Chiral and $U(1)_A$ restoration in Effective Field Theories

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A. Gómez Nicola, J. R. Peláez, JRE, PRD82 074012 (2010), PRD87 016001 (2013),

A. Gómez Nicola, R. Torres Andrés, JRE, PRD88 076007 (2013),

A. Gómez Nicola, JRE, JHEP 03 186 (2016), PRD97 074016 (2018), PRD98 014020 (2018),

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QCD and chiral symmetry

- Low-energy regime of QCD

$$\mathcal{L}_{QCD} = \sum_{q=u,d,s} i\bar{q}_L \not{D} q_L + i\bar{q}_R \not{D} q_R - m_q \left(\bar{q}_R q_L + \bar{q}_L q_R\right)$$

[Fritzsch, Gell-Mann, Leutwyler (1973)]

$\hookrightarrow$ QCD invariant under **flavor rotations** of the q_L and q_R fields in the **massless limit**

- Symmetry group: $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$
- $N_f = 2$, m_u and m_d **very small**

$\hookrightarrow$ $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ should be a nearly **perfect symmetry** [Nambu (1960)]

- $N_f = 3$, $m_s \sim \Lambda_{QCD}$

$\hookrightarrow$ $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ larger corrections

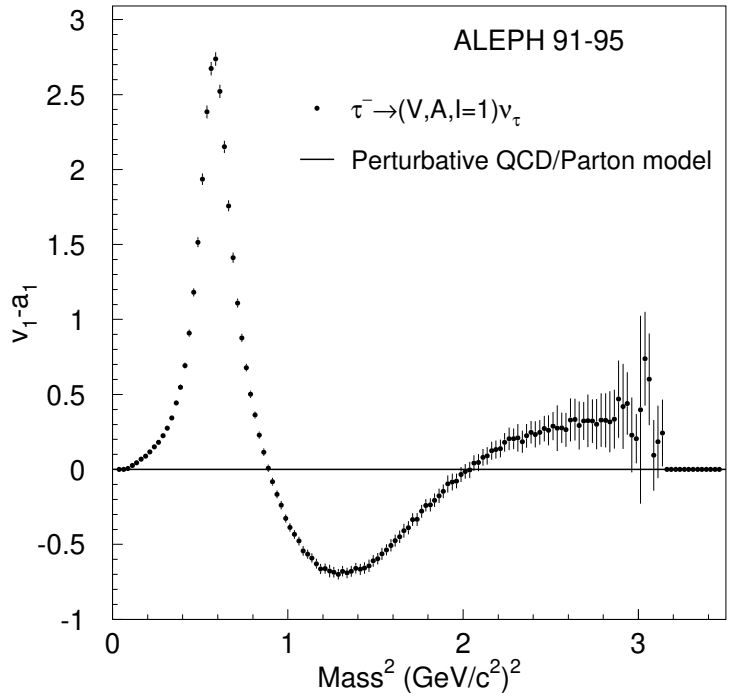
Spontaneous chiral symmetry breaking

- Chiral symmetry is not "visible" in the spectrum
  - ▷ no parity doublets
  - ▷ $\langle 0|AA|0\rangle \neq \langle 0|VV|0\rangle$
- **Spontaneous breaking** of **chiral symmetry**

  $\hookrightarrow$ π, K, η are the **Goldstone Bosons** from Chiral Symmetry Spontaneous Breakdown

$$\langle 0|\partial^\mu J_\mu^{5\,a}(0)|\pi^b(p)\rangle = \delta^{ab} f_\pi m_\pi^2$$

$$\langle 0|[Q^{A,a}, \bar{q}\gamma_5 t^b q]|0\rangle = -\frac{2}{3}\delta_{ab}\langle 0|\bar{q}q|0\rangle$$



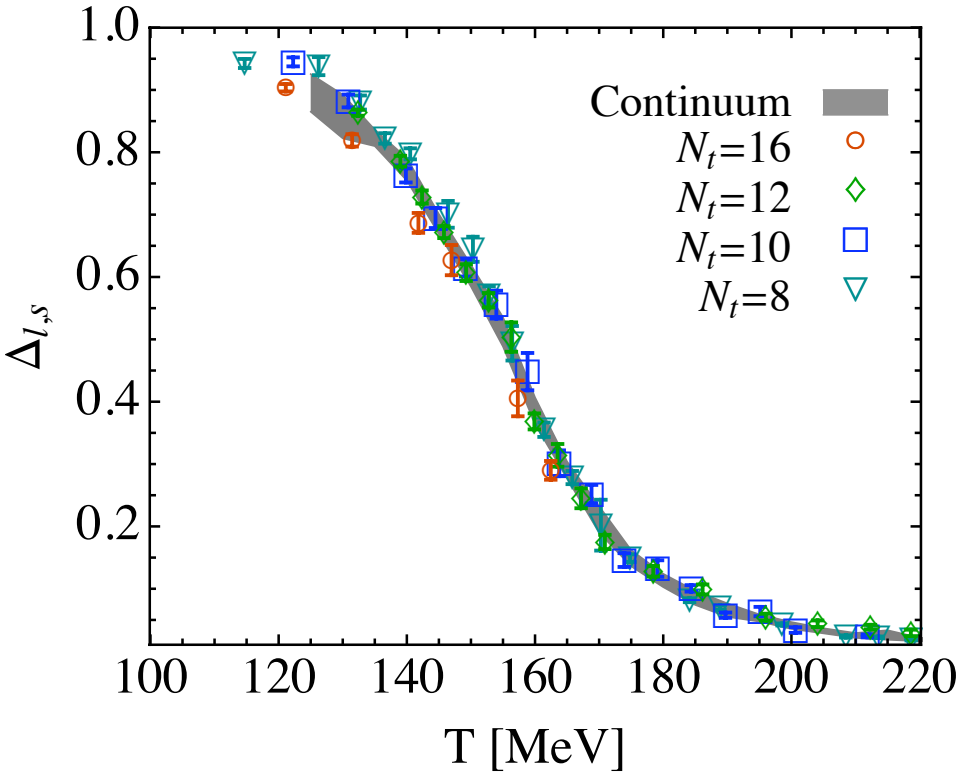
[Schael et al. (ALEPH Collaboration) (2005)]

- Chiral QCD Lagrangian invariant under $U(1)_L \times U(1)_R$

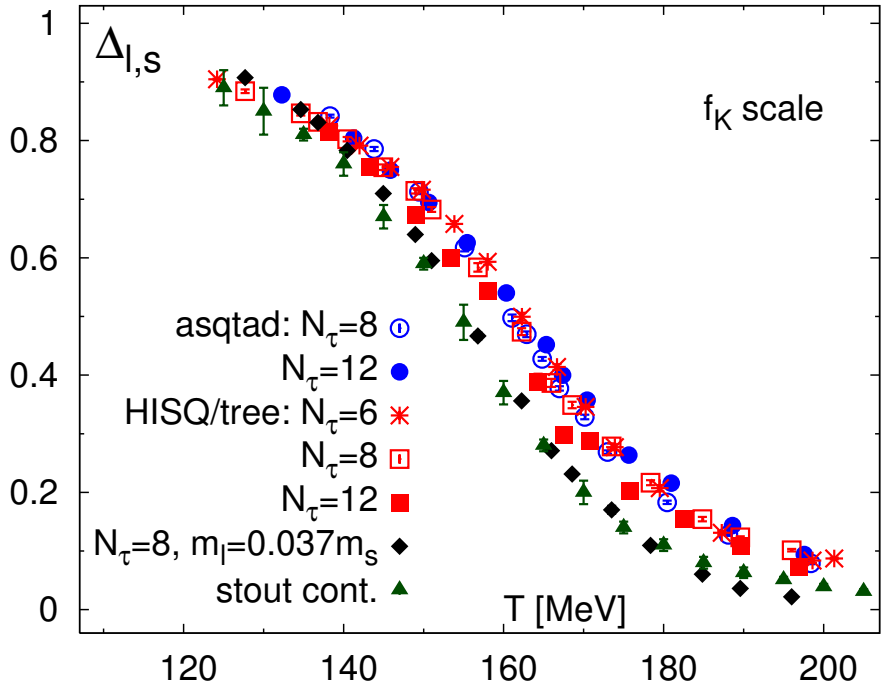
$$\langle 0|\partial^\mu J_\mu^{5,0}|\eta'(p)\rangle = f_\pi m_{\eta'}^2 = \langle 0|\frac{3g^2}{32\pi^2} F^{a\mu\nu} F^a_{\mu\nu}|\eta'(p)\rangle$$

$\hookrightarrow$ does not vanish in the chiral limit

Chiral symmetry restoration: quark condensate



[Borsányi et al. (Wuppertal-Budapest Collaboration) (2010)]

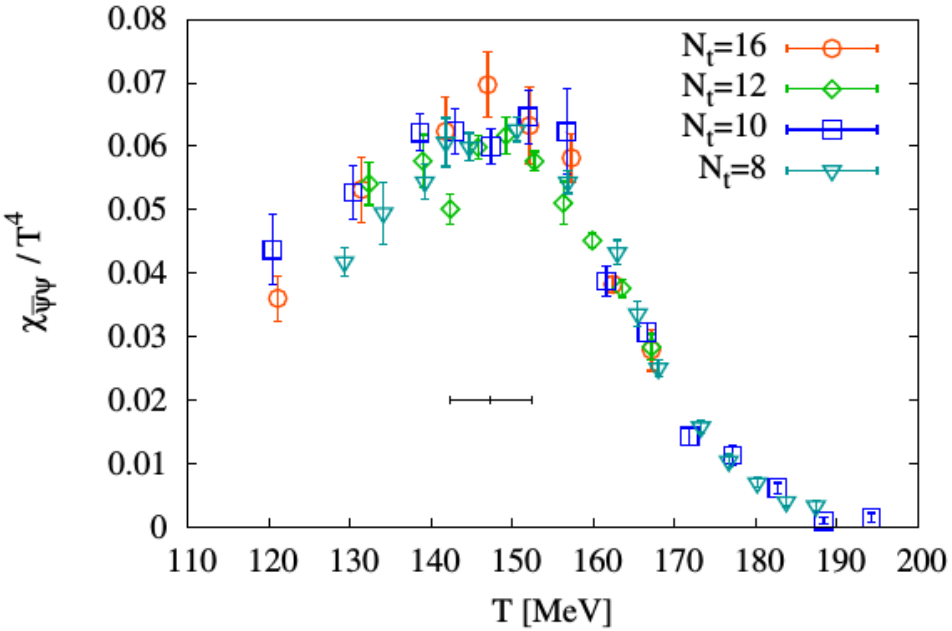


[Bazavov et al. (HotQCD Collaboration) (2012)]

$$\Delta_{ls} = \frac{\langle \bar{q}q \rangle_T - (2m_l/m_s)\langle \bar{s}s \rangle_T}{\langle \bar{q}q \rangle_0 - (2m_l/m_s)\langle \bar{s}s \rangle_0} \quad \text{for } N_f\text{=2+1 and } \textbf{physical masses}$$

- Restoration temperature?

