Departamento de Física Teórica. Institute of Particle & Cosmos Physics (IPARCOS)
Universidad Complutense de Madrid

Unveiling femtoscopic correlations of light hadrons

Alejandro Canoa Monsalve

In collaboration with:
M. Albaladejo, J.M. Nieves, J.R. Peláez, E. Ruiz Arriola and J. Ruiz de Elvira

Excited QCD 2026, Granada, January 13th 2026

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Grant PID2022-136510NB-C31 funded by MICIN

OUTLINE

- Introduction to femtoscopic correlations
- $\pi^+ K_S$ femtoscopic correlations

  M. Albaladejo *et al.*, *Phys. Lett. B* **866** (2025) 139552
- Ongoing work
  - Predictions for other light hadron channels
  - EFT-like approach to femtoscopy

INTRO: FEMTOSCOPIC CORRELATIONS

Formalism originally devised to estimate the size of the source

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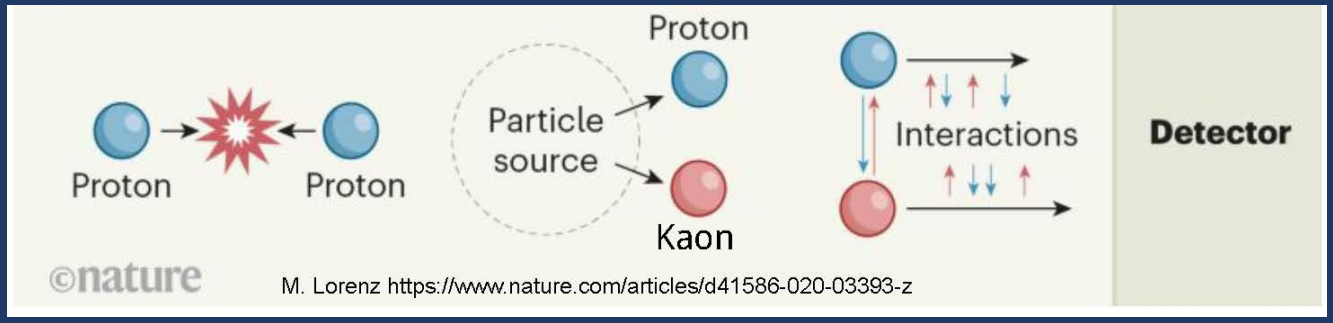
In nucleus-nucleus collisions (pp,Pb-Pb…)



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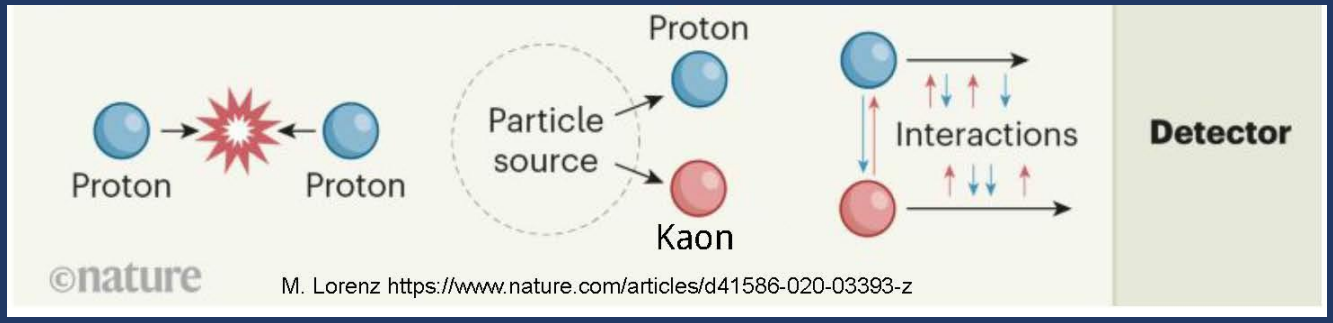

Observed correlations

$$\overbrace{C(k^*) = \xi(k^*)\frac{N_{\text{same}}(k^*)}{N_{diff}\ (k^*)}}^{\text{experiment}} = \overbrace{\int \mathrm{d}^3\boldsymbol{r}^*\ S(\boldsymbol{r}^*)|\psi(\boldsymbol{k}^*, \boldsymbol{r}^*)|^2}^{\text{theory}}$$

source wave function

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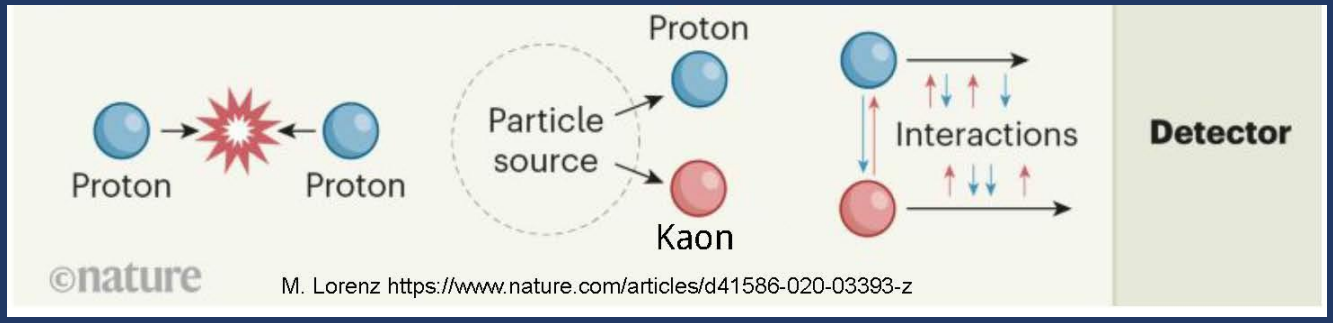

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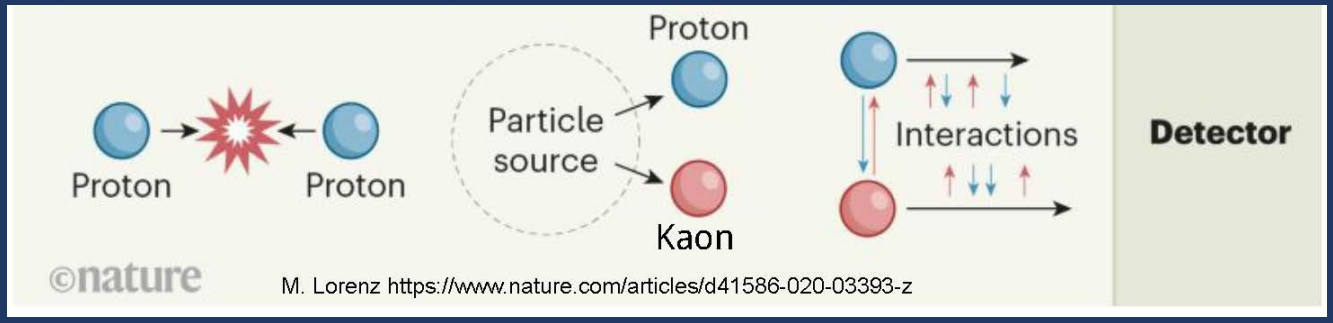

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source wave function

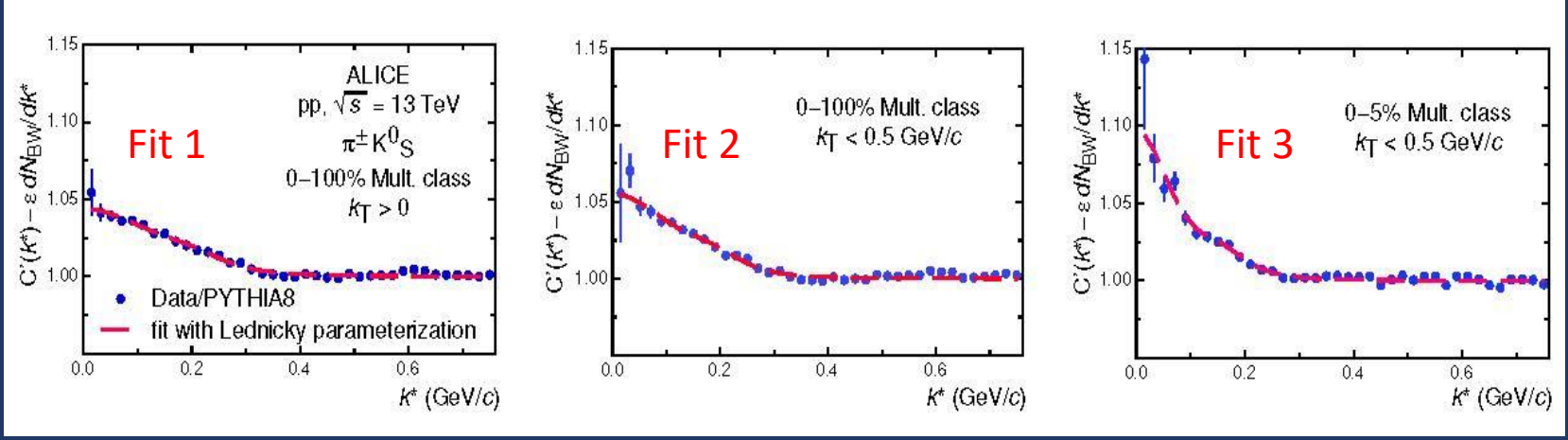
Experimental effects

$$S(r^*) = \left(4\pi r_0^2\right)^{-3/2} \cdot \exp\left(-\frac{r^{*2}}{4r_0^2}\right),$$

Example: Phys. Lett. B 856 (2024) 138915

CERN-EP-2023-287
11 December 2023

Investigating the nature of the $K_0^*(700)$ state with $\pi^{\pm}K_S^0$ correlations at the LHC



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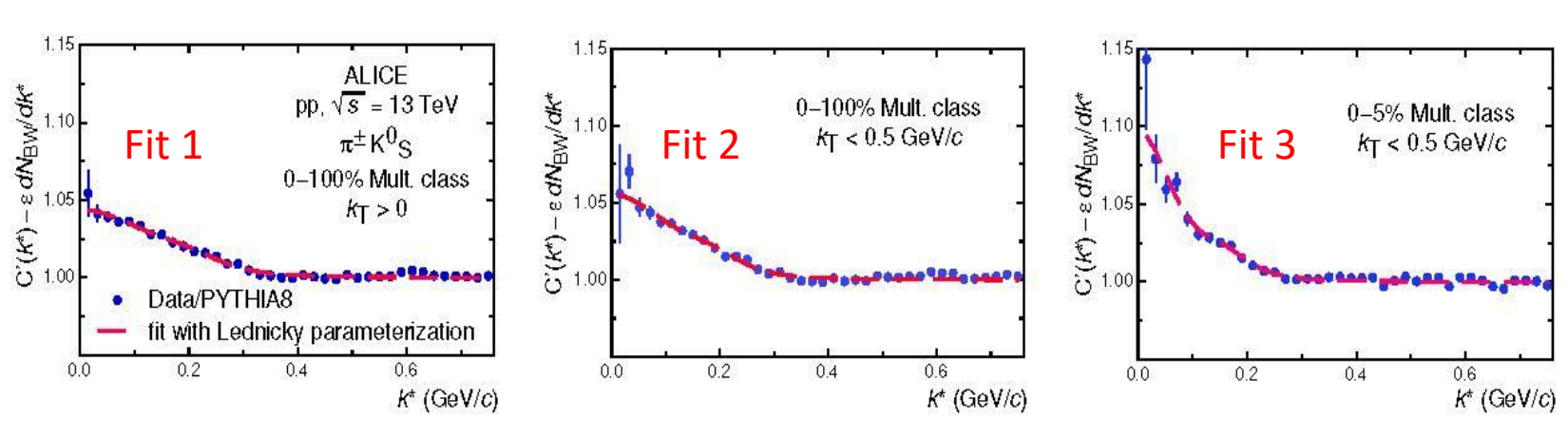

Three different sets (different cuts)

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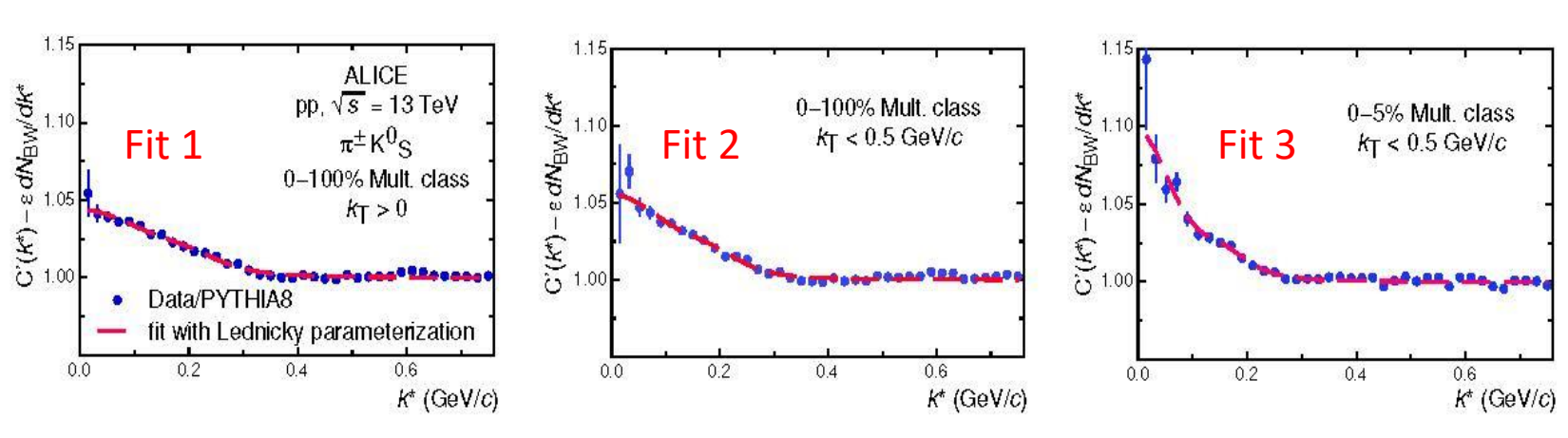

Three different sets (different cuts)

- P-wave and other effects removed with PYTHIA
- Basically S-wave.

