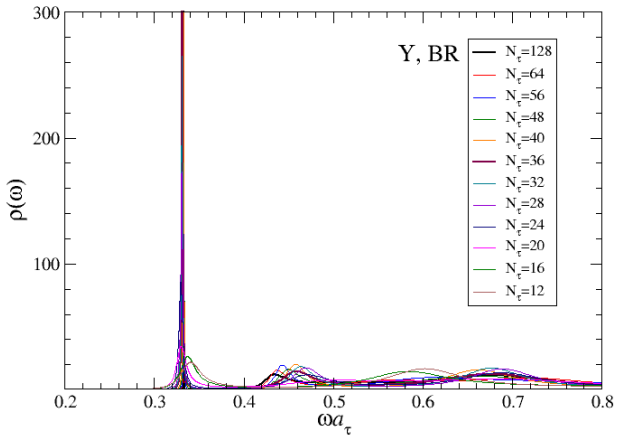
Spectral function $\rho(\omega)$ is expressed in terms of default model $m(\omega)$

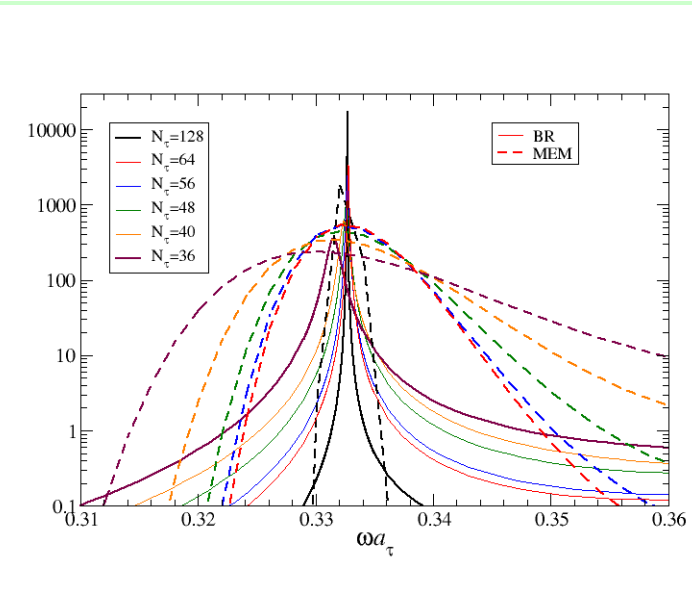
$$\rho(\omega) = m(\omega)\exp[\sum_{k=1}^{N_b} b_k u_k(\omega)]$$

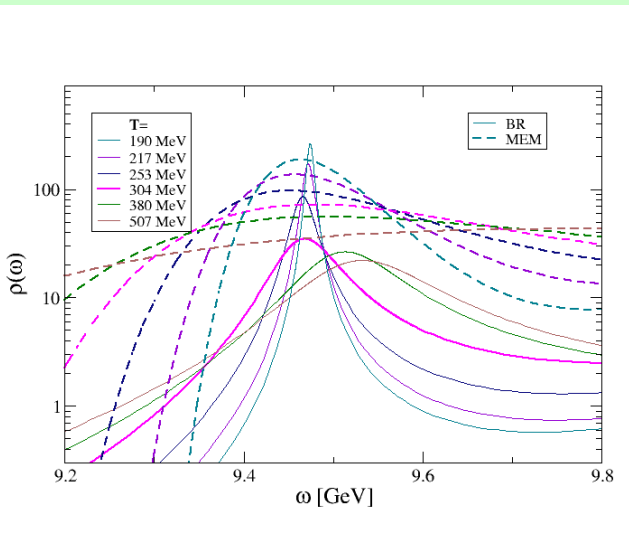
Different prescriptions for S, u_k: MEM and BR

