Chirally and chiral spin symmetric hot matter in a model with manifest confinement

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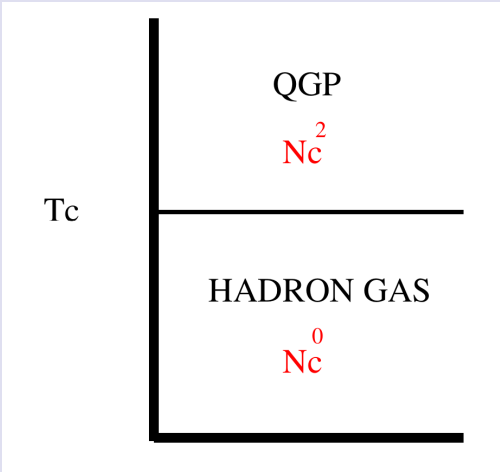
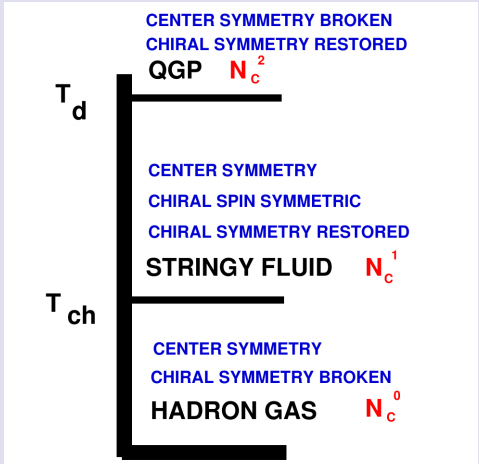
Outline

- Introduction of the model
- Gap equation at $T = 0$ and $T > 0$
- Bethe-Salpeter equation for mesons at $T = 0$ and $T > 0$
- Results for Bethe-Salpeter equation for mesons at $T = 0$ and $T > 0$
- Summary and conclusions

Motivation

Phase diagram of QCD





L.Ya. Glozman, talk presented at

Real-Time and Non-Equilibrium Quantum Field Theory. Aug 31 – Sep 5, 2025, Oberwölz, Austria

QCD-inspired confining and chirally symmetric model for 2 flavors as testing ground for QCD

QCD-inspired confining and chirally symmetric model for 2 flavors

- $H = \int d^3x\, \psi^\dagger(\vec{x},t)\left(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m_q\right)\psi(\vec{x},t) + \frac{1}{2}\int d^3x\, d^3y\, \rho^a(\vec{x}) K_{ab}(|\vec{x}-\vec{y}|)\rho^b(\vec{y})$
- interaction: $\rho^a = \psi^\dagger \frac{\lambda^a}{2}\psi$, $K_{ab}(|\vec{x}-\vec{y}|) = \delta_{ab} V_0(|\vec{x}-\vec{y}|)$

  linearly rising potential $\frac{N_c^2-1}{2N_c} V_0(r) = \sigma r$

  $N_c = 3$ but essentially a large-N_c model

  has higher symmetry $SU(4) \times SU(4)$ (L.Ya. Glozman, Prog. Part. Nucl. Phys., 131 (2023) 104049)
- infrared regularization: introduce $\mu_{\rm IR} > 0$, let $\mu_{\rm IR} \to 0$

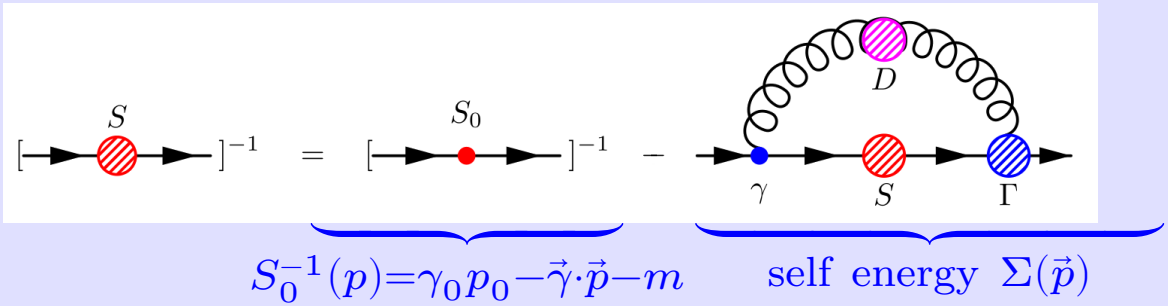
  $V(p) = \frac{8\pi\sigma}{(p^2+\mu_{\rm IR}^2)^2}$

