A precision test of First Row CKM Unitarity from Lattice QCD

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Fermilab Lattice and MILC collaboration
Universidad de Granada

Current situation

First row CKM unitarity

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

We will explore the unitarity in the first row

$$\Delta_u = |V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} \sim 0 - 1$$

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$$\Delta_u = |V_{ud}|^2 + |V_{us}|^2 + \cancelto{\sim 0}{|V_{ub}|^2} - 1 \overset{\text{SM}}{=} 0$$

First row CKM unitarity

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$$\Delta_u = |V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2}^{\sim 0} - 1 \overset{\text{SM}}{=} 0$$

$$\hookrightarrow \Gamma_{K_{\ell 3}} \propto |V_{us}|^2 \quad f_+(q^2 = 0)^2$$

$$K_{\ell 3} \equiv K \to \pi \ell \nu_\ell$$

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$$\Gamma_{K_{\ell 3}} \propto |V_{us}|^2 \quad f_+(q^2=0)^2$$

$$\Gamma_{K_{\ell 2}}/\Gamma_{\pi_{\ell 2}} \propto \frac{|V_{us}|^2}{|V_{ud}|^2} \quad \frac{f_{K^\pm}^2}{f_{\pi^\pm}^2}$$

$K_{\ell 3} \equiv K \to \pi \ell \nu_\ell \qquad K_{\ell 2}(\pi_{\ell 2}) \equiv K(\pi) \to \ell \nu_\ell$

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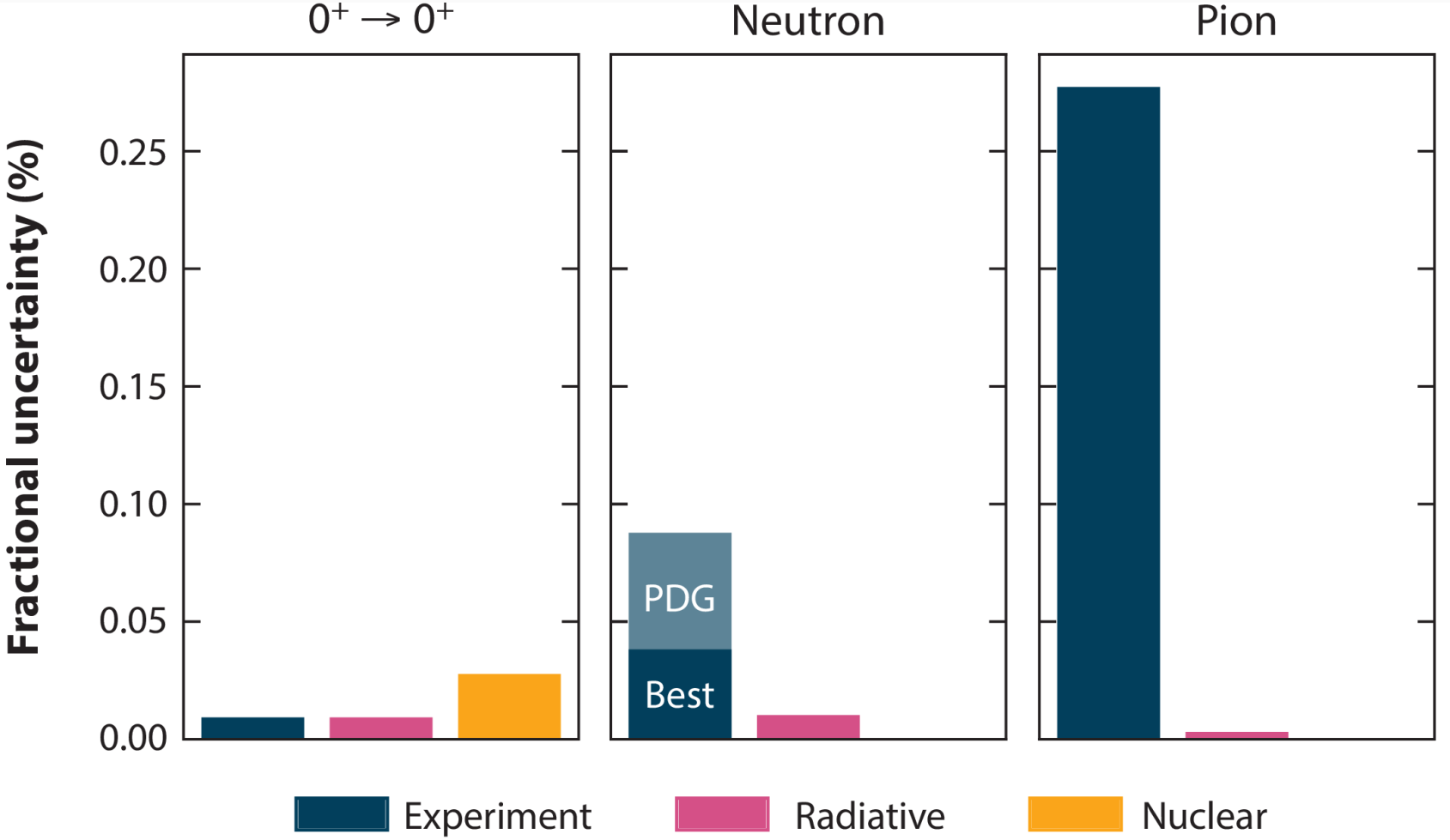
$K_{\ell 3} \equiv K \to \pi \ell \nu_\ell \qquad K_{\ell 2}(\pi_{\ell 2}) \equiv K(\pi) \to \ell \nu_\ell$

*QED corrections needed to match to experiment: Progress on replacing ChPT estimates by lattice results, see Matteo Di Carlo talk at KAON2025

$|V_{ud}|$ determinations

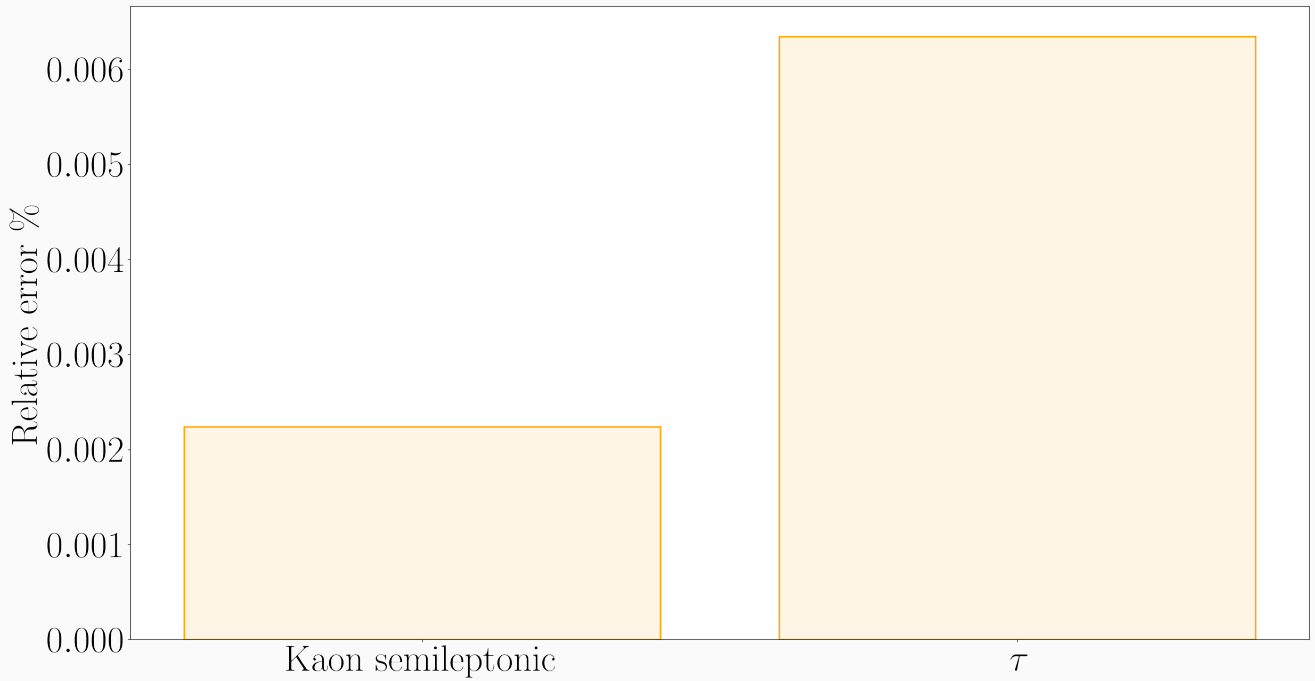
- Superallowed beta decays $0^+ \to 0^+$. Concerns about Nuclear Structure uncertainty
- Neutron decays
- Pion β decays.



V_{us} and V_{us}/V_{ud} determinations

V_{us}

- Semileptonic Kaon decays
- τ decays



V_{us}/V_{ud}

- Ratio between $K \to \ell\nu$ and $\pi \to \ell\nu$

V_{us} and V_{us}/V_{ud} determinations

V_{us}

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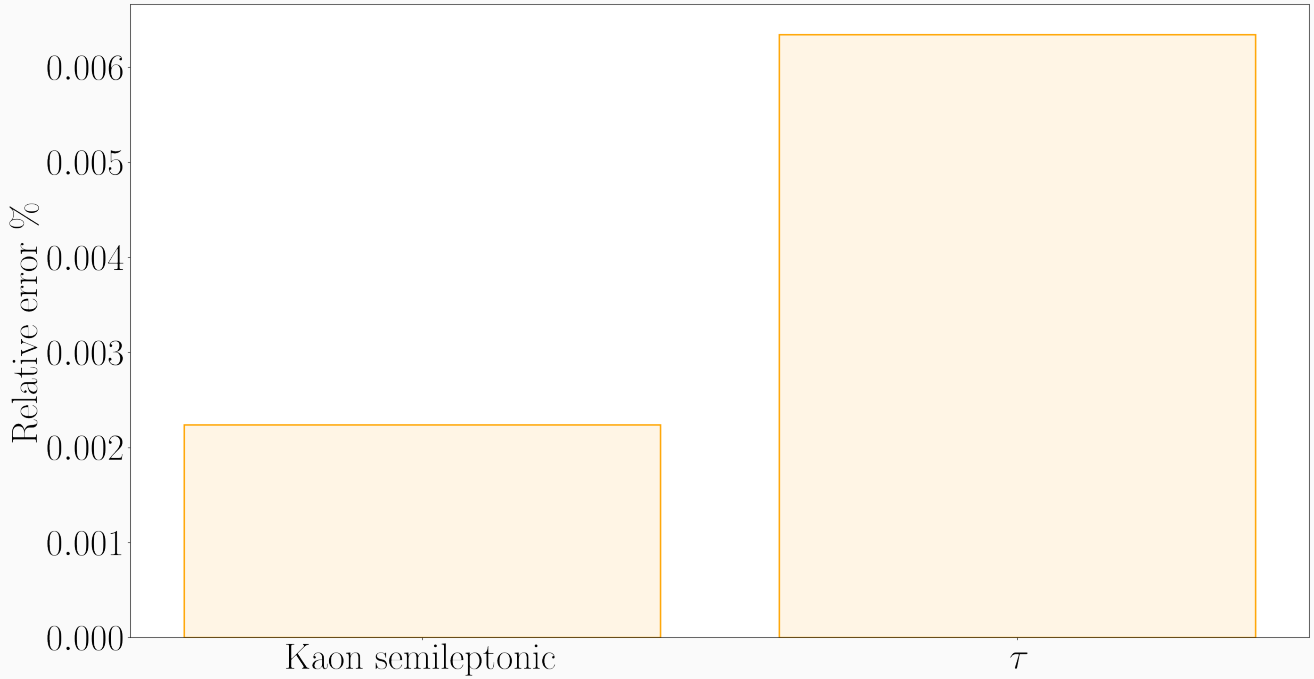
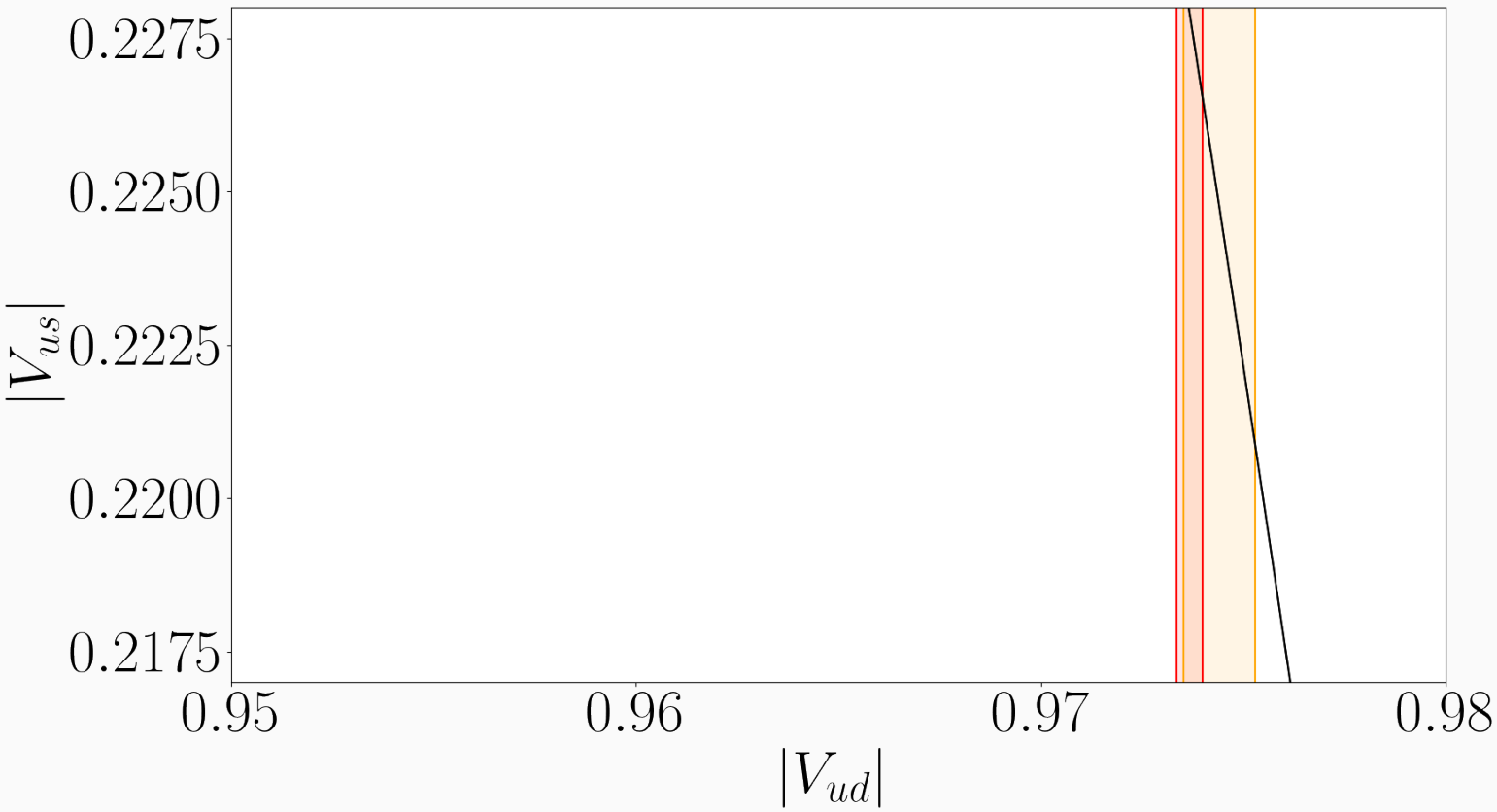