Usual (and ALICE) approach

Theoretical model: Lednicky-Lyuboshits (LL) formalism

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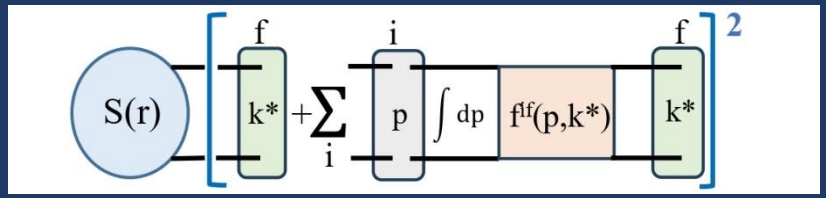

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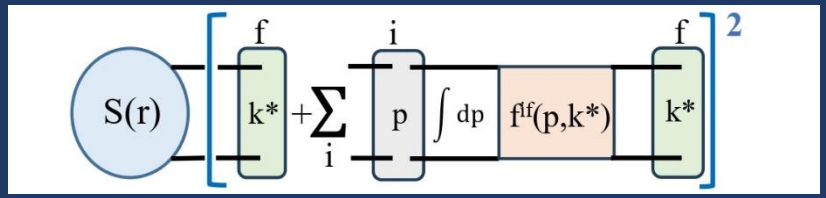

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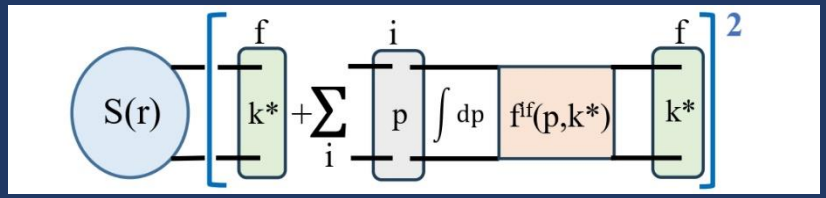

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- R radius of the source
- λ correlation strength parameterizes other sources of the pair

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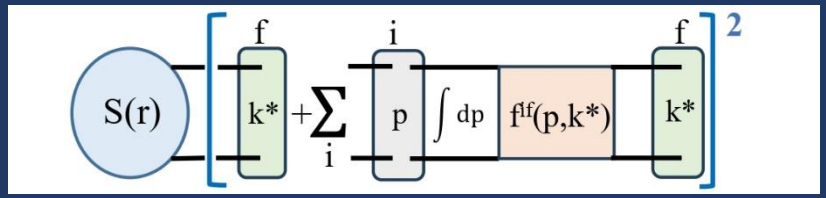

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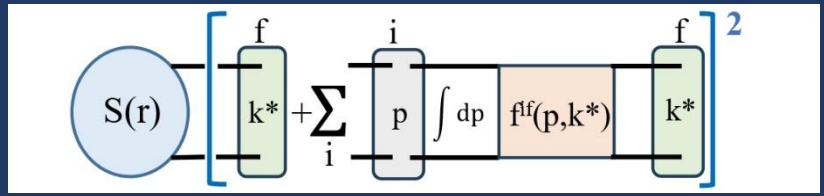

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- ΔC is a correction to the free wavefunction

$$\Delta C = -d_0|f(k^*)|^2/2\sqrt{\pi}R^3$$

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ALICE approach for the πK interaction

LL formalism + an elastic Breit Wigner (BW) !!

$$f(k^*) = \gamma/(M_R^2 - s - i\gamma k^*)$$

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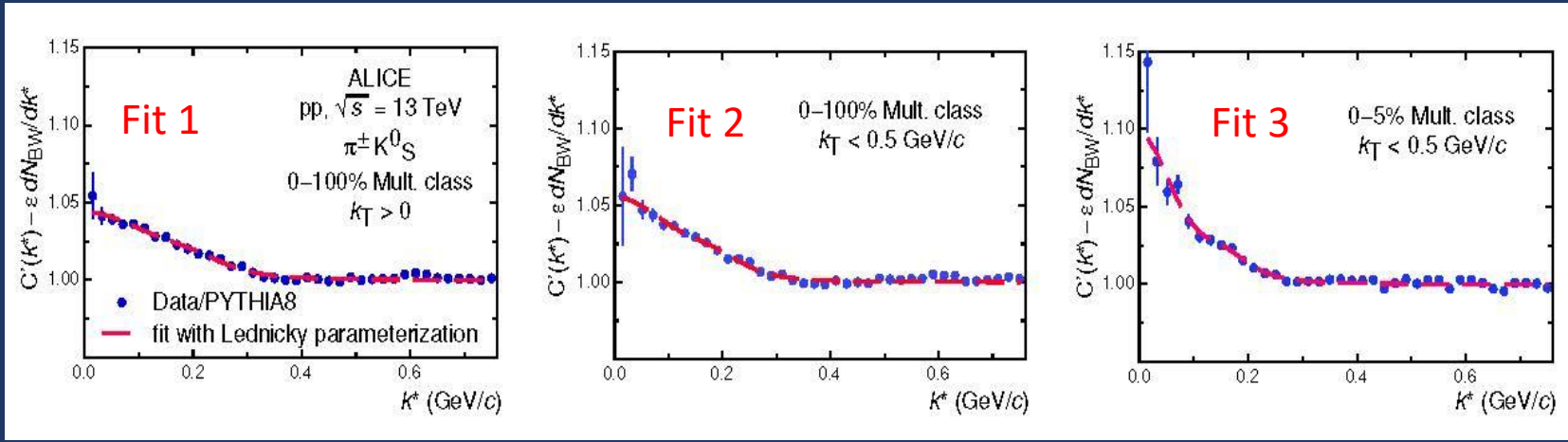

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Fit 1	0.833 ± 0.015	0.890 ± 0.025
Fit 2	0.804 ± 0.014	0.801 ± 0.033
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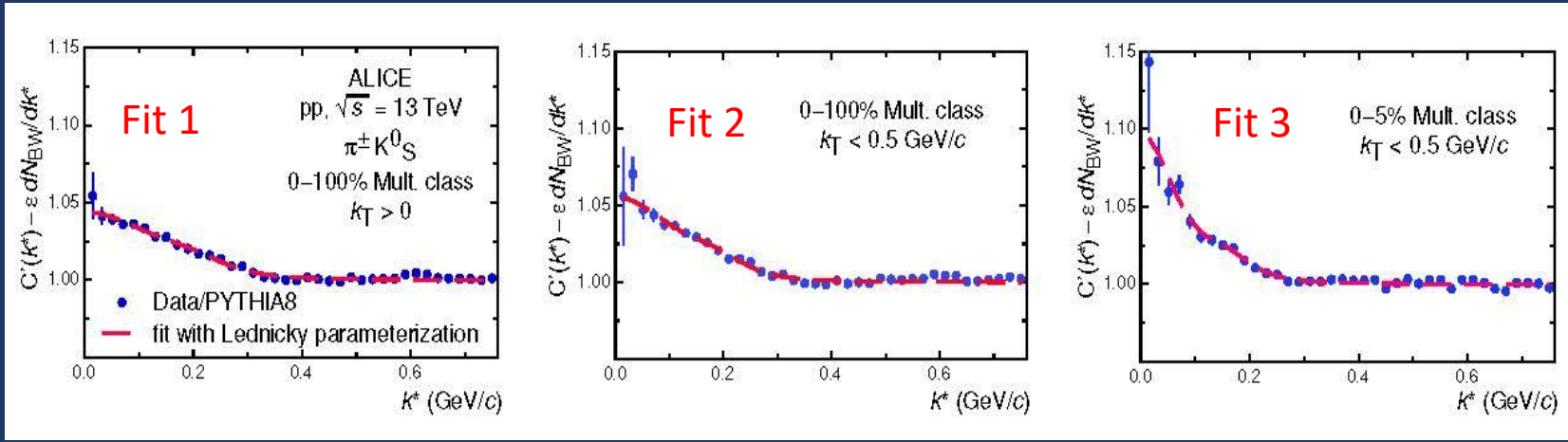

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Imcompatible with each other!!

Our approach

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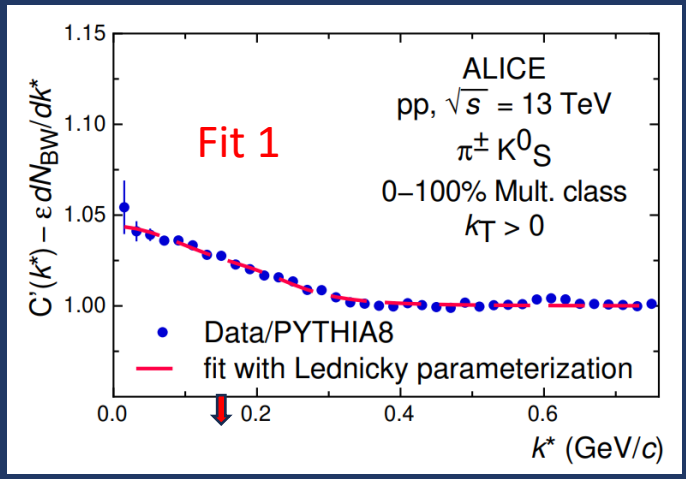
   - $M_\pi \approx 0.14$ GeV, but k^* up to 0.76 GeV



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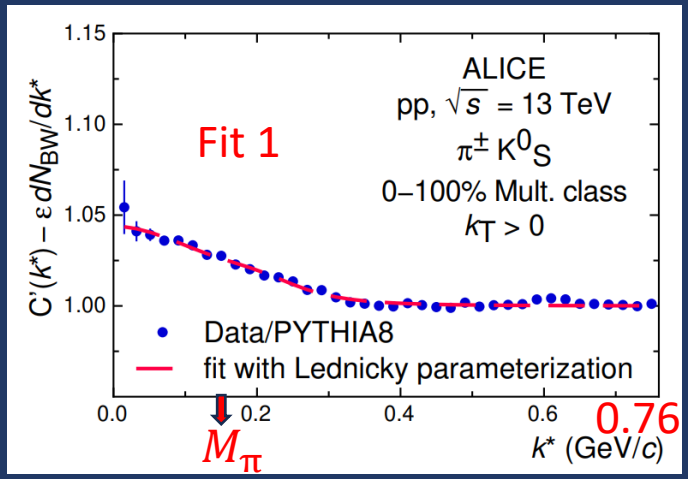

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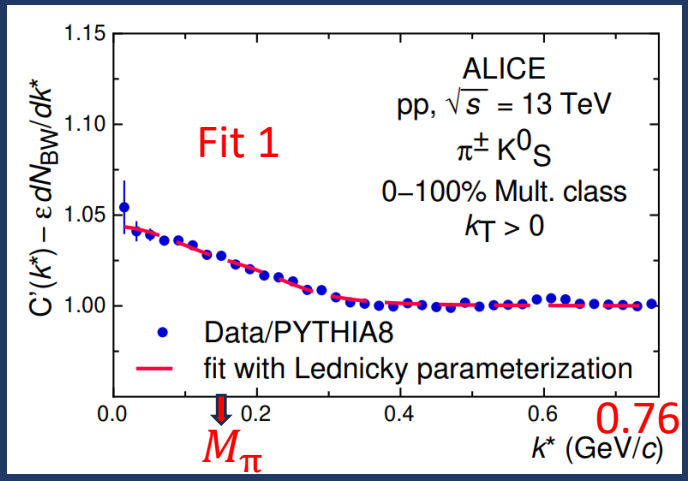

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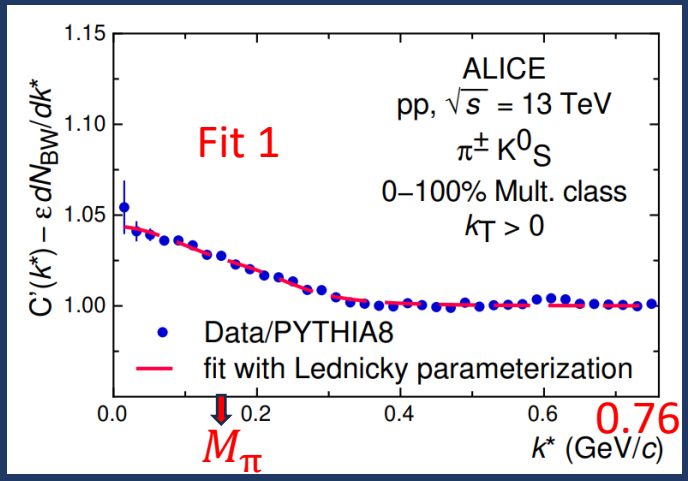

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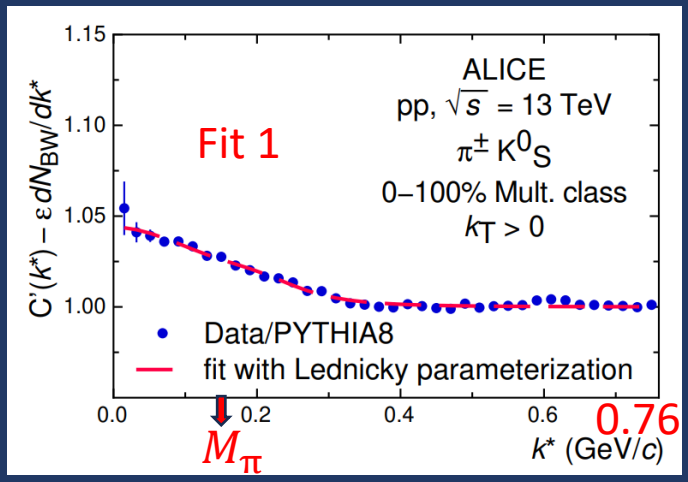

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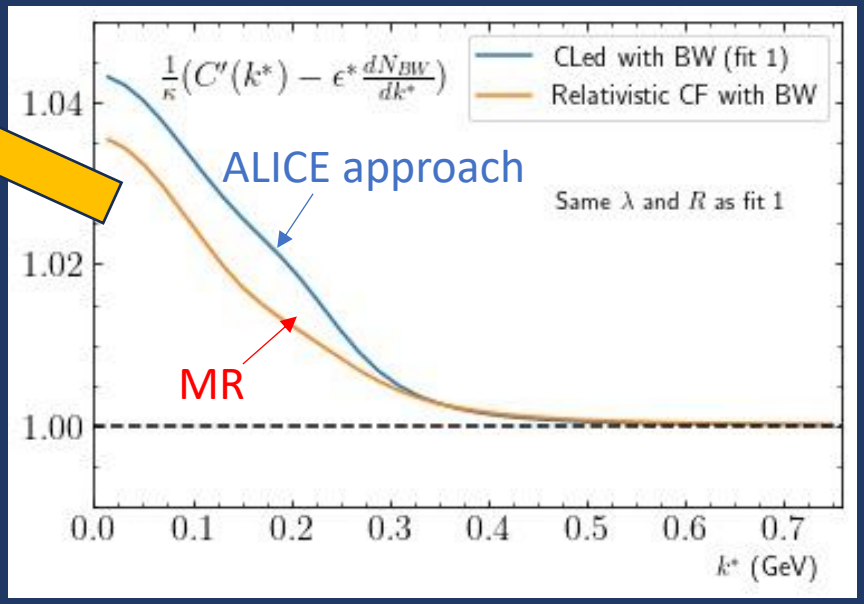

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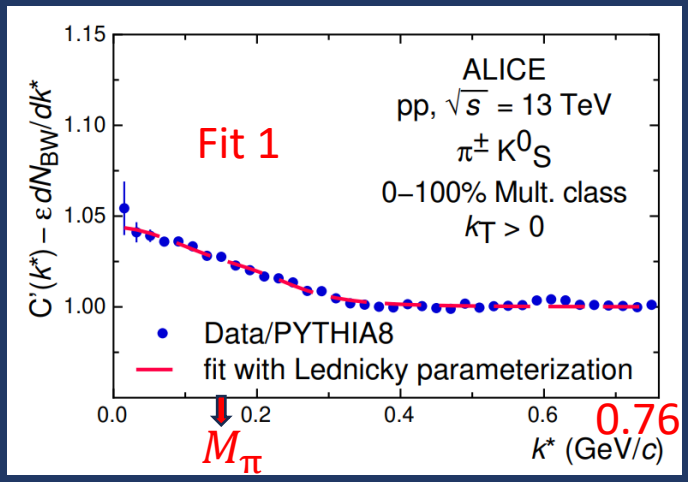
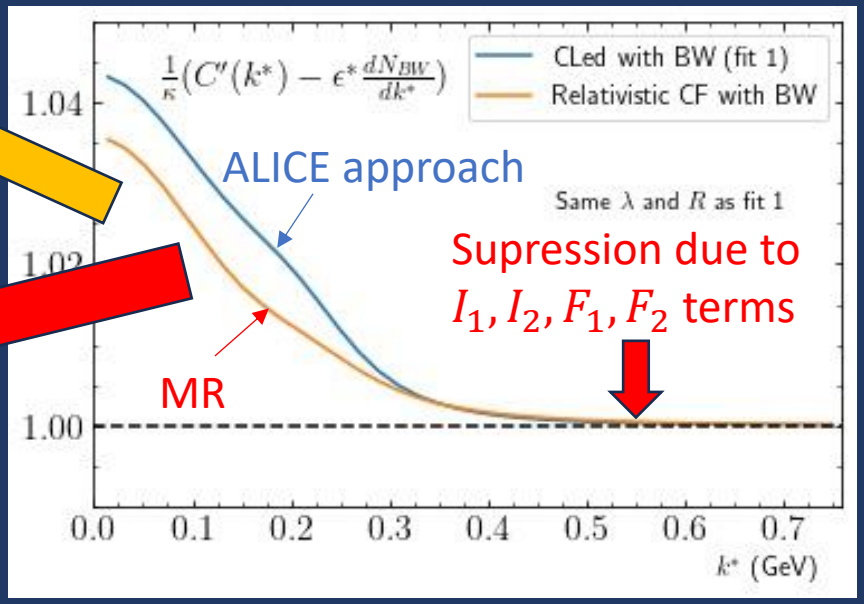


Need for relativistic corrections!!

