KAON-DEUTERON CORRELATION FUNCTION FROM AN EFT APPROACH

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Universitat de Barcelona & Institut de Ciències del Cosmos

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Excited QCD 2026 Workshop

Universidad de Granada.

Carmen de la Victoria.

KAON-DEUTERON CORRELATION FUNCTION FROM AN EFT APPROACH

This talk summarizes our first approach to
3-body hadron dynamics
using chiral effective interactions:

A. Ramos, JMT-R, A. de Fagoaga, E. Cabré
2507.22593 [hep-ph]

Saint Surrounded by Three Pi-Mesons (1956). S. Dalí

Meson-baryon chiral effective theory

$$\mathcal{L}_{\Phi B} = \mathrm{Tr}\left[\bar{B}\left(i\not{D} - M_0\right)B + \frac{D}{2}\left(\bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\right) + \frac{F}{2}\left(\bar{B}\gamma^\mu\gamma_5\left[u_\mu, B\right]\right)\right]$$

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B] \qquad \Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] = \frac{1}{2}\left(u^\dagger\partial_\mu u + u\partial_\mu u^\dagger\right) \qquad u_\mu = \frac{i}{2}\{u^\dagger, \partial_\mu u\} = \frac{i}{2}\left(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right)$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Ecker, *Prog. Part. Nucl. Phys., 35*:1-80 (1995), Bernard, Kaiser, Meißner, *Int. J. Mod. Phys. E, 4*:193-346 (1995), Oset, Ramos, *Nucl.Phys.A 635* (1998) 99-120, Hyodo, Jido, *Prog. Part. Nucl. Phys. 67*(1):55-98 (2012)

Meson-baryon chiral effective theory

$$\mathcal{L}_{\rm WT} = \frac{1}{4f^2}{\rm Tr}\ \left[\bar{B}i\gamma^\mu\left[\Phi\partial_\mu\Phi - (\partial_\mu\Phi)\Phi, B\right]\right]$$

$$V_{ij} = \frac{-C_{ij}}{4f_i f_j}\bar{u}(j)\gamma_\mu u(i)(k_i + k_j)^\mu$$

$Q = -1$ $\quad K^-n, \pi^0\Sigma^-, \pi^-\Sigma^0, \pi^-\Lambda, \eta\Sigma^-, K^0\Xi^-$

$Q = 0$ $\quad K^-p, \bar{K}^0n, \pi^0\Lambda, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^-\Sigma^+, \eta\Lambda, \eta\Sigma^0, K^+\Sigma^-, K^0\Sigma^0$

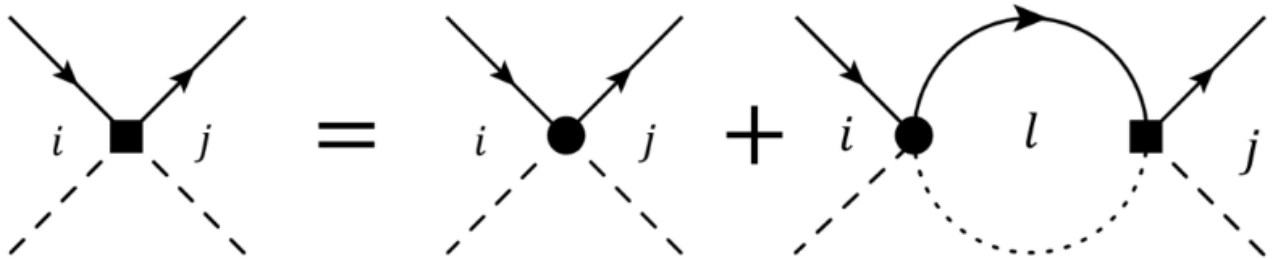
$$V_{ij}^{s-\text{wave, on-shell}} = -\frac{C_{ij}}{4f_i f_j}\sqrt{\frac{E_j + M_j}{2M_j}}\sqrt{\frac{E_i + M_i}{2M_i}}\left(2\sqrt{s} - M_i - M_j\right)$$

Isospin limit: $f_i = f_j = 1.123 f_\pi$

E. Oset, A. Ramos, *Nucl.Phys.A 635* (1998) 99-120, Hyodo, Jido, *Prog. Part. Nucl. Phys. 67*(1):55-98 (2012)

T-matrix equation with on-shell factorization

$$t_{ij} = V_{ij} + \sum_l V_{il} G_l t_{lj}$$



$$G_l(P) = i \int \frac{d^4q}{(2\pi)^4} \frac{2M_l}{(P-q)^2 - M_l^2 + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