V_{us}/V_{ud}

- Ratio between $K \to \ell\nu$ and $\pi \to \ell\nu$

Current status (using 2208.11707, 1712.09262, 1809.02827)



• Unitarity band • $0^+ \to 0^+$ β decays • Neutron decay

2208.11707 Cirigliano *et al.*, Physics Letters B, Volume 838

1712.09262 FNAL/MILC, PRD98. 1809.02827 FNAL/MILC, PRD99

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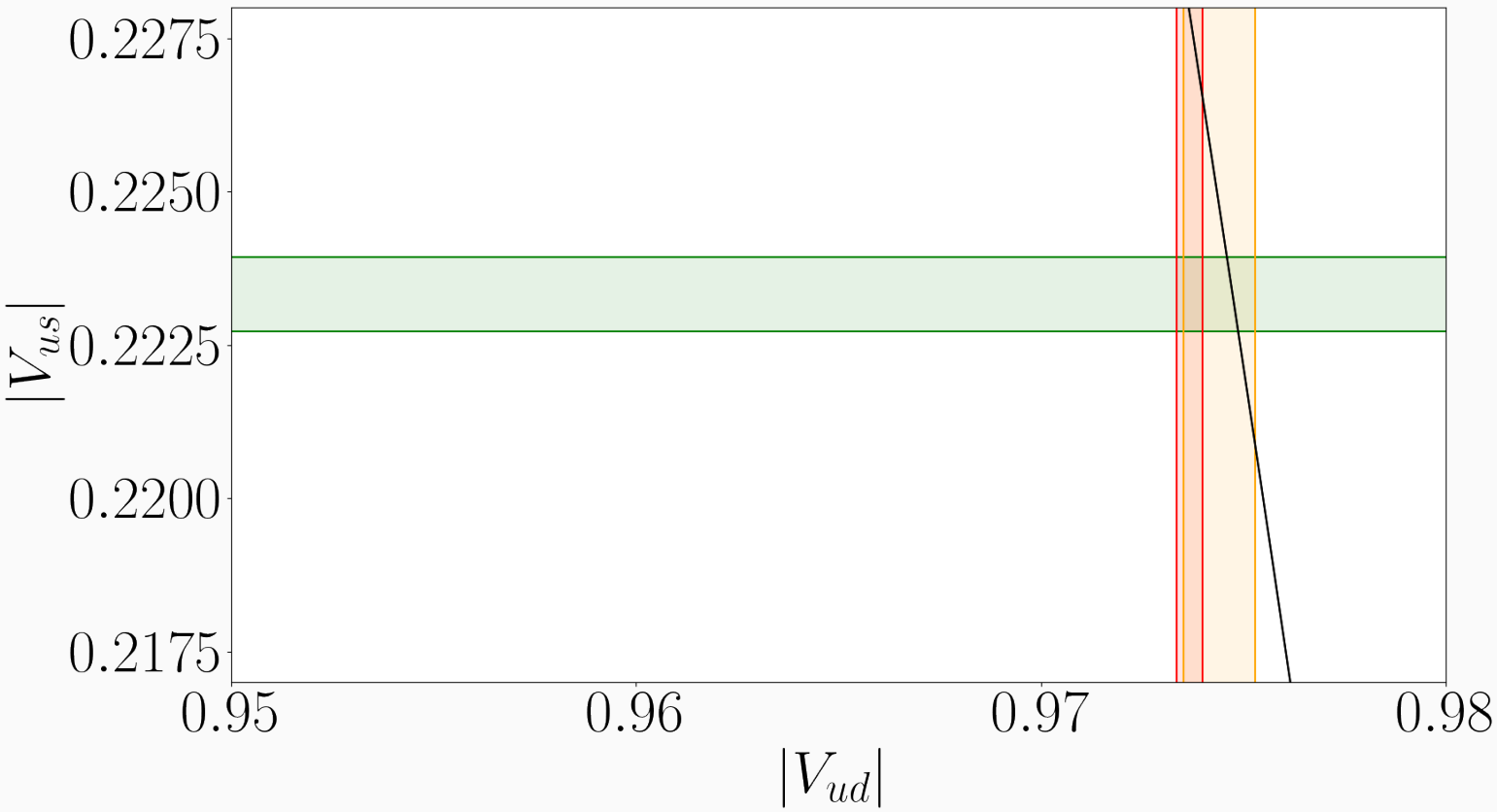

- Unitarity band
- $0^+ \to 0^+$ β decays
- Neutron decay
- $K_{\ell 3}$

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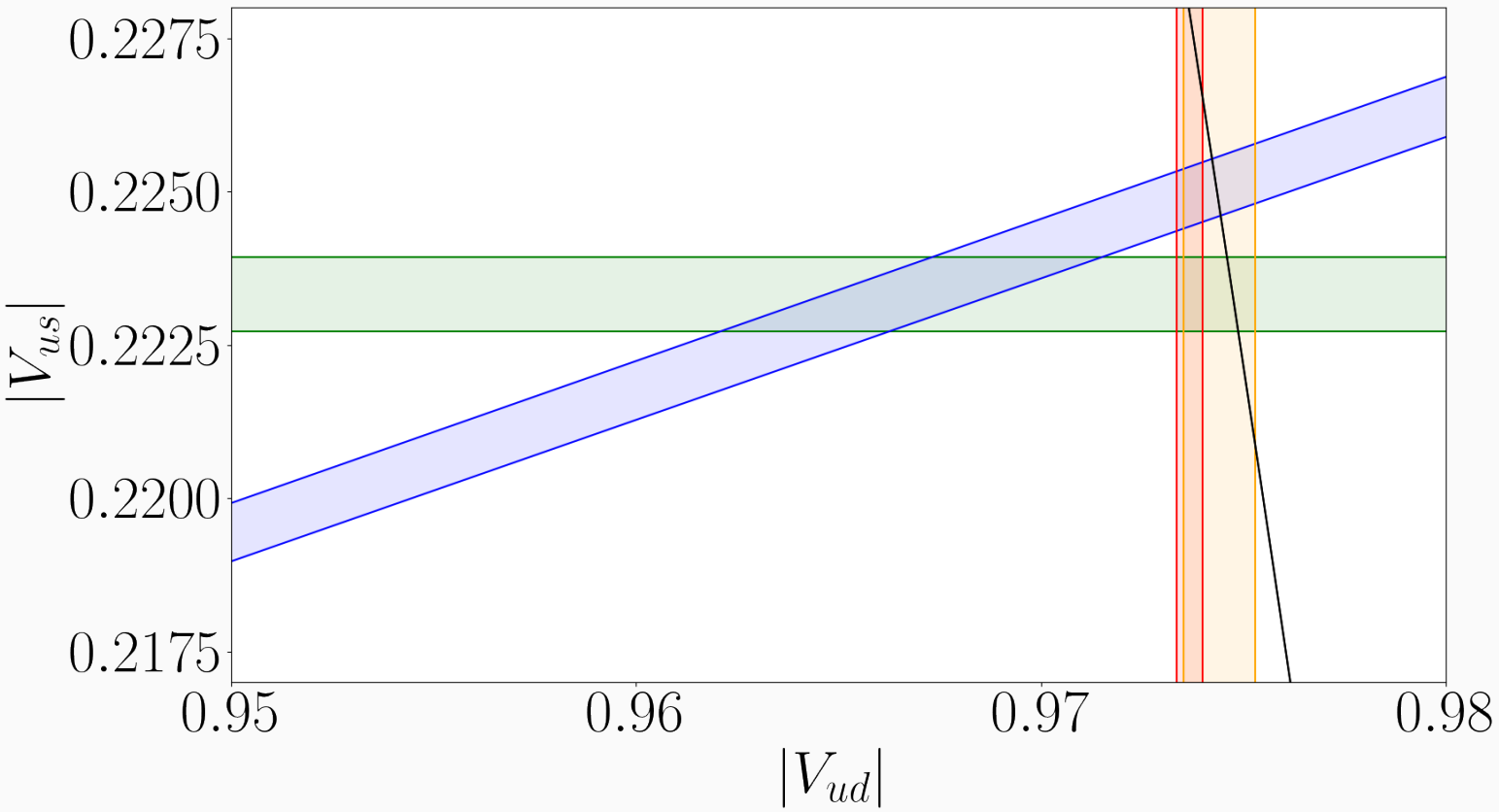

- Unitarity band
- $0^+ \to 0^+$ β decays
- Neutron decay
- $K_{\ell 3}$
- $K_{\ell 2}/\pi_{\ell 2}$