Chiral symmetry restoration: scalar susceptibility



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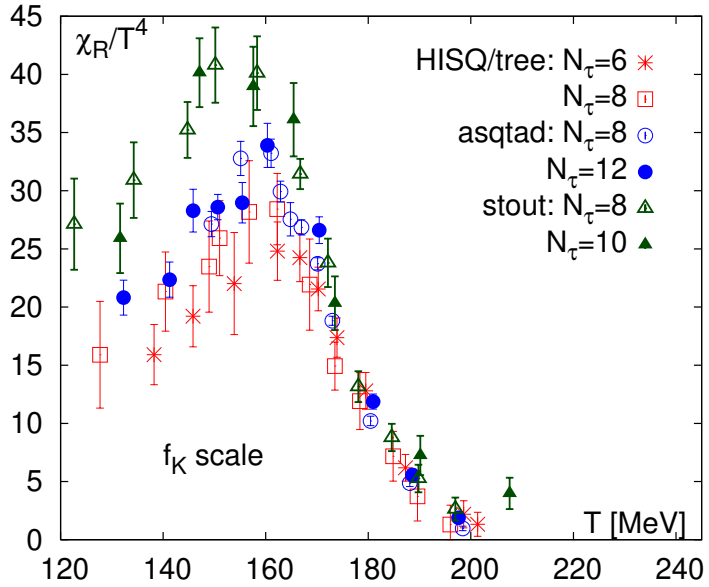


[Bazavov et al. (HotQCD Collaboration) (2012)]

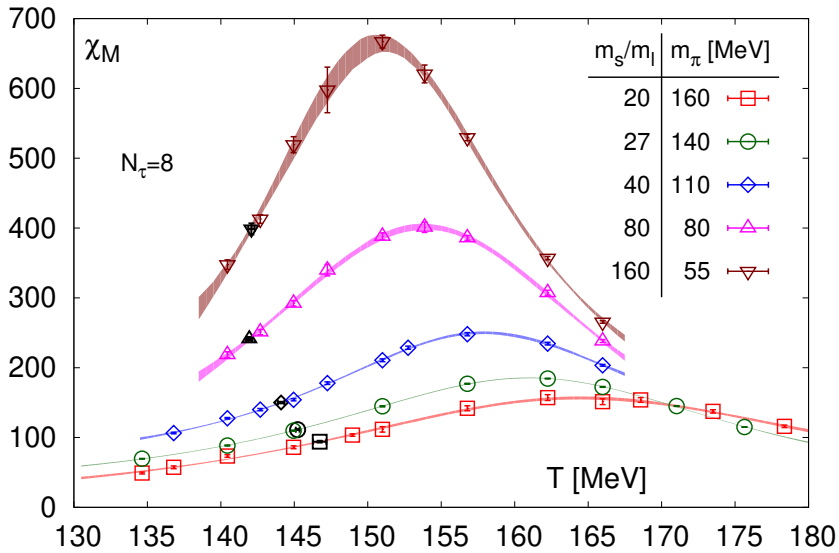
$$\chi_l(T) = -\frac{\partial}{\partial m}\langle(\bar{q}q)_l\rangle_T = \int_E d^4x \left[\langle \mathcal{T}(\bar{q}q)_l(x)(\bar{q}q)_l(0)\rangle_T - \langle(\bar{q}q)_l\rangle_T^2\right]$$

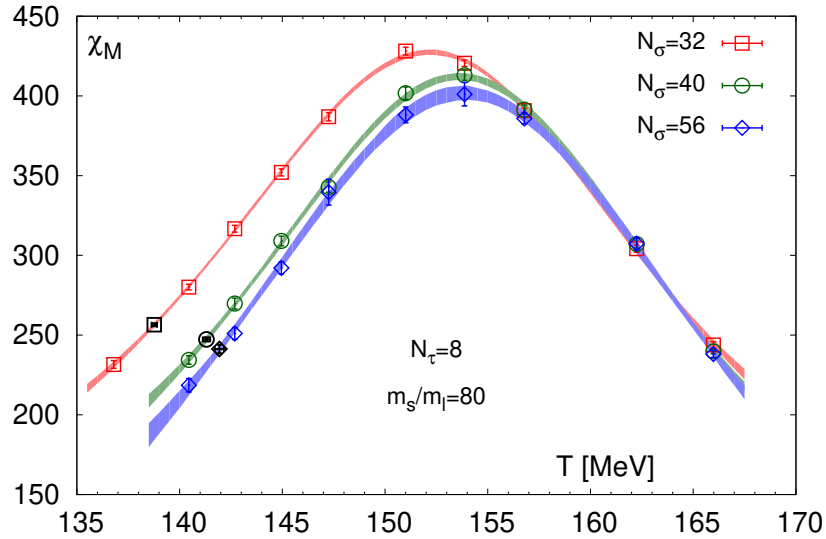
- **Crossover transition** at $T_c \sim 155$ MeV for N_f **= 2+1** and **physical masses**

Chiral symmetry restoration: scalar susceptibility close to the chiral limit

- Exact restoration?

  results close to the chiral limit





$\hookrightarrow T_c\big|_{m_l \to 0} = 132^{+3}_{-6}$ MeV [Ding et al. (HotQCD Collaboration) (2019)]

Possible **second order** $O(4)$ phase **transition** for $N_f = 2$ in **chiral limit** [Pisarski and Wilczek (1984)]

$U(1)_A$ restoration

$U(1)_A$ restoration?

- $U(1)_A$ **asymptotic restoration** suggested by the vanishing of the instanton density

  [Gross, Pisarski, Yaffe (1981), Shuryak (1994), Cohen (1996), Lee-Hatsuda (1996)]

- Observed $M_{\eta'}$ **reduction** at finite temperature [Csorgo, Vertesi, Sziklai (2010)]
- Increase of η' **production** for dileptons and diphotons experiments [Kapusta, Kharzeev, McLerran (1996)]
- Is the $U(1)_A$ **effectively restored** close to T_c?

  $\hookrightarrow$ $O(4) \times U(1)_A$ pattern instead of $O(4) \equiv SU(2)_L \times SU(2)_R$

  $\hookrightarrow$ first order phase transition at $m_l \to 0$ [Pisarski, Wilczek, (1984), Pelissetto, Vicari (2013), Eser, Grahl, Rischke (2015)]

  $\hookrightarrow$ critical end point at finite temperature and baryon density [Mitter, Schaefer (2014)]

- How should one study $U(1)_A$ restoration?

  $\hookrightarrow$ study chiral and $U(1)_A$ restoration from the **particle spectrum**

Chiral bilinears

- Define scalar and pseudoscalar quark bilinears

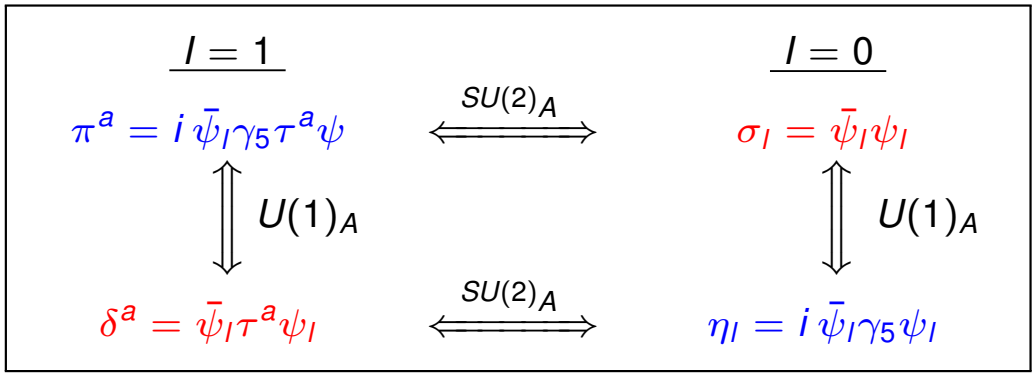
$$\begin{array}{lll} & \underline{P} & \underline{S} \\ \underline{\text{I=1}} & \pi^a = i\bar{\psi}_l\gamma_5\tau^a\psi, & \delta^a = \bar{\psi}_l\tau^a\psi_l \quad \sim \; a_0(980) \\ \underline{\text{I=0}} & \eta,\, \eta' \sim \left\{ \begin{array}{ll} \eta_l = i\bar{\psi}_l\gamma_5\psi_l, & \sigma_l = \bar{\psi}_l\psi_l \\ \eta_s = i\bar{\psi}_s\gamma_5\psi_s, & \sigma_s = \bar{\psi}_s\psi_s \end{array} \right\} \sim f_0(500),\, f_0(980) & \\ \underline{\text{I=1/2}} & K^a = i\bar{\psi}\gamma_5\lambda^a\psi, & \kappa^a = \bar{\psi}\lambda^a\psi \end{array}$$

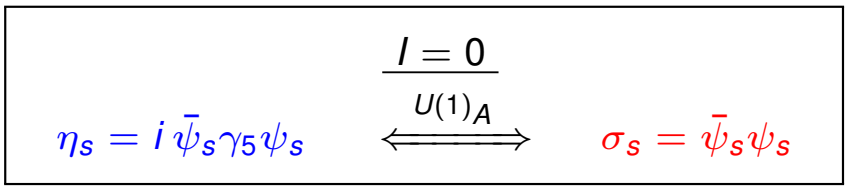
- Or their corresponding susceptibilities

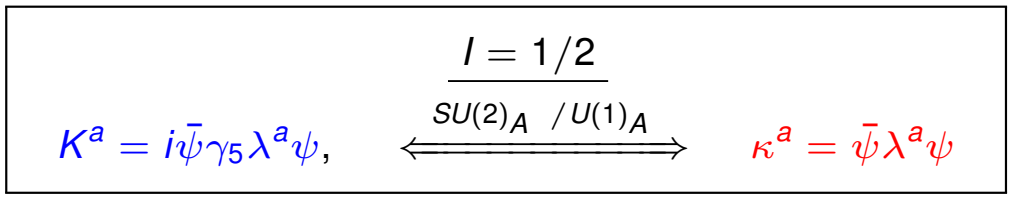
$$\chi_P^{ab} \equiv \int_T dx\, \langle P^a(x)P^b(y)\rangle = K_P^{ab}(p=0), \quad \chi_S^{ab} \equiv \int_T dx\, \langle S^a(x)S^b(y)\rangle = K_S^{ab}(p=0)$$