MEM vs BR method

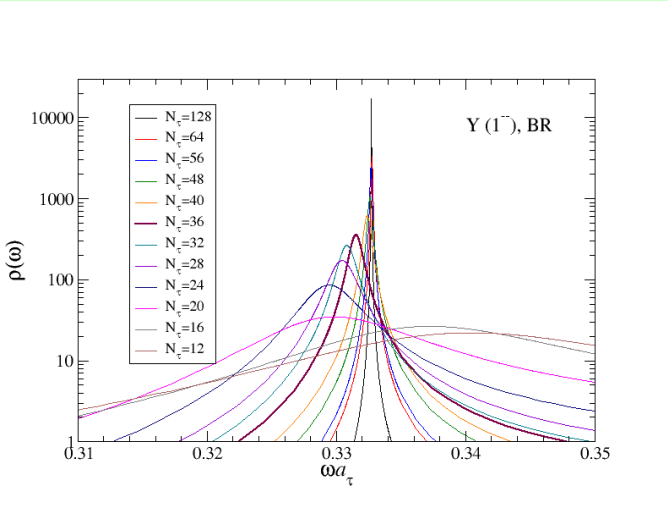


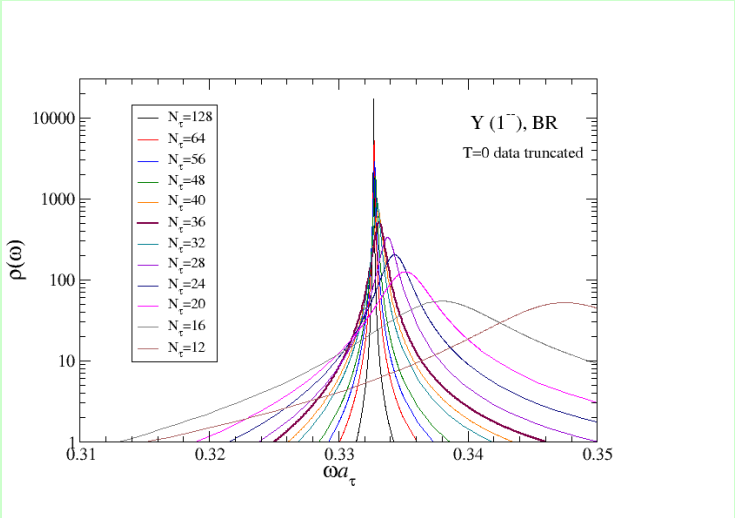






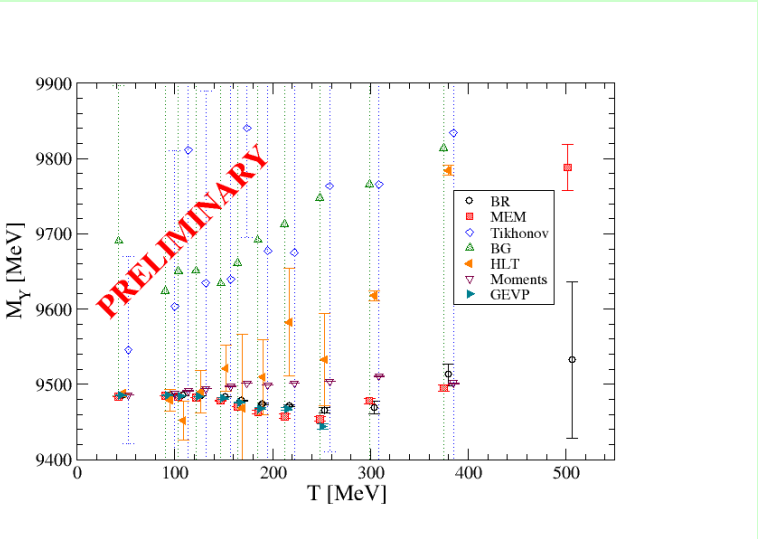
BR results: Υ

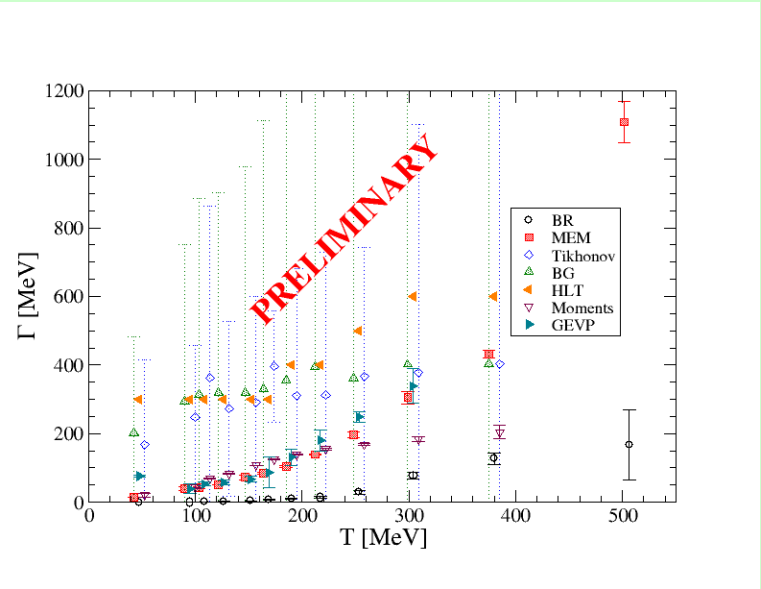


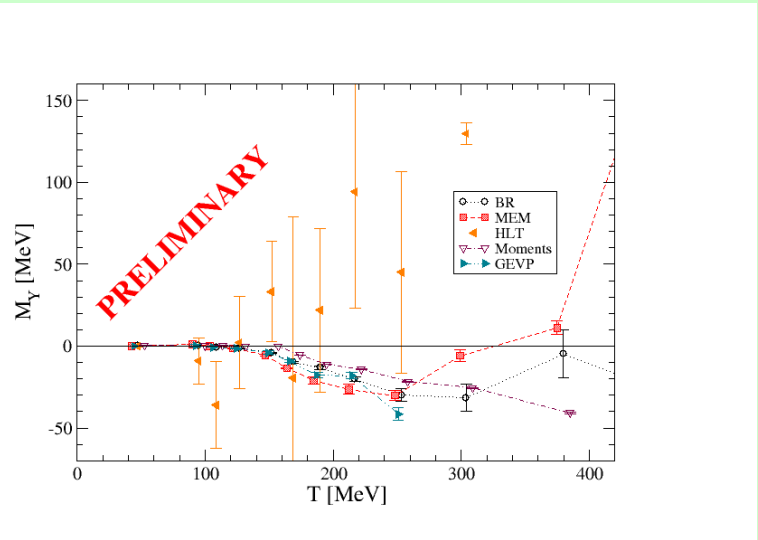


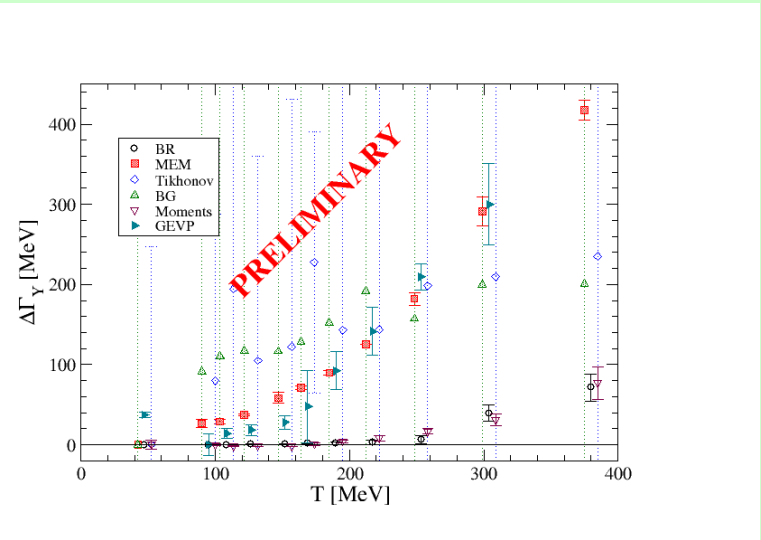
- Clear evidence of negative mass shift
- Finite N_τ artefacts would give positive mass shift
- Broadening similar in magnitude to $T = 0$ truncated data