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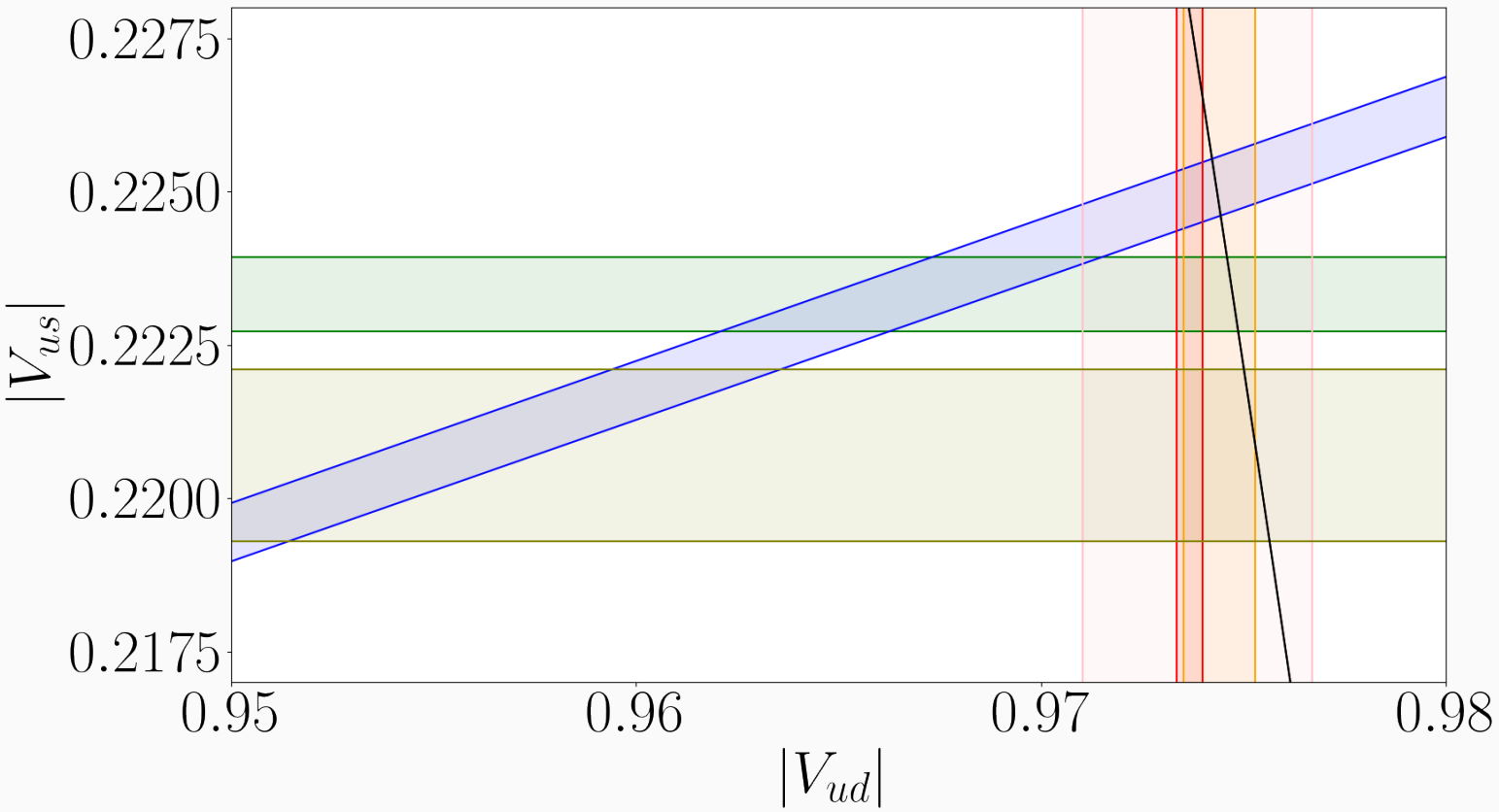

• Unitarity band • $0^+ \to 0^+$ β decays • Neutron decay

• $K_{\ell 3}$ • $K_{\ell 2}/\pi_{\ell 2}$ • τ • $\pi\beta$

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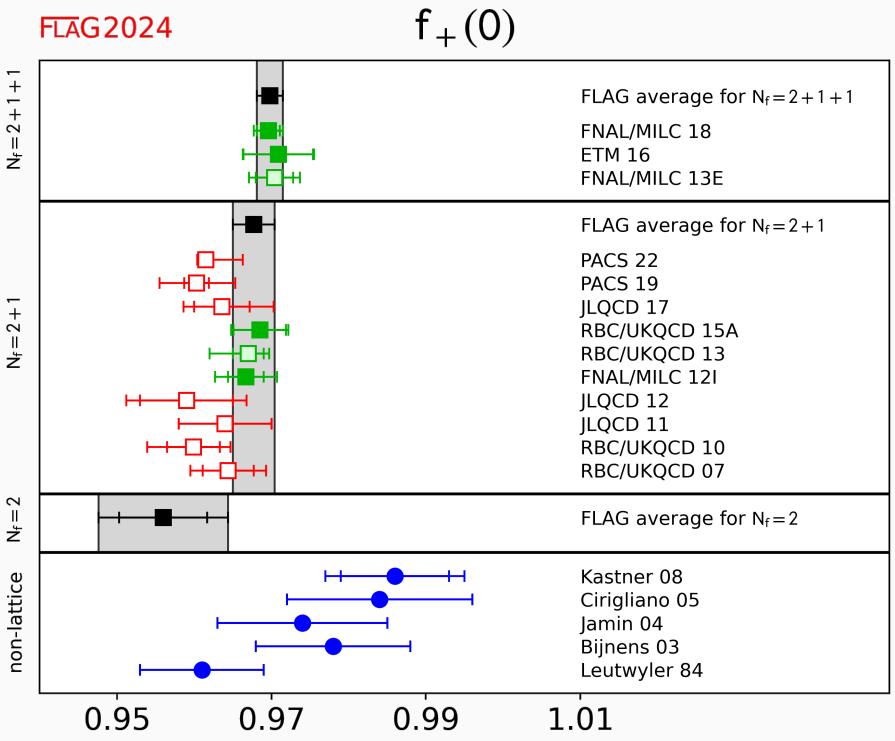
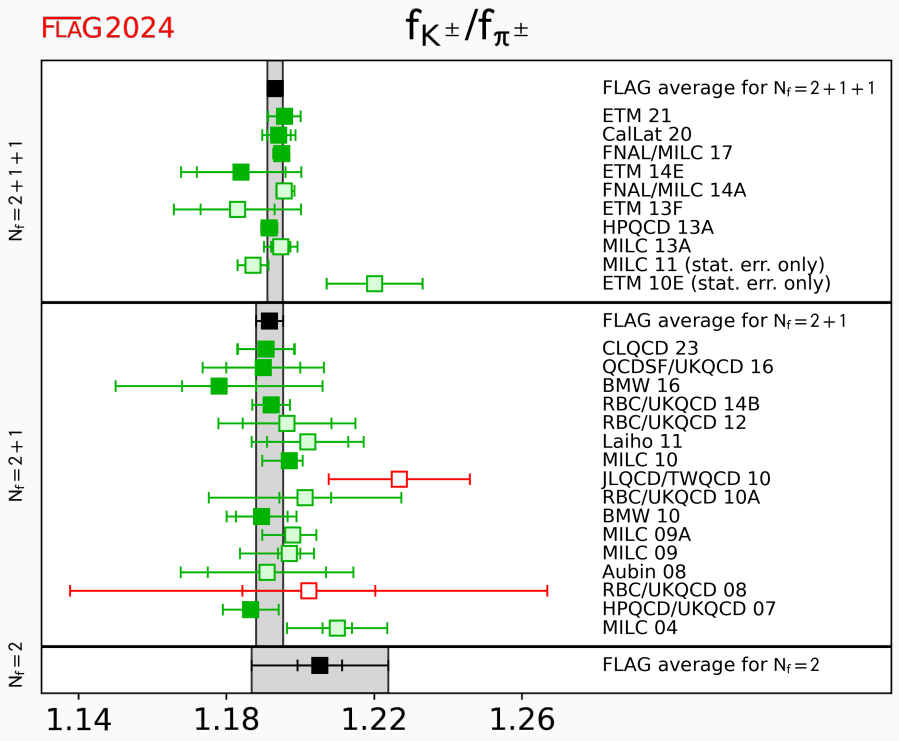
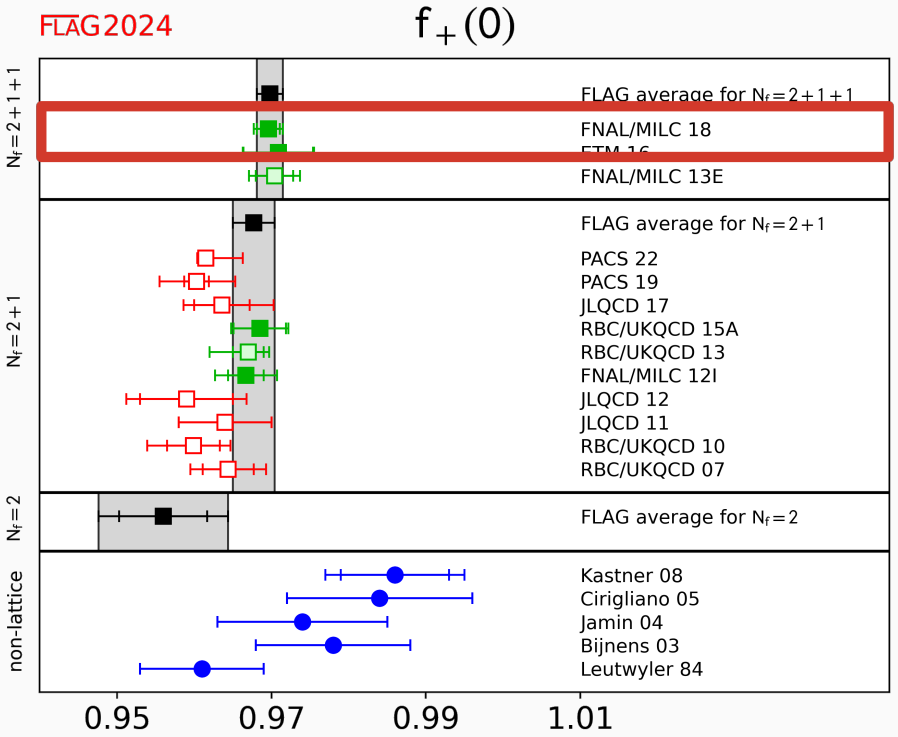
Current status: FLAG24

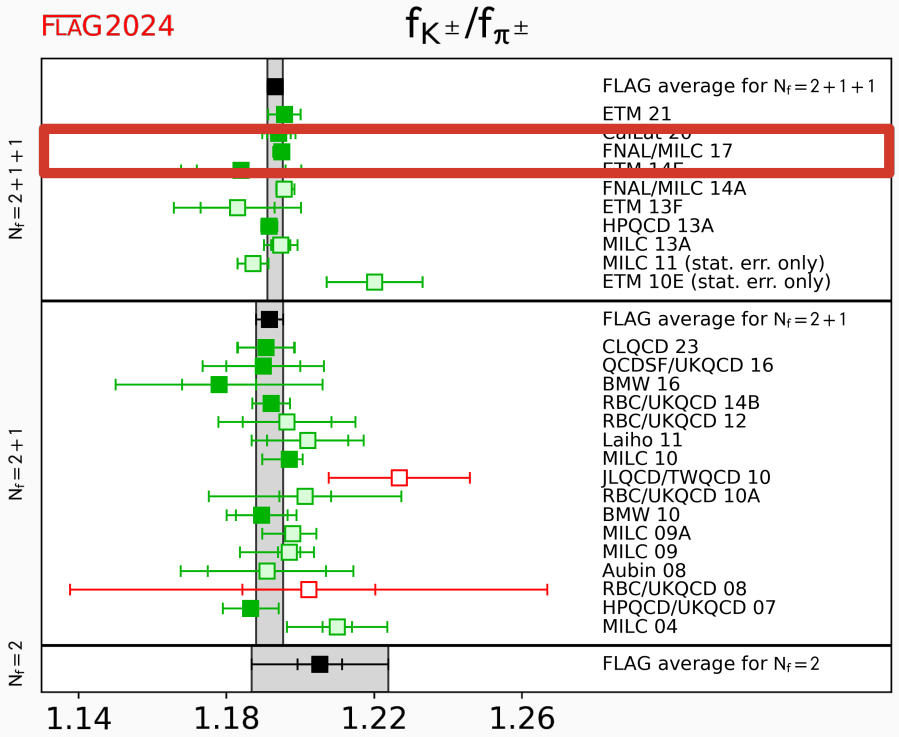




New $N_f = 2 + 1$ $f_{K^\pm}/f_{\pi^\pm}$ computation by A. Conigli et al. (2512.19294)

Current status: FNAL/MILC Collaboration





$$f_+(0) = 0.9696(15)(12)$$

1809.02827

FNAL/MILC, PRD99

$$f_K/f_\pi = 1.1980(12)(^{+5}_{-15})$$

1712.09262

FNAL/MILC, PRD98

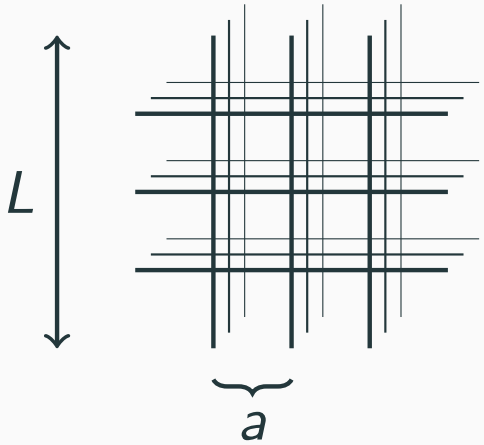
Our strategy

Non perturbative computation: Lattice QCD

Path integral formulation

$$\langle \mathcal{O} \rangle = \int \mathcal{D}[\Phi, U] e^{-S[\Phi,U]} \mathcal{O}[\Phi, U]$$

- Discrete Euclidean Space-Time (spacing a)



  - Fermionic (Φ) fields live in the sites
  - Gluonic fields (U) live in the links

- Solve Integrals using Monte Carlo methods

Lattice QCD Workflow

- Generate ensembles of configurations of gauge fields.