  $-V(r) = -\int \frac{d^3p}{(2\pi)^3} V(p) e^{i\vec{p}\cdot\vec{r}} = \frac{\sigma}{\mu_{\rm IR}} e^{-\mu_{\rm IR} r} = -\frac{\sigma}{\mu_{\rm IR}} + \sigma r + \ldots$

  R. Alkofer et al., Phys. Rev. Lett. 96 (2006), 022001

Gap equation in rainbow approximation

Gap equation for dressed quark propagator $S(p)$



- for $T = 0$ first solved by S.L. Adler and A.C. Davis Nucl. Phys. B 244 (1984), 469
  and many others since then
- for $T > 0$: Fermi-Dirac distribution functions

$$n_p = \left(1 + e^{(\omega_p - \mu)/T}\right)^{-1} = \left(1 + e^{(\sqrt{p^2+M_p^2} - \mu)/T}\right)^{-1}$$

$$\bar{n}_p = \left(1 + e^{(\omega_p + \mu)/T}\right)^{-1} = \left(1 + e^{(\sqrt{p^2+M_p^2} + \mu)/T}\right)^{-1}$$

where μ is the chemical potential (from here on: $\mu = 0$)

Results for the gap equation for $m_q = 0$

- infrared convergent:

  chiral angle φ_p

  dynamical quark mass $M_p = p \tan \varphi_p$

  infrared-finite quark energy $\omega_p = \sqrt{p^2 + M_p^2}$

  condensate $\langle \bar{\psi}\psi \rangle = -\frac{N_c}{\pi^2} \int_0^\infty dp\, p^2 \sin \varphi_p$

- infrared divergent:

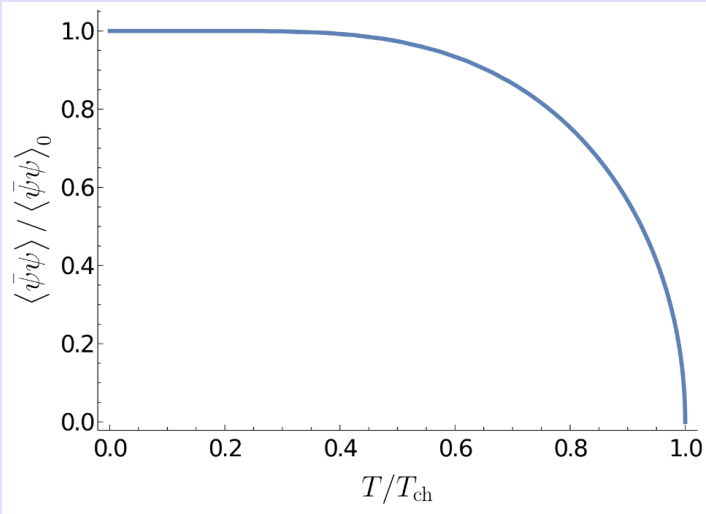
  energy of the dressed quark: $E_p = A_p \sin \varphi_p + B_p \cos \varphi_p$

  with $A_p \cos \varphi_p - B_p \sin \varphi_p = 0$

- For $T > 0$ equations were already derived by A. Kocic, Phys. Rev. D, 33 (1986), 1785
  but he used E_p instead of ω_p in distribution functions

Results for the gap equation for $m_q = 0$

Temperature dependence of the condensate $\langle\bar{\psi}\psi\rangle$



$\langle\bar{\psi}\psi\rangle_{T=0} \approx -(250\ \text{MeV})^3$ for QCD is used to fix σ

there is a phase transition:

$\langle\bar{\psi}\psi\rangle$ vanishes at $T_{\rm ch} \approx 0.084\sqrt{\sigma} \approx 90\ \text{MeV}$