- Replace propagators and the integral measure (Relativistic CF)
  - I_1, I_2 play the role of F_1, F_2

15-20 % difference!!



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2. BW vs. realistic interactions

- $\pi^+ K_S$ combination of I=1/2 and I=3/2 isospin πK states
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Reminder!

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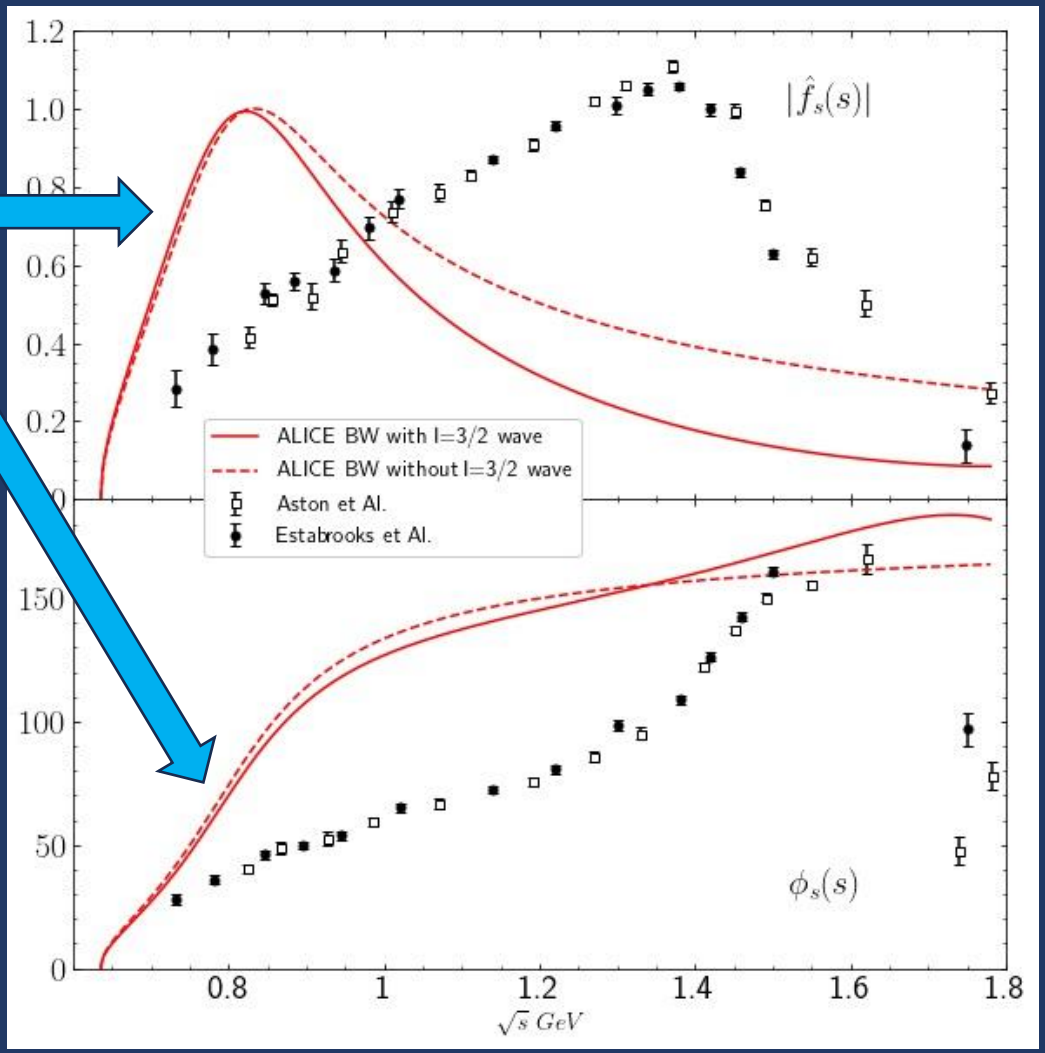

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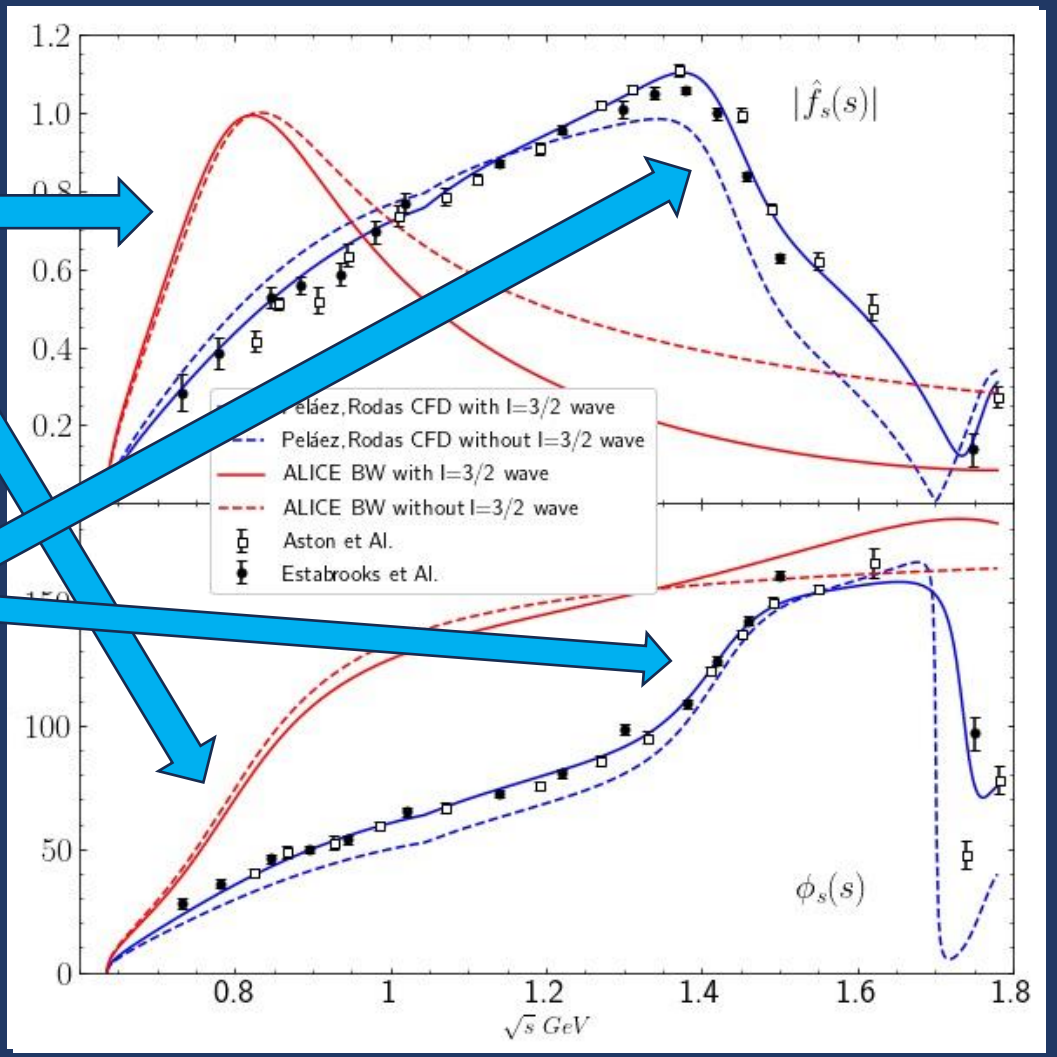

*Peláez, Rodas Phys. Rept. 969 (2022) 1-126

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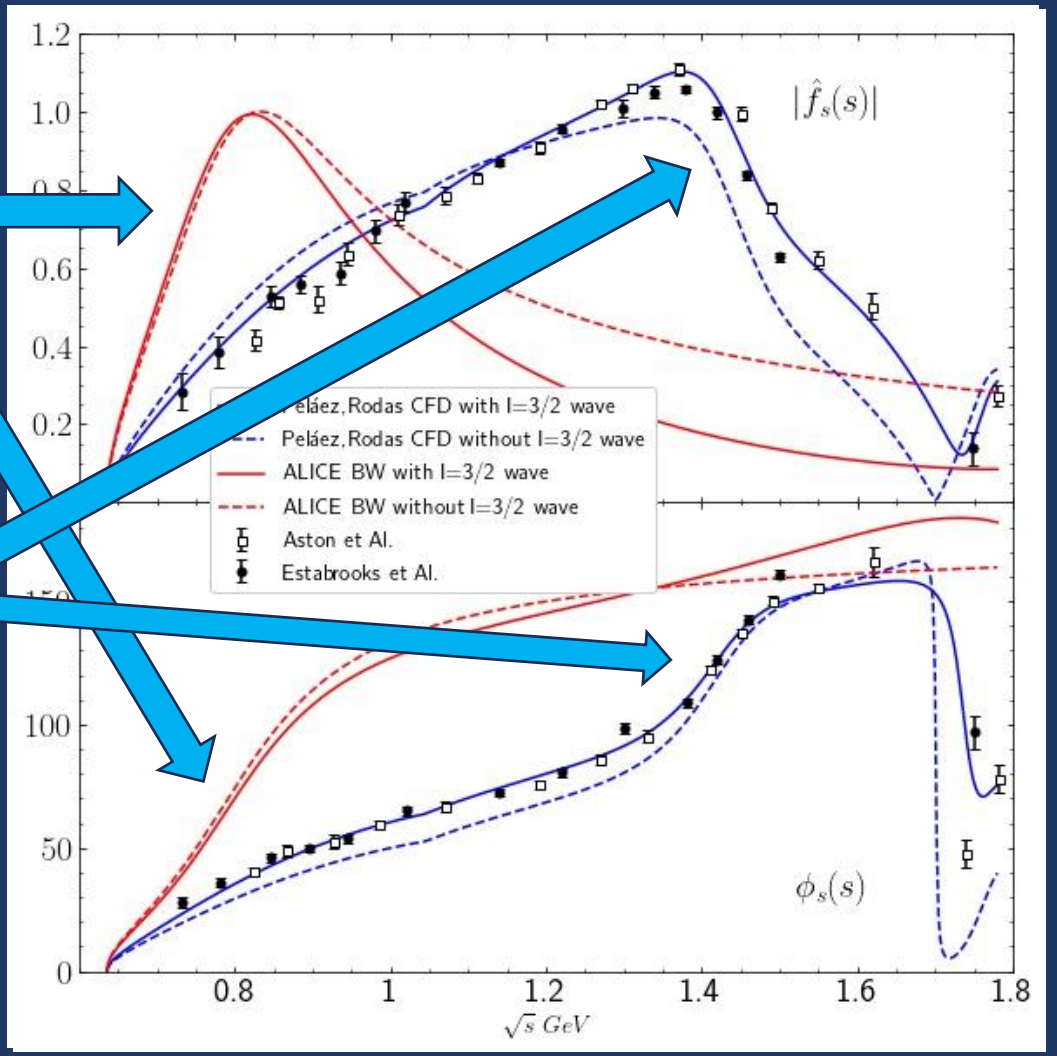

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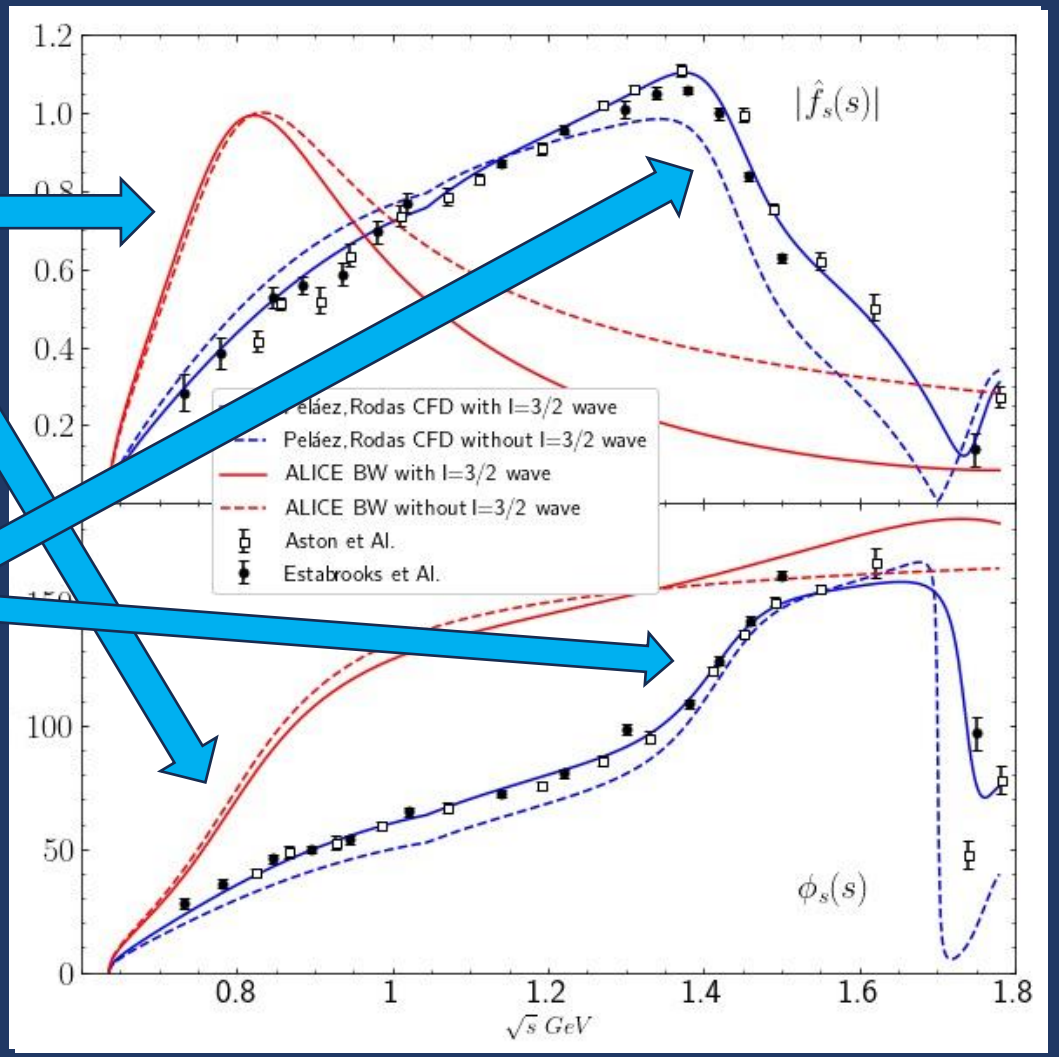
- Effect of the I=3/2 noticeable

x4 bigger in $\pi^+ K_S$!!