  $\rightarrow$ Our ensembles were generated by the MILC collaboration

Lattice QCD Workflow

 $\Rightarrow \quad \langle \mathcal{O} \rangle$

- Measure correlation functions on configurations

  $\rightarrow$ We will compute 2pt and 3 point functions

Lattice QCD Workflow

 $\Rightarrow \quad \langle \mathcal{O} \rangle \quad \Rightarrow \; f_+(a),\, f_K/f_\pi(a)$

- Extract desired quantities from corr. func.

  $\rightarrow$ Repate these for different ensembles with different a, L, m_i

Lattice QCD Workflow

$\Rightarrow \quad \langle \mathcal{O} \rangle \quad \Rightarrow \; f_+(a),\, f_K/f_\pi(a) \qquad \Rightarrow \; f_+^{\text{phys}},\, f_K^{\text{phys}}/f_\pi^{\text{phys}}$

- Take continuum + infinite volume + physical masses limit

  $\rightarrow$ We will use Staggered Chiral Perturbation Theory

Semileptonic form factor

$$\langle \pi | V^\mu | K \rangle = f_+(q^2) \left[p_K^\mu + p_\pi^\mu - \frac{m_K^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_K^2 - m_\pi^2}{q^2} q^\mu$$

Semileptonic form factor

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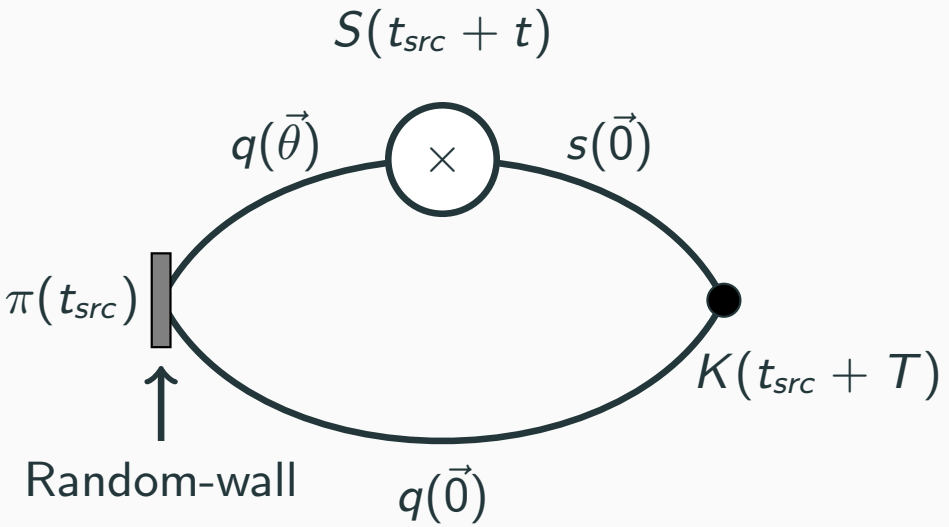
Ward-Takahashi Identity

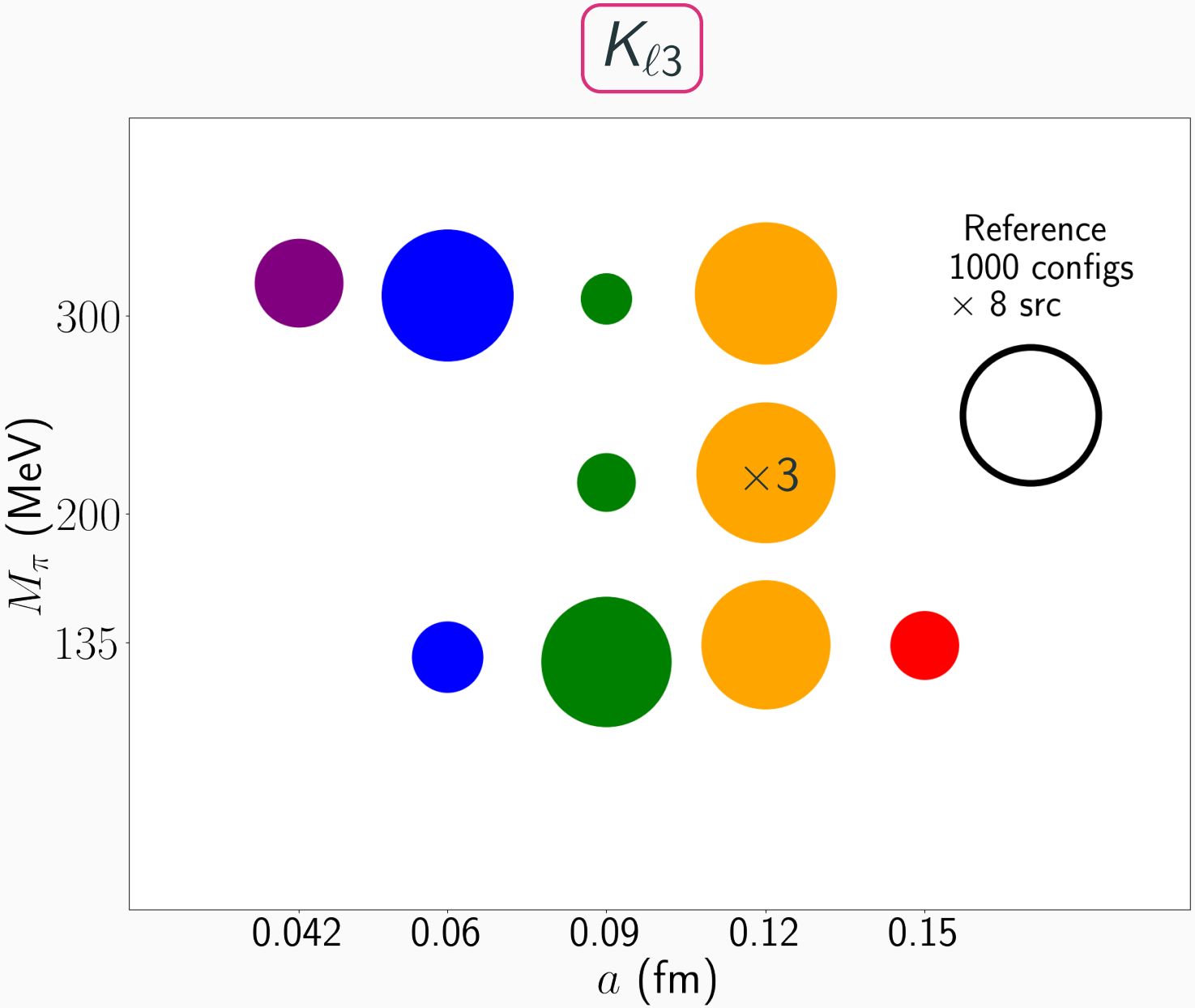
$$f_+^{K\pi}(0) = f_0^{K\pi}(0) = \frac{m_s - m_\ell}{m_K^2 - m_\pi^2} \langle \pi | S | K \rangle_{q^2=0}$$



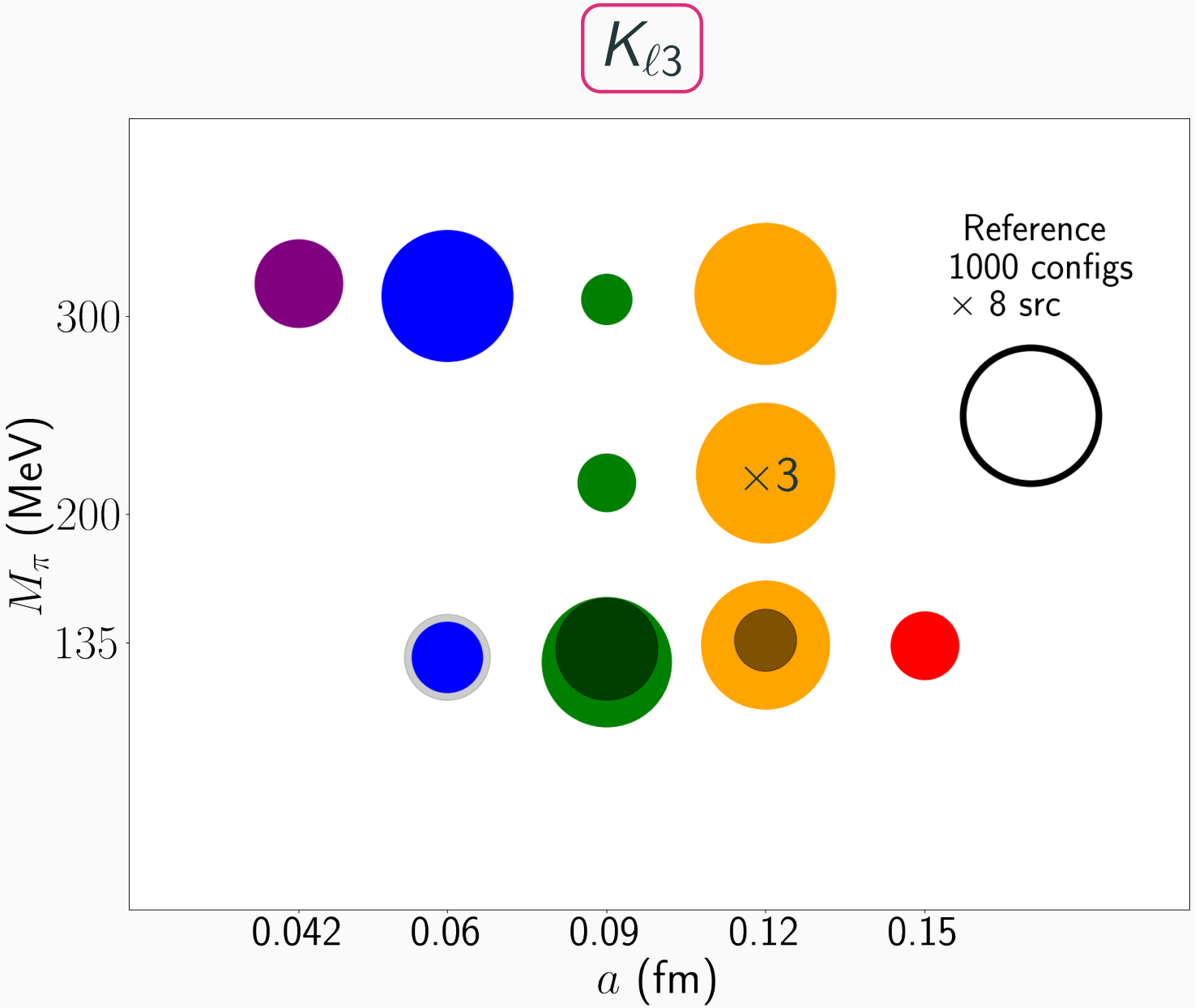
Twisted Boundary Conditions:

tune θ to get $q^2 = 0$

FNAL/MILC 2018: Ensembles HISQ 2+1+1

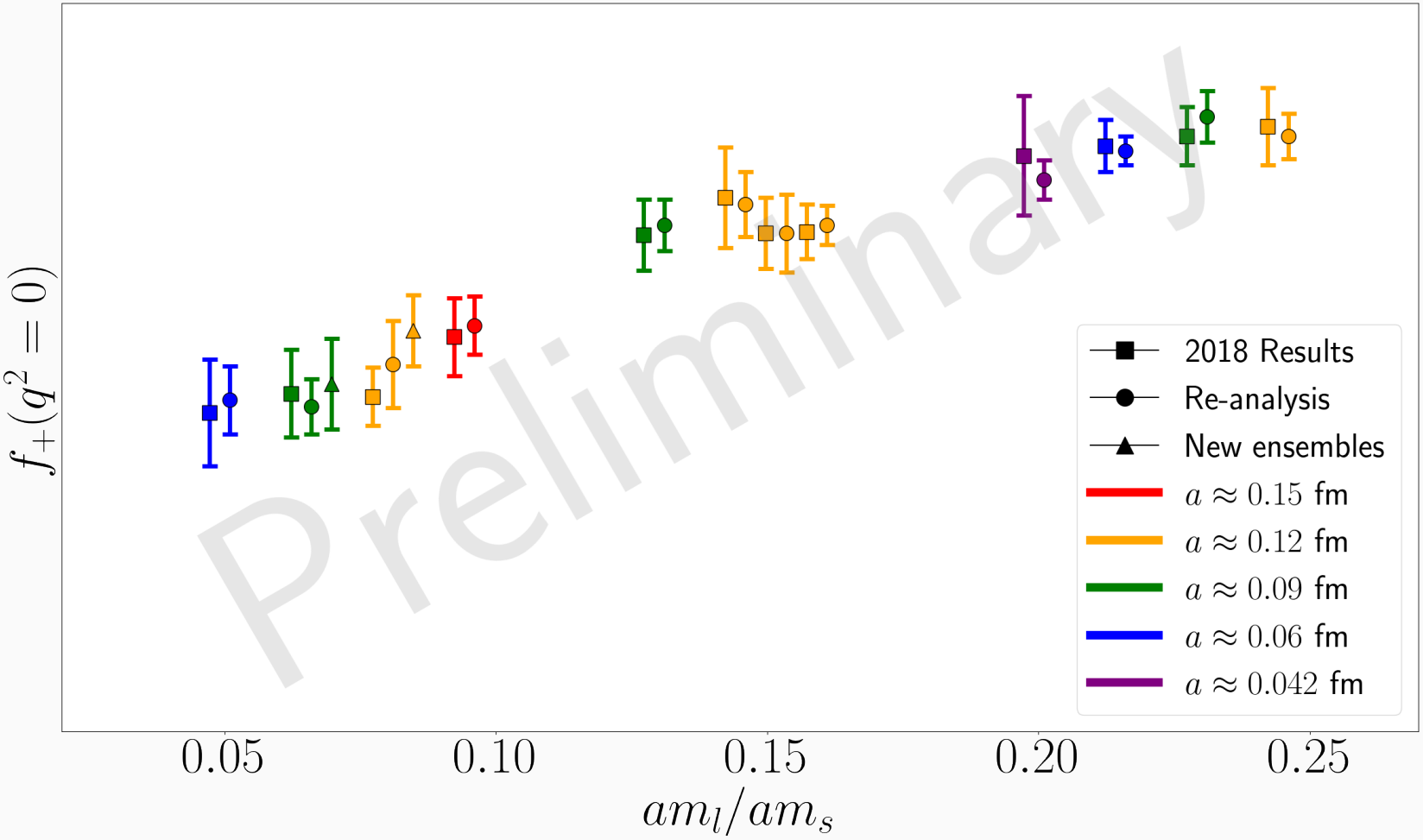


FNAL/MILC 2025: Ensembles HISQ 2+1+1



Semileptonic form factor: Preliminary results

More statistics + improvements in the data analysis



Staggered Chiral Perturbation Theory (SChPT)

Chiral Perturbation Theory + Lattice discretization effects

$$f_+^{K\pi}(0) = 1 + f_2^{\mathrm{SChPT}} + f_4^{\mathrm{cont}} + g_{1,a} + (m_\pi^2 - m_K^2)^2 \left[C_4 + g_{2,a} + h_{m_\pi} \right]$$

Staggered Chiral Perturbation Theory (SChPT)

Chiral Perturbation Theory + Lattice discretization effects

$(\mu,\ \delta_V,\ \delta_A)$

NLO

$$f_+^{K\pi}(0) = 1 + f_2^{\mathrm{SChPT}} + f_4^{\mathrm{cont}} + g_{1,a} + (m_\pi^2 - m_K^2)^2\,[C_4 + g_{2,a} + h_{m_\pi}]$$

$\delta_{A,V}$: hairpin parameters (Lattice)

$$M_{xy} = \mu(m_x + m_y)$$

$\mathcal{O}(p^2)$ LEC

Staggered Chiral Perturbation Theory (SChPT)

Chiral Perturbation Theory + Lattice discretization effects

$(\mu,\ \delta_V,\ \delta_A)$

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NNLO

(L_{1-8})

L_{1-8}: $\mathcal{O}(p^4)$ LECs

Staggered Chiral Perturbation Theory (SChPT)

Chiral Perturbation Theory + Lattice discretization effects

$(\mu,\ \delta_V,\ \delta_A)$

NLO

Higher order discretization effects

$$f_+^{K\pi}(0) = 1 + f_2^{\mathrm{SChPT}} + f_4^{\mathrm{cont}} + g_{1,a} + (m_\pi^2 - m_K^2)^2\,[C_4 + g_{2,a} + h_{m_\pi}]$$

NNLO

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Staggered Chiral Perturbation Theory (SChPT)

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NNLO

(L_{1-8})

Higher order chiral effects

Staggered Chiral Perturbation Theory (SChPT)

Chiral Perturbation Theory + Lattice discretization effects

$(\mu,\ \delta_V,\ \delta_A)$

NLO

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$$f_+^{K\pi}(0) = 1 + f_2^{\mathrm{SChPT}} + f_4^{\mathrm{cont}} + g_{1,a} + (m_\pi^2 - m_K^2)^2\,[C_4 + g_{2,a} + h_{m_\pi}]$$

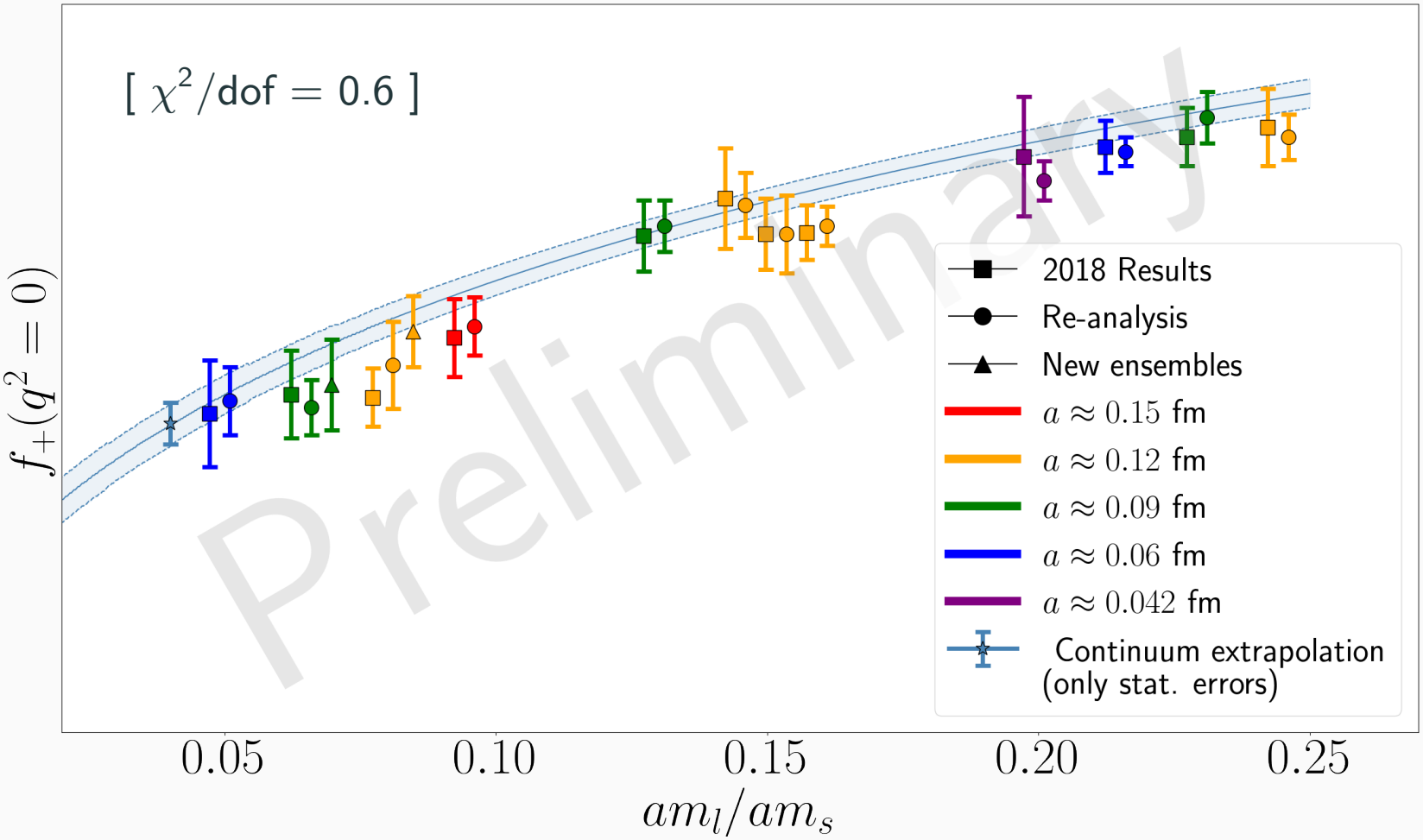
NNLO

(L_{1-8})

Higher order chiral effects

Some Low Energy Constants (LECs) are also present in the decay constants analysis

Semileptonic form factor: Central fit



Includes N3LO, N4LO chiral terms $+\mathcal{O}(\alpha_s a^2, \alpha_s^2 a^2, a^4)$

Decay constants: 2017 analysis

Data

- Partially quenched pseudoscalar meson masses and decay constants
- $>$ 20 ensembles with 6 lattice spacings $a \in [0.15, 0.03]$ fm

Continuum fit for f_K/f_π

- Continuum extrapolation of only physical mass ensembles
- Corrections due to quark mass mistunings

Scale-setting

- p4s method and f_π

Decay constants:

Data

New Analysis

- New light-m_s ensembles at $a \simeq 0.12$, 0.09 and 0.06 fm
- New physical mass ensembles at $a \simeq 0.12$ and 0.09 fm

Continuum fit for f_K/f_π

- Continuum extrapolation of only physical mass ensembles
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Decay constants:

Data

New Analysis

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- New physical mass ensembles at $a \simeq 0.12$ and 0.09 fm

Continuum fit for f_K/f_π

- NLO SU(3) SChPT + NNLO ChPT + higher order chiral and disc.
- We can use the whole set of ensembles

Scale-setting

- p4s method and f_π

Decay constants:

Data

New Analysis

- New light-m_s ensembles at $a \simeq 0.12$, 0.09 and 0.06 fm
- New physical mass ensembles at $a \simeq 0.12$ and 0.09 fm

Continuum fit for f_K/f_π

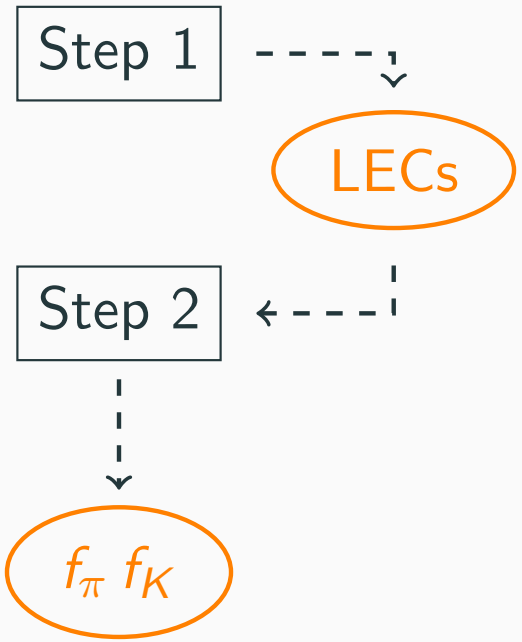
- NLO SU(3) SChPT + NNLO ChPT + higher order chiral and disc.
- We can use the whole set of ensembles

Scale-setting

- We plan on updating to w_0 and M_Ω (2509.14367 Bazavov *et al.*)

What's the plan?