lattice estimate for QCD: $T_{\rm ch} \approx 130\ \text{MeV}$

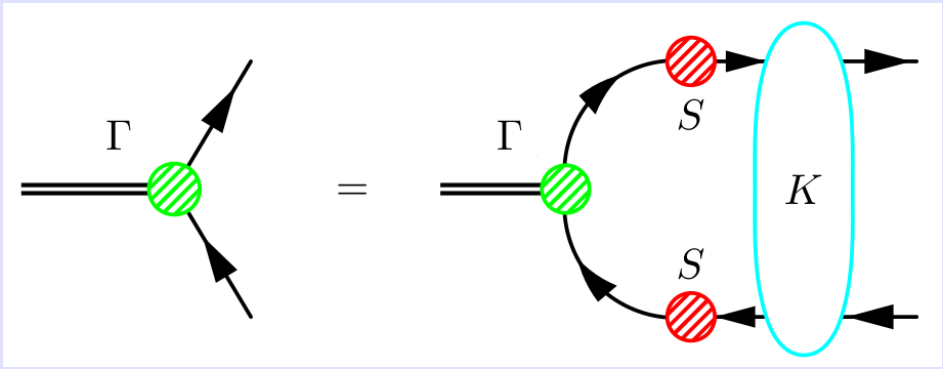
Mechanism responsible for chiral symmetry restoration

Pauli blocking by thermal excitations of the quarks and antiquarks

damping factor $1 - n_p - \bar{n}_p$ in Gap equation

Bethe-Salpeter equation in ladder approximation

Bethe-Salpeter equation for meson vertex function $\Gamma(p)$



Bethe-Salpeter kernel

- use solution for S of gap equation and same interaction
- parametrization of $\Gamma(p)$ with functions $g(p)$ and $h(p)$
- 3 different types of mesons according to their quantum numbers I, J^{PC}
- model has no isospin dependence, $0, J^{PC}$ and $1, J^{PC}$ states with same J, P, C and n are degenerate

Equations for $J = 0$

Equations for pseudoscalar mesons with mass m

$$h(p)\,E_p = \tfrac{1}{2}\int \tfrac{d^3k}{(2\pi)^3}\; V(\vec{p}-\vec{k})(1-n_k-\bar{n}_k)\left[h(k)+\tfrac{m^2}{4\,E_k}g(k)\right]$$

$$g(p)\,[E_p - \tfrac{m^2}{4\,E_p}] = h(p) + \tfrac{1}{2}\int \tfrac{d^3k}{(2\pi)^3}\; V(\vec{p}-\vec{k})(1-n_k-\bar{n}_k)$$
$$\left[\sin\varphi_p \sin\varphi_k + \hat{p}\cdot\hat{q}\cos\varphi_p\cos\varphi_k\right] g(k)$$

Equations for scalar mesons with mass m

$$h(p)\,E_p = \tfrac{1}{2}\int \tfrac{d^3k}{(2\pi)^3}\; V(\vec{p}-\vec{k})(1-n_k-\bar{n}_k)$$
$$\left[\cos\varphi_p \cos\varphi_k + \hat{p}\cdot\hat{q}\sin\varphi_p\sin\varphi_k\right]\left[h(k)+\tfrac{m^2}{4\,E_k}g(k)\right]$$

$$g(p)\,[E_p - \tfrac{m^2}{4\,E_p}] = h(p) +$$
$$\tfrac{1}{2}\int \tfrac{d^3k}{(2\pi)^3}\; V(\vec{p}-\vec{k})(1-n_k-\bar{n}_k)\,\hat{p}\cdot\hat{q}\,g(k)$$

Equations for $J = 0$

Equations with $\varphi_p = 0$ for pseudoscalar and scalar mesons

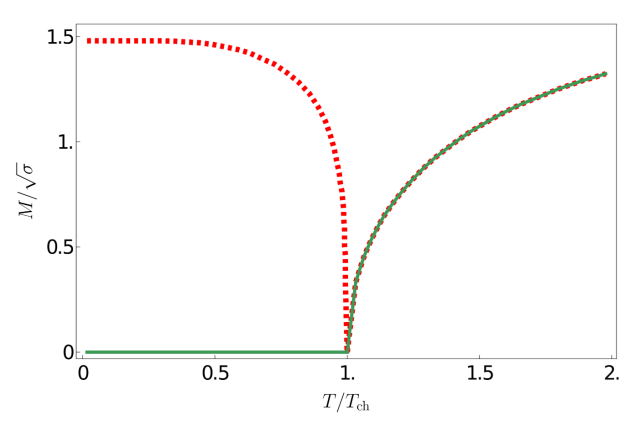
$$h(p)\, E_p = \tfrac{1}{2} \int \tfrac{d^3k}{(2\pi)^3}\; V(\vec{p} - \vec{k})(1 - n_k - \bar{n}_k) \left[h(k) + \tfrac{m^2}{4\, E_k} g(k) \right]$$

$$g(p)\, [E_p - \tfrac{m^2}{4\, E_p}] = h(p) + \tfrac{1}{2} \int \tfrac{d^3k}{(2\pi)^3}\; V(\vec{p} - \vec{k})(1 - n_k - \bar{n}_k)\, \hat{p} \cdot \hat{q}\, g(k)$$