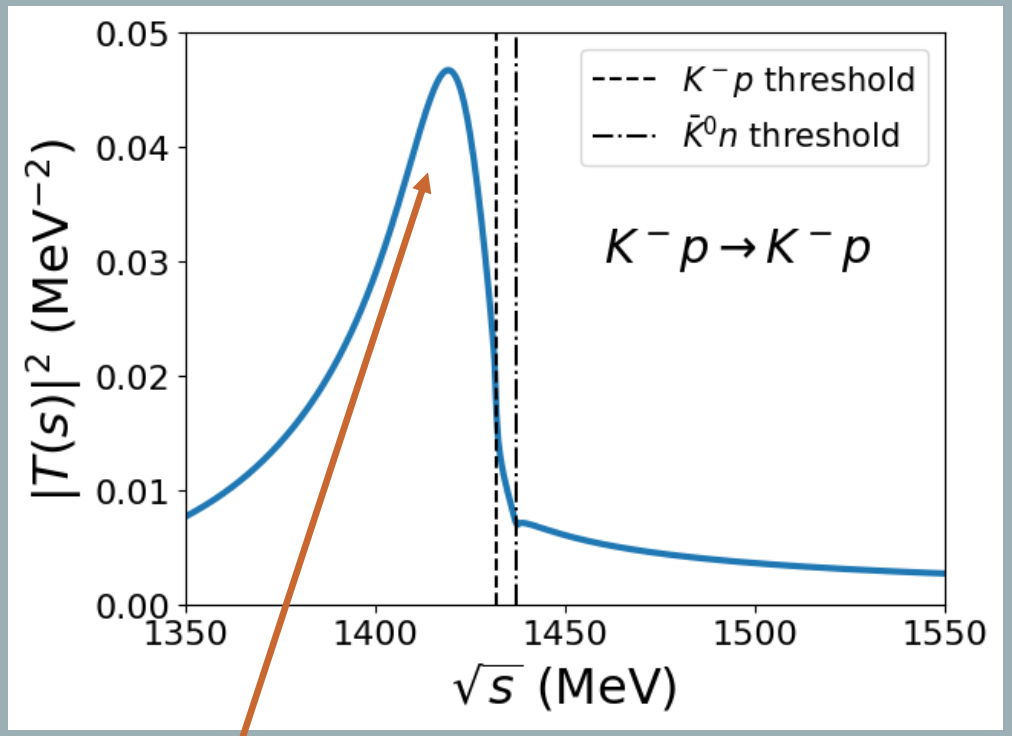
$$\boxed{t = [1 - VG]^{-1} V}$$

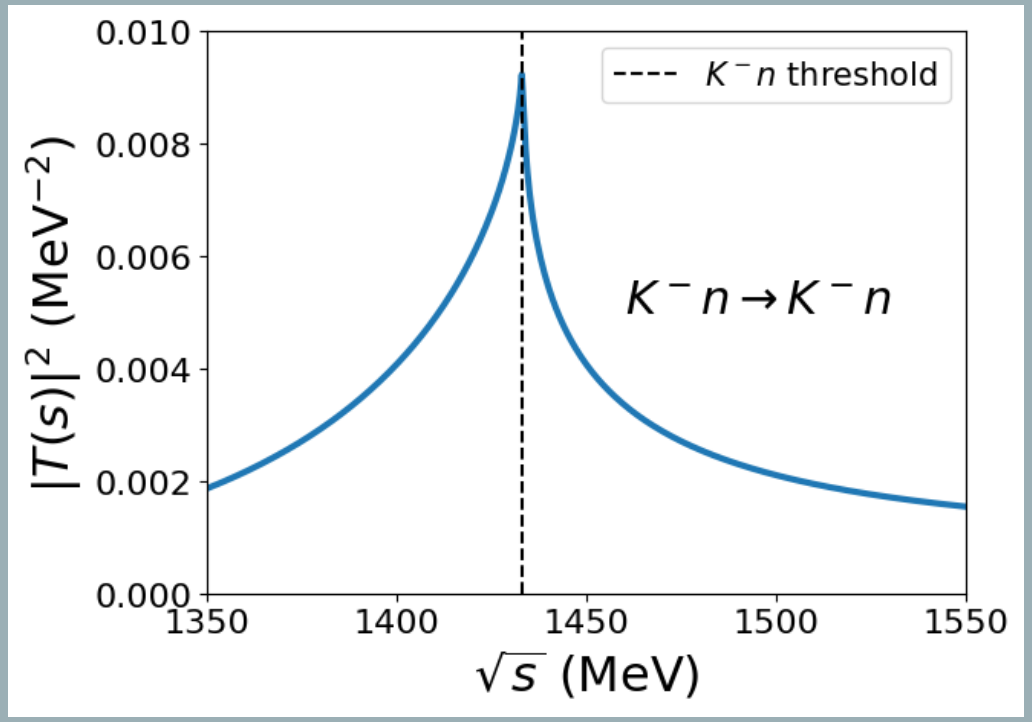
Dimensional regularization for loop functions at μ=630 MeV

E. Oset, A. Ramos, *Nucl.Phys.A 635* (1998) 99-120
J.A. Oller, E. Oset, *Nucl.Phys.A 620* (1997) 438-456, J.A. Oller, U.G. Meißner, *Phys.Lett.B* 500 (2001) 263-272

T matrices for (elastic) K^-p and K^-n

t_p





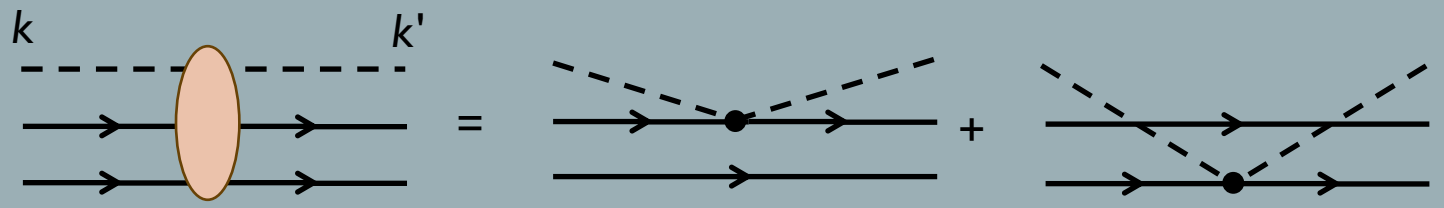
t_n

$\Lambda(1405)$ resonance below K^-p threshold (but above $\pi\Sigma$)

E. Oset, A. Ramos, *Nucl.Phys.A 635* (1998) 99-120; J.A. Oller, U.G. Meißner, *Phys.Lett.B* 500 (2001) 263-272,;
S. Navas et al. (Particle Data Group), *Phys. Rev. D* 110, 030001 (2024)

Kaon-Deuteron interaction

Impulse Approximation

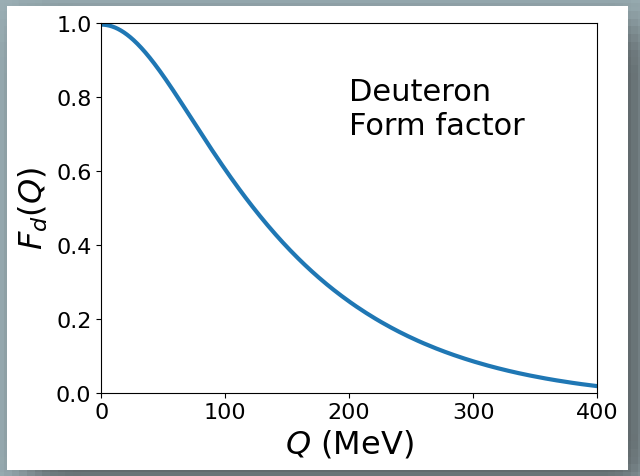


$$T_{Kd}(k',k) = [t_p(k',k) + t_n(k',k)]\, F_d(Q)$$

Chiral Kp interaction

Chiral Kn interaction

$$F_d(Q) = \int e^{-i\mathbf{Q}\cdot\mathbf{r}} \mid \phi_d(\mathbf{r}) \mid^2 d\mathbf{r}$$

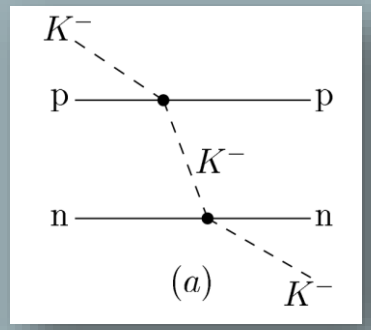


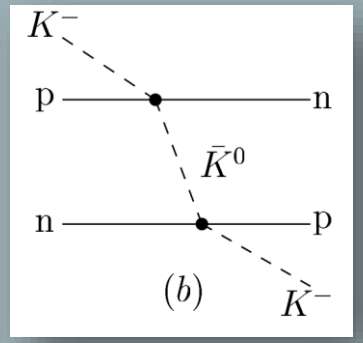
Argonne AV18 wave functions

S.S. Kamalov, E. Oset, A. Ramos, *Nucl.Phys.A 690* (2001) 494-508

Kaon-Deuteron interaction

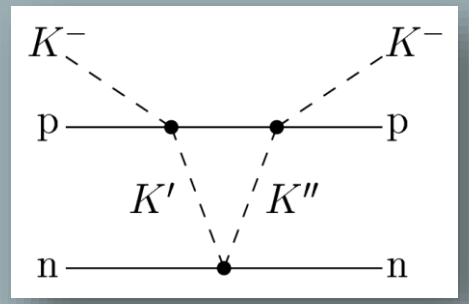
Double scattering approximation

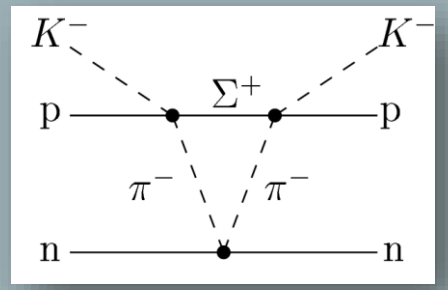




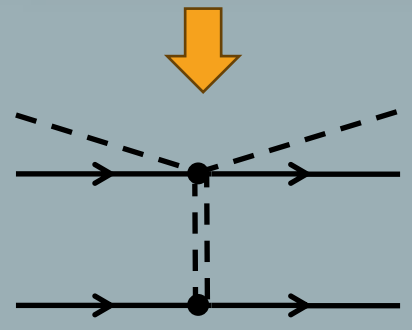
Cancellation of off-shell 2-body T operators with chiral 3-body contact terms