Chiral partners and patterns

- Study their behavior under $O(4)$ and $U(1)_A$ transformations





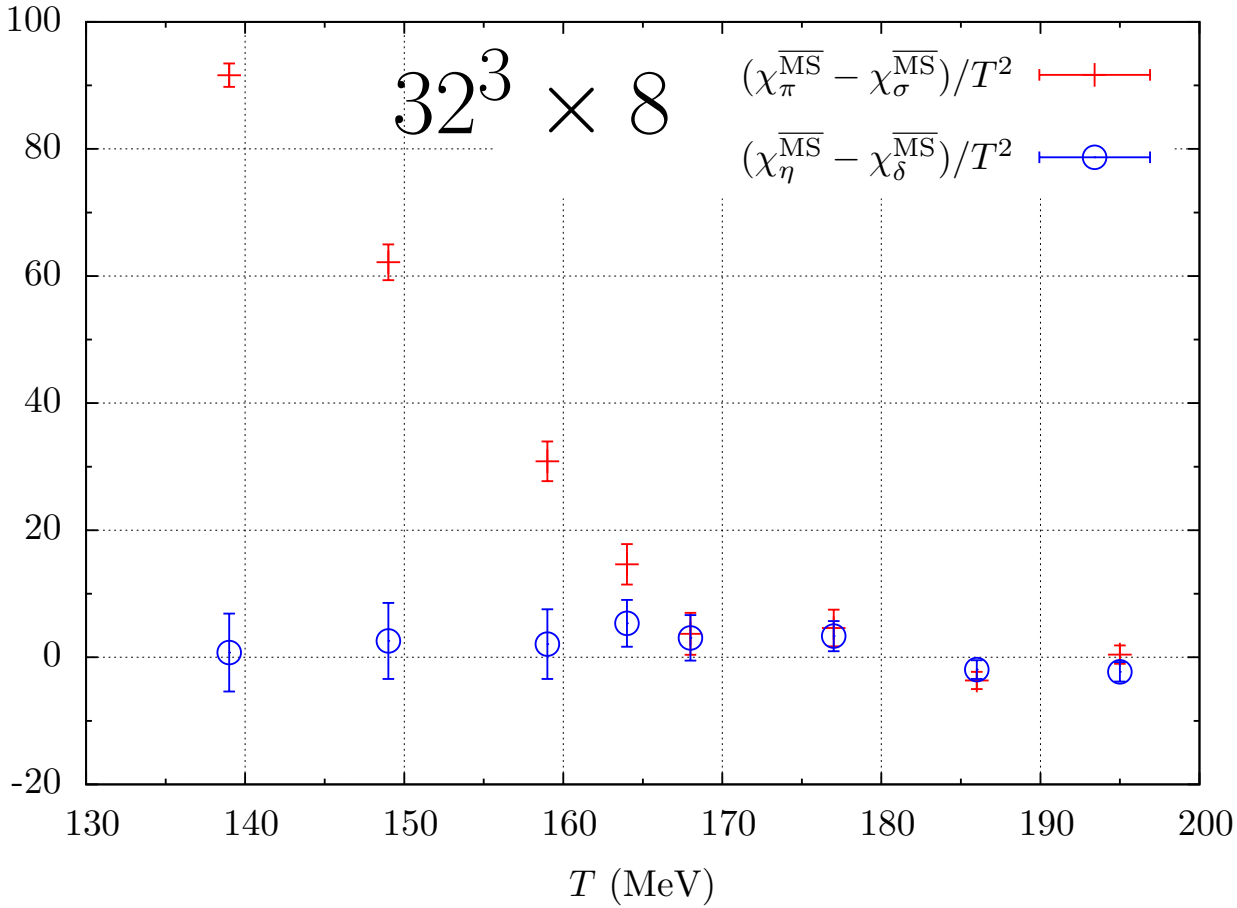


$$\chi_P^{\pi} \overset{O(4)}{\sim} \chi_S^{\sigma}, \qquad \chi_P^{\eta_l} \overset{O(4)}{\sim} \chi_S^{\delta}, \qquad \chi_P^{ls} \overset{O(4)}{\sim} 0, \qquad \chi_S^{ls} \overset{O(4)}{\sim} 0$$

$$\chi_P^{\pi} \overset{U(1)_A}{\sim} \chi_S^{\delta}, \qquad \chi_P^{\eta_l} \overset{U(1)_A}{\sim} \chi_S^{\sigma}, \qquad \chi_P^{\eta_s} \overset{U(1)_A}{\sim} \chi_S^{\sigma_s}, \qquad \chi_P^{ls} \overset{U(1)_A}{\sim} \chi_S^{ls}$$

Chiral partner degeneration

- Susceptibilities for $N_f = 2 + 1$ and physical masses, LLNL/RBC Collaboration



[Buchoff et al. (LLNL/RBC Collaboration) (2014)]

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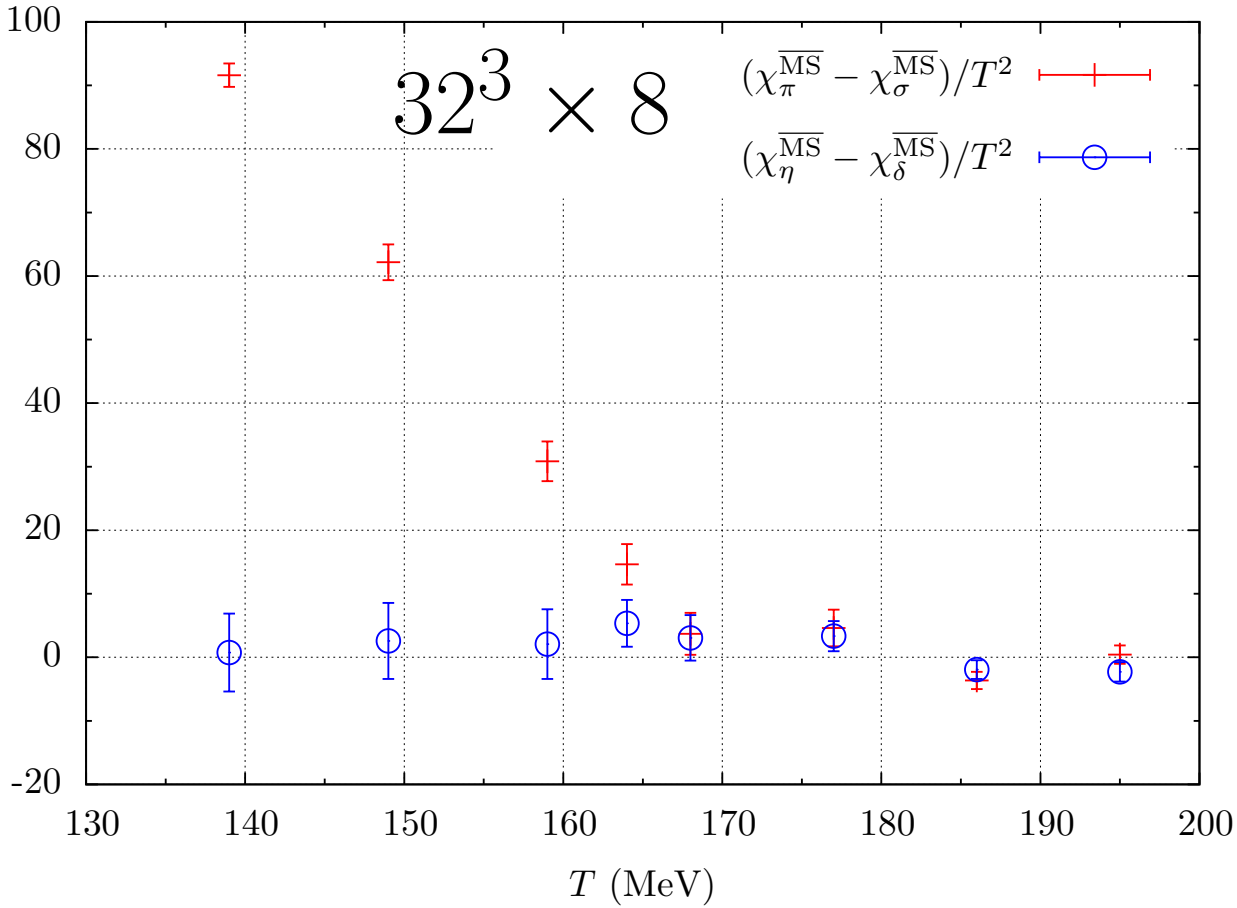

[Buchoff et al. (LLNL/RBC Collaboration) (2014)]

- π, σ **degeneration** around $T \sim T_c$

  $\hookrightarrow$ **O(4) restoration**

$U(1)_A$ partner degeneration: $N_f = 2 + 1$

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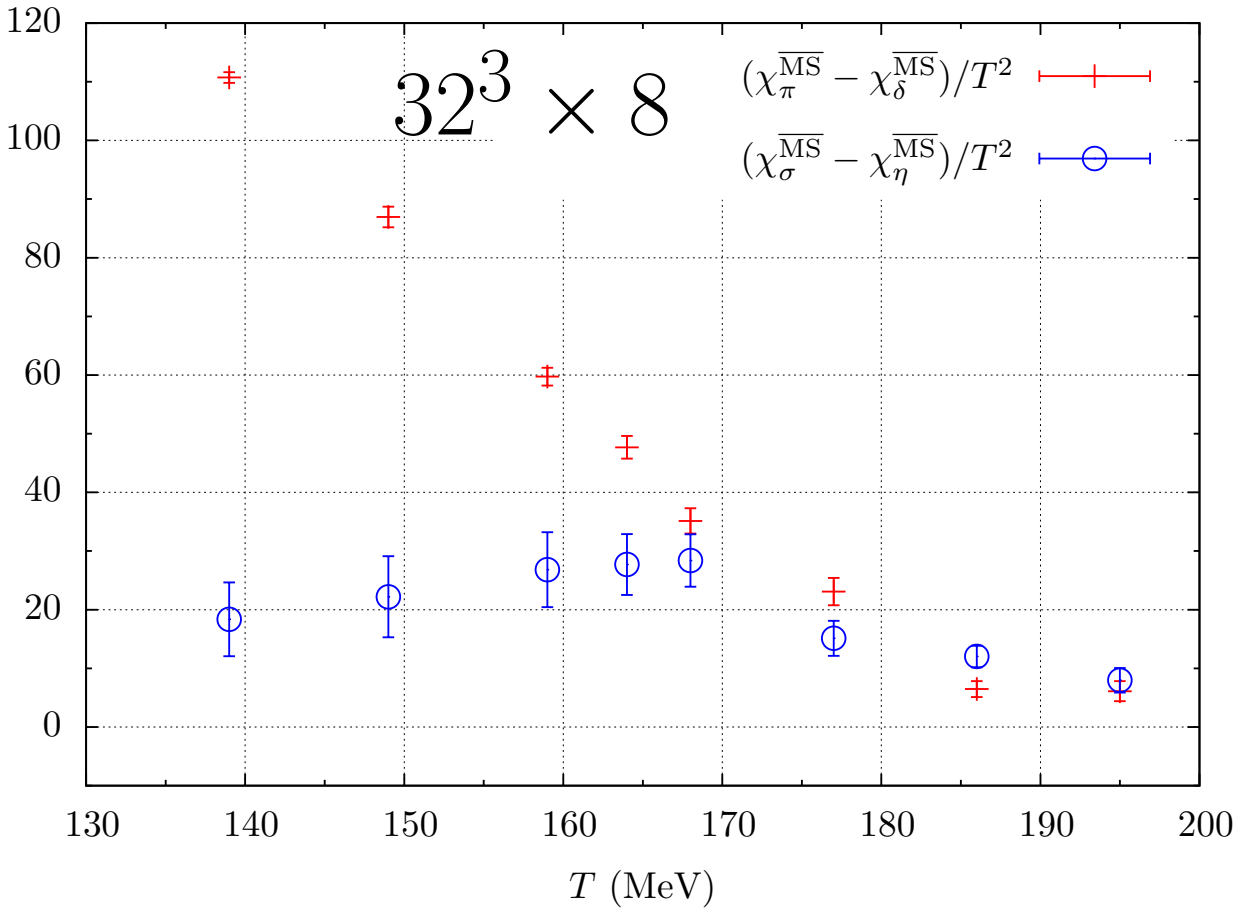