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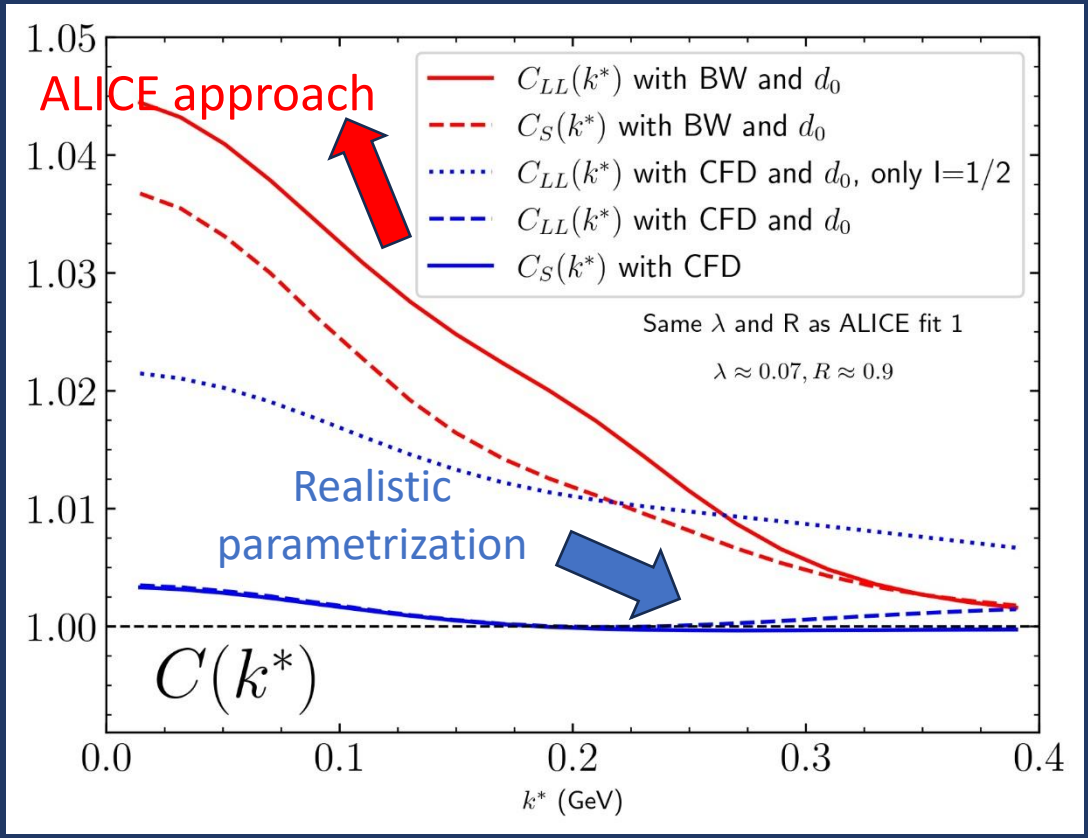

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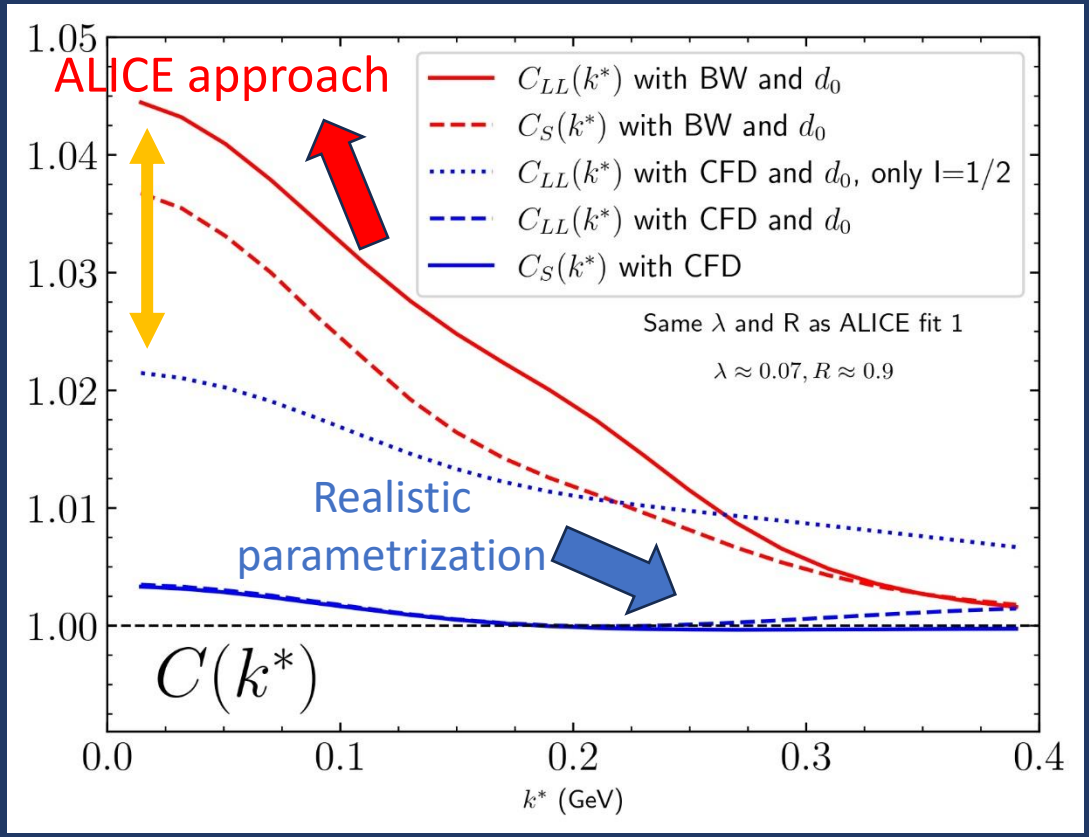

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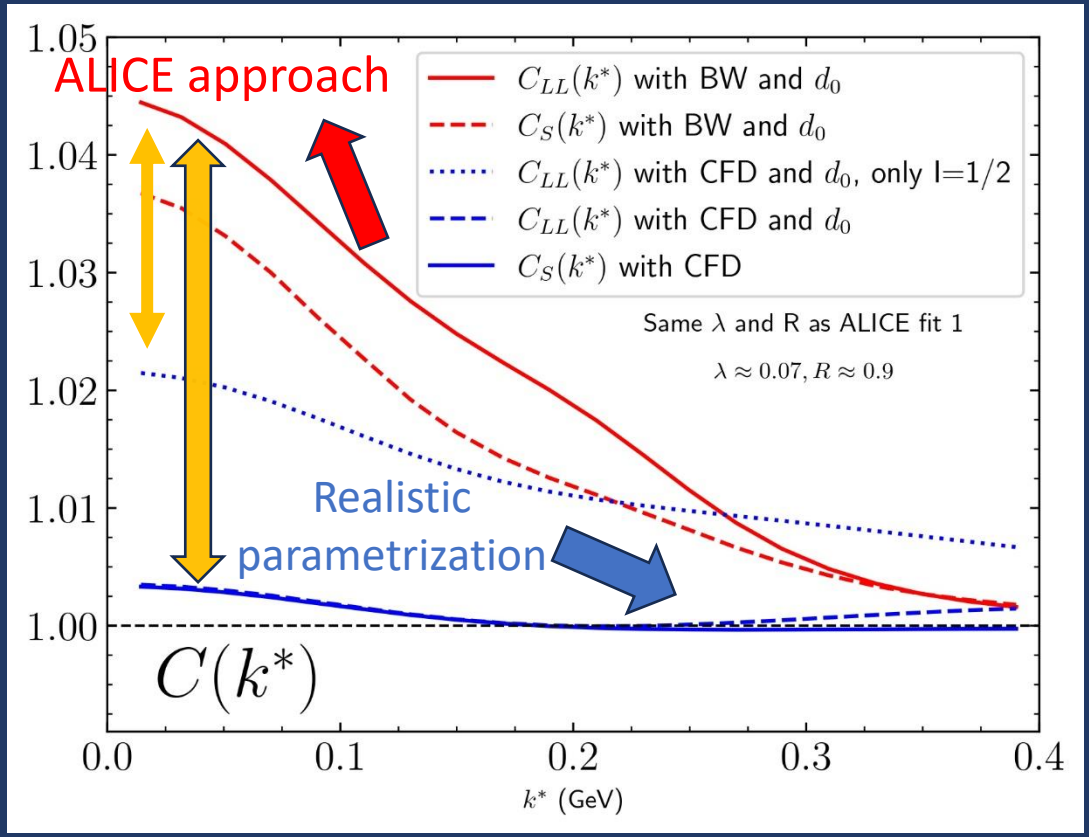
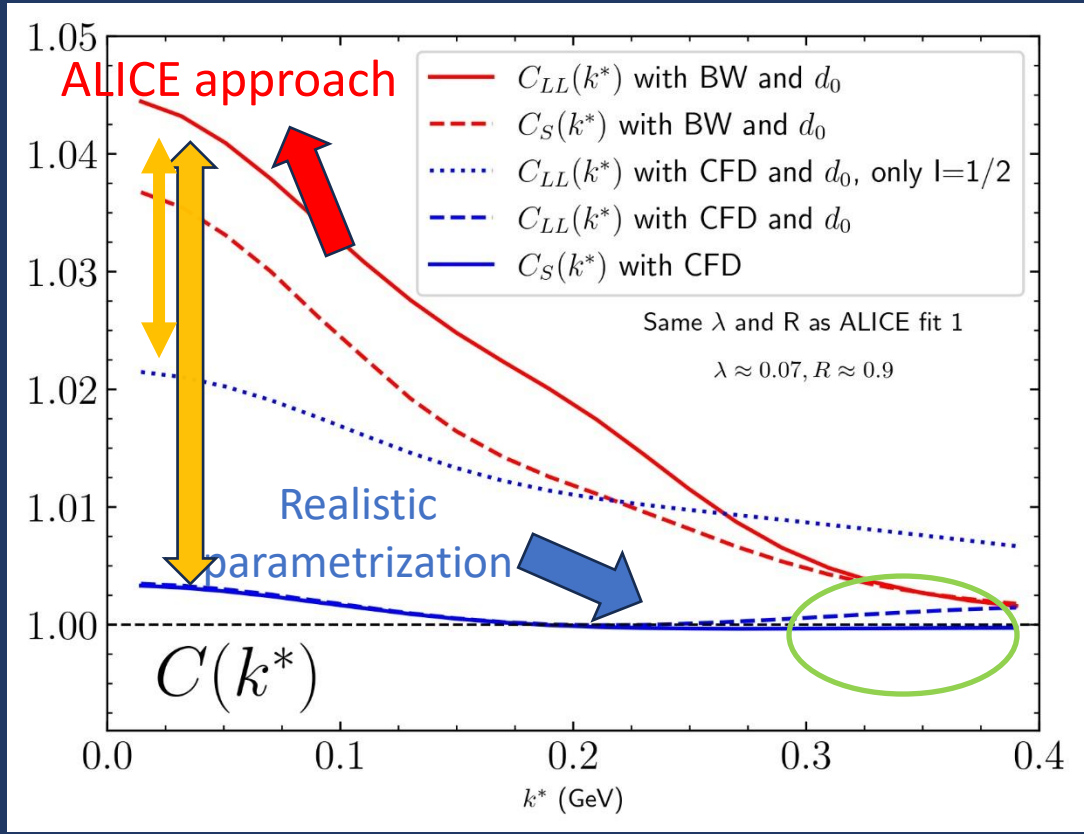


Our approach

3. Redefinition of ΔC

$$\frac{d_0}{2} \Rightarrow \frac{\mathrm{d}\left(\mathrm{Re}\frac{1}{\mathrm{f}_0^{(\mathrm{I})}(k^*)}\right)}{\mathrm{d}\,(\mathrm{k}^*)^2}$$

- Smaller, but noticeable effect between LL and MR with realistic interaction and redefinition of ΔC
- Relevant when fitting to data



Our approach on $\pi^+ K_S$

3. Our complete formula

$$C_S(k^*) = 1 + \frac{\lambda}{2}\left(\frac{1}{3}\delta C^{(1/2)}(k^*) + \frac{2}{3}\delta C^{(3/2)}(k^*)\right),$$

$$\delta C^{(I)}(k^*) = \frac{\mathrm{Re}\, f_0^{(I)}(k^*)}{R} I_1(s,R) - \frac{\mathrm{Im}\, f_0^{(I)}(k^*)}{R} I_2(s,R) + \left|\frac{f_0^{(I)}(k^*)}{R}\right|^2 I_3(s,R) + \Delta C^{(I)},$$

$$\Delta C^{(I)}(k^*) = -\frac{|f_0^{(I)}(k^*)|^2}{2R^3\sqrt{\pi}} \frac{d\,\mathrm{Re}\left(1/f_0^{(I)}(k^*)\right)}{d(k^{*2})}.$$

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- Coupled physical channels

$\pi^+ K_S, \pi^+ K_L, \pi^0 K^+$

2 elastic $f_0^I \rightarrow 2\,\delta\, C^I$ in the isospin basis

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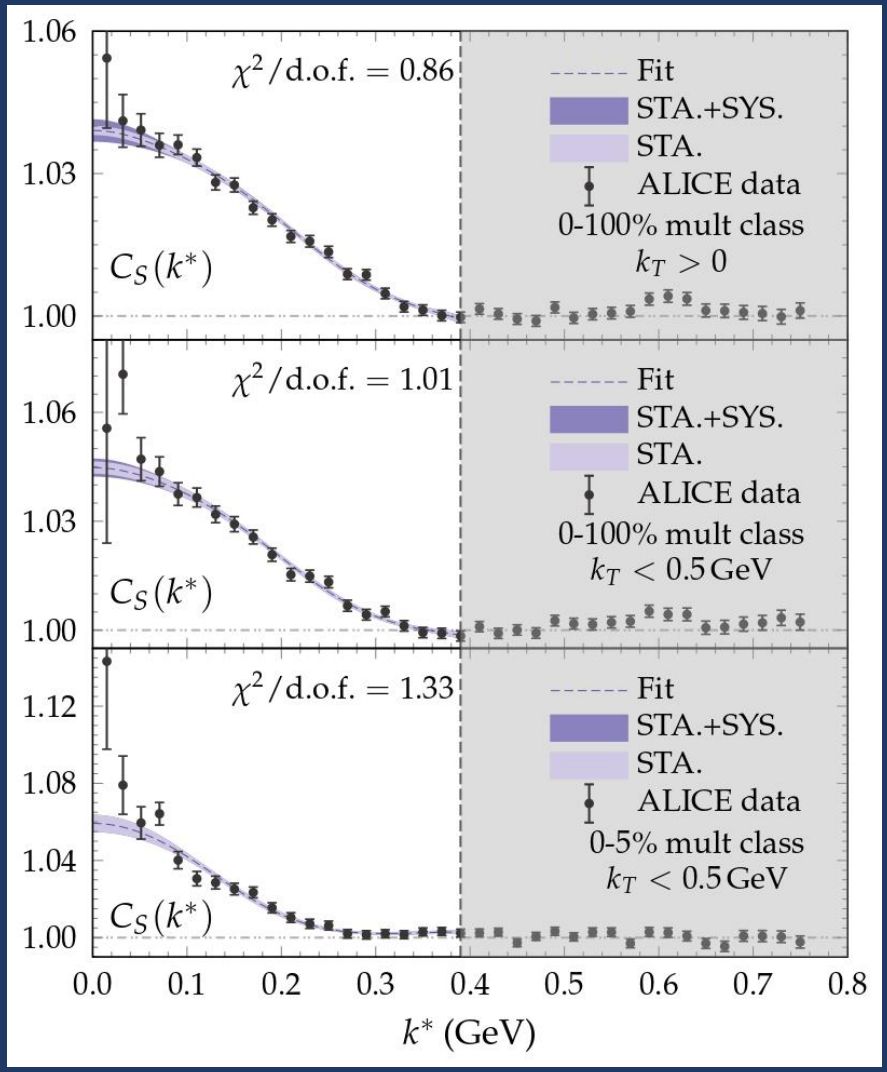
When fitting, normalization factor N

κ, ϵ absorbed in ALICE data

$$C'(k^*) = \kappa\left[C_{\mathrm{Lednicky}}(k^*) + \varepsilon \frac{dN_{BW}}{dm}\frac{dm}{dk^*}\right]$$