Triple scattering approximation





Only 1% contribution to the K^-d amplitude



Fixed-Center Approximation

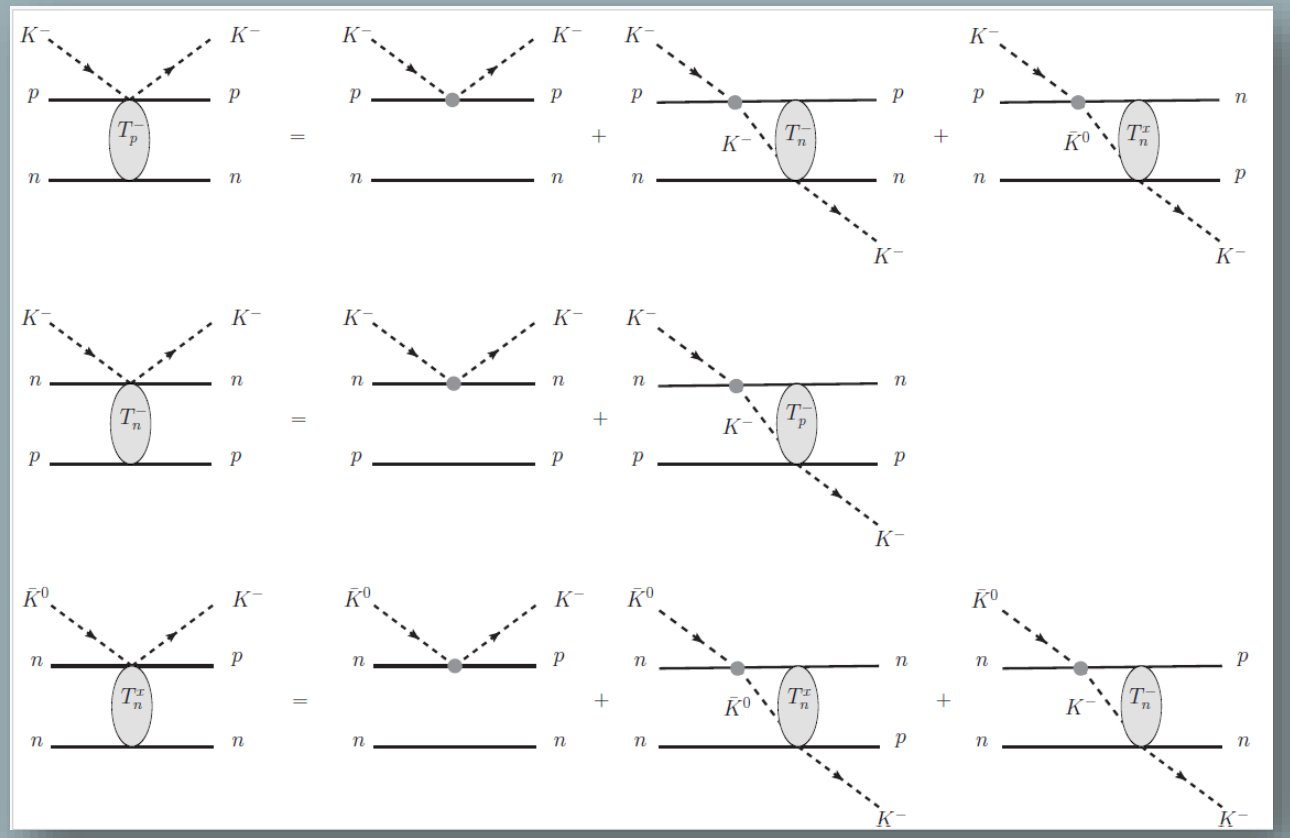
S.S. Kamalov, E. Oset, A. Ramos, *Nucl.Phys.A 690* (2001) 494-508;
E. Oset et al., *Nucl.Phys.A 881* (2012) 127-140;
A. Martinez-Torres et al. *Phys.Rev.D* 78 (2008) 074031; A. Martinez-Torres, Ph.D. thesis, IFIC (2009)

Kaon-Deuteron interaction

Faddeev equations in the
Fixed Center Approximation

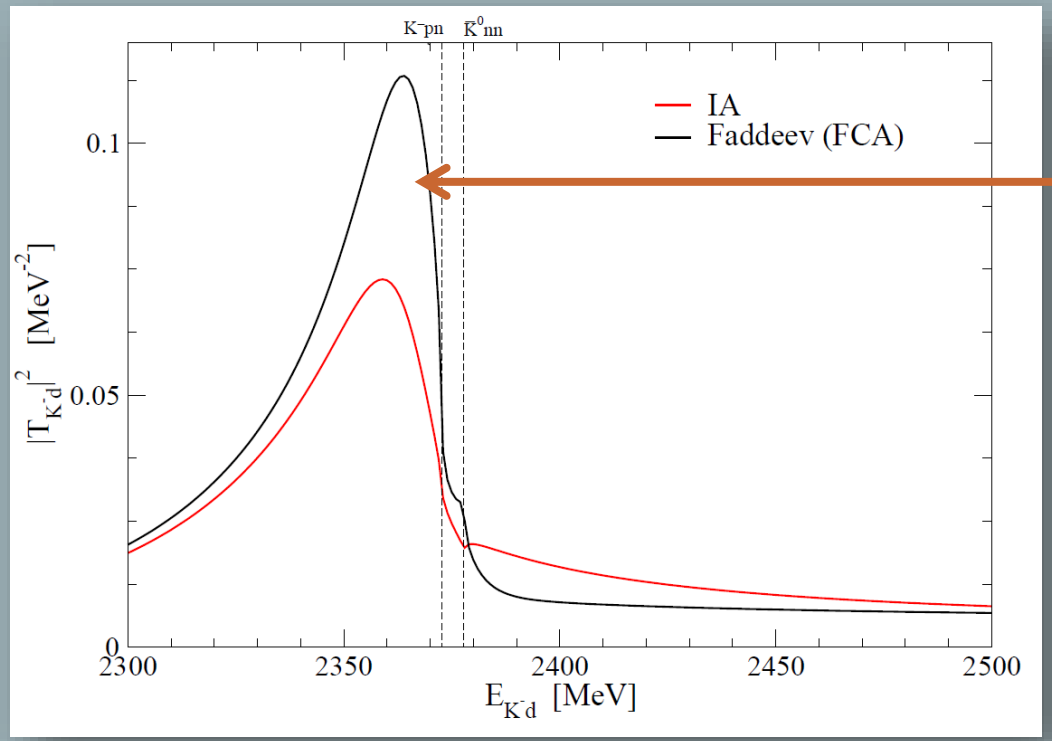
$$T_{Kd} = T_p + T_n$$

$$\begin{aligned} T_p &= t_p + t_p\, G_0\, T_n + t_p^x\, G_0\, T_n^x\,, \\ T_n &= t_n + t_n\, G_0\, T_p\,, \\ T_n^x &= t_n^x + t_n^0\, G_0\, T_n^x + t_n^x\, G_0\, T_n\,, \end{aligned}$$