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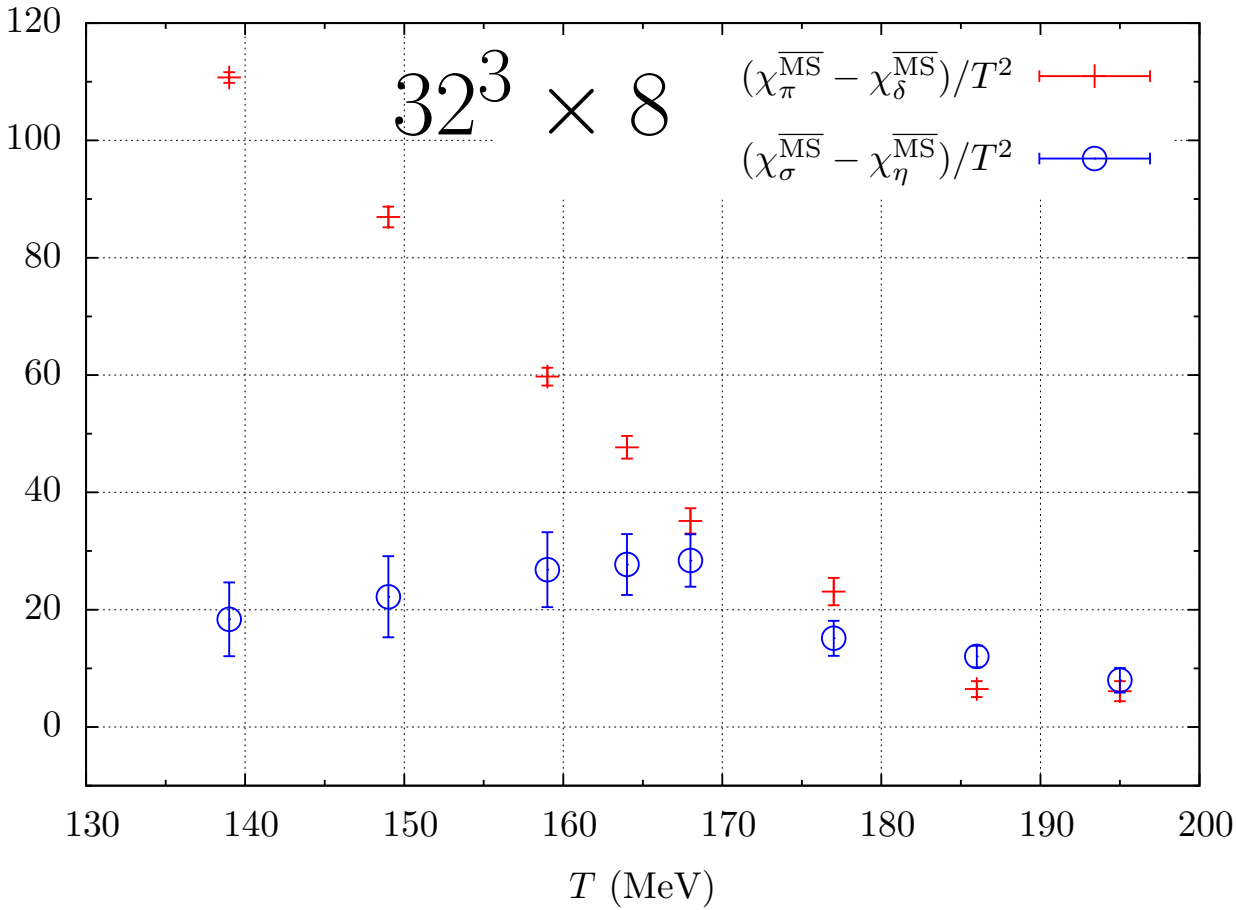


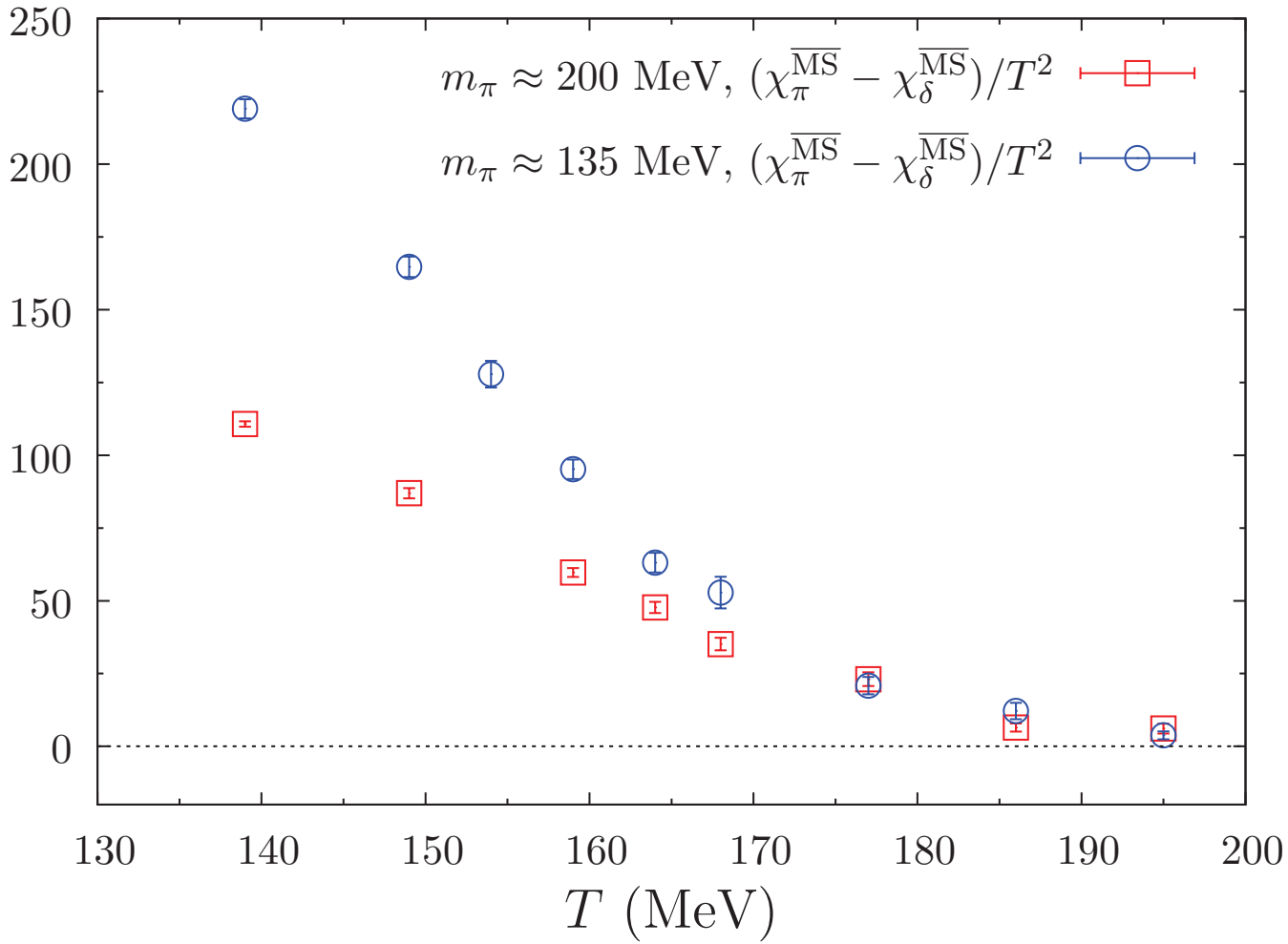
[Buchoff et al. (LLNL/RBC Collaboration) (2014)]

- **No** π, δ and η, σ **degeneration** around $T \sim T_c$

  $\hookrightarrow$ $U(1)_A$ is **not restored** at $T_c \rightarrow$ O(4) pattern

$U(1)_A$ partner degeneration: $N_f = 2 + 1$

- Susceptibilities for $\mathbf{N_f = 2 + 1}$ at different pion masses, HotQCD (2012-2025)



[Bhattacharya et al. (HotQCD Collaboration) (2014)]

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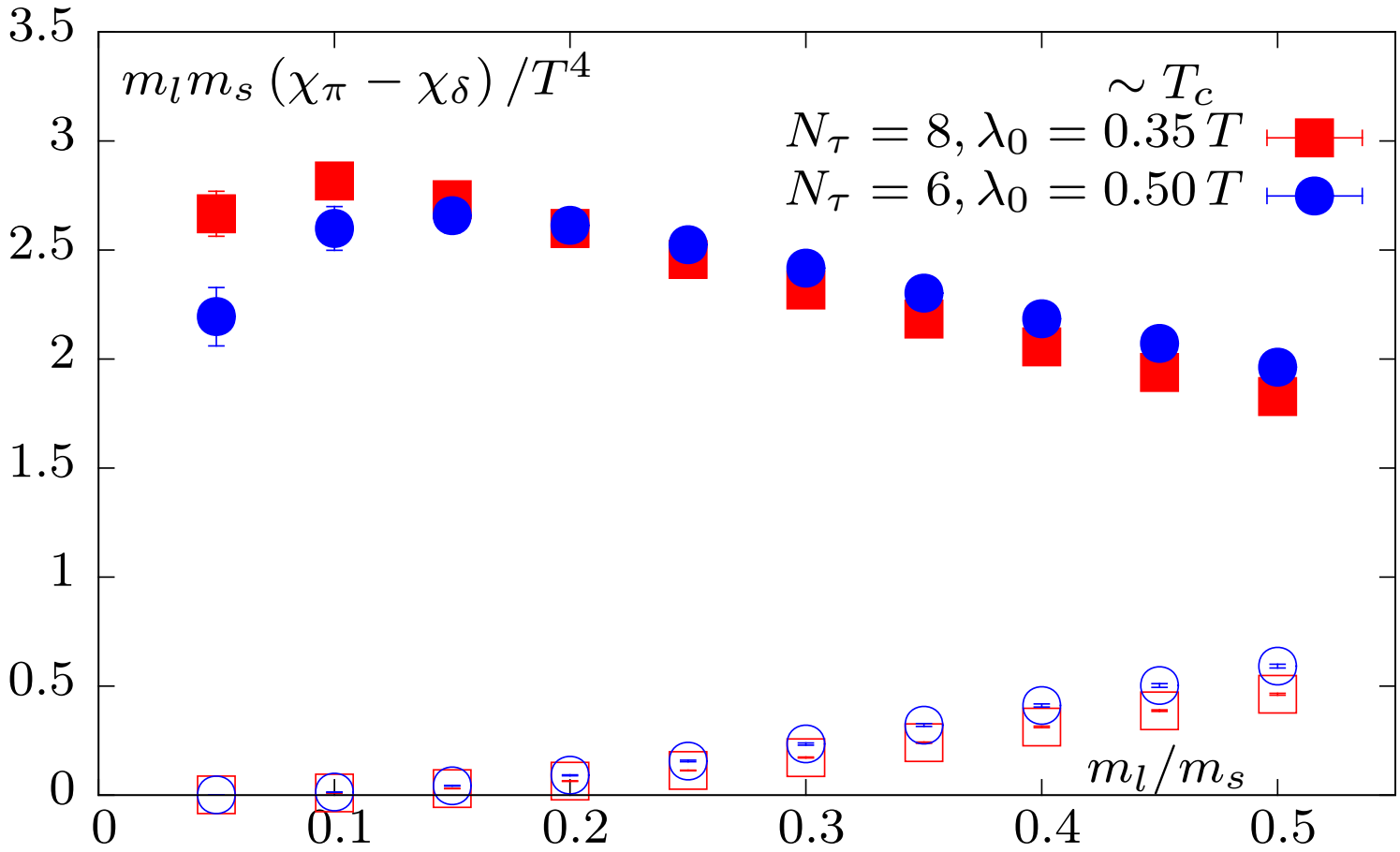