Leptonic analysis



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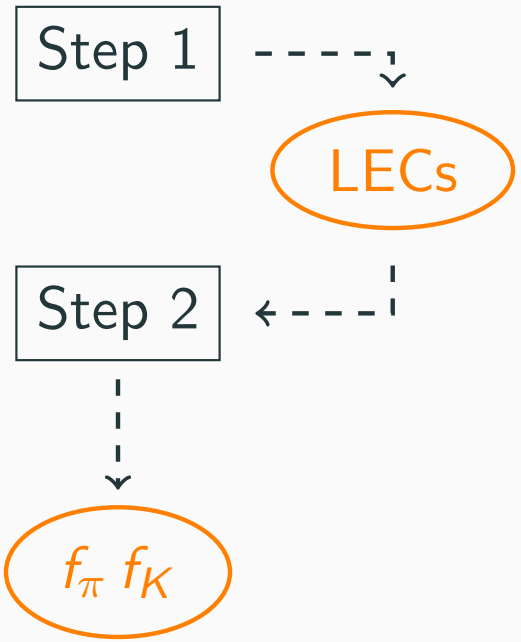
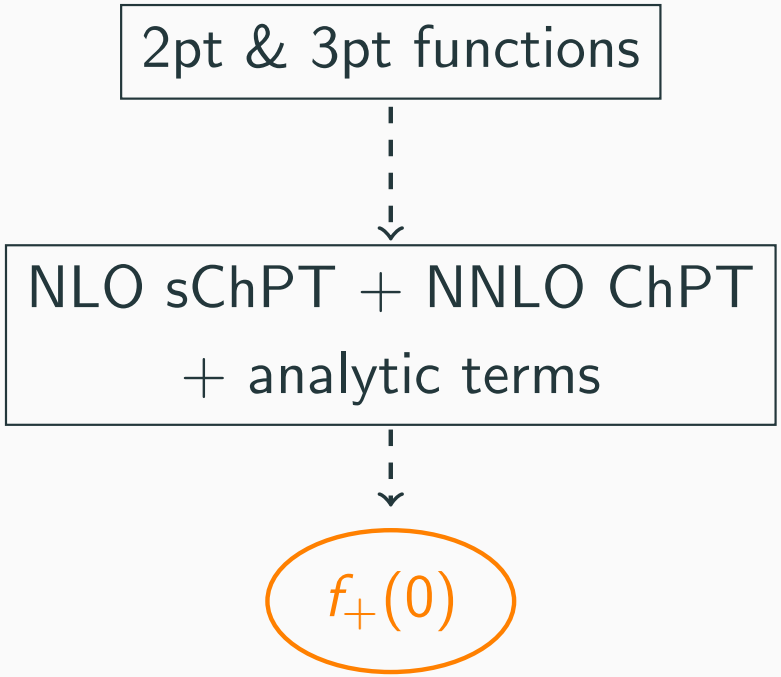
Leptonic analysis



Semileptonic analysis



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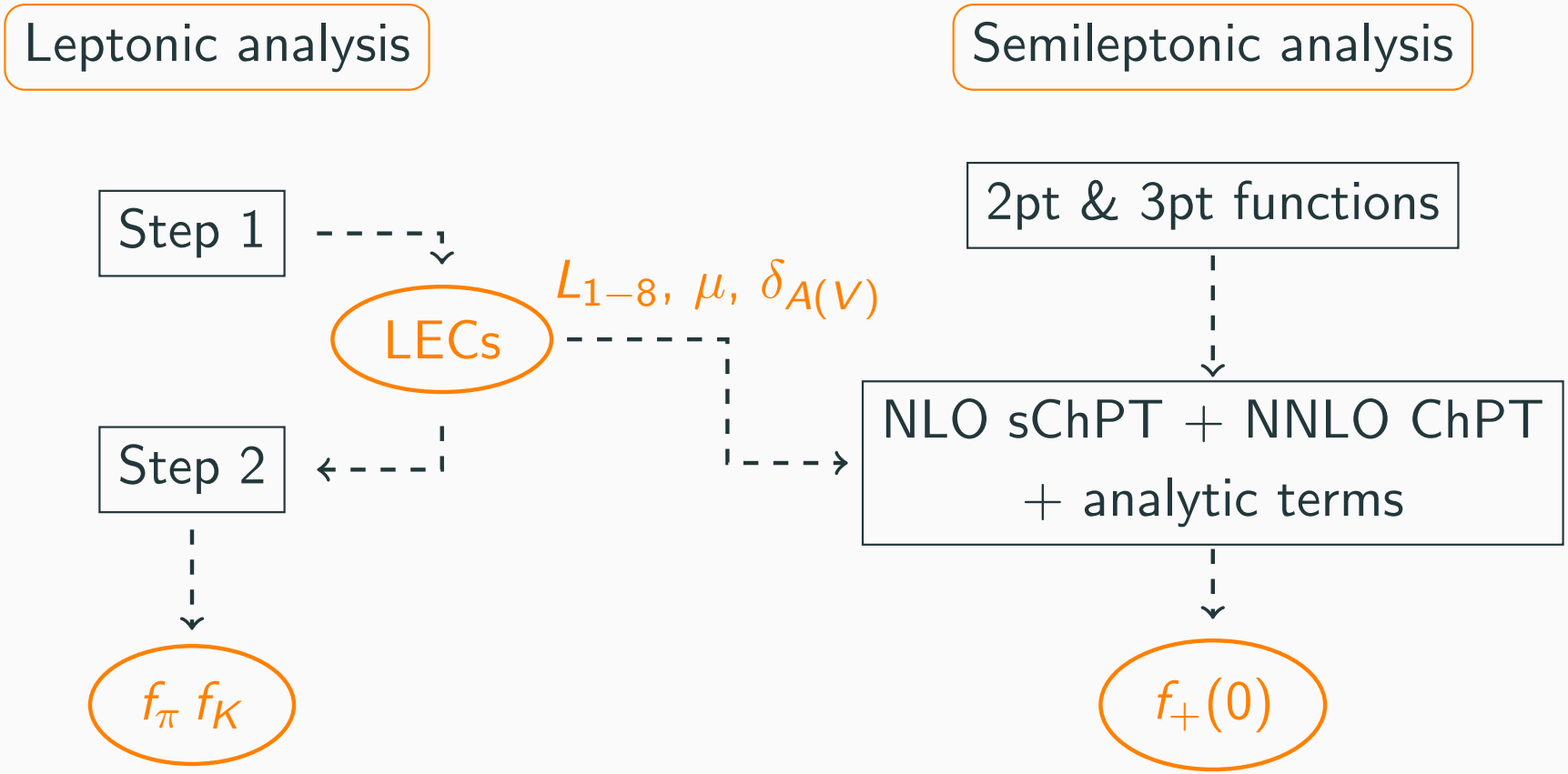

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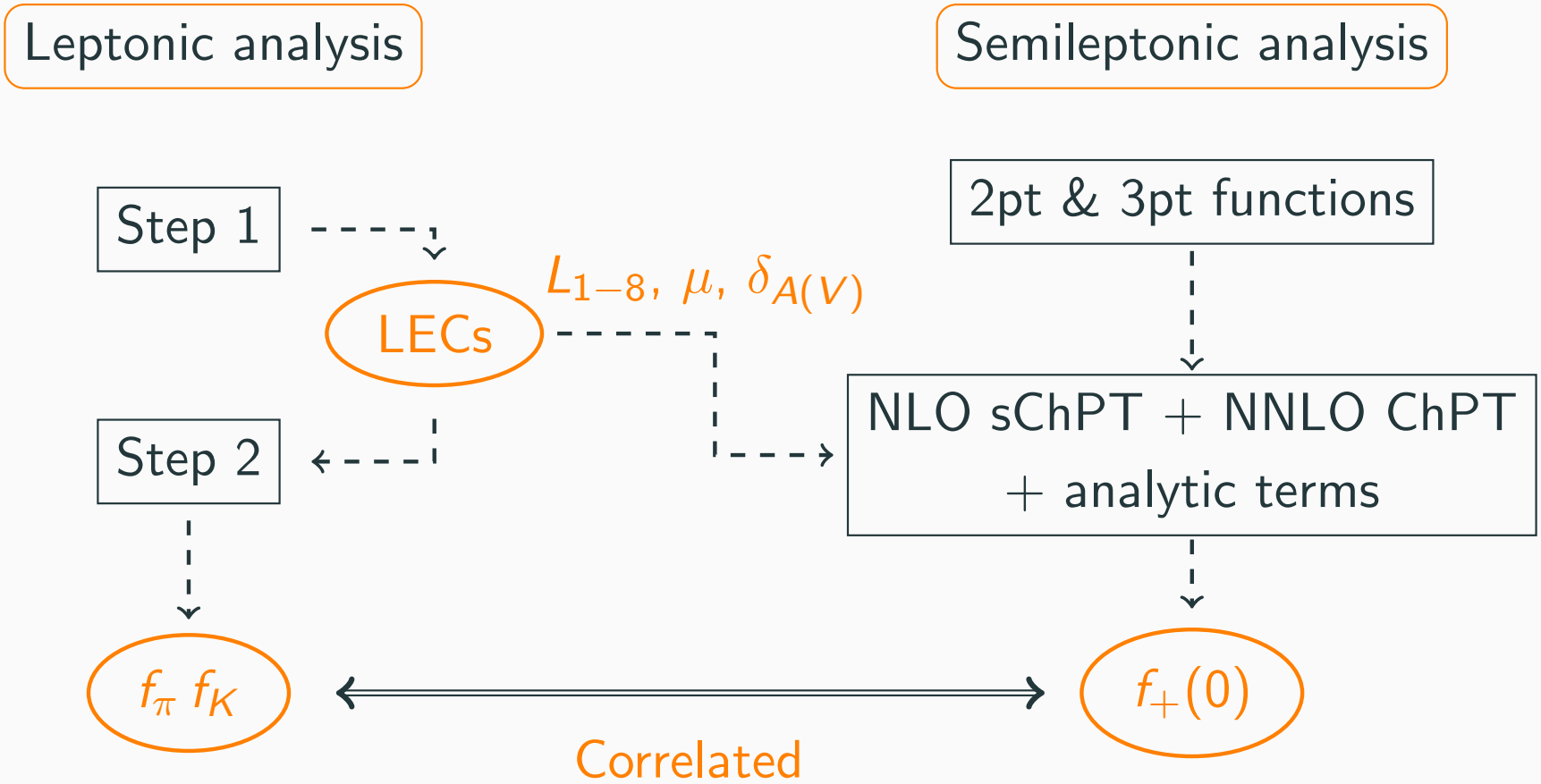
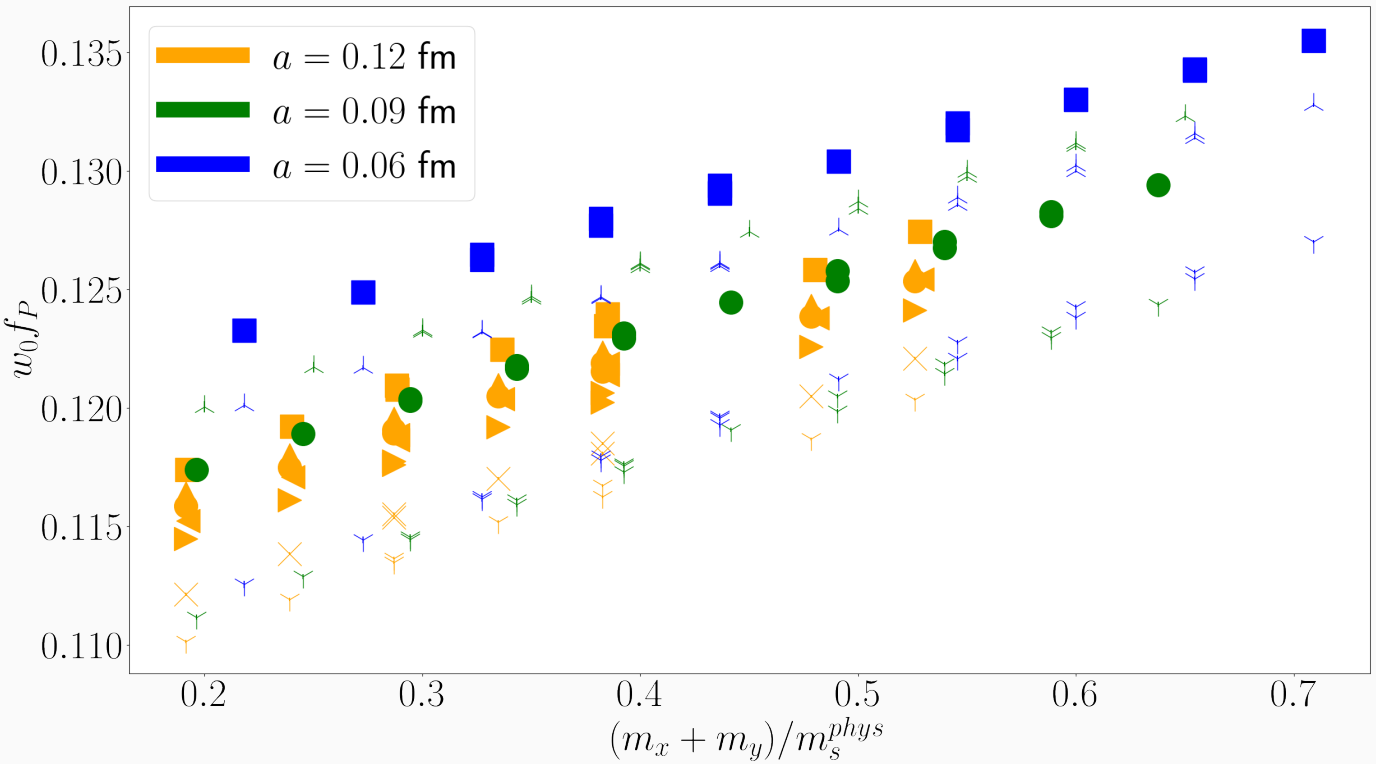