Summary of ϒ results



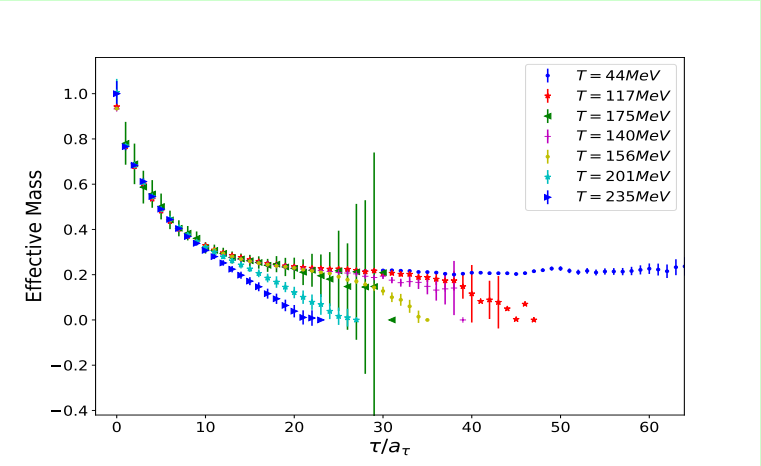


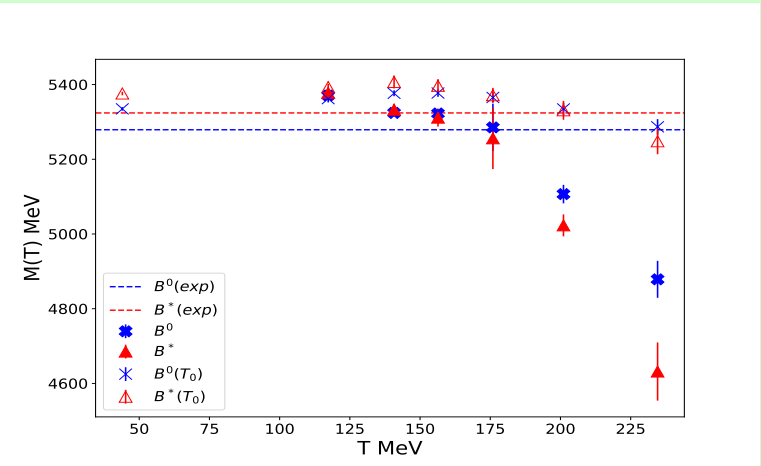




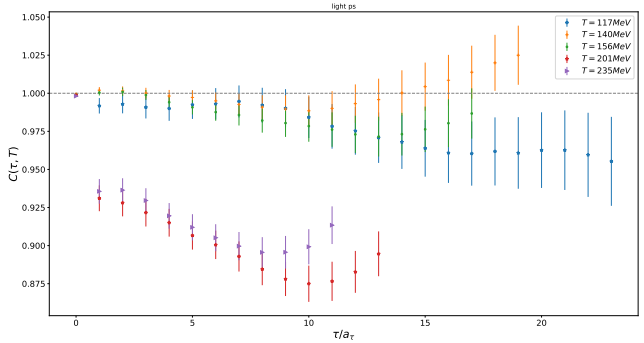
B mesons

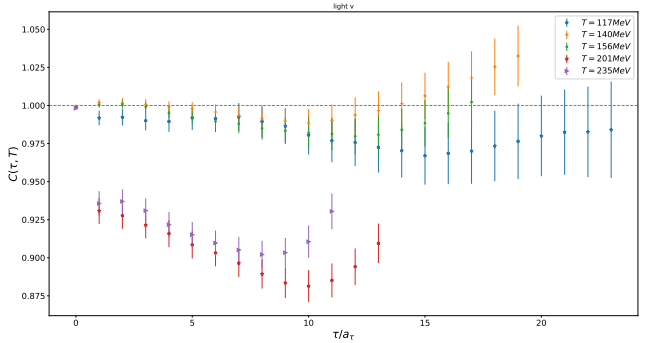
- b quarks using NRQCD
- relativistic light quarks
- transform to Dirac basis, truncate to upper components
- Correlator fits