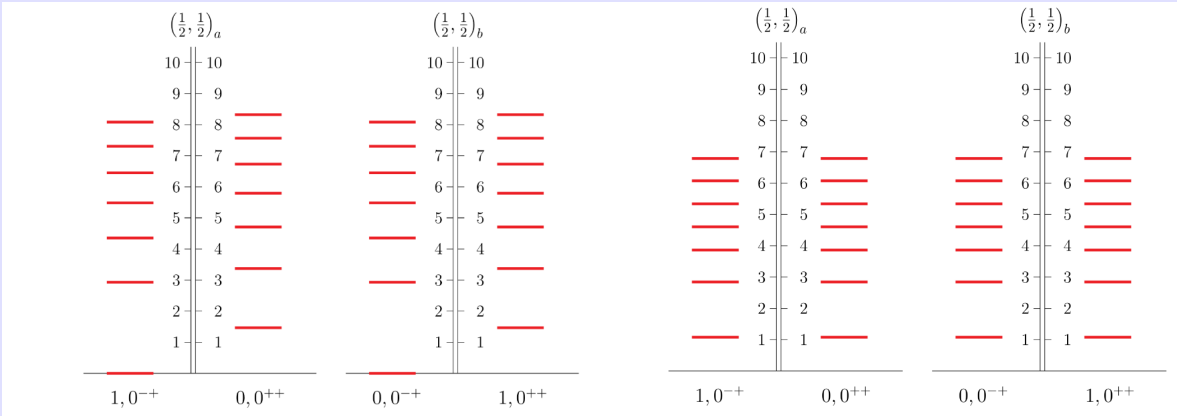
- IR-divergent terms on both sides mutually cancel
- when SBCS: $m_\pi = 0$ due to axial-vector Ward-Takahashi identity
- when no SBCS: masses of pseudoscalar and scalar mesons are degenerate

Scalar and pseudoscalar mesons

Mass of scalar and pseudoscalar meson as function of T



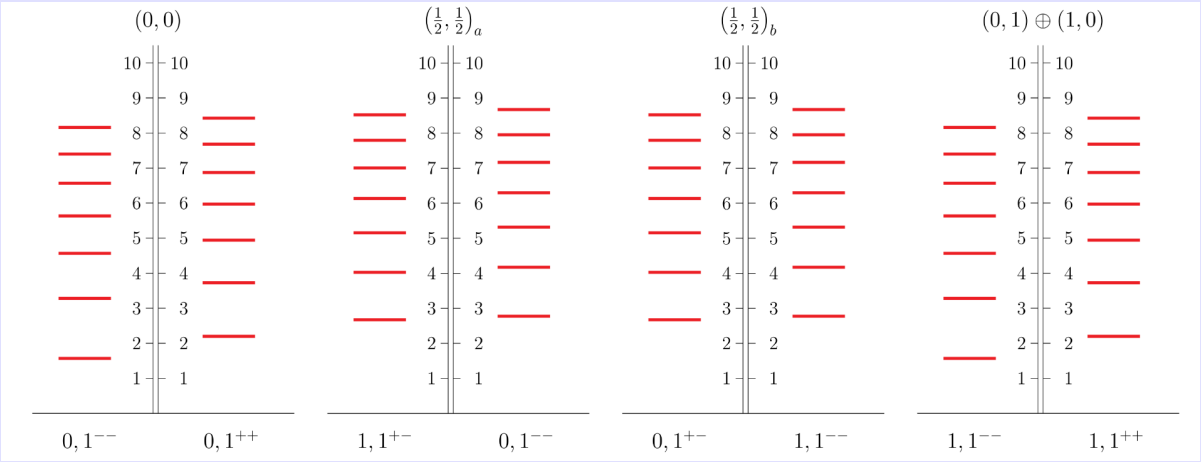
Spectra of pseudoscalar and scalar mesons