L.D. Faddeev, *Sov. Phys. JETP* 12 (1961) 1014, *Zh. Eksp. Teor. Fiz.* 39 (1960) 1459.
S.S. Kamalov, E. Oset, A. Ramos, *Nucl.Phys.A 690* (2001) 494-508
E. Oset et al., *Nucl.Phys.A 881* (2012) 127-140

Kaon-Deuteron interaction



T-matrix of K^- d scattering

3-body bound state originated from the Λ (1405)

S.S. Kamalov, E. Oset, A. Ramos, *Nucl.Phys.A 690* (2001) 494
E. Oset et al., *Nucl.Phys.A 881* (2012) 127
A. Martinez-Torres et al. *Phys.Rev.D* 78 (2008) 074031

$$G_0(q^0) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{F_d(q)}{(q^0)^2 - \boldsymbol{q}^2 - m_K^2 + i\eta}$$

$$T^{\mathrm{FCA}}_{K^-d} = \frac{T^{\mathrm{IA}}_{K^-d} + \left(2t_{K^-p\to K^-p}t_{K^-n\to K^-n} - \tilde{t}^2_{K^-p\to \bar{K}^0 n}\right) G_0 - 2\tilde{t}^2_{K^-p\to \bar{K}^0 n} t_{K^-n\to K^-n} G_0^2}{1 - t_{K^-p\to K^-p}t_{K^-n\to K^-n}G_0^2 + \tilde{t}^2_{K^-p\to \bar{K}^0 n} t_{K^-n\to K^-n} G_0^3}$$

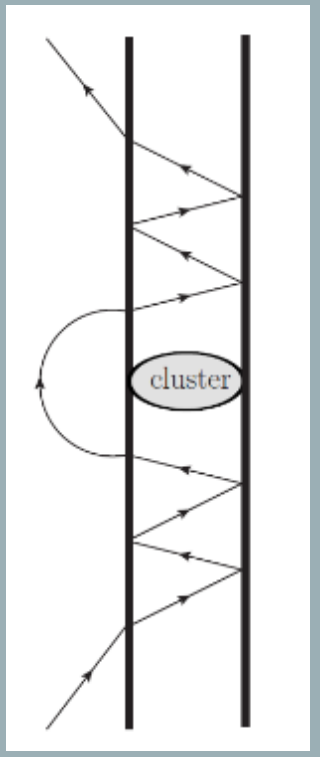
Kaon-Deuteron interaction

There are unitarity concerns in FCA raised by

- P. Encarnación, A. Feijoo, and E. Oset, *Phys. Rev. D* 111, 114023 (2025),
- N. Ikeno and E. Oset, *Phys. Rev D 111* (2025) 114023

in the particle-cluster scattering. It was solved in

- B. Agatão, P. Brandão, A. Martínez Torres, K. P. Khemchandani, L. M. Abreu, and E. Oset, *Eur. Phys. J. C* 85, 1136 (2025)



Unitarity condition not fully realized at the particle-cluster threshold, but only at the 3-particle threshold.

In the deuteron case this effect is small (binding energy only of 2.2 MeV), but fixing this would fulfill exact unitarity, and (perhaps) improve the real part of amplitudes

Deuterons in RHICs

- Deuteron production in HICs still under debate (coalescence versus thermal models)

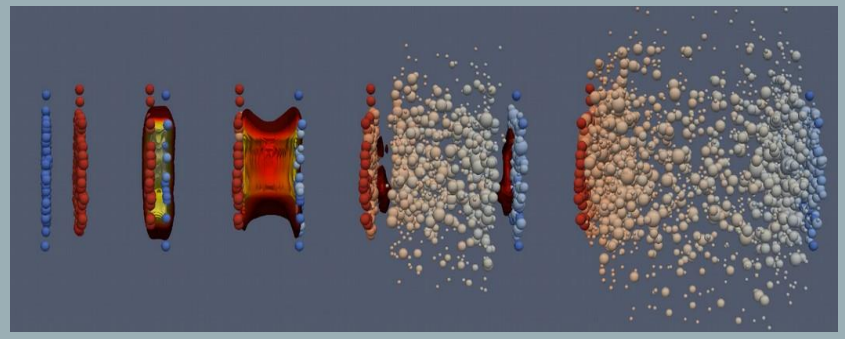
L. P. Csernai and J. I. Kapusta, *Phys. Rept.* 131, 223 (1986); P. Danielewicz and G. F. Bertsch, *Nucl. Phys. A* 533, 712 (1991). , R. Scheibl and U. W. Heinz, *Phys. Rev. C* 59, 1585 (1999), A. Andronic, P. Braun-Munzinger, and J. Stachel, *Nucl. Phys. A* 772, 167 (2006), A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel, *Nature* 561, 321 (2018), V. Vovchenko et al. Phys. Lett. B 800, 135131 (2020); T. Neidig, et al. *Phys. Lett. B* 827, 136891 (2022); J. Rais, H. van Hees and C. Greiner, Phys. Rev.C 106 (2022), 064004…