- BR spectral functions
- Gen2 (Gen2L in progress)





B meson correlator ratios, spectral functions





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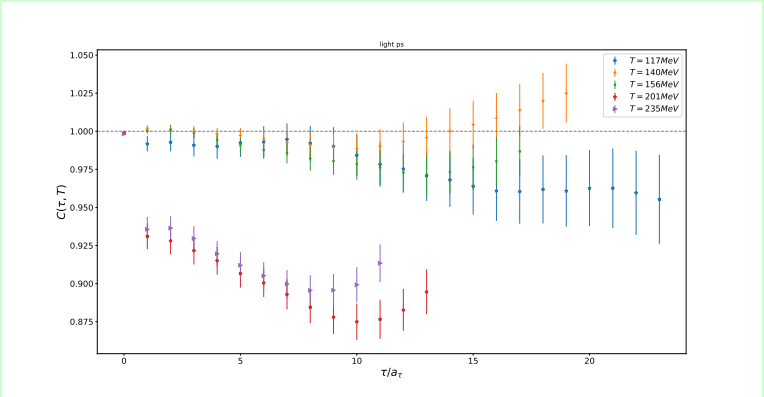
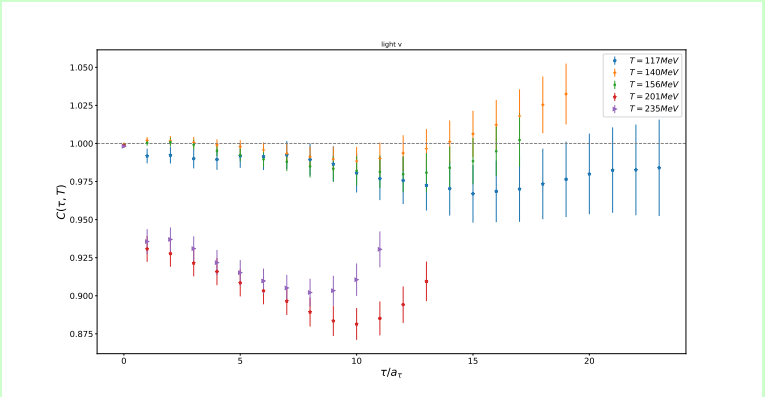
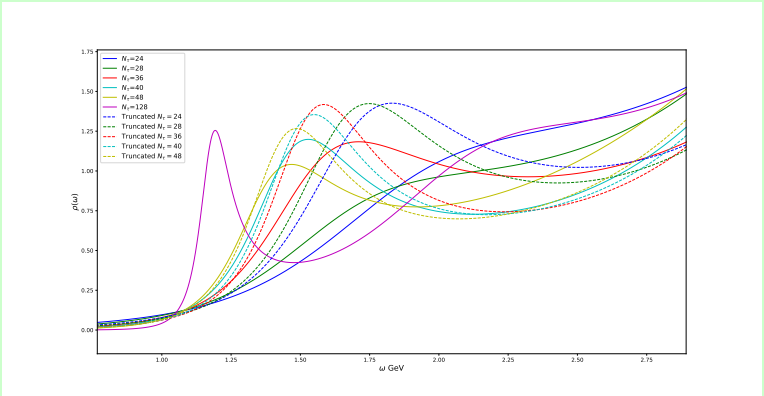
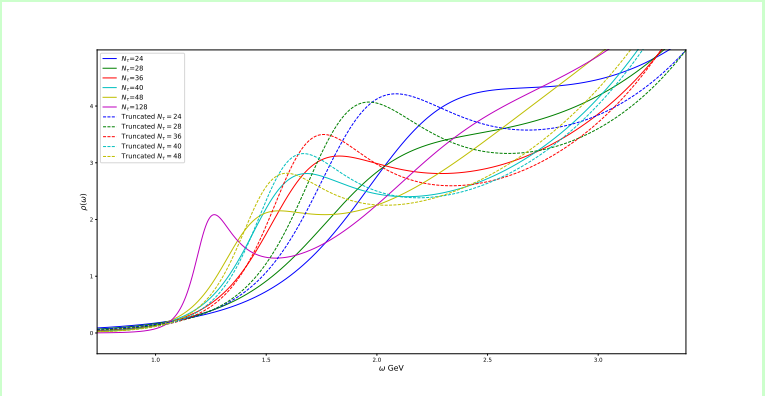