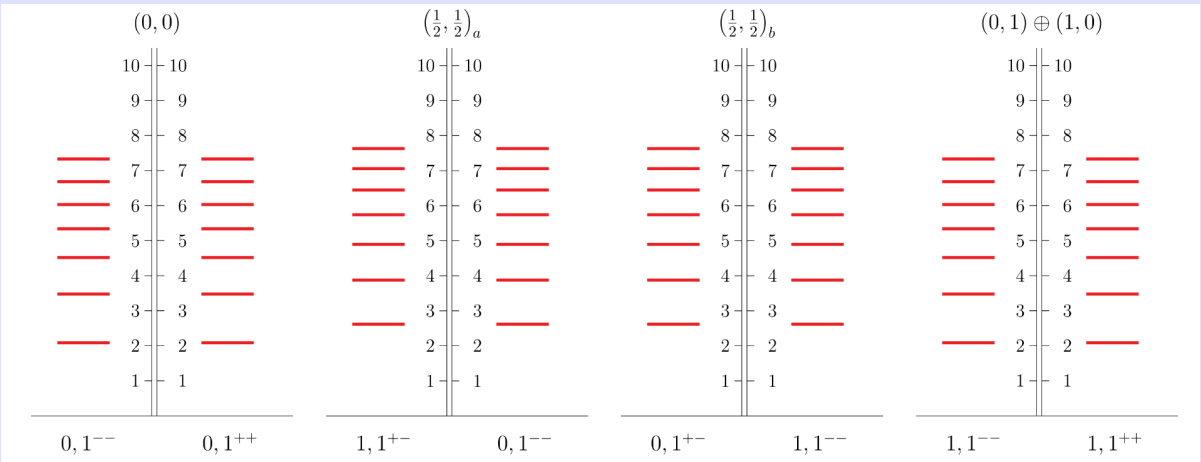
$T = 0.5T_{\rm ch}$ $T = 1.5T_{\rm ch}$

Mesons with $J = 1$

Spectra of mesons with $J = 1$ at $T = 0.5T_{\rm ch}$

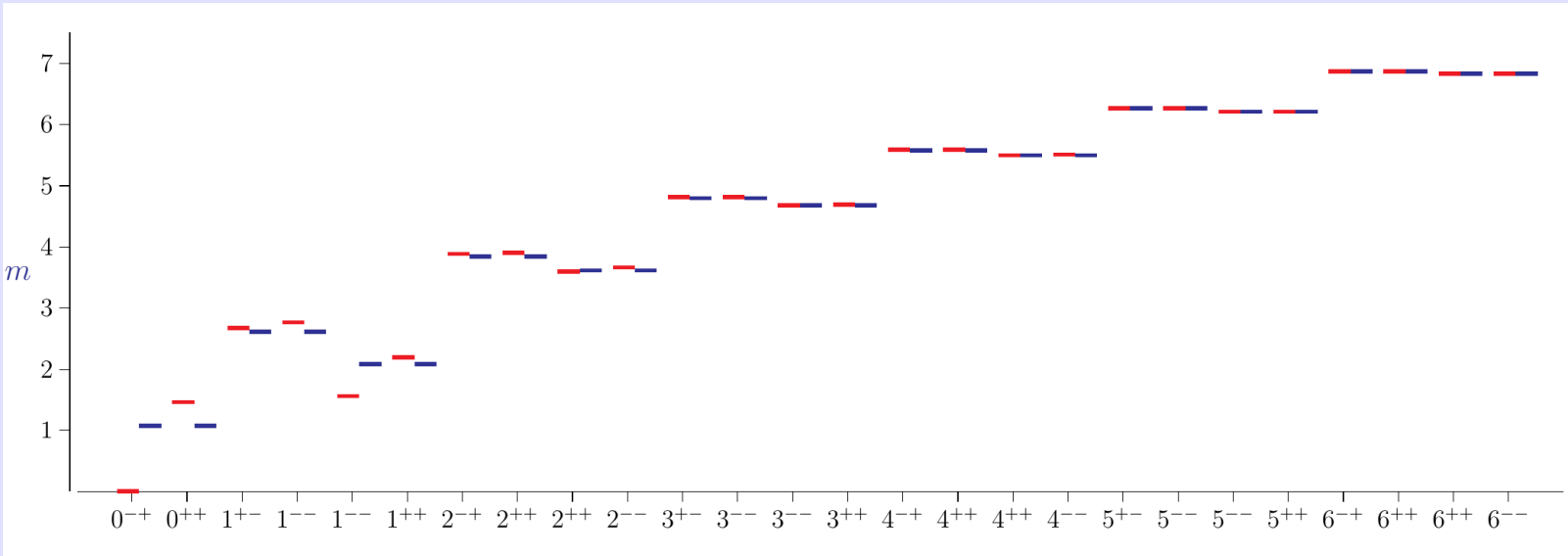


Spectra of mesons with $J = 1$ at $T = 1.5T_{\rm ch}$



Mesons with higher spin

Masses of mesons with $J = 0, \ldots, 6$ and $n = 0$