- Deuteron **femtoscopy** might help to distinguish between them

S. Mrówczýnski and P. Słoń, *Acta Phys. Polon. B* 51, 1739 (2020)

- Hints of deuteron early formation via deuteron-pion femtoscopy

O. Vázquez Doce, D. Mihaylov, L. Fabbietti, *Eur. Phys. J.A* 61 (2025) 3, 53



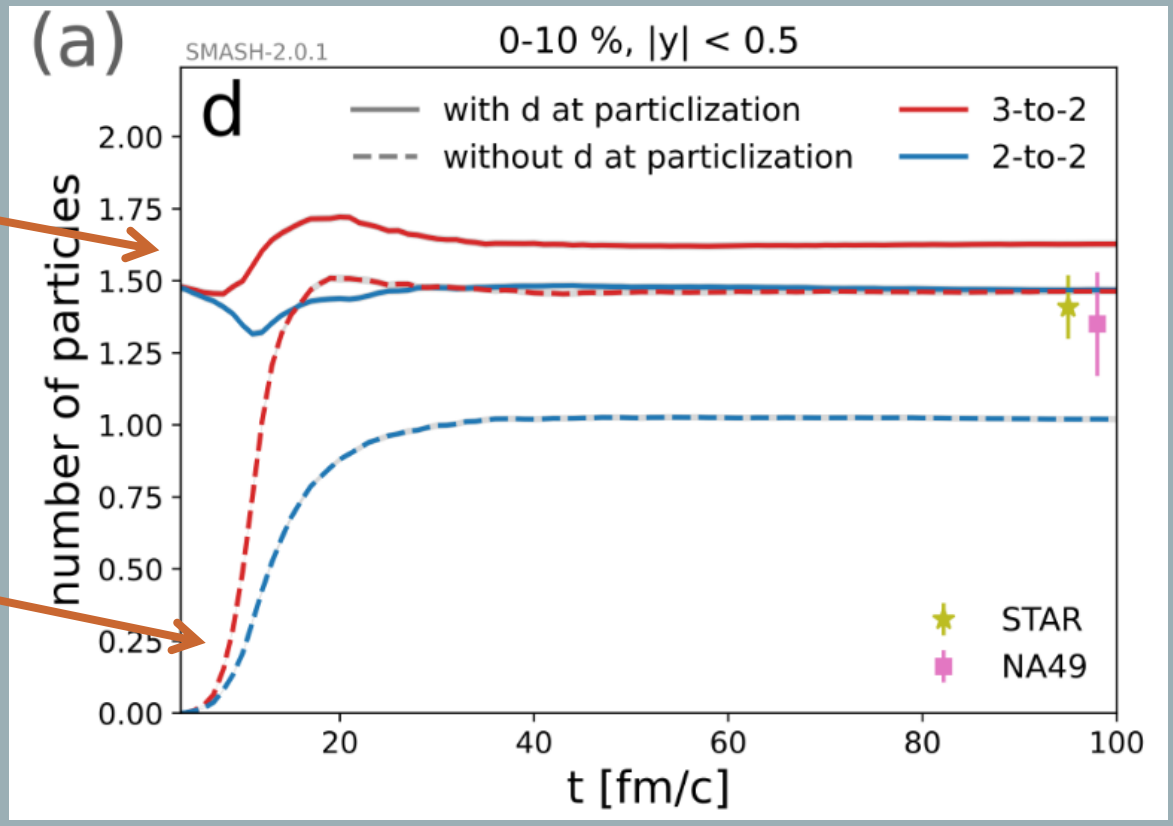
MADAI collaboration,
Hannah Petersen and Jonah Bernhard

Deuterons in RHICs

Time evolution of deuteron yield in Au Au collisions at $s^{1/2}_{NN}$ =7.7 GeV

Statistical Thermal Model:
Production at freeze out

Coalescence Model:
Dynamical production of composed hadrons



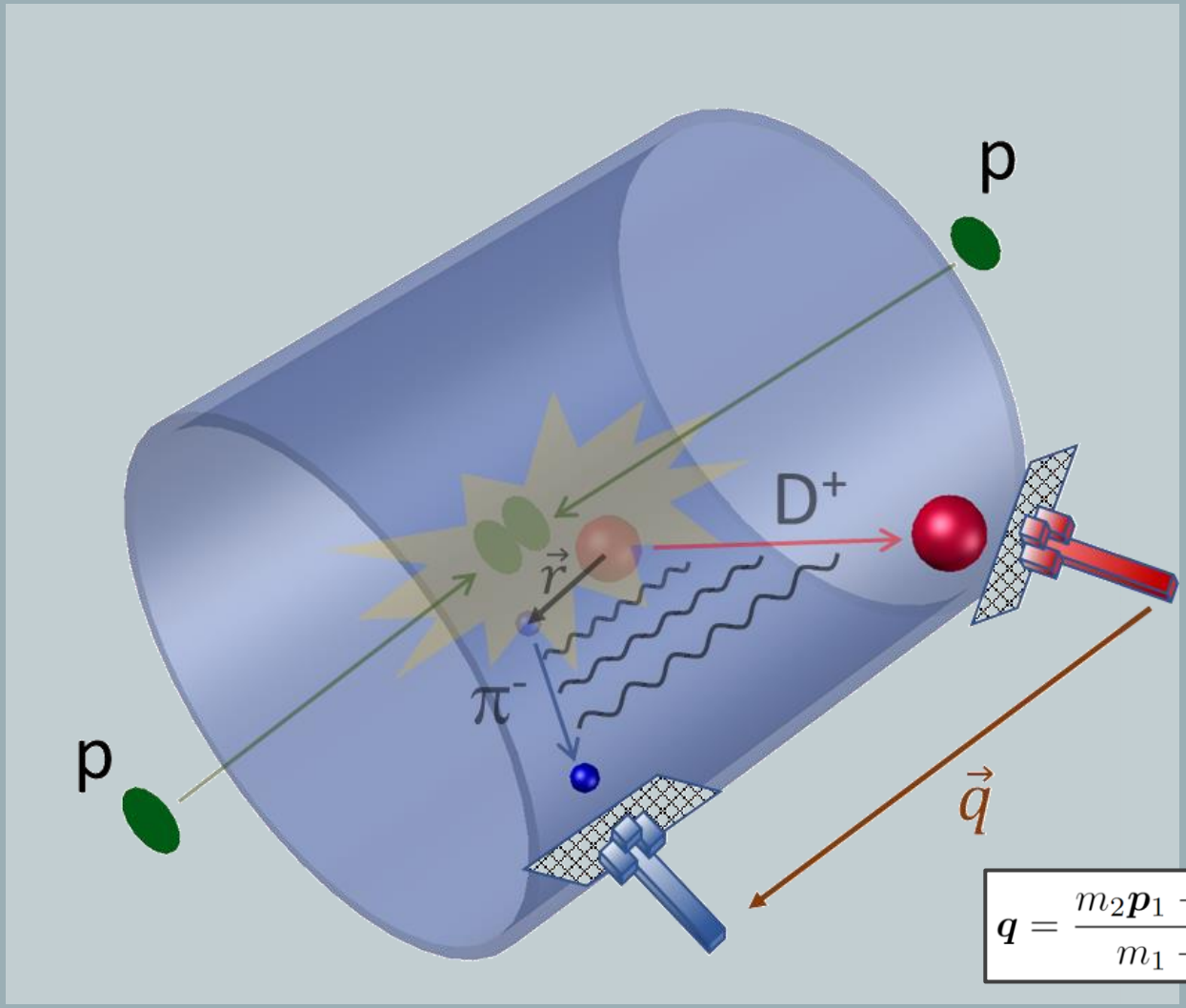
"Stochastic rates" to account for boost-invariant, 3-to-2 reactions:

$N\ p\ n \leftrightarrow N\ d$

$\pi\ p\ n \leftrightarrow \pi\ d$

J. Staudenmaier, D. Oliinychenko, JMT-R and H. Elfner
Phys.Rev.C 104 (2021) 3, 034908

Femtoscopy in RHICs



$$\boldsymbol{q} = \frac{m_2\boldsymbol{p}_1 - m_1\boldsymbol{p}_2}{m_1 + m_2}$$

U. Heinz, B. Jacak, *Ann.Rev.Nucl.Part.Sci.* 49 (1999) 529
M. Lisa, S. Pratt, U. Wiedemann,
Ann.Rev.Nucl.Part.Sci. 55 (2005) 357