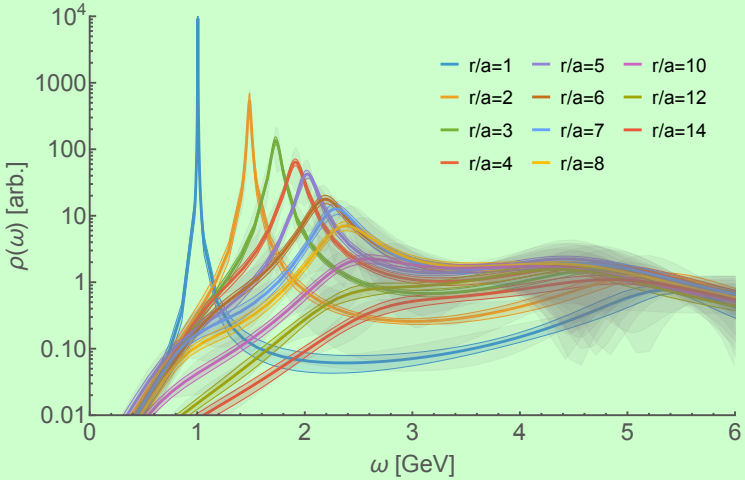
Big change around $T_c \rightarrow$ no surviving state above T_c?

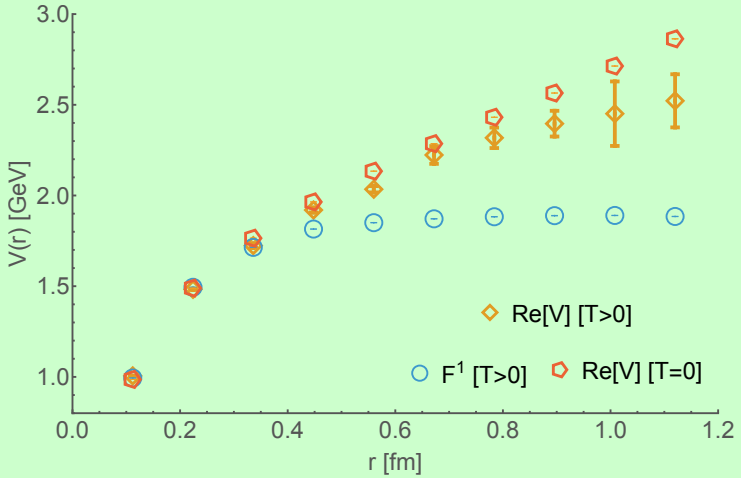
Static quark potential

- Long (pre-)history of potential models
- Definition of real-time potential, relation to Wilson loop/lines
- Real and imaginary parts (peak position and width)
- Screening or no screening: different methods give different results