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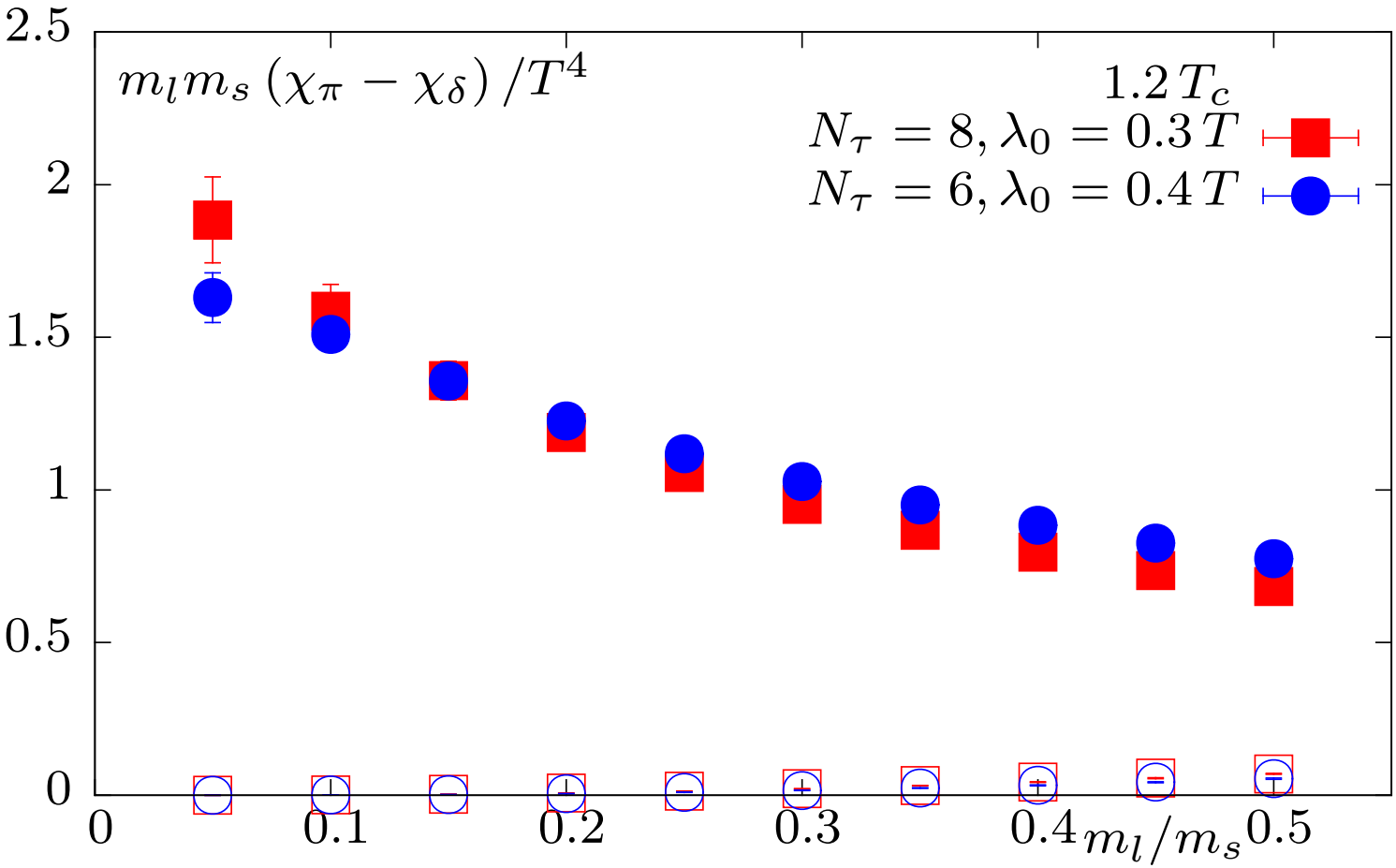

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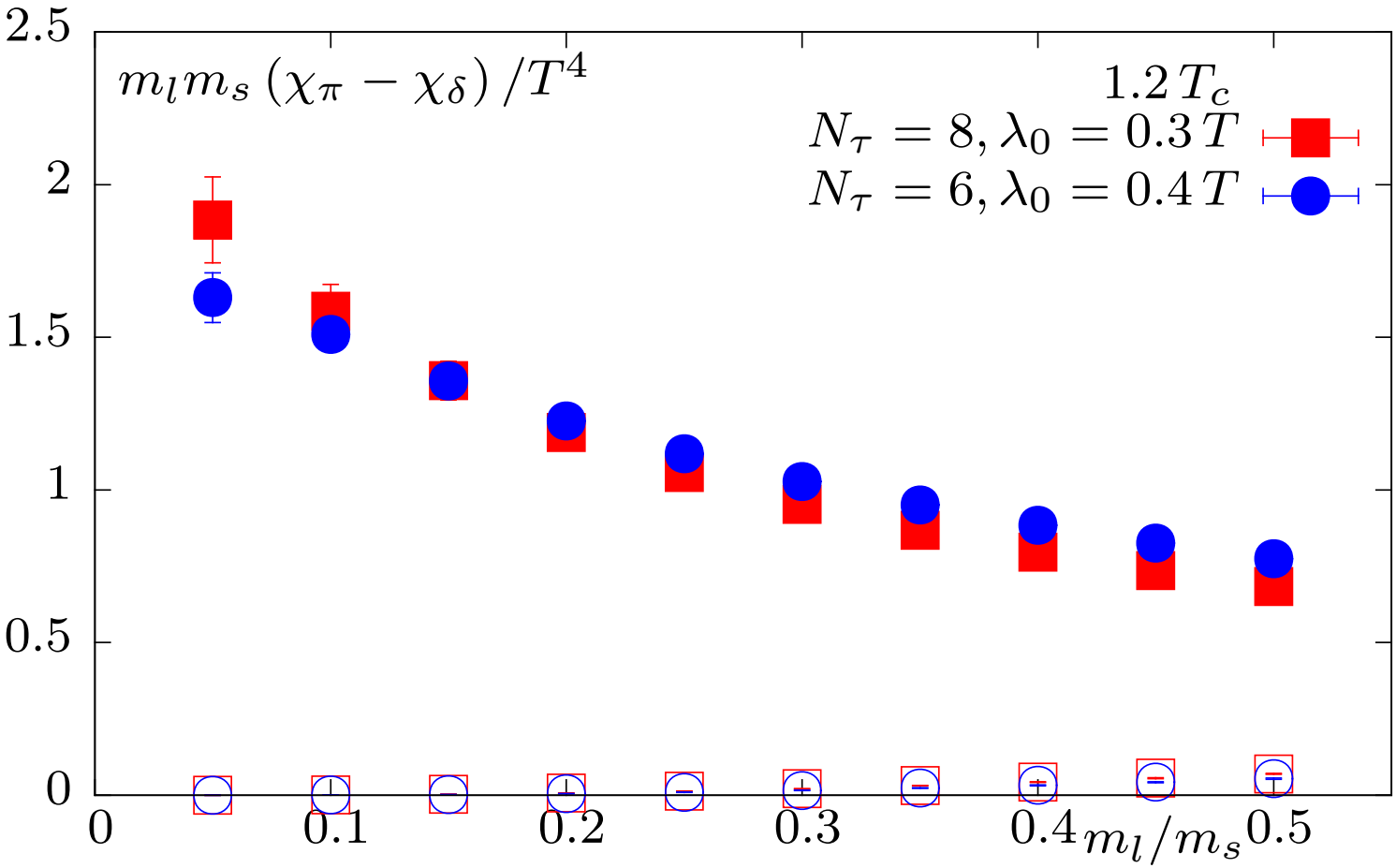
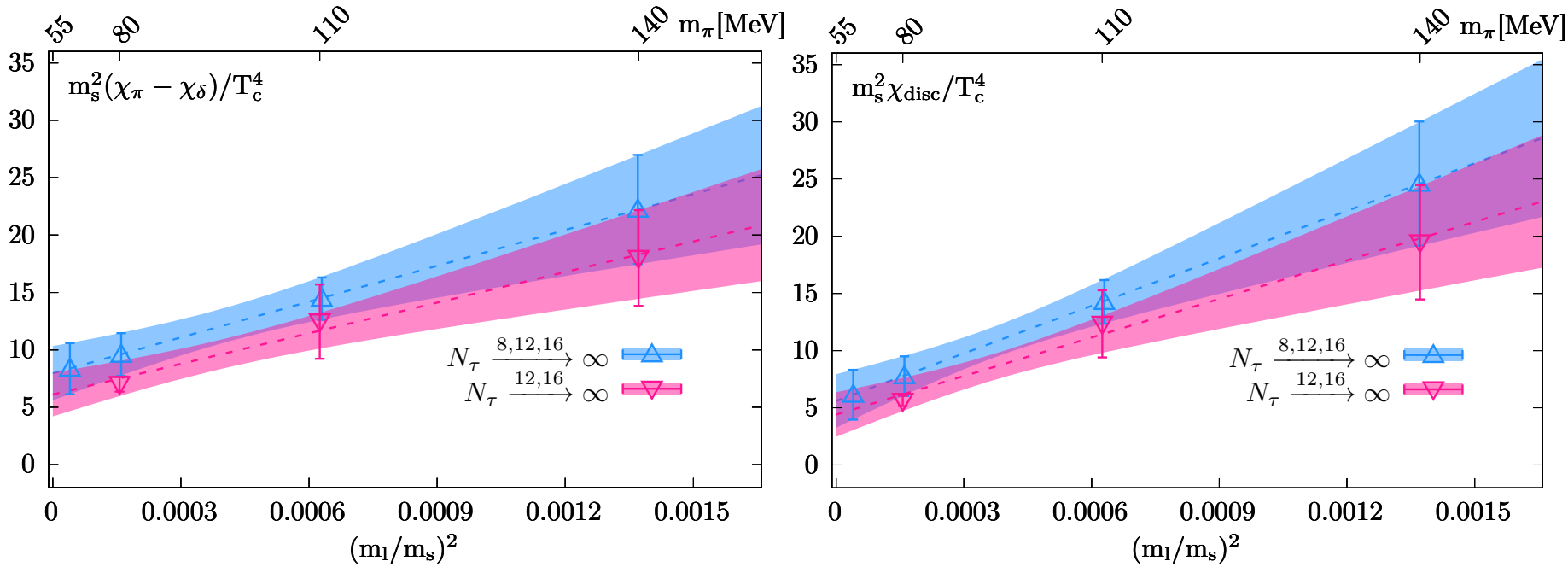


[Dick, Karsch et al. (2015)]

- $U(1)_A$ is **not restored** at T_c even in the **chiral limit**

$U(1)_A$ partner degeneration: $N_f = 2 + 1$

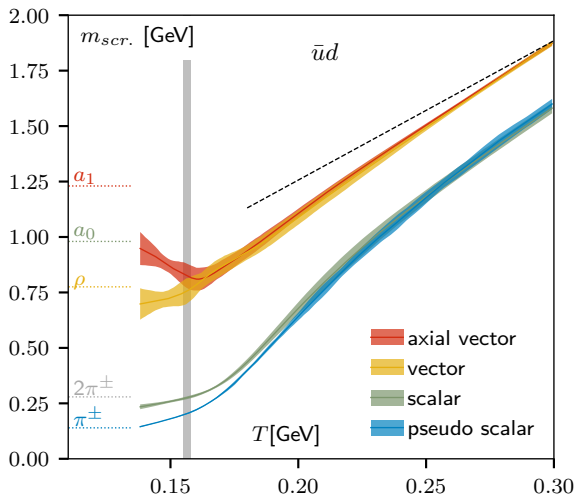
- Susceptibilities for $\mathbf{N_f = 2 + 1}$ close to the **chiral limit**, HotQCD (2012-2025)