Pair Correlation Function

$$C(\boldsymbol{q}) = \mathcal{N}\,\frac{N_{\text{same}}(\boldsymbol{q})}{N_{\text{mixed}}(\boldsymbol{q})}$$

$C(\boldsymbol{q}) > 1$: attraction
$C(\boldsymbol{q}) < 1$: repulsion

L. Fabbietti, V. Mantovani Sarti, O. Vazquez Doce, *Ann. Rev. Nucl. Part. Sci,* 71, 377 (2021)

Koonin-Pratt formula

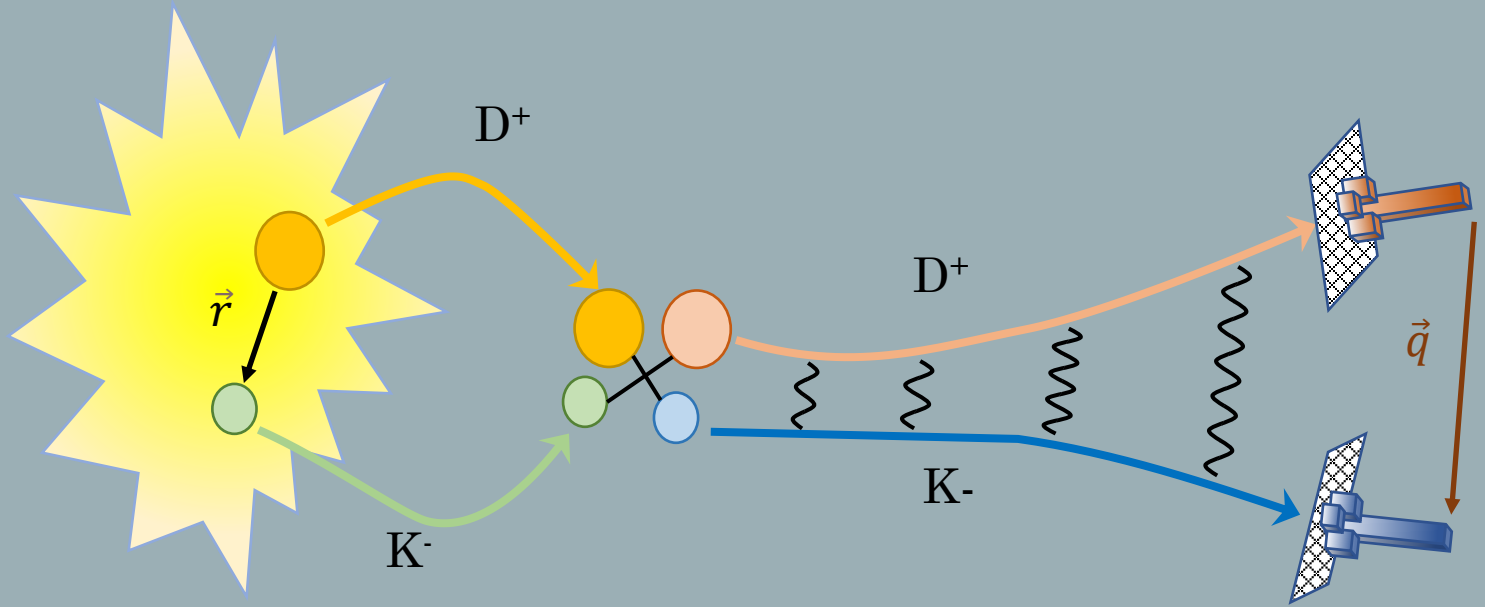
Koonin, *Phys.Lett.B*, 70, 43 (1977)
Pratt, Csorgo, Zimanyi, *Phys.Rev.C*, 42, 2646(1990)

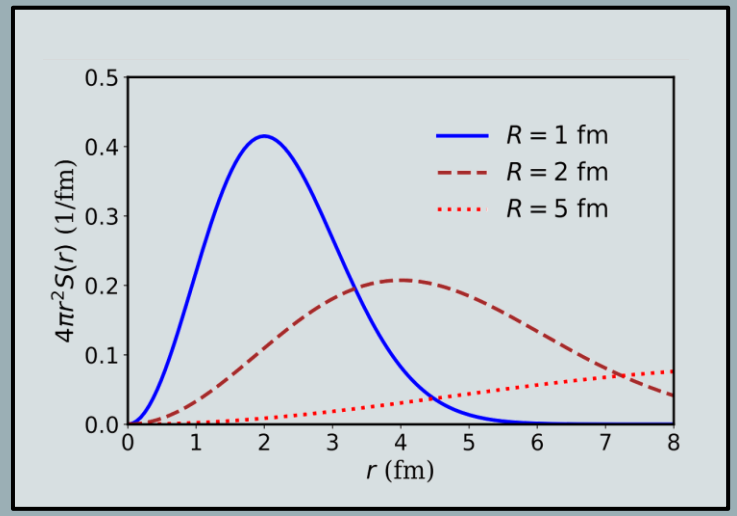
Single-channel KP formula

$$C(\boldsymbol{q}) = \int d^3r\ S(\boldsymbol{r})|\Psi(\boldsymbol{q};\boldsymbol{r})|^2$$

Pair wave function with CM momentum ***q***

Source function





Gaussian source function

$$S(r) = \frac{1}{(2\sqrt{\pi}R)^3}\exp\left(-\frac{r^2}{4R^2}\right)$$

Correlation function

KP Correlation function with Coulomb interaction

$$C(q) = \int d^3r\, S(r)|\Phi^{\rm C}(\boldsymbol{q};\boldsymbol{r})|^2 + 4\pi \int r^2 dr\, S(r) \sum_\ell (2\ell+1)\left[|\varphi_\ell(q;r)|^2 - |\Phi^{\rm C}_\ell(qr)|^2\right]$$

Complete Coulomb wave function

ℓ-wave strong + Coulomb wf

ℓ-wave Coulomb wf

As in 2-body, we keep only s-wave interactions

Kaon-Deuteron Correlation Function

We use T-matrix to compute the wave function (Lippmann-Schwinger equation)

$$\varphi_i(q;r) = j_0(qr)\delta_{if} + \int_0^\infty \frac{4\pi q'^2\, dq'}{(2\pi)^3} \frac{T_{if}(q', q; \sqrt{s})\, j_0(q'r)}{2\omega_{H,i}\, 2\omega_{\phi,i}(\sqrt{s} - \omega_{H,i} - \omega_{\phi,i} + \mathrm{i}\eta)}$$

$$T(q', q; \sqrt{s}) = T^{\mathrm{S}}(\sqrt{s}) + T^{\mathrm{C}}(q', q; \sqrt{s})$$