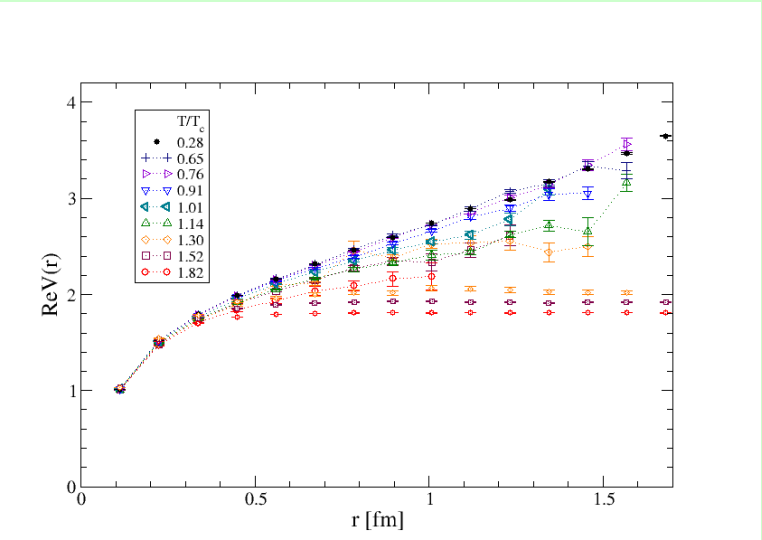
BR results in detail

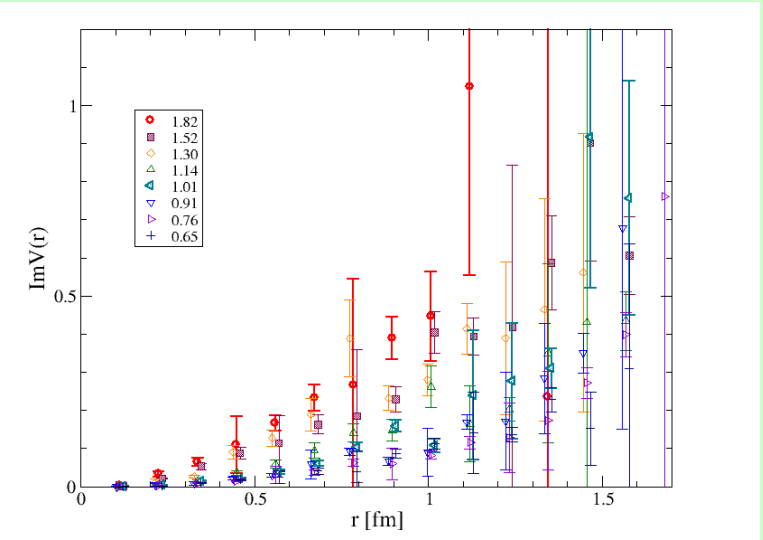
High statistics: ~ 3000 configs at $N_\tau = 24 (T/T_c = 1.5)$





BR results: summary





UV subtraction method

- Subtract T-independent "UV" correlator
- UV corr taken to be $T = 0$ corr minus lowest state(s)
- Fit UV subtracted m_{eff} to straight line (assume gaussian $\rho(\omega)$)
- Consider ground and first excited state

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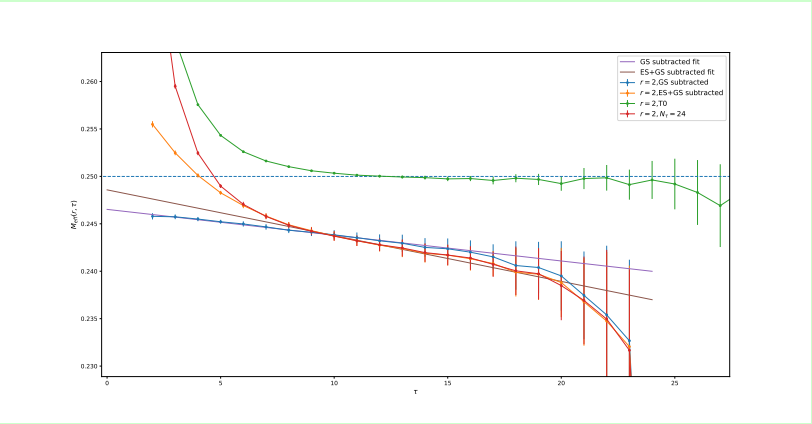

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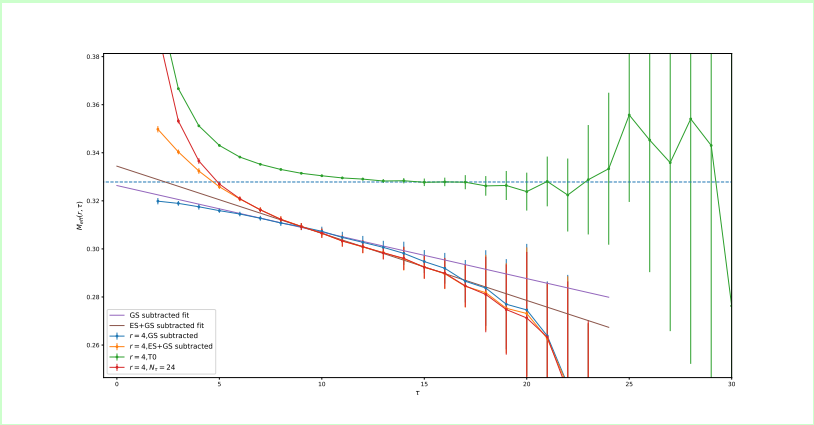