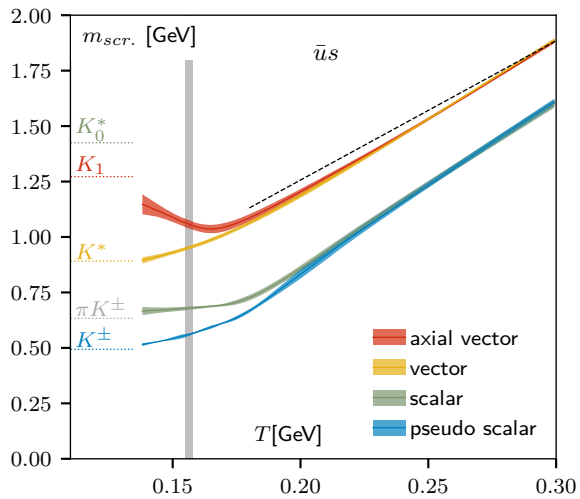


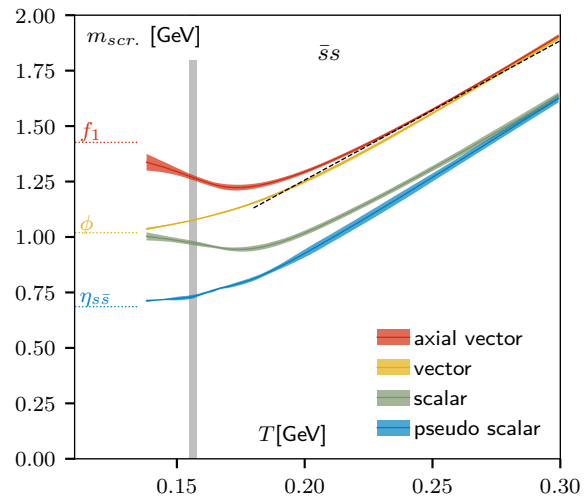
[H.-T. Ding et al. (2021)]

$U(1)_A$ partner degeneration: $N_f = 2 + 1$

- Screening masses for $\mathbf{N_f = 2 + 1}$ and physical masses, HotQCD (2012-2025)





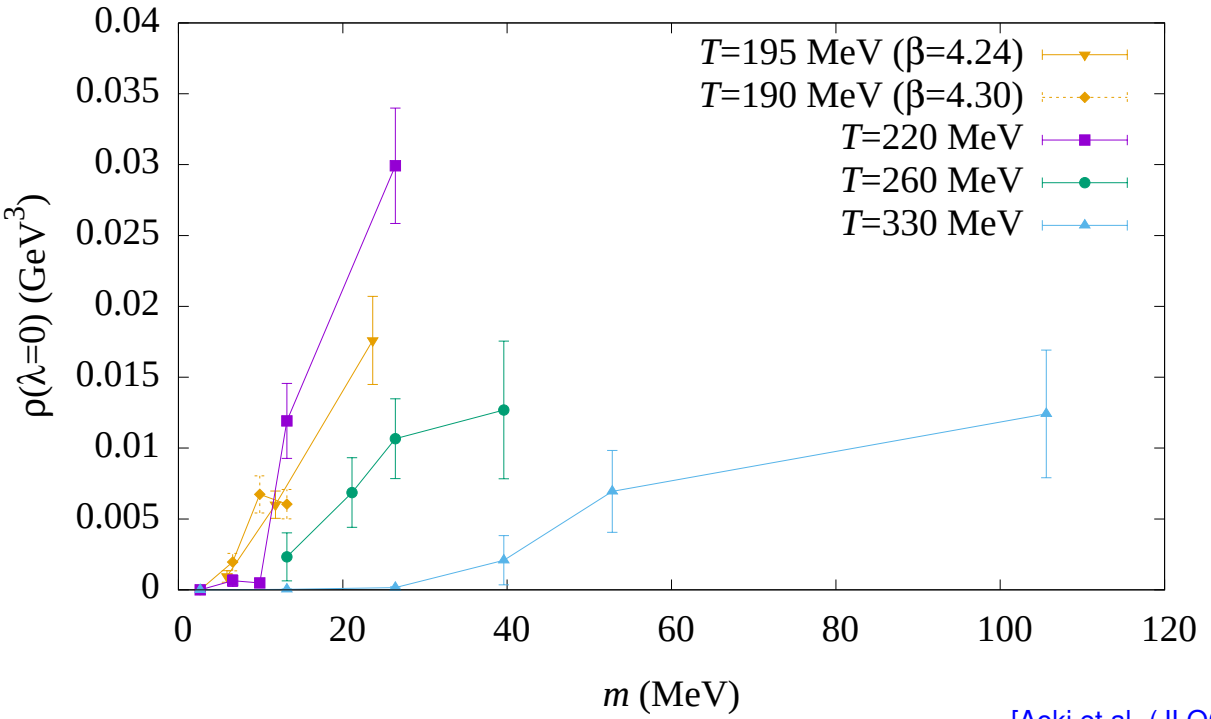


[Bazavov et al. (HotQCD Collaboration) (2019)]

$U(1)_A$ partner degeneration: $N_f = 2$

- Dirac operator, topological susceptibility and screening masses

  for $N_f = 2$ for different pion masses, JLQCD Collaboration (2012-2025)



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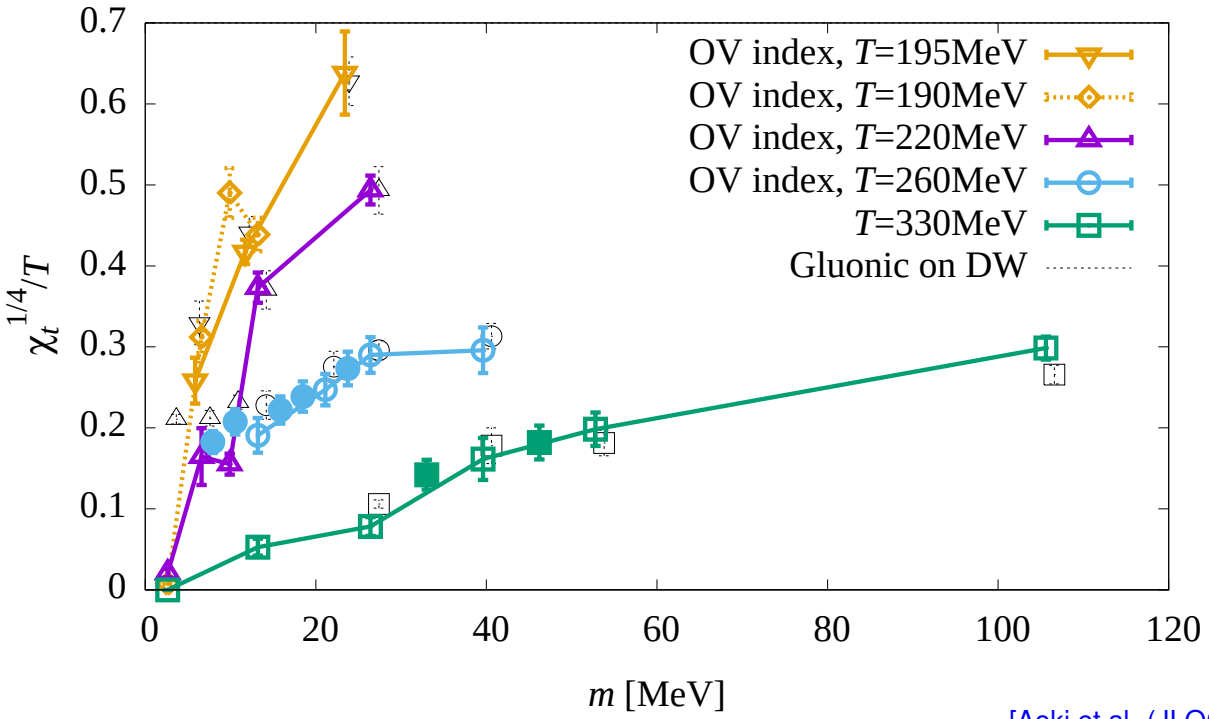

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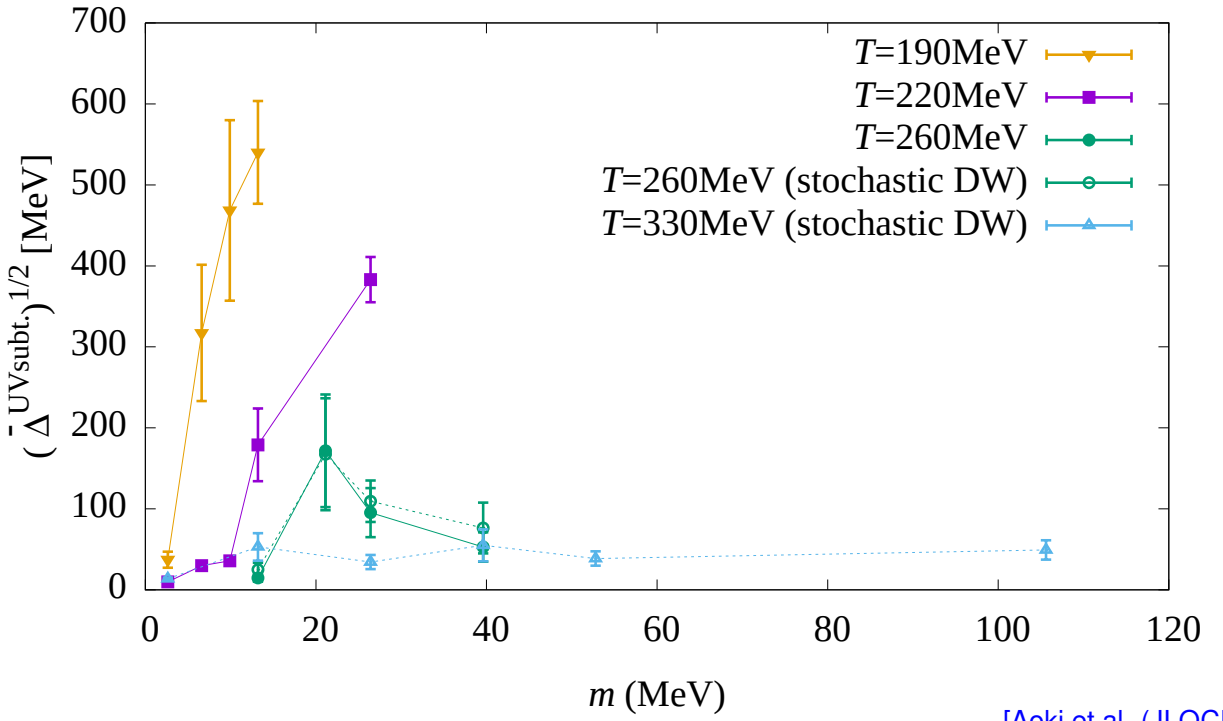