Decay constants: Step 1



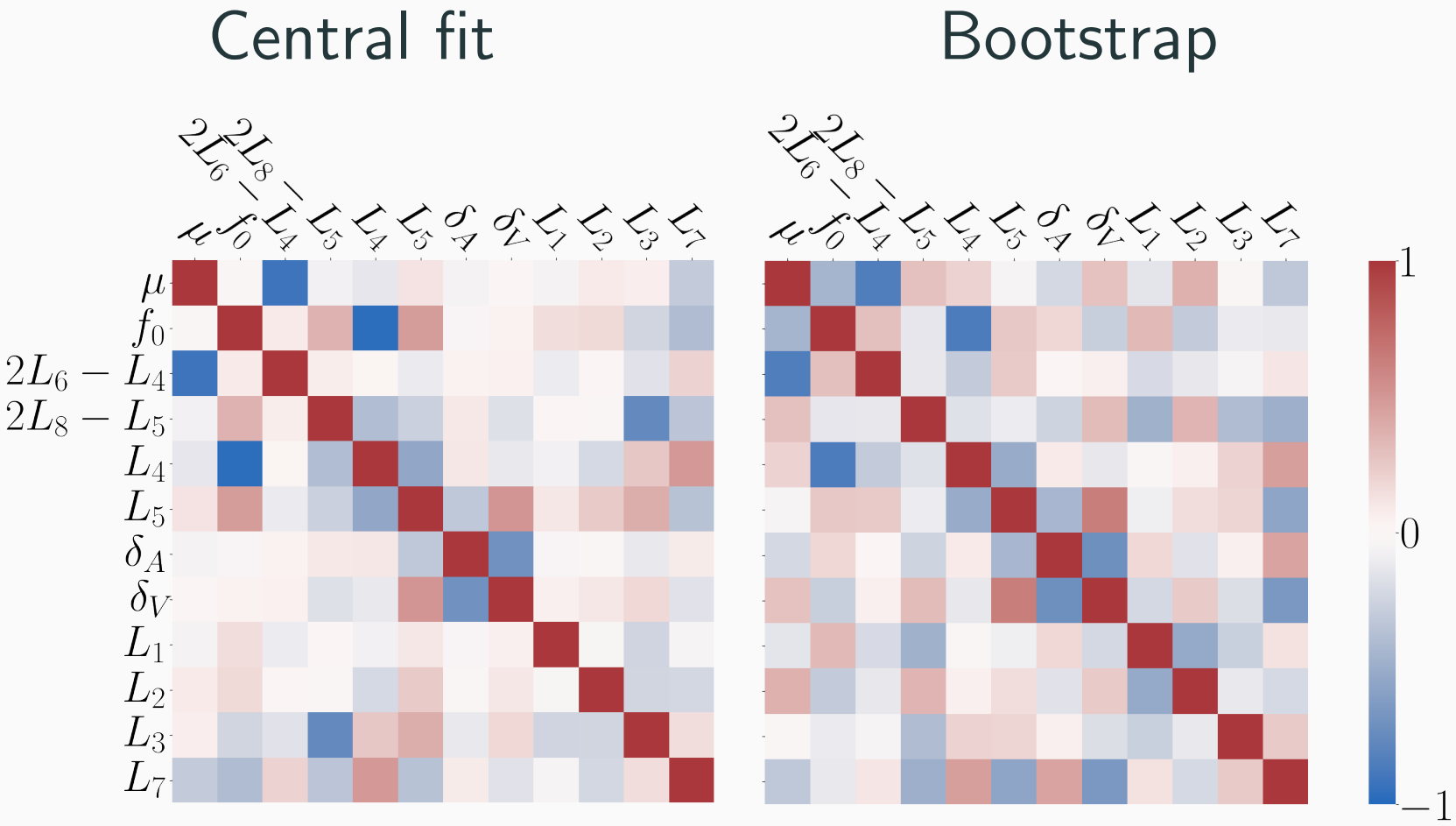
$(m_l/m_s^{phys}, m_s/m_s^{phys})$

- $(0.1, 0.6)$
- $(0.2, 0.6)$
- $(0.25, 0.25)$
- $(0.1, 0.25)$
- $(0.1, 0.45)$
- $(0.175, 0.45)$
- $(0.1, 0.1)$

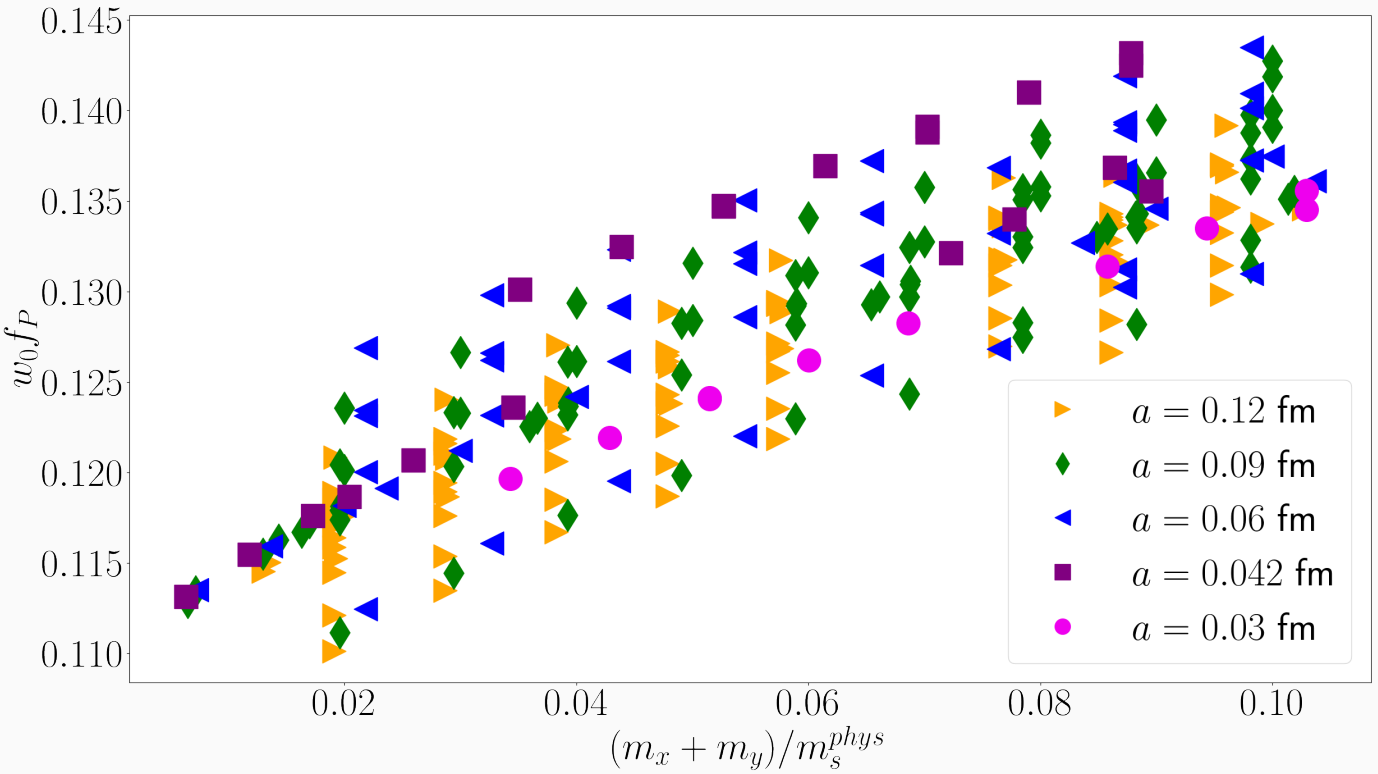
Step 1: 13 ensembles

- 123 masses + 123 decay constants
- Only data with $m_s < m_s^{\text{phys}}$
- NLO SU(3) SChPT + NNLO ChPT + $\mathcal{O}(\alpha_s a^2)$
- 47 parameters

Decay constants: Step 1 LECs correlation matrix



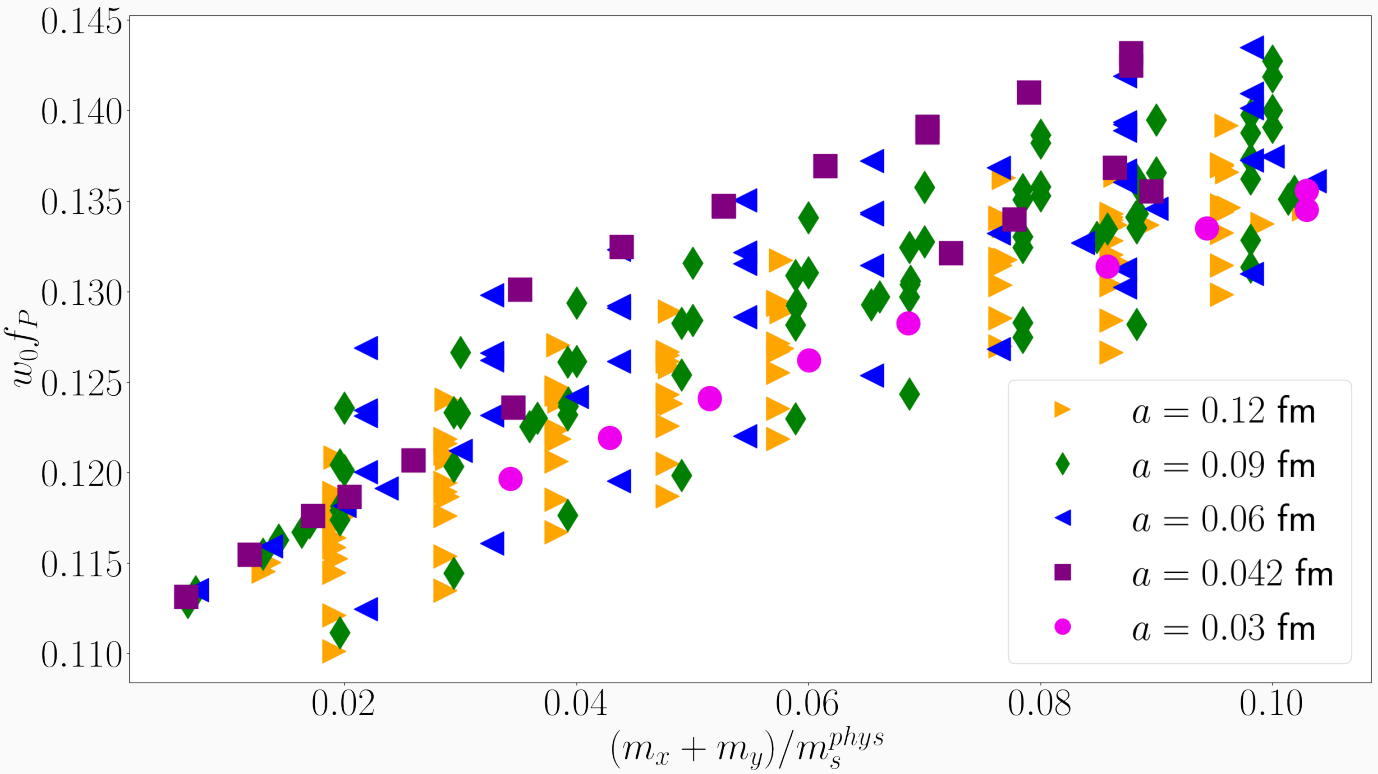
Decay constants: Step 2



Step 2: 29 ensembles

- 258 masses + 258 decay constants
- Physical and unphysical data
- Add N^{3}LO + N^{4}LO + $\mathcal{O}(a^4)$ + $\mathcal{O}(\alpha_s a^2)$
- 131 parameters

Decay constants: Step 2

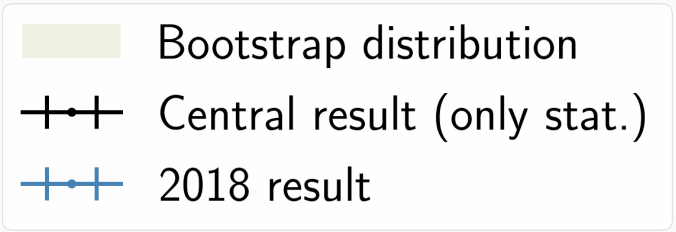


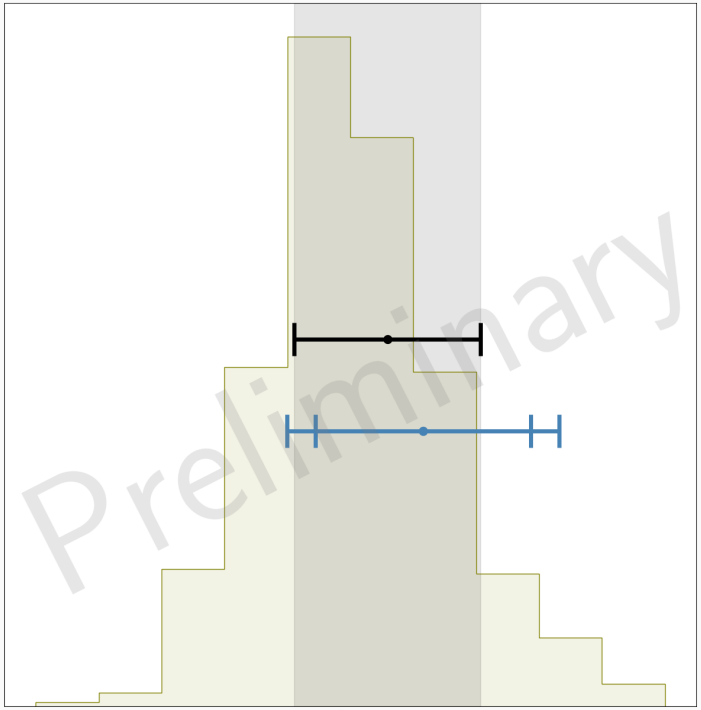
Working on best strategy to perform bootstrap