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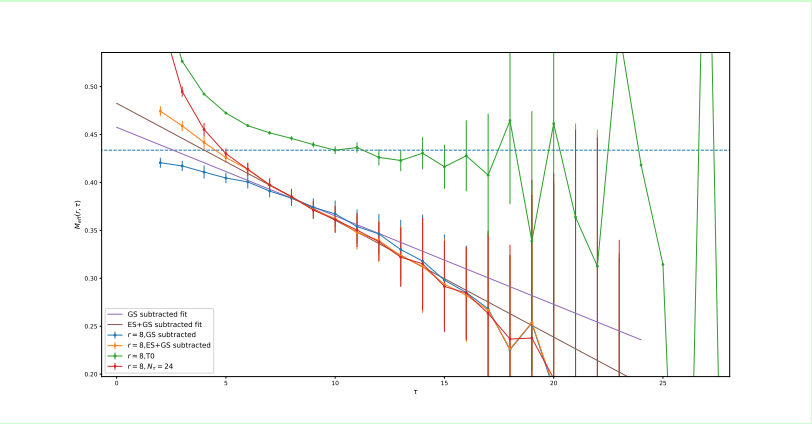

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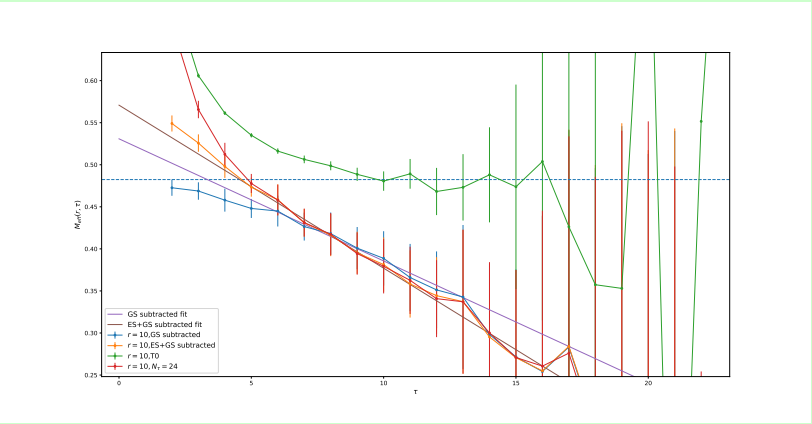

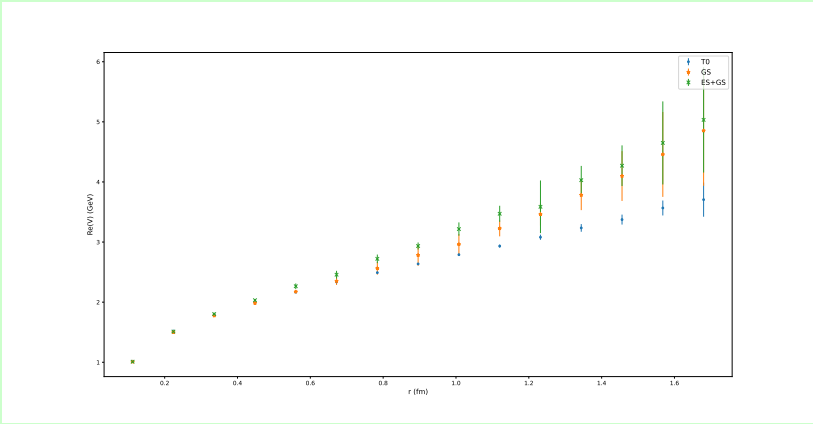

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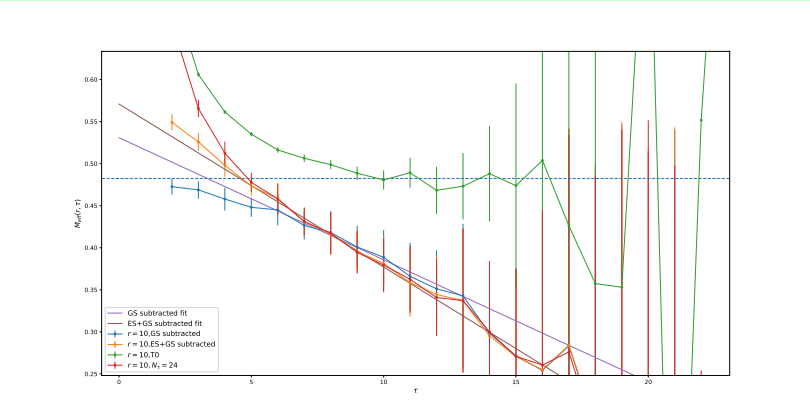