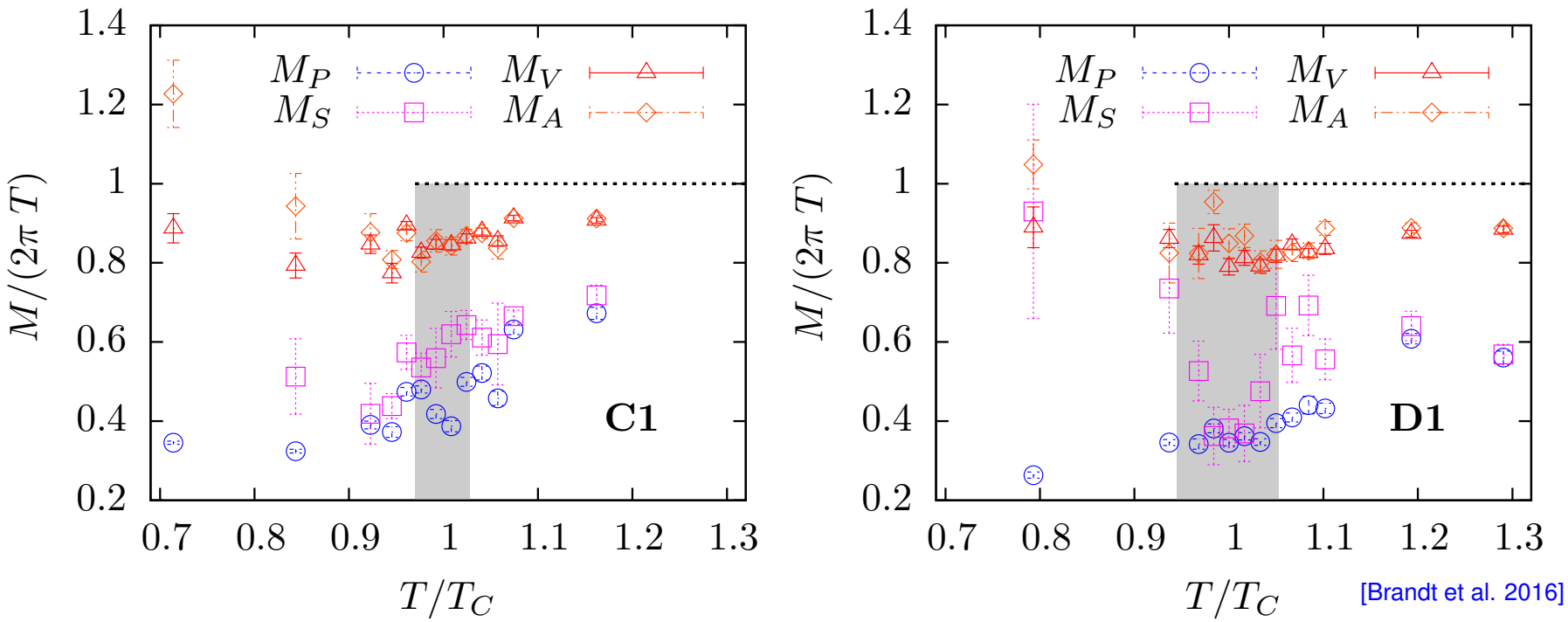
[Aoki et al. (JLQCD Collaboration) (2021)]

- π, δ **degeneration** around $T_c \sim 190 - 200$ MeV for this ensemble in the chiral limit

  $\hookrightarrow$ $U(1)_A$ **restoration** close to T_c in the chiral limit $\rightarrow$ $O(4) \times U(1)_A$ pattern

$U(1)_A$ partner degeneration

- Screening masses for $N_f = 2$, $m_\pi = \{200, 300\}$ MeV, Frankfurt/Mainz

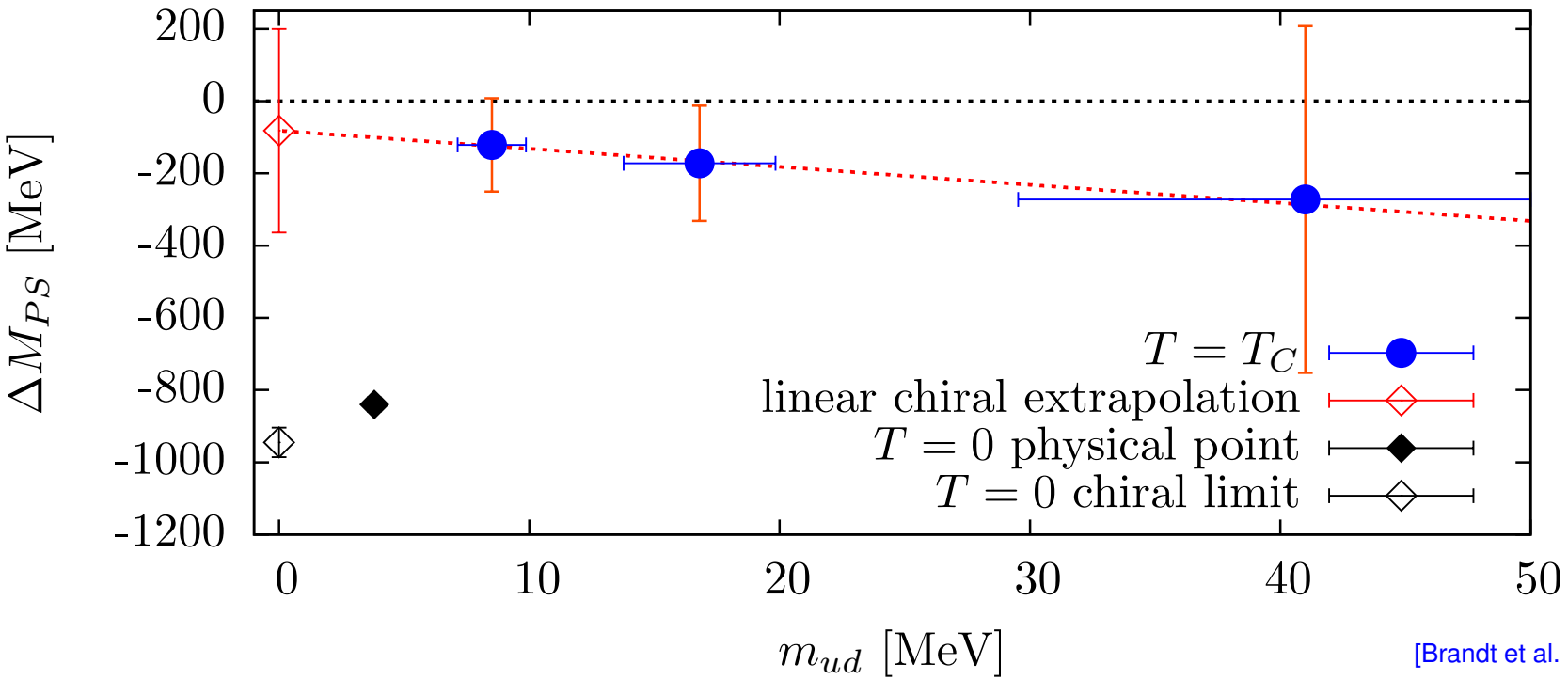


[Brandt et al. 2016]

- π, δ **degeneration** around $T \sim T_c$ compatible within uncertainties

$U(1)_A$ partner degeneration: $N_f = 2$

- Screening masses for $N_f = 2$, $m_\pi = \{200, 300\}$ MeV, Frankfurt/Mainz



[Brandt et al. 2016]

- π, δ **degeneration** around $T \sim T_c$ in the chiral limit

  $\hookrightarrow$ $U(1)_A$ **restoration** around $T_c \to O(4) \times U(1)_A$ pattern

ChPT and chiral symmetry restoration

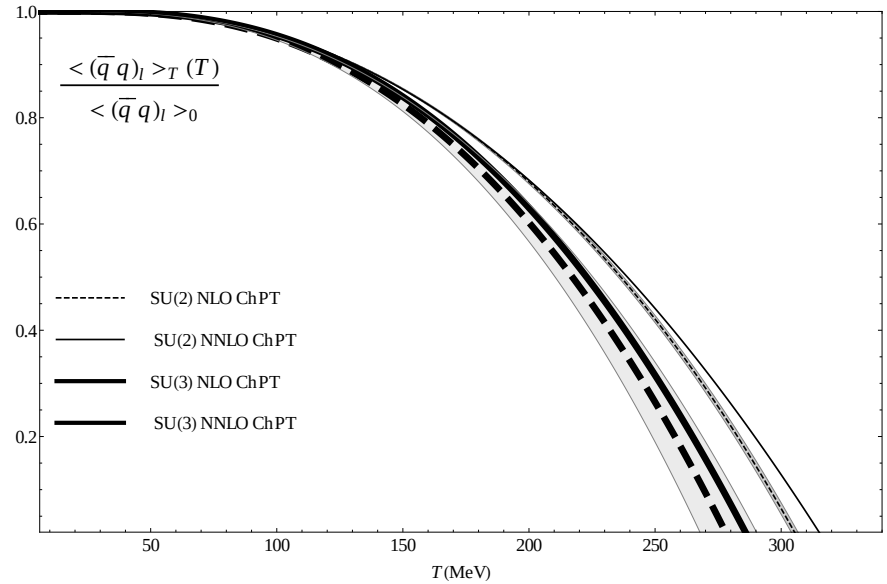
- **Most general theory** compatible with **QCD symmetries**
- **Degrees of freedom**: Goldstone Bosons of the spontaneous chiral symmetry breaking
- **Power counting**: based on the scale separation

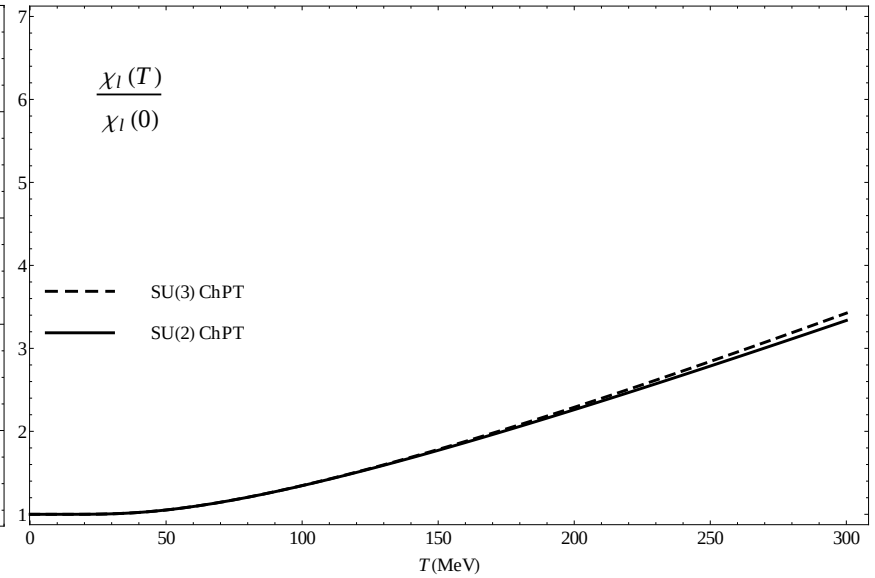
  $\hookrightarrow$ dimensional counting in p, m_q and T
- **Breakdown scale**: $\Lambda_{\chi l} = \dfrac{m_\pi^2}{(4\pi F_\pi)^2} \sim 0.014, \quad \Lambda_{\chi s} = \dfrac{m_k^2}{(4\pi F_\pi)^2} \sim 0.18$

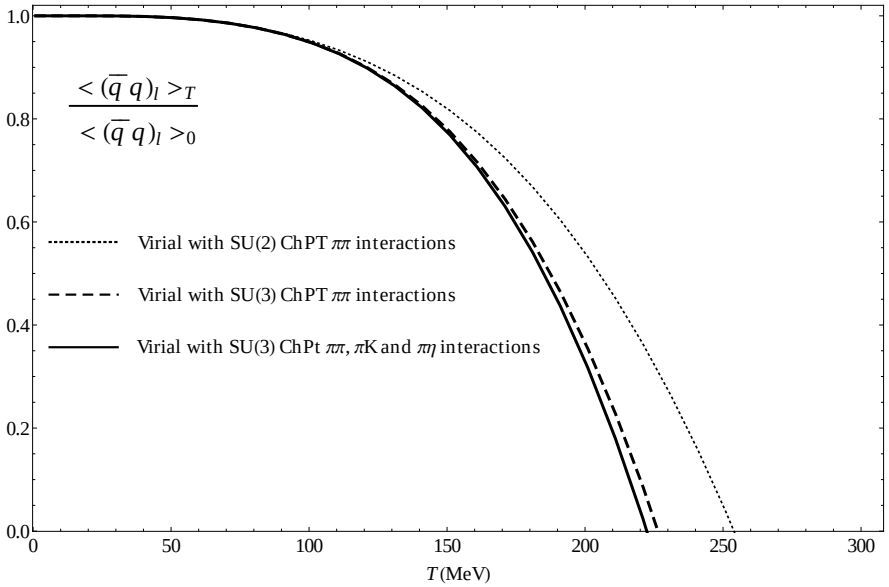
  $\hookrightarrow$ non-perturbative effects: resonances, **phase transition** $\cdots$
- What does **ChPT** say about the phase transition?