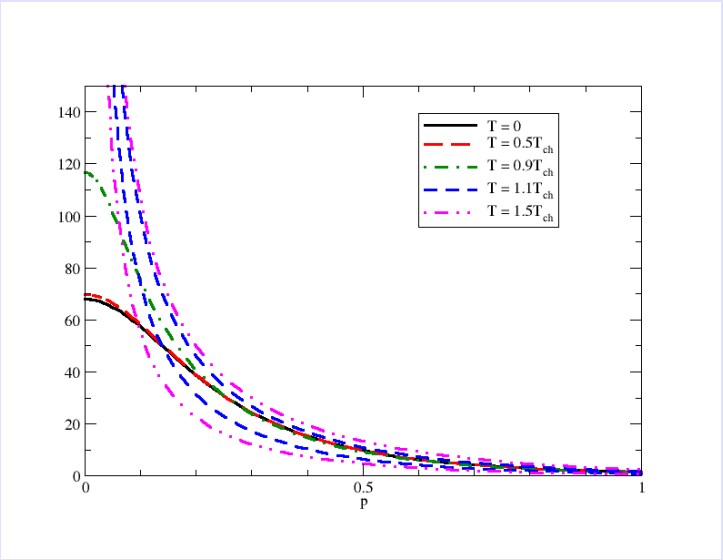
red: $T = 0.5 T_{\mathrm{ch}}$, blue: $T = 1.5 T_{\mathrm{ch}}$

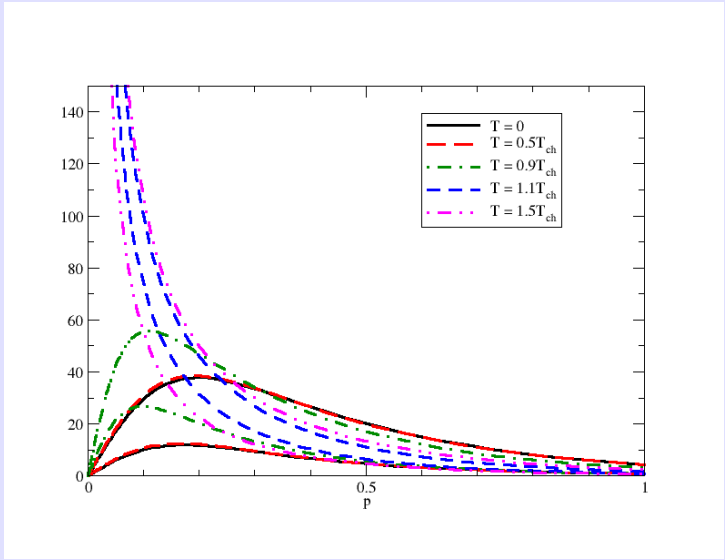
For higher spins:

- chiral symmetry approximately restored even at $T < T_{\mathrm{ch}}$
- approximate chiral spin symmetry and its flavor extension $SU(4) \times SU(4)$ of confinement