Total Coulomb wave function and $\ell = 0$ partial wave

$$\Phi^{\mathrm{C}}(r, z; k) = e^{-\pi\gamma/2}\Gamma(1 + i\gamma)e^{ikz}\ {}_1F_1(-i\gamma; 1; ik(r - z))$$

$$\gamma = Z_1 Z_2 \mu\alpha/k$$

$$\Phi^{\mathrm{C}}_{0f}(qr) = (qr)^{-1}e^{i\sigma_0}F_0(\gamma; qr)$$

$$\sigma_l = \arg\Gamma(1 + l + i\gamma)$$

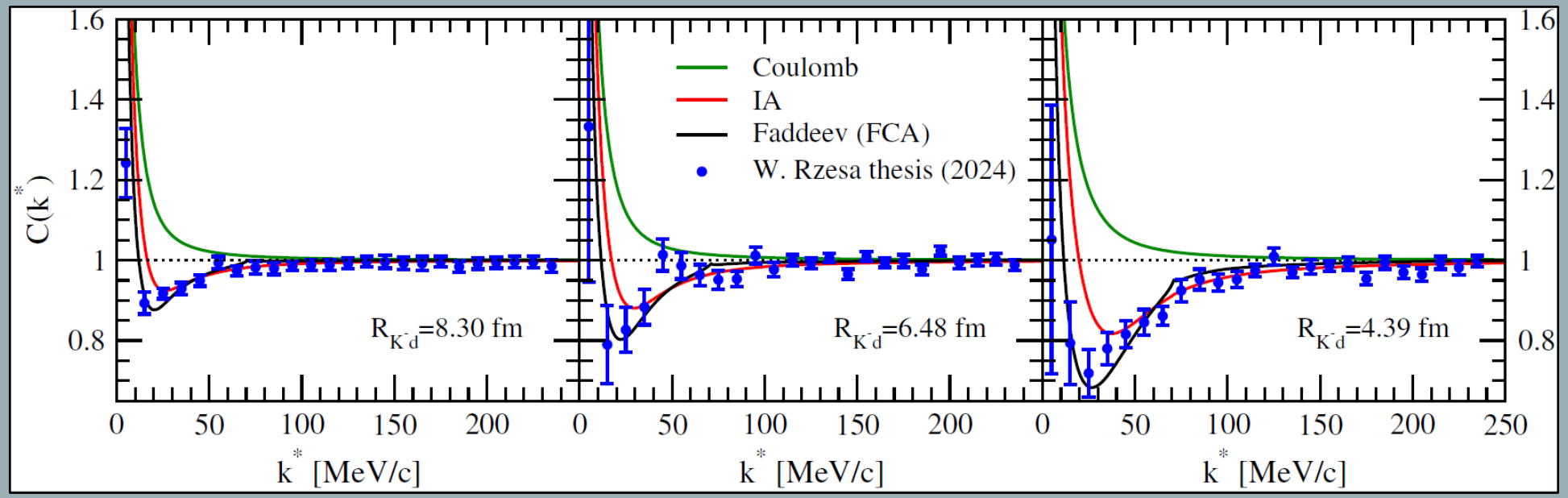
C.J. Joachain, *Quantum Collision Theory* (1975)

K^- d correlation function

Pb-Pb collisions @ $s^{1/2}_{NN}$=5.02 TeV

W. Rzesa, Ph.D. thesis,
Warsaw U. of Tech. (2024) (ALICE Coll.)

Centrality: 0-10% 10-30% 30-50%



Source radii only **indicative** from experimental fits

Signal enhanced in small systems. No results for pp collisions.

K^+d correlation function

Pb-Pb collisions @ $s^{1/2}_{NN}$=5.02 TeV

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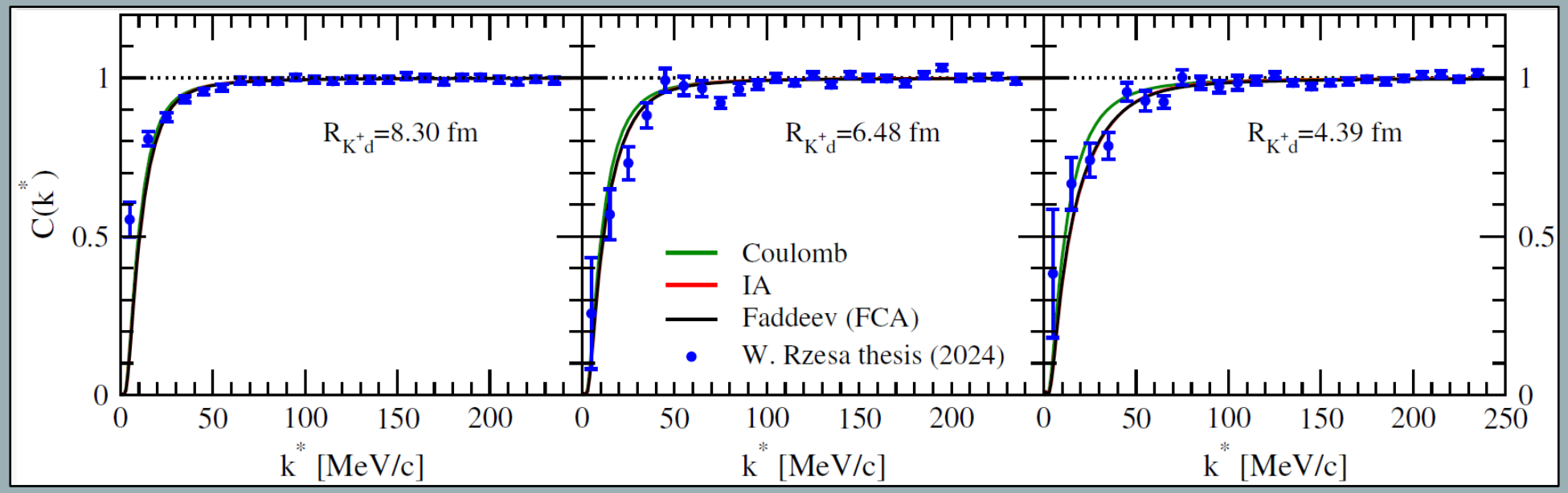
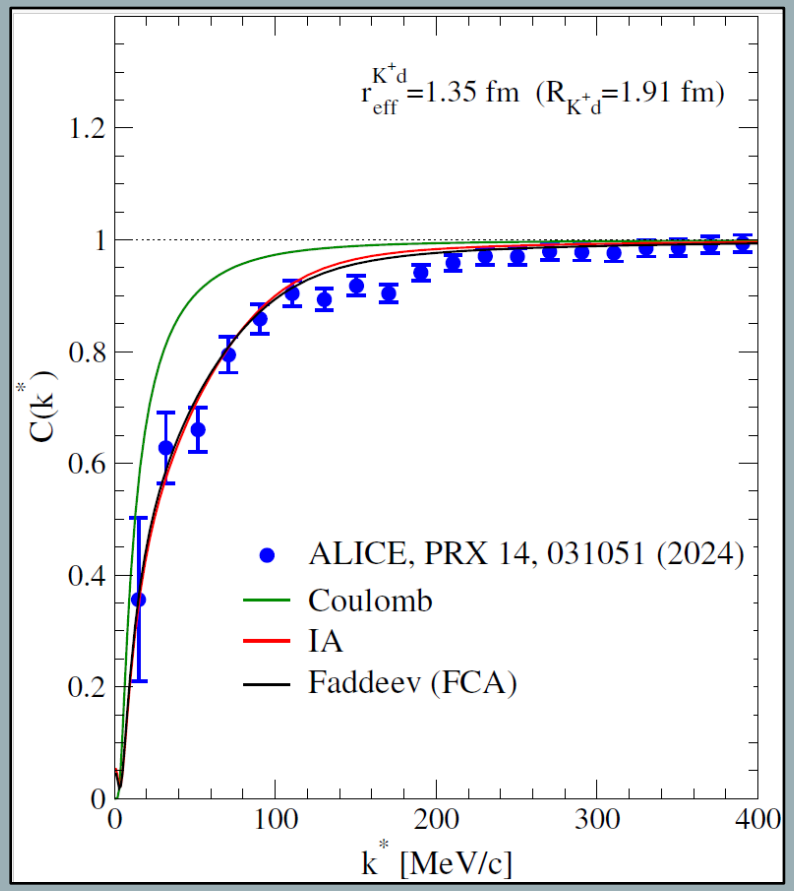