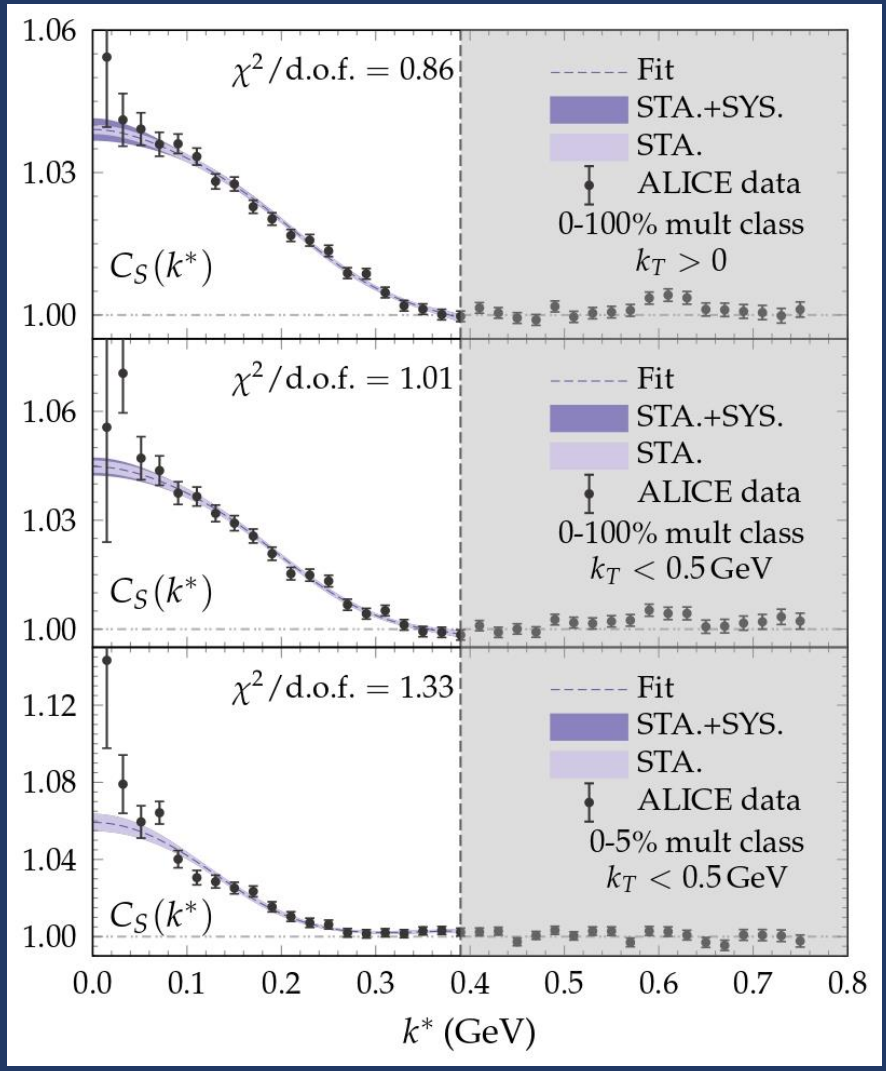
Our results on $\pi^+ K_S$



M. Albaladejo, AC *et al.*, *Phys. Lett. B* **866** (2025) 139552

Our results on $\pi^+ K_S$

- Just 3 parameters (R, λ, N) vs 6 fit parameters in ALICE for each plot



M. Albaladejo, AC *et al.*, *Phys. Lett. B* **866** (2025) 139552

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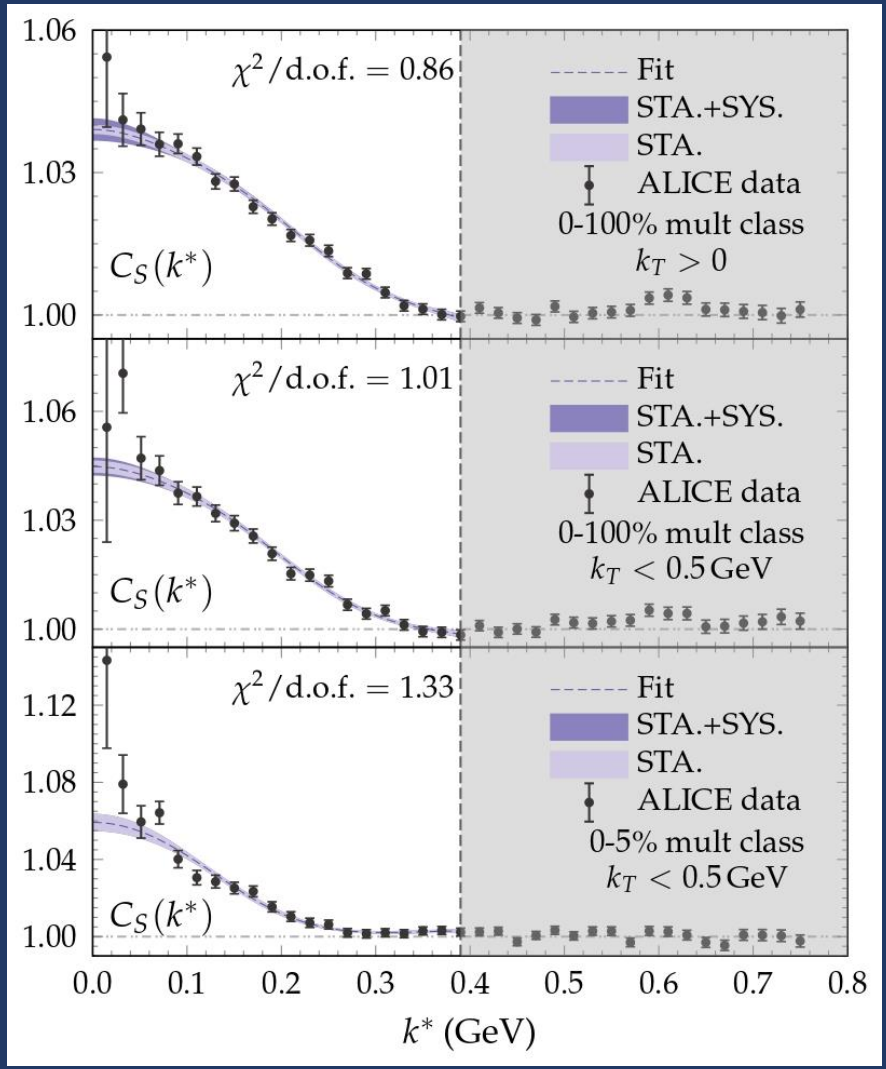

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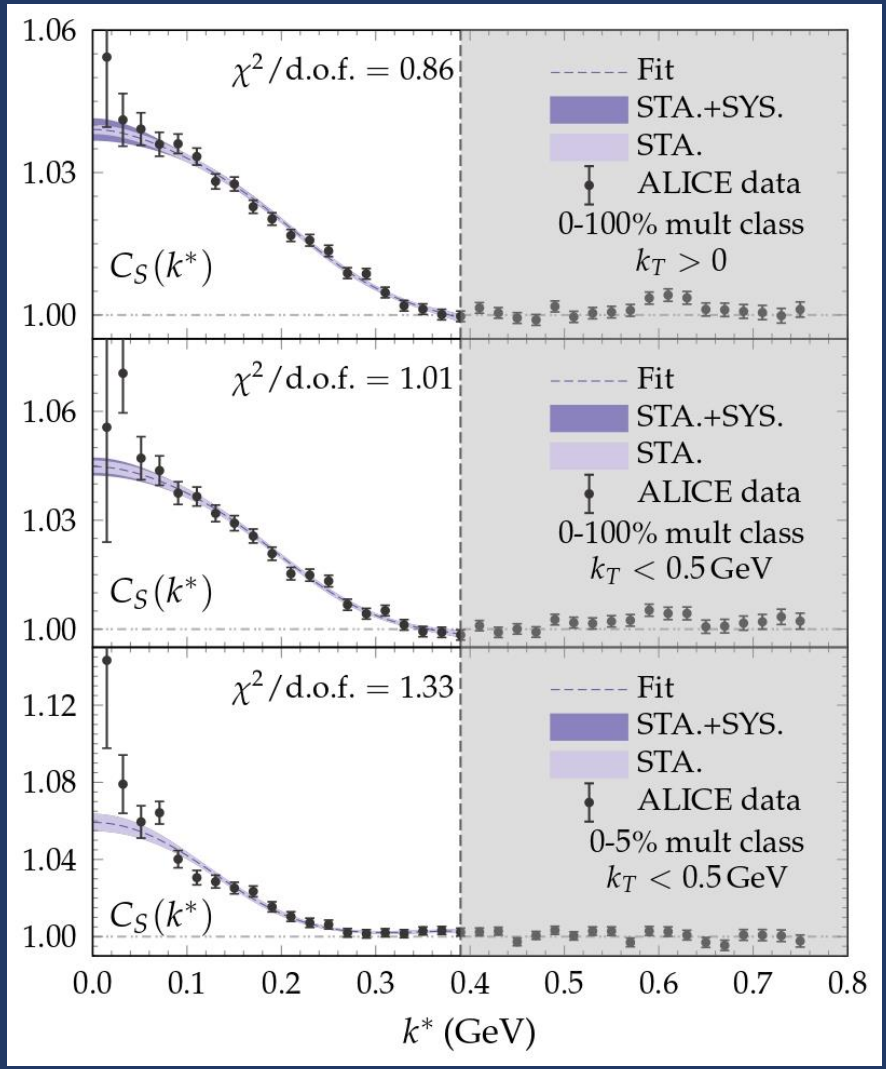

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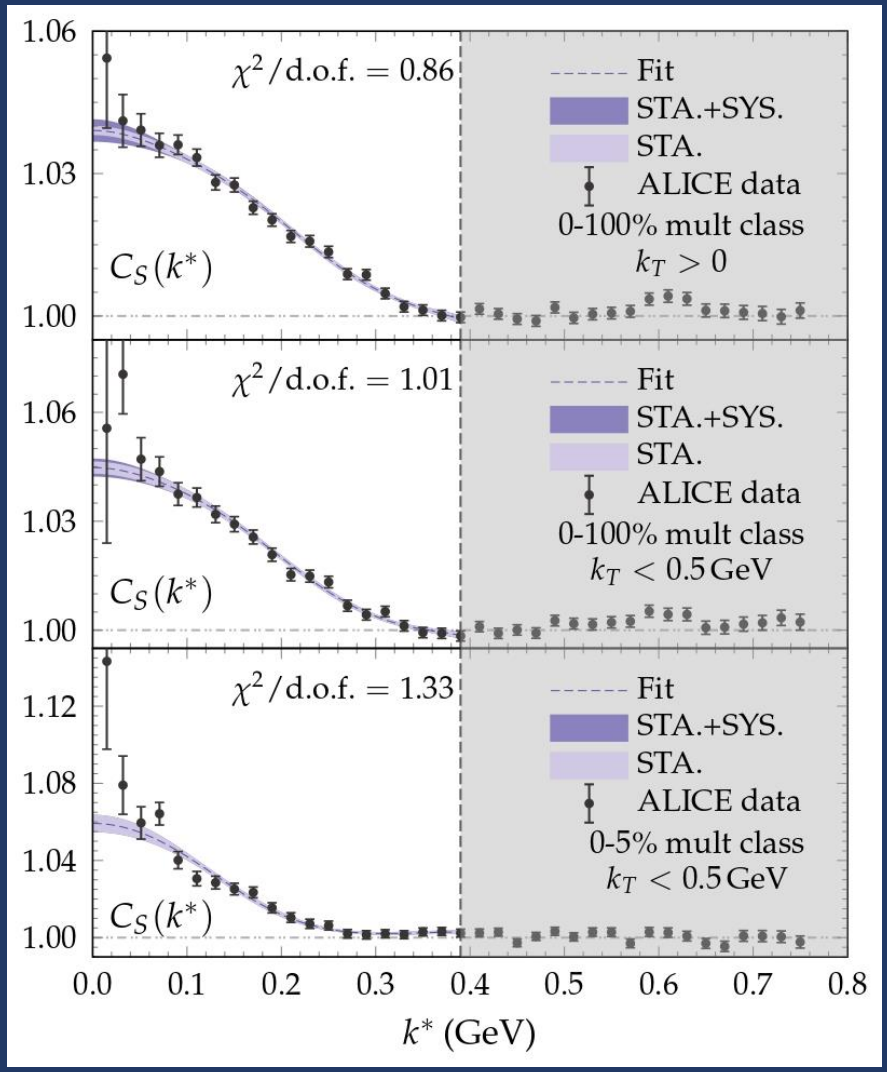

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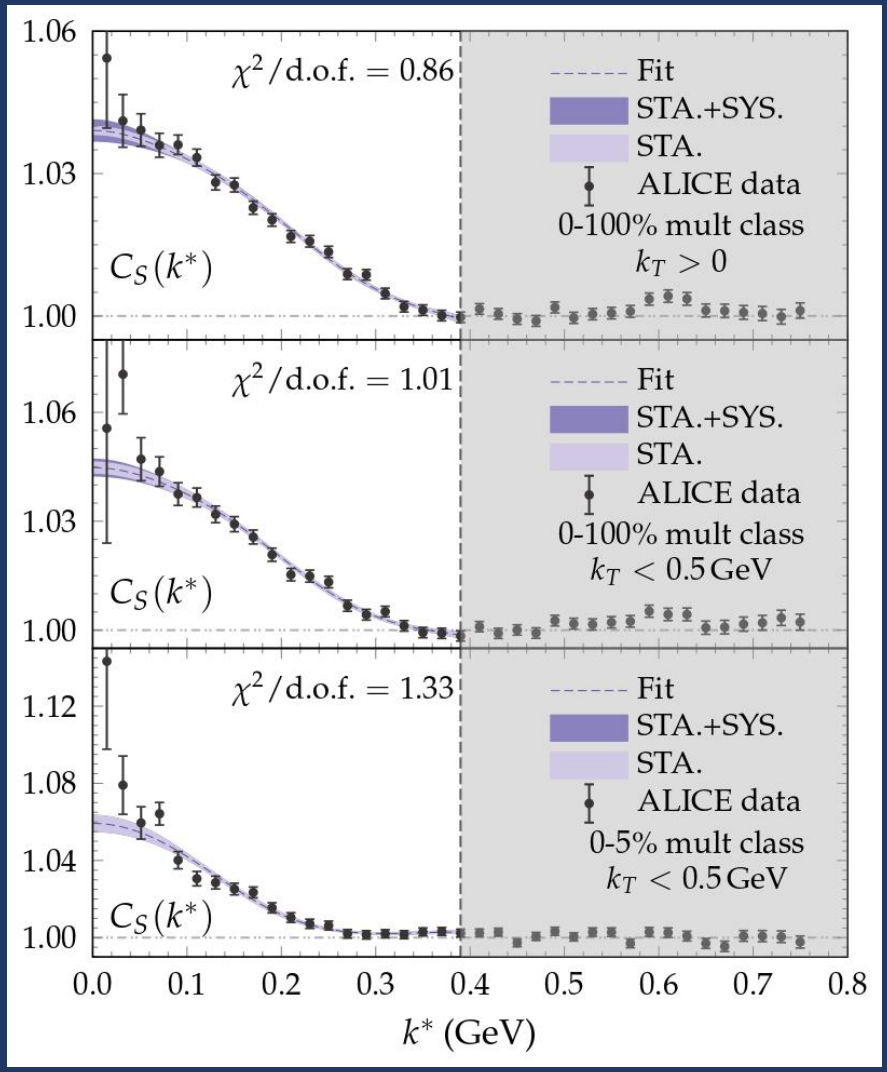
- The interaction is fixed and the same for the three fits
- Inconsistent (M_R, γ) pairs from each fit in ALICE analysis