Step 2: 29 ensembles

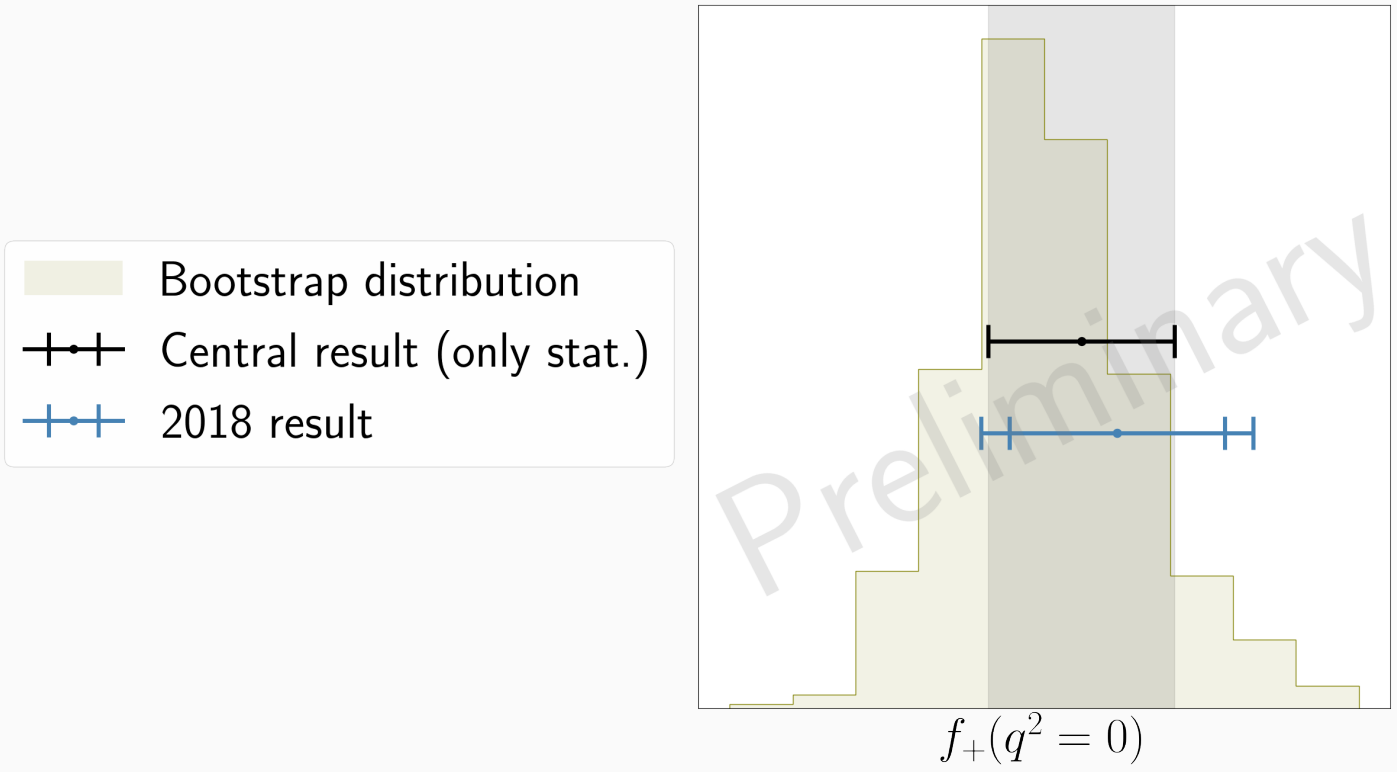
- 258 masses + 258 decay constants
- Physical and unphysical data
- Add $\mathrm{N^3LO} + \mathrm{N^4LO} + \mathcal{O}(a^4) + \mathcal{O}(\alpha_s a^2)$
- 131 parameters

Semileptonic bootstrap with leptonic inputs

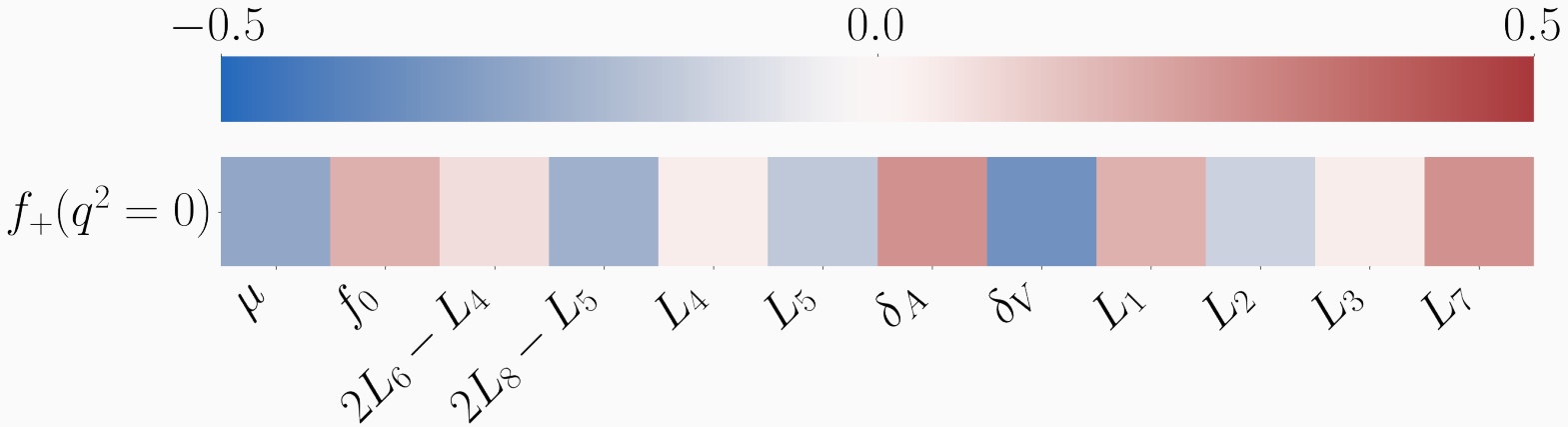




Semileptonic bootstrap with leptonic inputs



Correlation between $f_+(q^2=0)$ and SChPT parameters



Applications

First row unitarity and LECs

- Slightly improved determination of $f_+(0)$
- Improved and more robust determination of f_K/f_π
- Goal: estimate correlation between both quantities
- This will improve the precision in the first row unitarity relation

- Bonus: obtain LECs values and correlations

Beyond the Standard Model: SMEFT

A more precise determination of $|V_{us}|$ and $|V_{ud}|$ would impact BSM searches

$$\mathcal{L}_{\mathrm{BSM}} = \mathcal{L}_{\mathrm{SM}} + \sum_i C_i \mathcal{O}_i$$

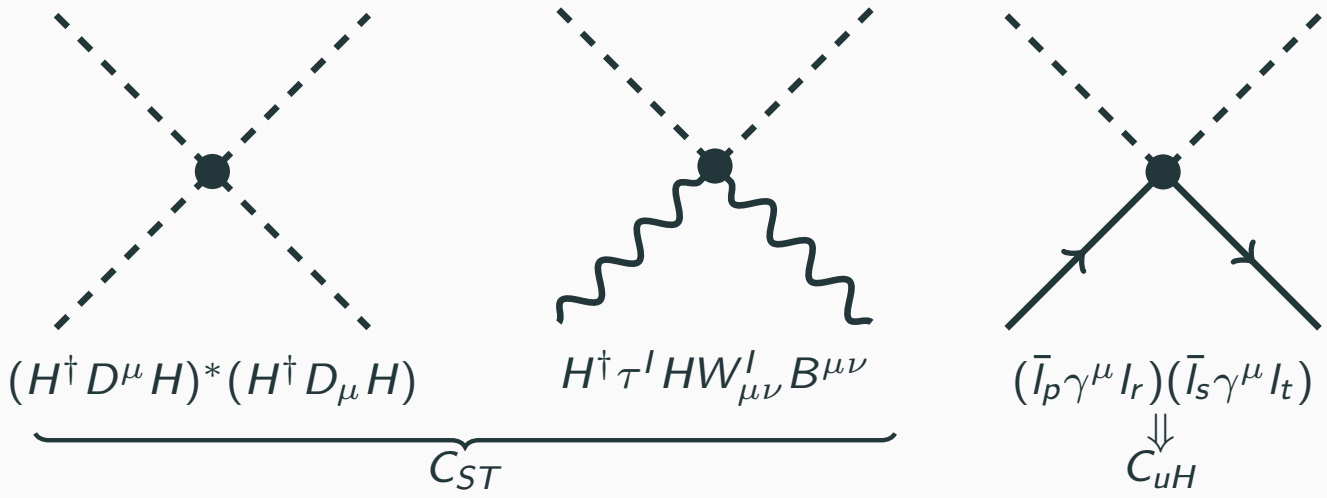
Different O_i would contribute differently to different processes.

Non-zero value of C_i associated to right handed charged currents could explain tension

Beyond the Standard Model: SMEFT fits (2311.00021)

Which combination of dim. 6 operators is preferred to explain tensions?

From Cirigliano et al. (2311.00021): a global fit was performed. Best model found was



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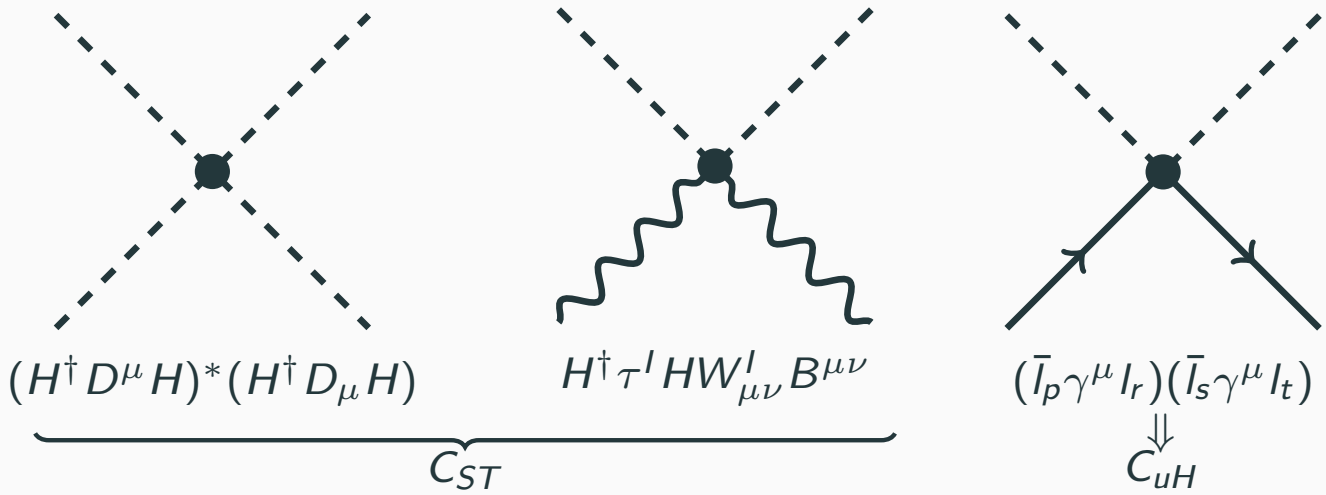

$$C_{ST} = -0.0038(22)\ \text{TeV}^2 \quad C_{uH}^{(1,1)} = -0.030(8)\ \text{TeV}^2 \quad C_{uH}^{(1,2)} = -0.0040(11)\ \text{TeV}^2$$

These operators could (for instance) be generated by vector like quarks

Our determination could help constrain this coefficients and thus BSM models

Thanks for your attention!