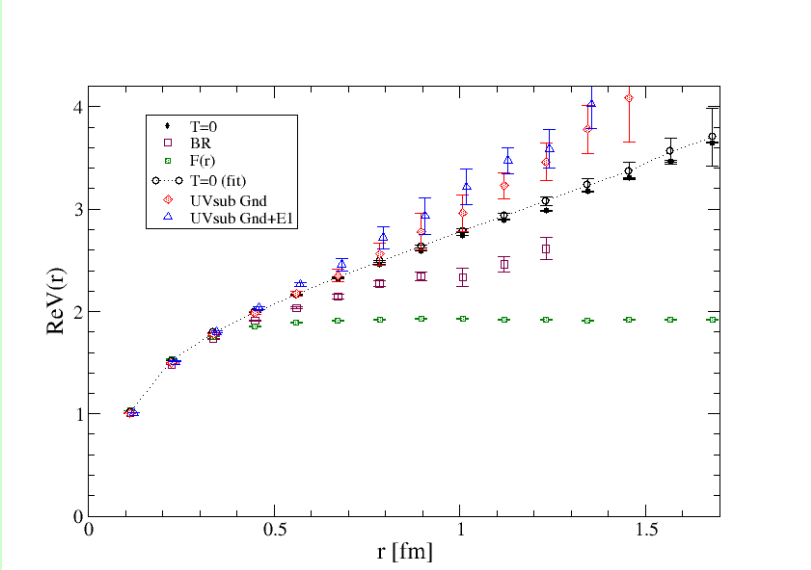

Summary

- Mass and width of ground state S- and P-wave bottomonium (Υ and χ_{b1}) studied with a range of different methods
  - Agreement between correlator and bayesian methods on a negative mass shift of up to 20–40 MeV at $T \sim 250$ MeV.
- First lattice study of B mesons at high T
  - Evidence of small negative mass shift
  - Spectral functions suggest no state above T_c
- Static quark potential from BR exhibits screening at high T
- UV subtraction method gives paradoxical results
  — based on invalid assumptions?

Outlook

- Complete study of systematics
  — including $T = 0$ subtraction for all methods
- Repeat with smaller a_τ: Gen3

Generalised eigenvalue problem

Correlator matrix

$$G_{ij}(\tau) = \langle \Omega | \mathcal{O}_i \mathcal{O}_j^\dagger | \Omega \rangle = \sum_\alpha \frac{Z_i^\alpha Z_j^{\alpha\,\dagger}}{2E_\alpha} e^{-E_\alpha \tau}$$

Generalised eigenvalue problem

$$G_{ij}(\tau_0 + \delta\tau) u_j^\alpha = e^{-E_\alpha \delta\tau} G_{ij}(\tau_0) u_j^\alpha$$
$$v_i^\alpha G_{ij}(\tau_0 + \delta\tau) = e^{-E_\alpha \delta\tau} v_i^\alpha G_{ij}(\tau_0)$$

Eigenstate-projected correlator

$$G_\alpha(\tau) = v_i^\alpha G_{ij}(\tau) u_j^\alpha$$

A tale of three methods

Coefficients g_τ will become exponentially large and oscillating.
Regularise by minimising functional $W[g_\tau] = A[g_\tau] + \lambda B[g_\tau]$

Tikhonov

$$A[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega (\omega - \omega_n)^2 [\overline{\Delta}(\omega, \omega_n)]^2 , \quad B[g_\tau] = \sum_{\tau_1, \tau_2} g_{\tau_1} g_{\tau_2} I(\tau_1, \tau_2)$$

Backus–Gilbert

$$A[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega (\omega - \omega_n)^2 [\overline{\Delta}(\omega, \omega_n)]^2 , \quad B[g_\tau] = \sum_{\tau_1, \tau_2} g_{\tau_1} g_{\tau_2} \operatorname{Cov}(\tau_1, \tau_2)$$

Hansen–Lupo–Tantalo

$$A[g_\tau] = \int_{\omega_{\min}}^{\infty} d\omega |\overline{\Delta} - \Delta_\sigma|^2 , \quad B[g_\tau] = \sum_{\tau_1, \tau_2} g_{\tau_1} g_{\tau_2} \operatorname{Cov}(\tau_1, \tau_2)$$

$$\Delta_\sigma(\omega, \omega_n) = \alpha \exp[-(\omega - \ \omega_n)^2 / 2\sigma^2]$$

Bayesian methods

Maximum entropy method (Bryan)

$$S = \int d\omega \rho(\omega) \log \frac{\rho(\omega)}{m(\omega)} \quad \text{(Shannon–Jaynes entropy)}$$

Singular value decomposition:

$$K(\omega, \tau) \to K(\omega_i, \tau_j) = K_{ij} = U \Xi V^T$$

u_k are column vectors of U: $N_b = N_s \leq N_{\text{data}}$

BR method

$$S = \int d\omega \left(1 - \frac{\rho(\omega)}{m(\omega)} + \ln \frac{\rho(\omega)}{m(\omega)}\right)$$

$N_b = N_\omega, u_k(\omega) = \delta_{k\omega}$