M. Albaladejo, AC *et al.*, *Phys. Lett. B* **866** (2025) 139552

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ALICE	R(fm)	λ
Fit 1	$0.912(65)$	$0.0783(^{+127}_{-121})$
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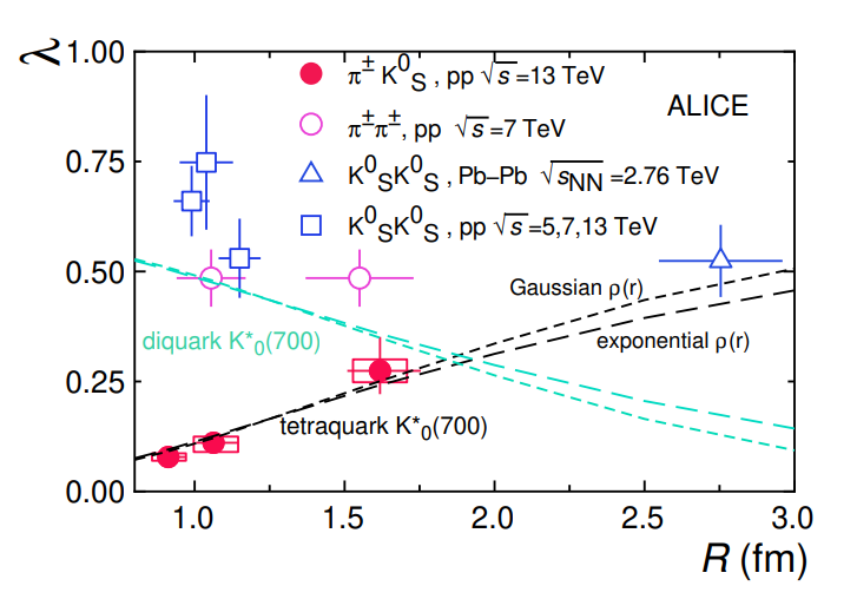
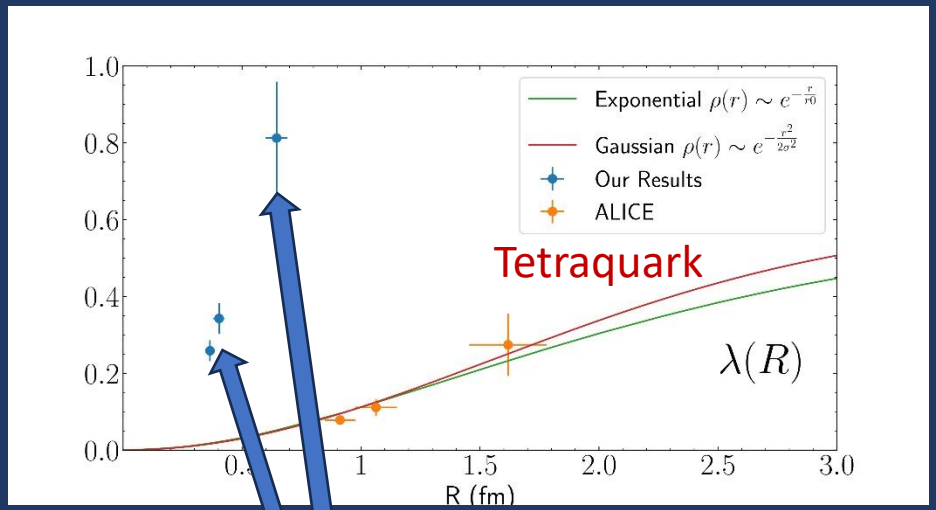
Different nature of the κ resonance?

Diquark-tetraquark ALICE model

P overlap probability

$\lambda = \lambda_0\, a\, P$ for $q\bar{q}$ meson
$\lambda = \lambda_0 (1 - a\, P)$ for tetraquarks





Our λs

Other channels: $\pi^+ \pi^0$

- $|\pi^+ \pi^0 >= \frac{1}{\sqrt{2}} |\pi\,\pi, I = 2, I_3 = 1 > + \frac{1}{\sqrt{2}} |\pi\,\pi, I = 1, I_3 = 1 >$
- We study up to P-wave
- Bose symmetry

I=2 -> S, D,… waves

I=1 -> P, F, … waves

- CFD dispersive parametrizations of f_0^2 and f_1^1

R. Garcia-Martín et al., Phys.Rev.D83:074004,2011

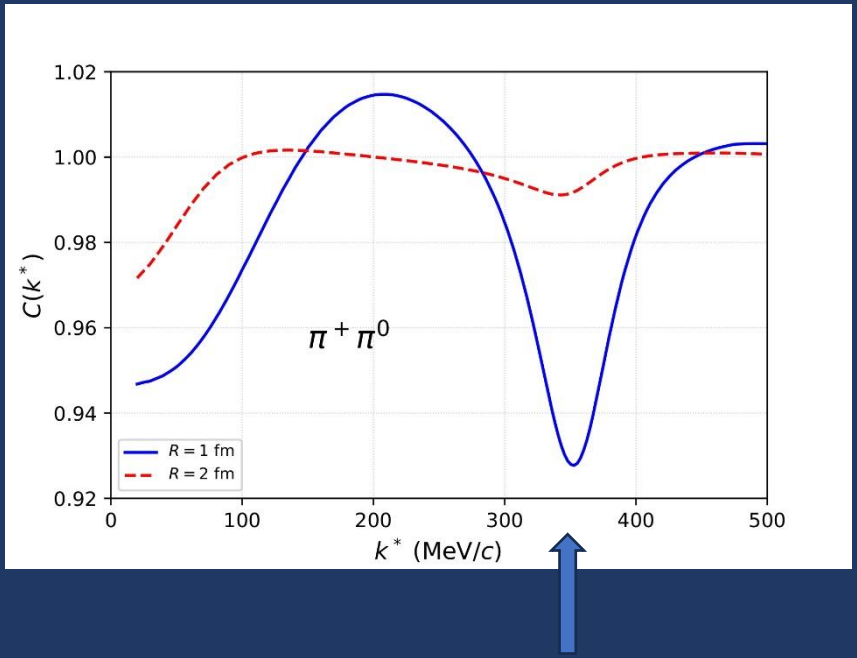
$$C(k) = 1 + \delta C_S\,(k) + 3\delta C_P\,(k) + \Delta\, C(k)$$

$C_s \Leftarrow f_{0,\pi\pi}^{(2)}$

$C_p \Leftarrow f_{1,\pi\pi}^{(1)}$

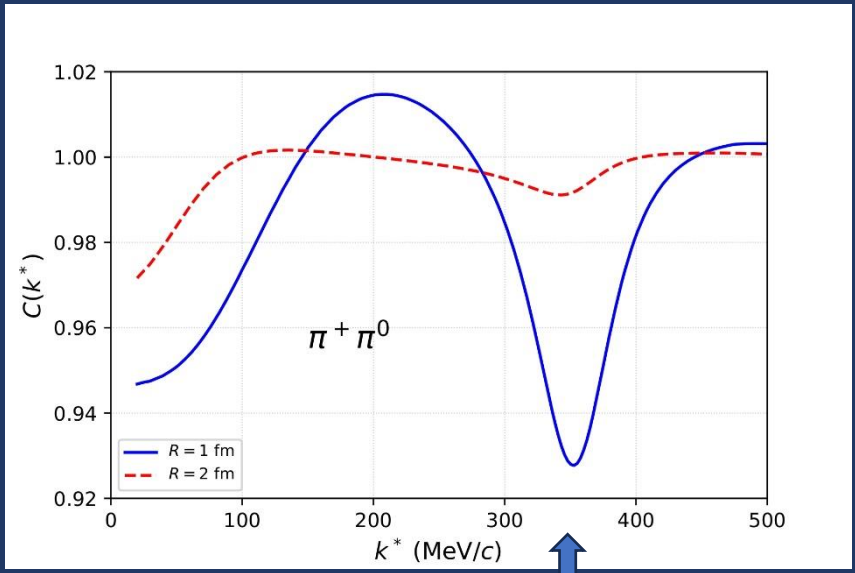
$$\Delta C = -\frac{1}{4\sqrt{\pi}\, R^3} \left| \mathrm{f}_{\mathbf{0},\pi\pi}^{(2)}(k) \right|^2 \frac{\mathrm{d}(\mathrm{Re\ f}(k))^{-1}}{dk^2}$$

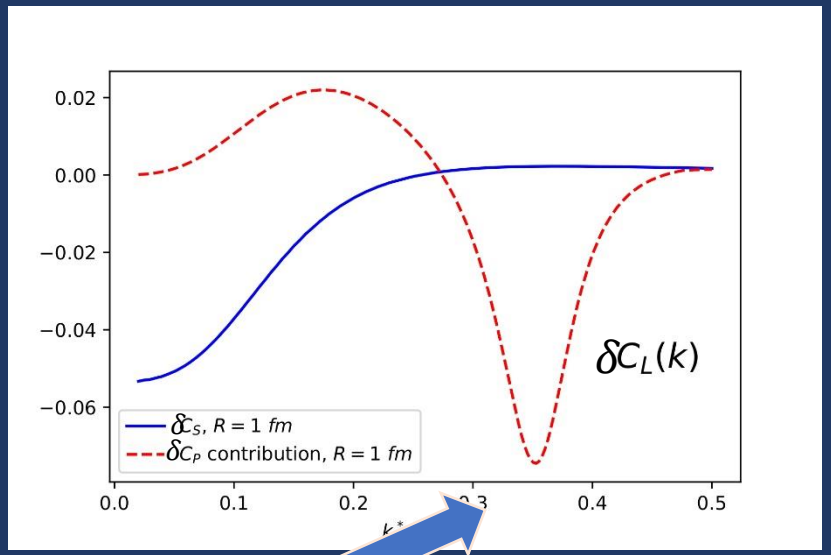
Other channels: $\pi^+ \pi^0$



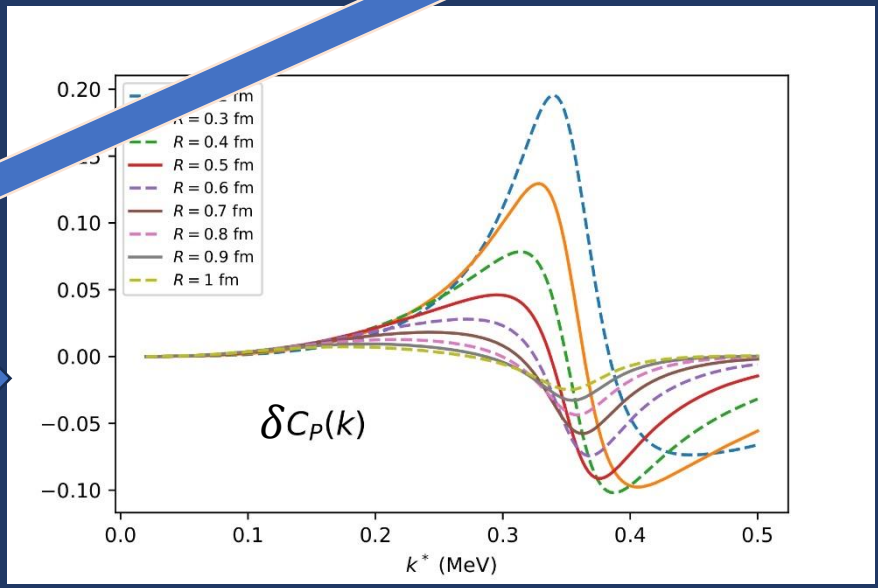
- Visible $\rho\ (770)$ ($k^* \sim 0.35$ GeV)

Other channels: $\pi^+ \pi^0$ and Δ C





- Visible ρ (770) ($k^* \sim 0.35$ GeV)
- Adequate partial wave origin (p-wave)
- Smaller radii increase observed correlations



Other channels: $\pi^+\eta, K^+ K_S$

- ALICE Data for $K^+ K_S$ s-wave femtoscopic correlations

PLB 833 (2022) 137335

$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

Phenomenological model for I=1 $\eta\pi$, $K\bar{K}$

- Elastic unitarity below $K\bar{K}$ threshold
- Two-channel description above $K\bar{K}$ threshold

$$\boldsymbol{T}(s) = (1 - \boldsymbol{K}(s)\boldsymbol{\Phi}(s))^{-1}\boldsymbol{K}(s)$$

$$\boldsymbol{K}(s) = \boldsymbol{K}_{(2)}(s) + \boldsymbol{K}_{(4)}(s) + \boldsymbol{K}_{(6)}(s)$$

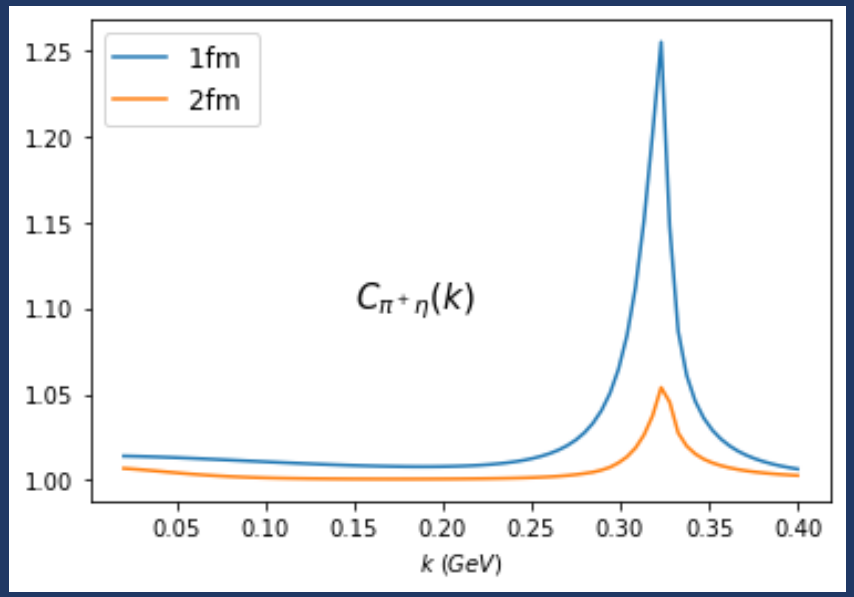
- Matching with NLO ChPT (approx $K\overline{K} \to K\overline{K}$)
- Realistic when $\sqrt{s} \leq 1.3\ GeV$ ($k^* \leq 0.41\ GeV$)

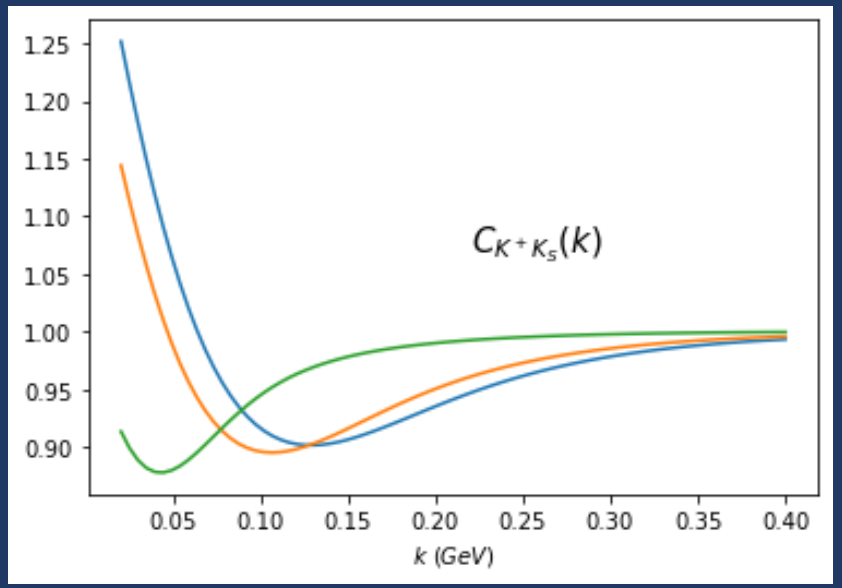
M.Albaladejo, B. Moussalam, Eur.Phys.J.C 75 (2015) 10, 488