Wave functions

Radial functions of pseudoscalar and scalar mesons

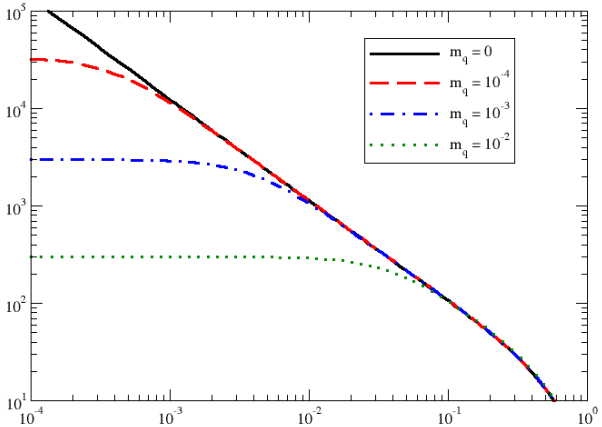


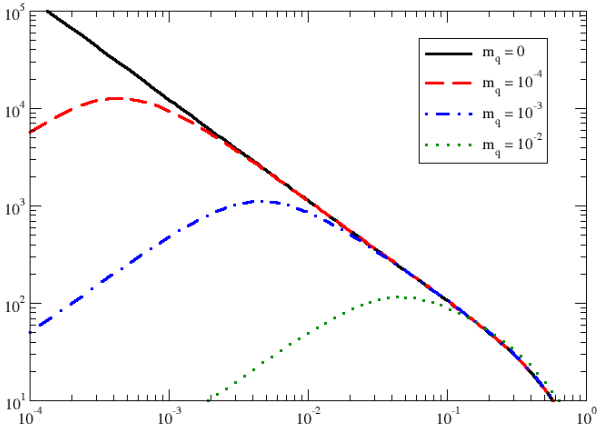


- when SBCS ($T < T_{\rm ch}$): pseudocsalar meson with $m = 0$ $\psi_+(0,p) = \psi_-(0,p)$
- for $T > T_{\rm ch}$:
  pseudoscalar and scalar mesons have same radial functions which are divergent at $p = 0$, what about normalization?

Wave functions

Radial functions of pseudoscalar and scalar mesons, dependence on m_q for $T = 1.5T_{\rm ch}$ (log-log plots)

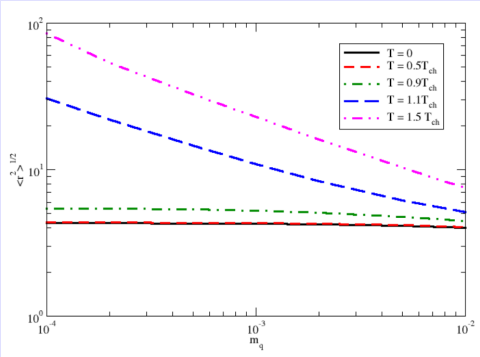


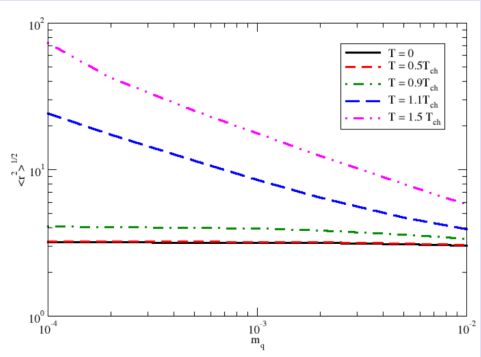


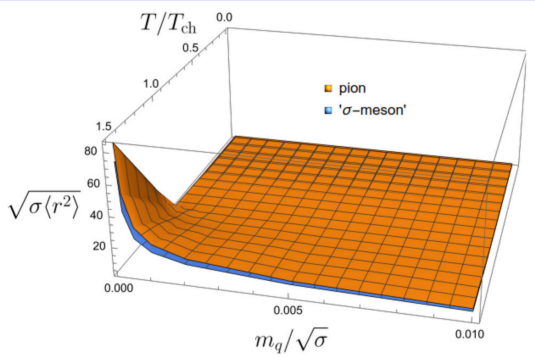
- for $m_q > 0$ no divergence at $p = 0$
- results for $m_q = 0$ as limit for $m_q \to 0$
- what about the size?

Radius

Charge radius of pseudocsalar and scalar mesons (log-log plot)







$T > T_{\mathrm{ch}}$

- radii of $q\bar{q}$ states with $J = 0, 1$ diverge when $m_q \to 0$ infinite swelling of these states is related with Pauli blocking
- dense medium of overlapping color-singlet $q\bar{q}$ systems

Summary and conclusions

- manifestly confining and chirally symmetric large-N_c model in 3+1 dimensions as simple testing ground for real QCD
- chrial symmetry broken at $T = 0$ but becomes restored at $T_{\rm ch} \approx 90$ MeV
- approximate $SU(4) \times SU(4)$ symmetry of confinement for "mesons" with higher spin
- mesons are located in small space at $T = 0$ but low-spin mesons become infinitely delocalized at $T > T_{\rm ch}$
- chiral symmetry restoration and delocalization related to Pauli blocking
- above $T_{\rm ch}$ dense medium of huge strongly overlapping color-singlet quark-antiquark low-spin systems (stringy fluid)