Source radii only **indicative** from experimental fits

Rescattering is not important in K^+d case. No bound state.

K^+d correlation function

High multiplicity p-p collisions @ $s^{1/2}_{NN}$=13 TeV data from ALICE



ALICE Coll. *Phys. Rev X14*, 031051 (2024)

Summary

Our first approach to 3-body dynamics in 2-body femtoscopy produce quite satisfactory results.

Next steps:

- Unitarity. Break-up, recombination, rearrangement processes in Faddeev equations.
- Implementation into TROY
  T-matrix-based Routine for hadrOn femtoscopy
- Coulomb - strong crossed terms
- Higher partial waves

- Tackle 3-body femtoscopy

KAON-DEUTERON CORRELATION FUNCTION FROM AN EFT APPROACH

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Granada, January 10, 2026,

Excited QCD 2026 Workshop

Universidad de Granada.

Carmen de la Victoria.

Kaon-Deuteron interaction

	A_{K^-d} [fm] ($\sqrt{s} = 2369.290$ MeV)	A_{K^+d} [fm] ($\sqrt{s} = 2369.290$ MeV)
IA	$-0.59 + \mathrm{i}2.15$	-0.47
FCA	$-2.06 + \mathrm{i}1.77$	-0.43

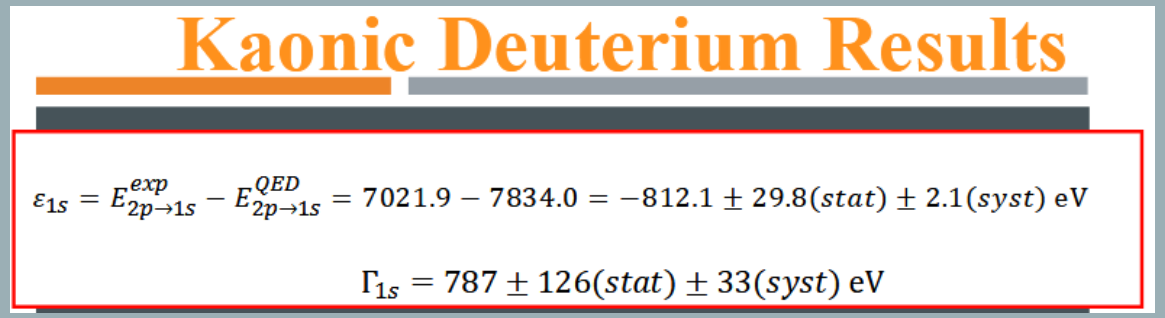
Scattering Lengths

Impulse Approximation
Fixed-Center Approximation

Energy shift and width of the *1s* atomic level in kaonic deuterium

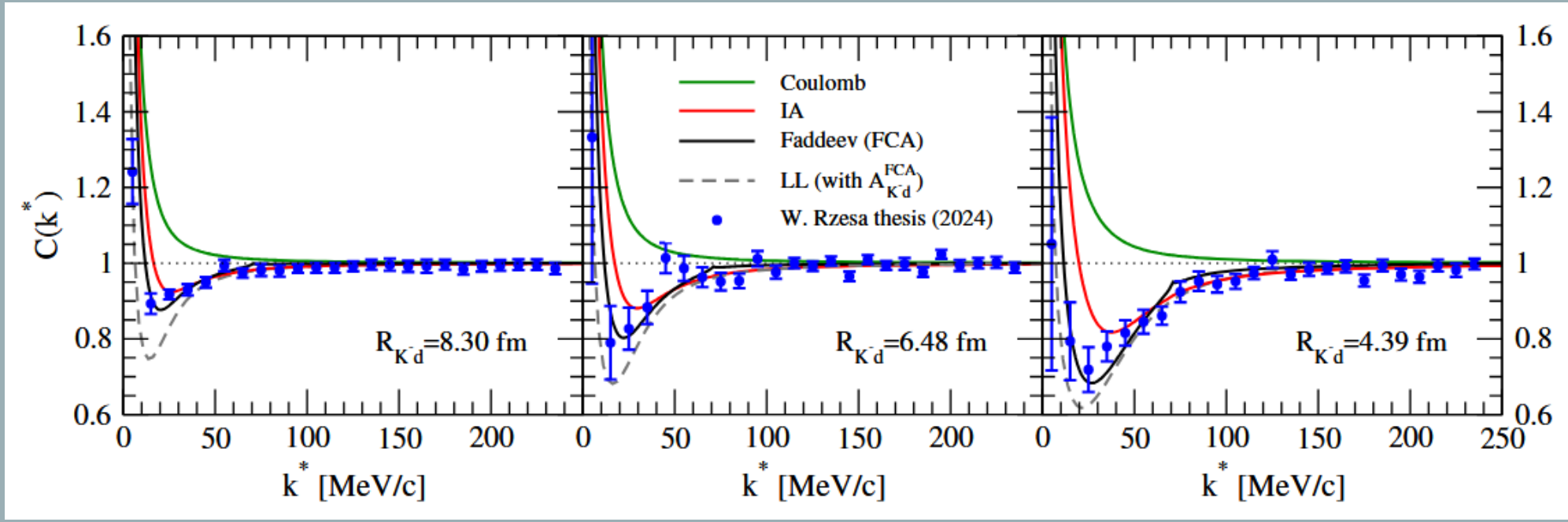
	ε_{1s} [eV]	Γ_{1s} [eV]
IA	791	2063
FCA	1124	626

Experimental Preliminary (SIDDHARTA-2)



Talk by F. Sgaramella at Workshop on Fundamental Physics with Exotic Atoms,INFN Frascati, Italy (2025).

ALICE results in PbPb collisions



$$|\psi(r^*, k^*)| = \sqrt{A_C(\eta)}\left[\exp(-ik^*r^*)F(-i\eta, 1, i\xi) + f_c(k^*)\frac{G}{r^*}\right]$$

$$S(\vec{r}) \sim \exp\left(-\frac{r^2}{2R_{Kd}^2}\right)$$

Asymptotic part of the wave function

W. Rzesa (ALICE Coll.) *PhD Thesis*