For $K^+ K^0$,

$$T_{K^+K^0} = (1 - VG)^{-1} V$$
$$V \sim O(p^2)$$

Other channels: $\pi^+\eta, K^+ K_S$





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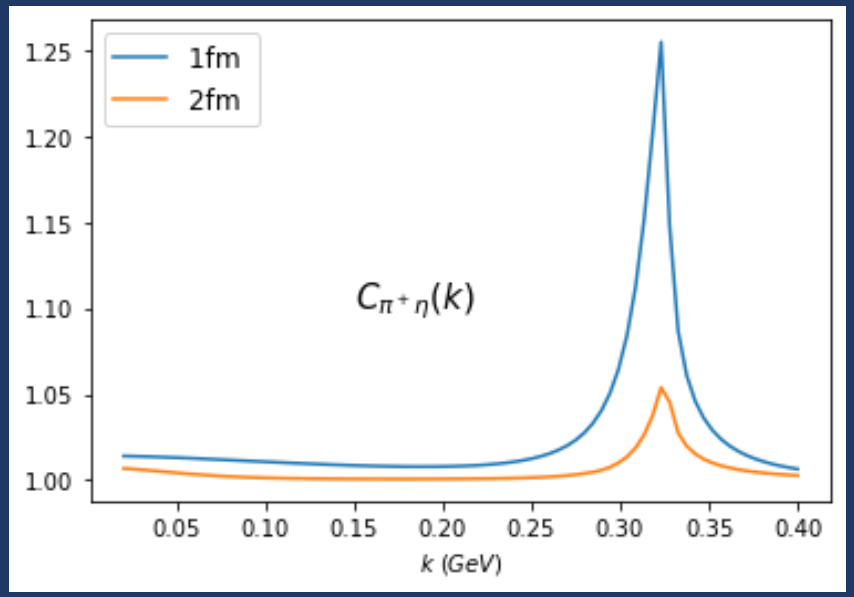
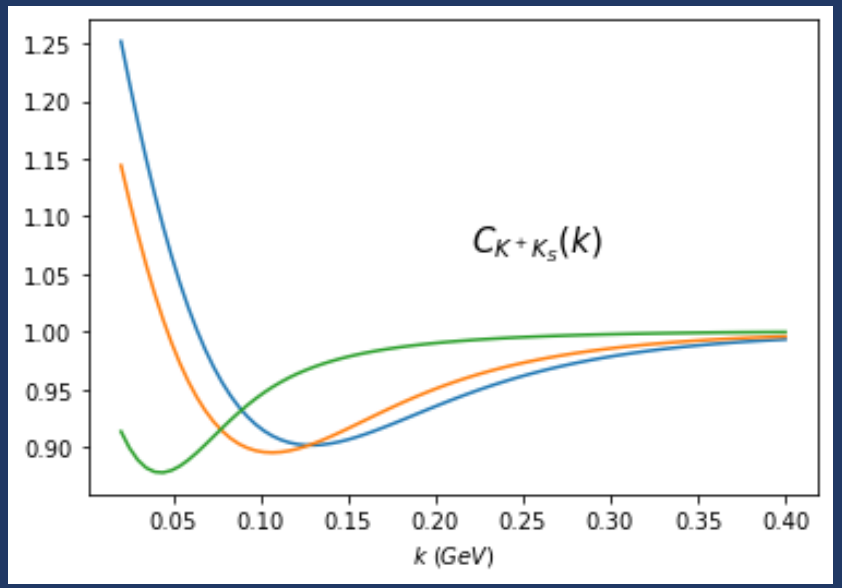
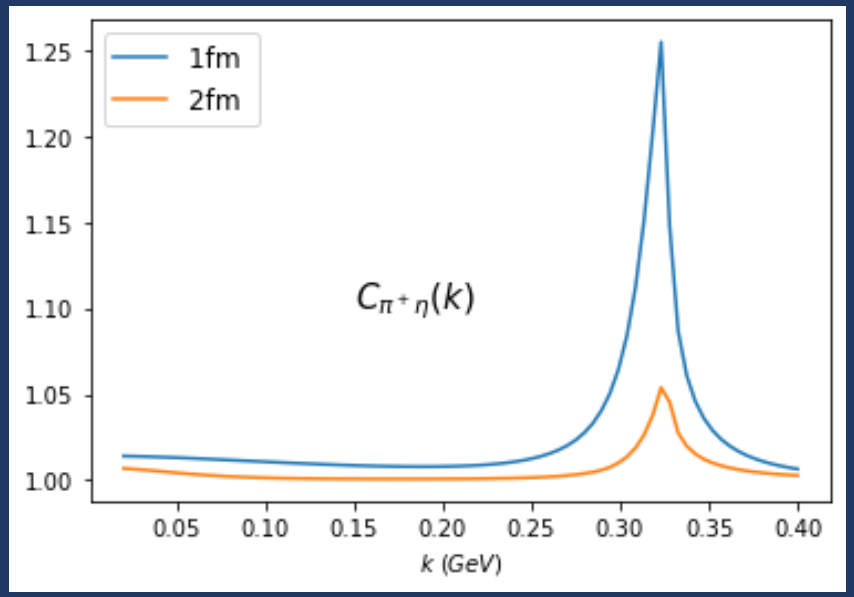
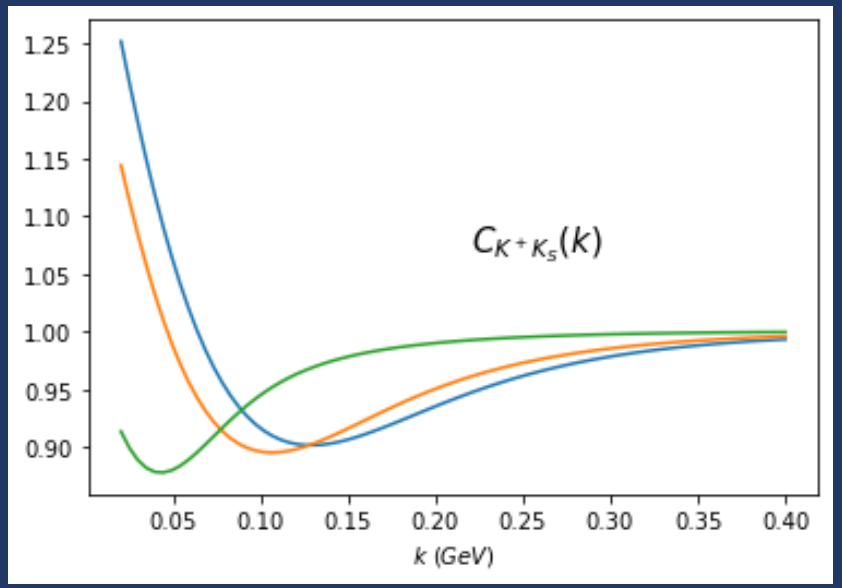




- $C_{\pi^+\eta}$ shows $a_0(980)$ resonance $k^* \approx 0.3 - 0.35$ GeV

Other channels: $\pi^+\eta, K^+ K_S$





- $C_{\pi^+\eta}$ shows $a_0(980)$ resonance $k^* \approx 0.3 - 0.35$ GeV

Work in progress

1) Include $\Delta\, C$ corrections (more complex, as the thresholds are different)
2) Comparison to data for K^+K_s (PLB 833 (2022) 137335)

Femtoscopy from a ChPT perspective. Motivation

- Caveats of previous approaches
  - Non-covariant approach of the KP formula
  - On-Shell factorization of the scattering amplitude

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- Key idea: The correlation function is an observable
  - Dependence only on on-shell quantities

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- Caveats of our approach
  - Non-covariant approach of the KP formula
  - On-Shell factorization of the scattering amplitude

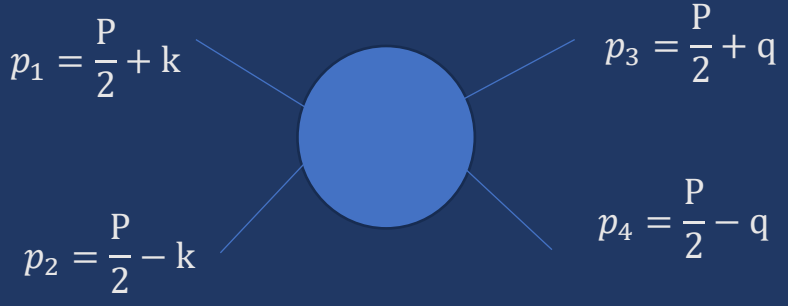
- Key idea: The correlation function is an observable
  - Dependence only on on-shell quantities

- Can we mimic an EFT approach for femtoscopy?

Femtoscopy from a ChPT perspective. Formalism

- Starting point: Koonin-Pratt formula

$$C(k) = \int d^4 x \, S(x,k) |\psi(x,k)|^2$$



$$\psi(x,k) = e^{-ikx} + \int \frac{d^4 q}{(2\pi)\,4} \frac{e^{-i\,q\,x}\, T(p_1,p_2,p_3,p_4)}{\left(\left(\frac{P}{2}-q\right)^2 - {m_1}^2\right)\left(\left(\frac{P}{2}+q\right)^2 - {m_2}^2\right)}$$

Femtoscopy from a ChPT perspective. Formalism

- Half-off-shellness (p_1 and p_2 on the mass shell)

- Chiral expansion of the amplitude

$$C(k) = S_k(0) + 2\,Re \int \frac{d^4q}{(2\pi)^4} \frac{S_k(k-q)T(p_1,p_2,p_3,p_4)}{\left(\left(\frac{P}{2}-q\right)^2 - {m_1}^2\right)\left(\left(\frac{P}{2}+q\right)^2 - {m_2}^2\right)} + \ldots,\ S_k(q) = \int e^{iqx}\, S_0(x,k)$$

- Momentum expansion of the source

$$S_k(k-q) = S_k(0) + S_k(0)'(k-q)^2 + \ldots,\ S_k(q) = \int e^{iqx}\, S_0(x,k)$$

NOTE: Expansion on $(k-q)^2$, not k^2

Femtoscopy from a ChPT perspective. $\pi\pi$ case

- Invariant $\pi\pi$ scattering amplitude (stereographic representation)

$$T(s,k,q) = \frac{s+2m^2-2(k^2+q^2)}{2f^2}$$

- Calculation involving Veltmann-Passarino A_0 and B_0 functions
- Fixing half-off-shellness (p_1 and p_2 on the mass shell),

$$C(k) = 1 + \bar{c} + 2\,T_{on}\,Re\,J(s), \qquad Re\,J(s) = \frac{1}{16\pi^2}\left(2 + \sigma(s)\,Log\,\frac{\sigma(s)-1}{\sigma(s)+1}\right.$$

Note: $\bar{c} = \overline{c(s,m^2)}$

- Same expresion holds for $O(4)$ representation

Femtoscopy from a ChPT perspective. $\pi\pi$ case

- Well-defined $\pi\pi$ isospin amplitudes

$$T_0(s,t,u) = 3\,T(s,t,u) + T(t,s,u) + T(u,t,s)$$
$$T_1(s,t,u) = \; T(t,s,u) - T(u,t,s)$$
$$T_1(s,t,u) = \; T(t,s,u) + T(u,t,s)$$

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- T_0 and T_2 reproduce the same structure as the invariant amplitude

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- Well-defined $\pi\pi$ isospin amplitudes

$$T_0(s,t,u) = 3\,T(s,t,u) + T(t,s,u) + T(u,t,s)$$
$$T_1(s,t,u) = \; T(t,s,u) - T(u,t,s)$$
$$T_1(s,t,u) = \; T(t,s,u) + T(u,t,s)$$

- T_0 and T_2 reproduce the same structure as the invariant amplitude

- $T_1 \sim t - u \sim p\,q$, so its contribution vanishes due to symmetry

$$\int \frac{d^4q}{(2\pi)^4} \frac{q^\mu}{\left(\left(\frac{P}{2}-q\right)^2 - m^2\right)\left(\left(\frac{P}{2}+q\right)^2 - m^2\right)} = 0$$

Femtoscopy from a ChPT perspective. πK case

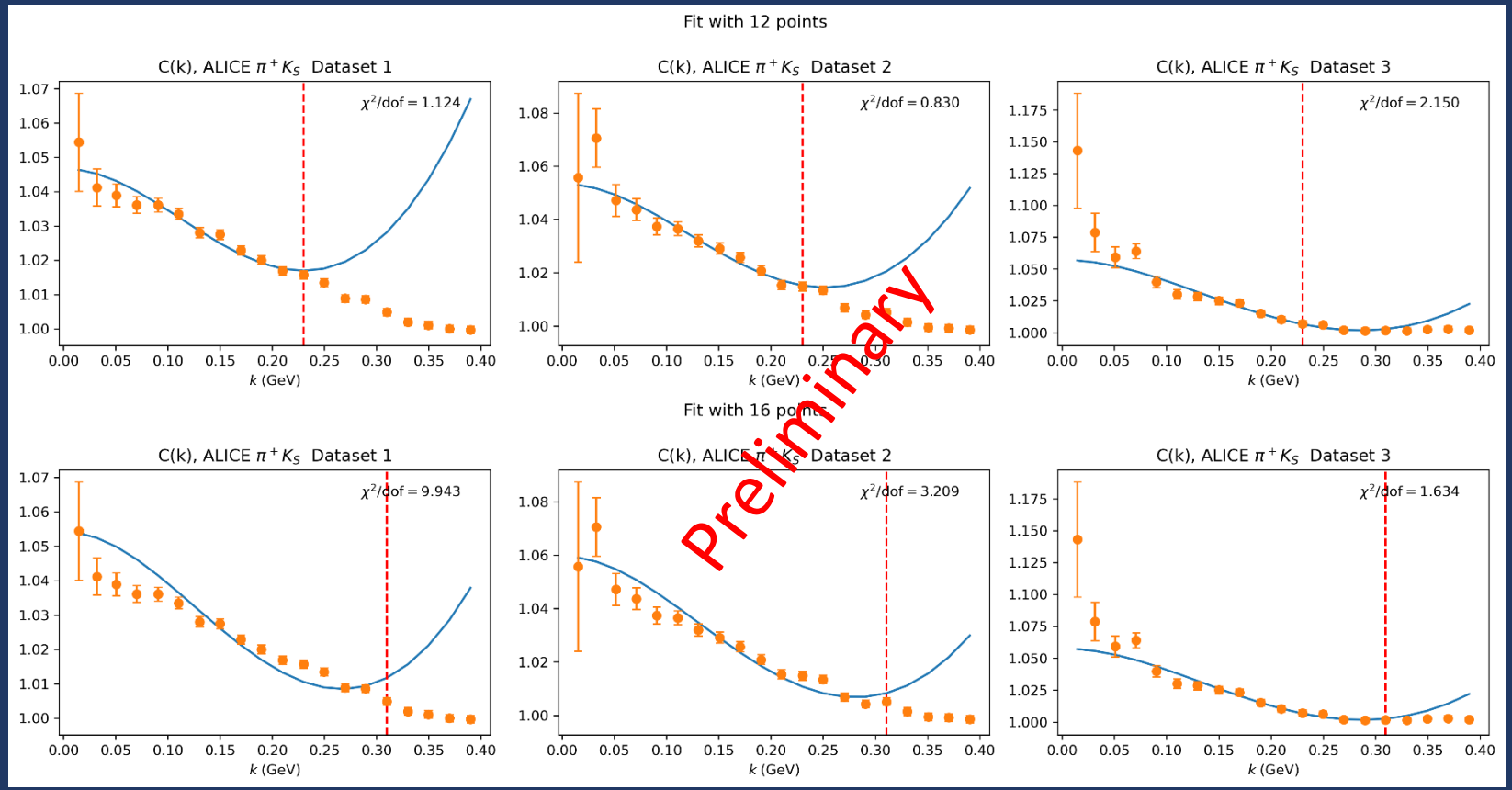
- Motivation: our recent $\pi\ K$ analysis
  - $I = \frac{1}{2}, \frac{3}{2}$ amplitudes studied separately
    - $I = \frac{1}{2}$: s-wave, analogous to equal-mass case
    - $I = \frac{3}{2}$: $p\ q$ dependence, which does not vanish

Partial wave decomposition

- p-wave contribution vanishes
- Just s-wave

Comparison with $\pi^+ K_S$ ALICE data

Femtoscopy from a ChPT perspective. πK case



- Fits from ALICE Datasets 1,2, $\frac{\chi^2}{dof} \lessapprox 1$ up to 0.2-0.25 GeV
- Fits from ALICE Dataset 3 $\frac{\chi^2}{dof} \gtrapprox 2$, but more stable

Proof of concept or practical use?

Summary

- $\pi^+ K_S$ femtoscopic correlations
  - ALICE provide high quality data and their hint of interesting feature in their femtoscopic correlations is very encouraging.
  - Improvement of their model taking into account relativistic corrections and realistic interactions.
  - Radii obtained are smaller than typically expected

  Concerns about the applicability of popular formalisms (LL) to light hadrons.

- Other light-hadron femtoscopic interactions
  - Visible resonances involved in CF.
  - Comparison to data in process

- Femtoscopy from a ChPT point of view
  - A series expansion of both the amplitudes and the source suffices to fit very low energy data
  - The method makes it explicit how the source absorbs the divergences

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Universidad Complutense de Madrid

Thanks for your time!

Excited QCD 2026, Granada, January 13th 2026

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Spare slides

$$T^{\mathrm{QM}}(z) = V^{\mathrm{QM}} + V^{\mathrm{QM}} \frac{1}{z - H_0} T^{\mathrm{QM}}(z)$$

Lippmann-Schwinger equation

	R(fm)	λ
Fit 1	$0.34703 \pm 0.00518(\text{stat}) \pm 0.00074(sys)$	$0.2347 \pm 0.0091(stat) \pm 0.0022(syst)$
Fit 2	$0.3749 \pm 0.059(stat) \pm 0.0016(sys)$	$0.2945 \pm 0.0139(stat) \pm 0.0012(sys)$
Fit 3	$0.6667 \pm 0.016(stat) \pm 0.0015(sys)$	$0.886 \pm 0.062(stat) \pm 0.027(sys)$

Other channels: $\pi^+\eta, K^+ K_S$ and the radius

- Femtoscopic $K^+ K_S$ source radii studied by ALICE Collaboration

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ALICE approach for the interaction:
Flatté-like parametrization

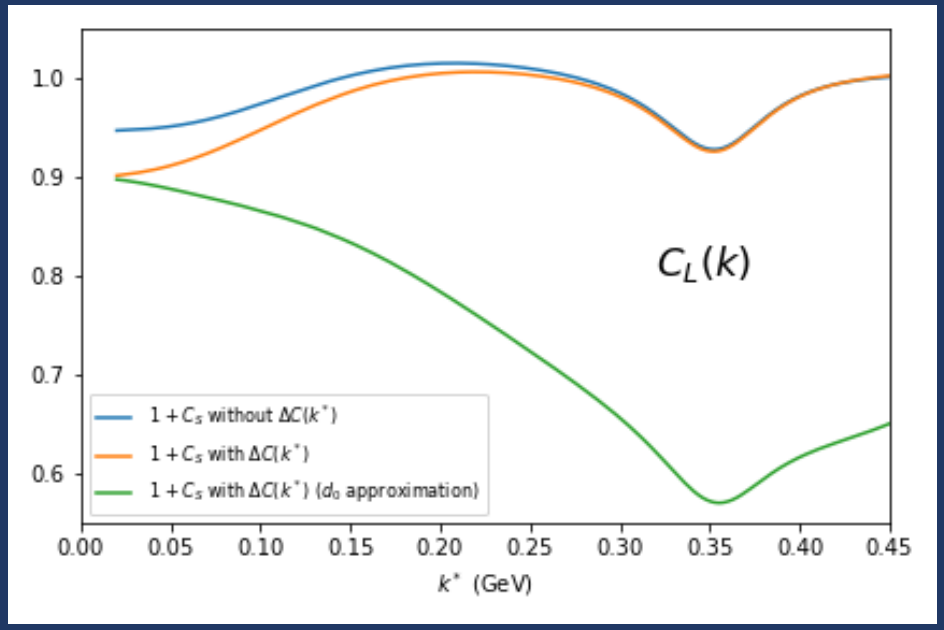
$$f(k^*) = \frac{\gamma_{a_0 \to K\bar{K}}}{m_{a_0}^2 - s - i\left(\gamma_{a_0 \to K\bar{K}}\, k^* + \gamma_{a_0 \to \pi\,\eta}\, k_{\pi\eta}\right)}$$

Summary

- ALICE provide high quality data and their hint of interesting feature in their femtoscopic correlations is very encouraging.
- We have improved their model taking into account relativistic corrections and realistic interactions
- The values of the radius obtained are smaller than typically expected, but non- relativistic frameworks dominate the literature.

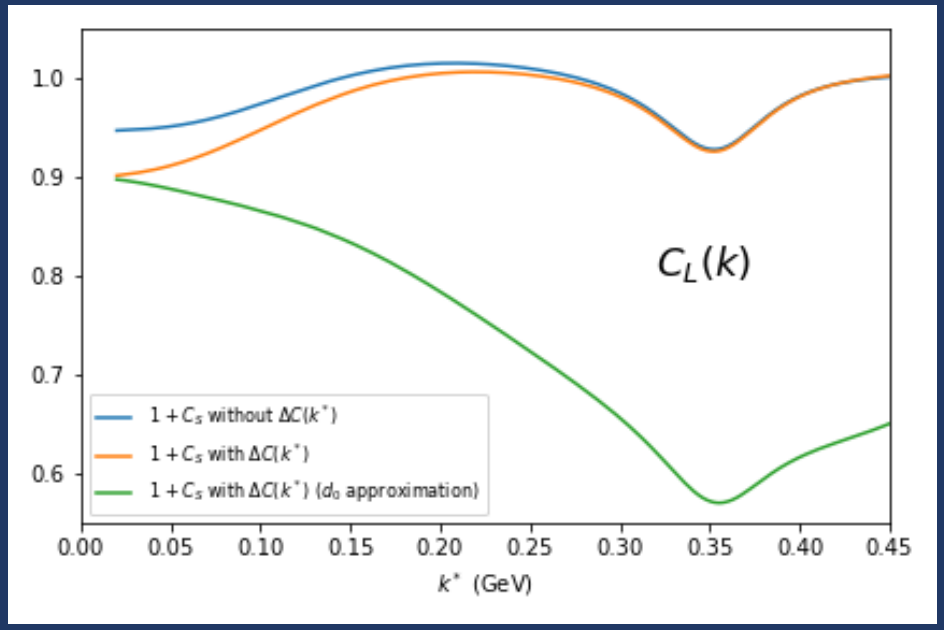
- Lots of work ahead:
    - Could we extract information of the interaction if the source parameters are known?
    - Analysis of other cases (2 Kaons?)
    - Could the femtoscopic formalism clarify the nature of the kappa?

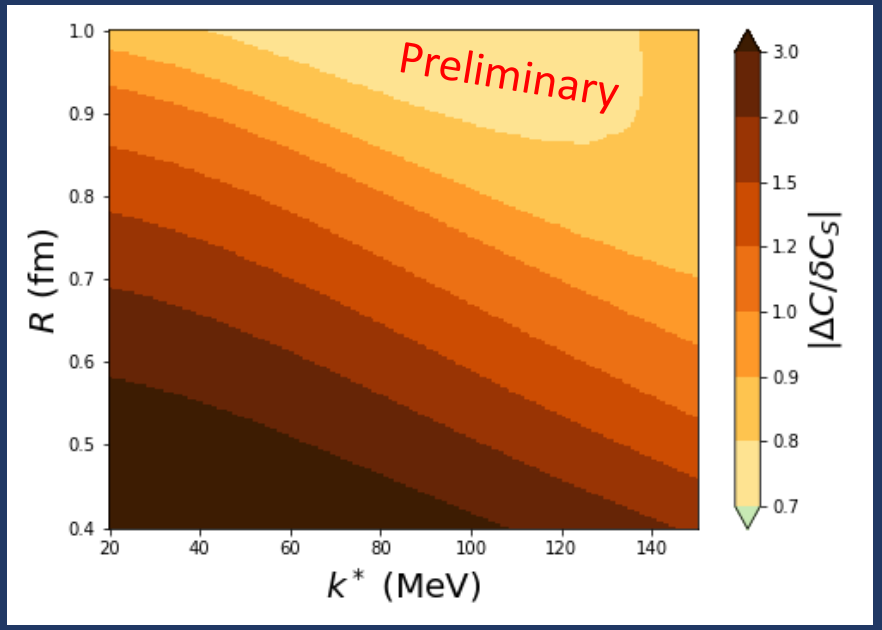
Other channels: $\pi^+ \pi^0$ and Δ C



- Huge effect for the ΔC with d_0

Other channels: $\pi^+ \pi^0$ and Δ C

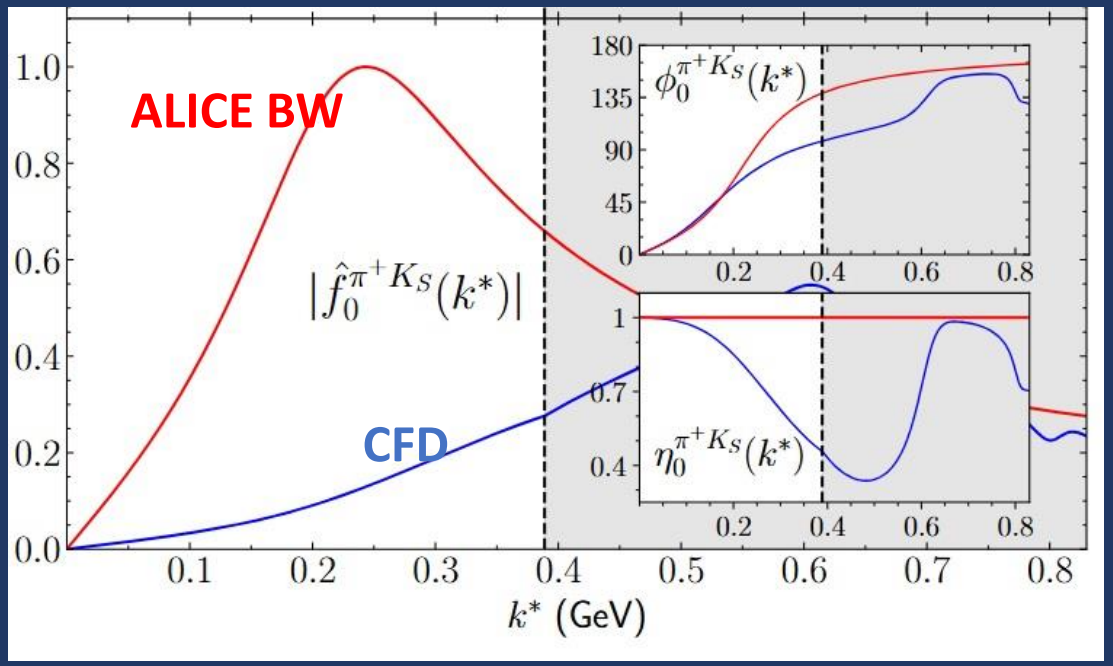




- Huge effect for the ΔC with d_0
- Prevalence of ΔC over partial-wave contributions

Our approach

2. Realistic interactions vs BW



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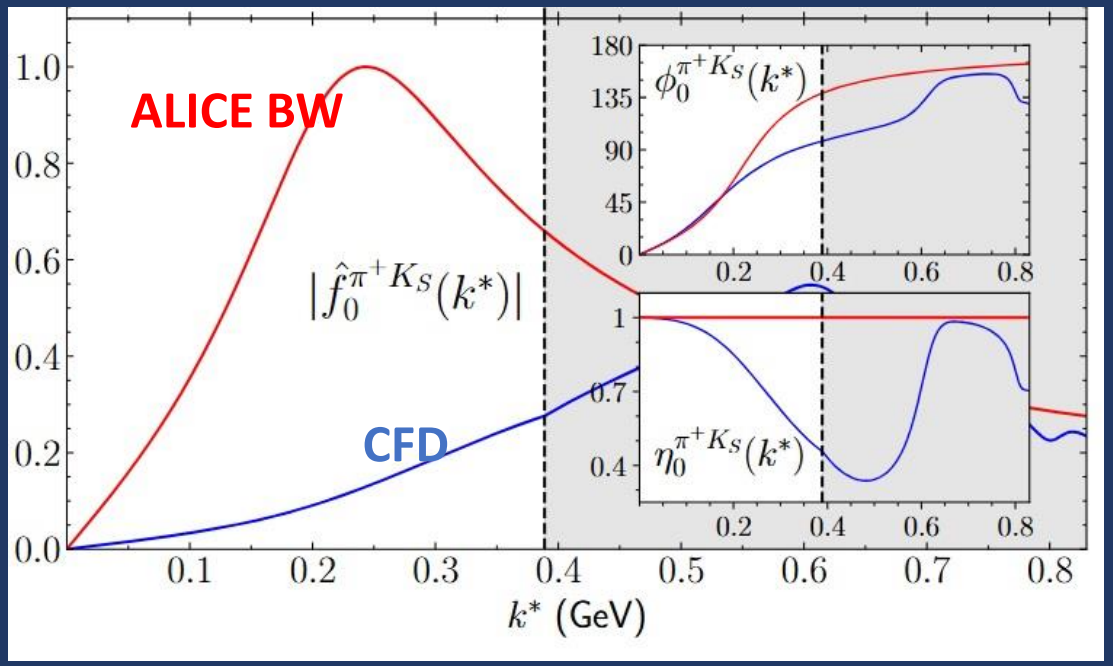

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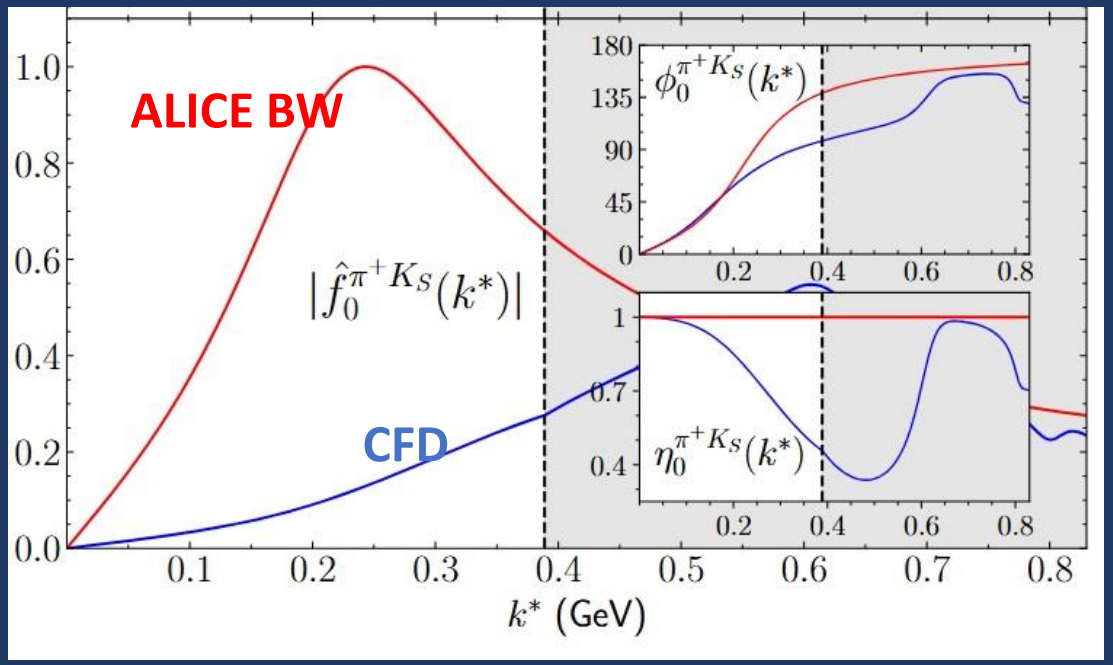
- BW saturates at much smaller energies
- BW is artificially elastic, $\eta = 1$

Our approach

2. Realistic interactions vs BW



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- BW saturates at much smaller energies
- BW is artificially elastic, $\eta = 1$

- Low Energy Parameters

	ALICE BW Fit 1	CFD
$m_\pi a_0$	0.43(4)	0.22(5)
$m_\pi d_0$	-1.83(5)	-3.8(4)