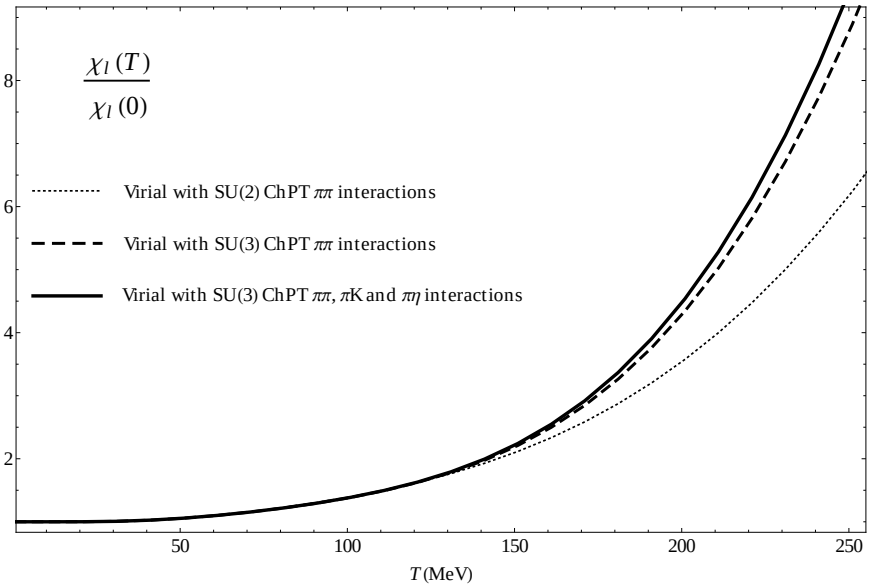
  $\hookrightarrow$ look at the ChPT prediction for the **quark condensate** and **scalar susceptibility**

ChPT and chiral symmetry restoration









[A. Gómez-Nicola, J. R. Peláez and JRE (2013)]

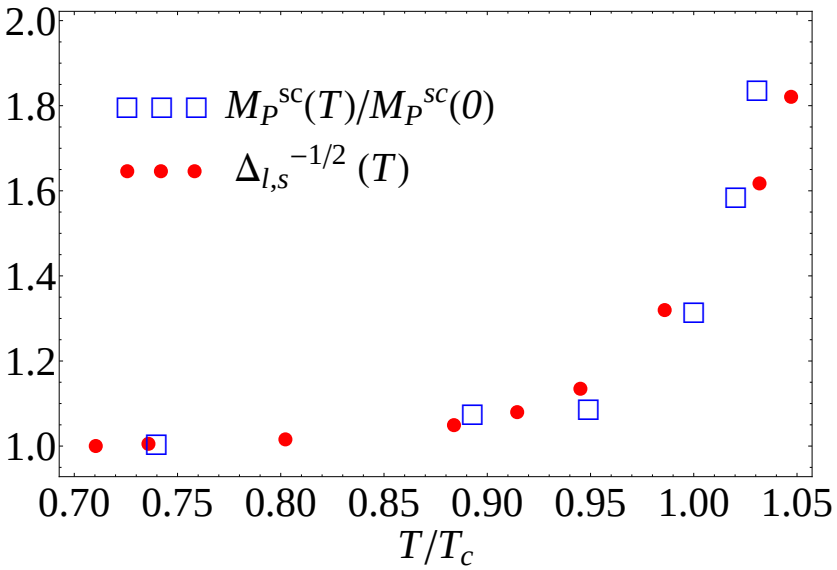
ChPT and partner degeneration

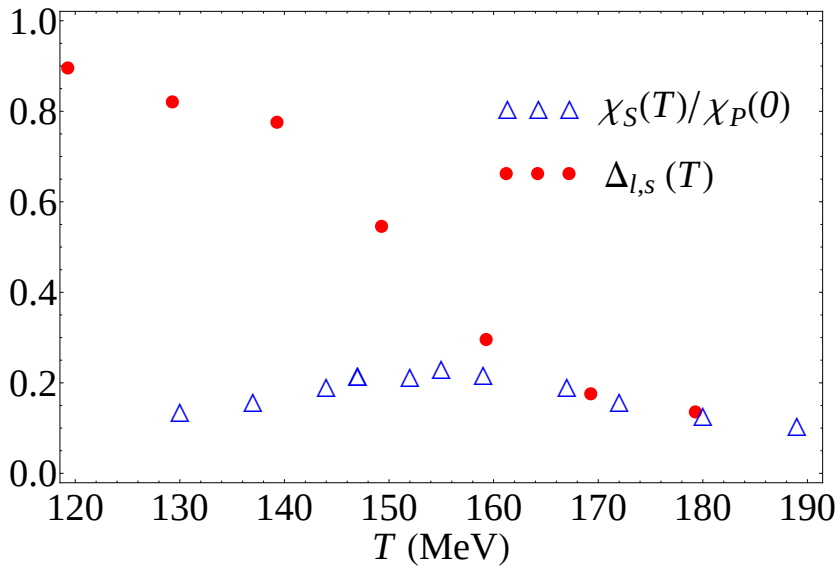
- Compare χ_π and χ_σ in ChPT

$$\chi_\pi^{\rm ChPT}(T) = -\frac{\langle \bar{q}q \rangle_T}{m_q} + \mathcal{O}(f_\pi^{-4})$$

[A. Gómez-Nicola, JRE and R. Torres-Andrés (2013)]

- Compare this relation in the lattice
  - ▷ $\chi_P(T)/\chi_P(0) \simeq (M_\pi^{sc}(0)/M_\pi^{sc}(T))^2$





[A. Bazavov et al. (HotQCD) (2009), M. Cheng et al. (HotQCD) (2011)]

Ward Identities

- Expected value of a local operator

$$\langle \mathcal{O}(y) \rangle = Z^{-1} \int [dG][d\bar{\psi}][d\psi] \mathcal{O}(y) e^{S_{QCD}}$$

- Consider an **axial transformation** on the fermion fields

$$\left\langle \frac{\delta \mathcal{O}(y)}{\delta \alpha_A^a(x)} \right\rangle = -\left\langle \mathcal{O}(y) \left[\bar{\psi}(x) \{ \frac{\lambda^a}{2}, \mathcal{M} \} \gamma_5 \psi(x) \right] \right\rangle + i \frac{\delta^{a0}}{\sqrt{6}} \langle \mathcal{O}(y) A(x) \rangle$$

with $A(x) = \frac{3g^2}{16\pi^2} \mathrm{Tr}_c G_{\mu\nu} \tilde{G}^{\mu\nu}$, the anomalous divergence of the $U(1)_A$ current

- For $O(y) = P^b(y) = i\,\bar{\psi}(y)\gamma_5 \lambda^b \psi(y)$

$$\frac{\delta P^b(y)}{\delta \alpha_A^a(x)} = -2\bar{\psi}(x) \left\{ \frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right\} \psi(x) \delta^{(4)}(x-y)$$

- Integrating over the Euclidean space

$$\left\langle \bar{\psi}(y) \left\{ \frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right\} \psi(y) \right\rangle = -\frac{1}{2} \int_T dx \left\langle P^b(y) \left[i\bar{\psi}(x) \{ \frac{\lambda^a}{2}, \mathcal{M} \} \gamma_5 \psi(x) \right] \right\rangle - \frac{1}{2} \int_T dx \left\langle P^b(y) A(x) \right\rangle \delta^{a\,0}$$

Ward Identities: susceptibilities and condensates

- π sector: $\langle \bar{q}q \rangle_l (T) = -m_q \, \chi_P^{\pi}(T)$
- K sector: $\langle \bar{q}q \rangle_l (T) + 2 \langle \bar{s}s \rangle (T) = -(m_q + m_s) \, \chi_P^{K}(T)$
- η and η' sectors: the $U(1)_A$ anomaly enters

$$\chi_P^{\eta_l}(T) = -\frac{\langle \bar{q}q \rangle_l (T)}{m_q} - \frac{4}{m_q^2} \chi_{top}(T)$$

$$\chi_P^{\eta_s}(T) = -\frac{\langle \bar{s}s \rangle (T)}{m_s} - \frac{1}{m_s^2} \chi_{top}(T)$$

$$\chi_P^{ls}(T) = -\frac{2}{m_q m_s} \chi_{top}(T)$$

- with

$$\chi_{top}(T) \equiv -\frac{1}{36} \chi_P^{AA}(T) = -\frac{1}{36} \int_T dx \langle \mathcal{T} A(x) A(0) \rangle$$

[A. Gómez-Nicola and JRE (2016)]

Chiral patterns and partners from WIs

- **Crossed ls** correlator **nonzero** due to η-η' **mixing** at T=0

$$\chi_P^{ls}(T) = -\frac{2}{m_q m_s}\chi_{top}(T)$$

Chiral patterns and partners from WIs

- **Crossed Is** correlator **nonzero** due to η-η' **mixing** at T=0

$$\chi_P^{ls}(T) = -\frac{2}{m_q m_s}\chi_{top}(T) = -\frac{m_q}{2m_s}\left(\chi_P^{\pi}(T) - \chi_P^{\eta_l}(T)\right)$$

Chiral patterns and partners from WIs

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Chiral patterns and partners from WIs

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where $\chi_{5,disc}(T) = \frac{1}{4}\left(\chi_P^{\pi}(T) - \chi_P^{\eta_l}(T)\right)$ measures of the $O(4) \times U(1)_A$

Chiral patterns and partners from WIs

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- In the O(4) phase:

$$\chi_P^{\eta_l} \overset{O(4)}{\sim} \chi_S^{\delta} \to \chi_P^{ls} \overset{O(4)}{\sim} 0$$

Chiral patterns and partners from WIs

- **Crossed ls** correlator **nonzero** due to η-η' **mixing** at T=0

$$\chi_P^{ls}(T) = -\frac{2}{m_q m_s}\chi_{top}(T) = -\frac{m_q}{2m_s}\left(\chi_P^{\pi}(T) - \chi_P^{\eta_l}(T)\right) = -\frac{2m_q}{m_s}\chi_{5,disc}(T)$$

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$$\chi_P^{\eta_l} \overset{O(4)}{\sim} \chi_S^{\delta} \to \chi_P^{ls} \overset{O(4)}{\sim} 0$$

- At exact O(4) restoration

$$\boxed{\chi_{5,disc} \overset{O(4)}{\sim} 0, \quad \chi_{top} \overset{O(4)}{\sim} 0}$$

Chiral patterns and partners from WIs

- **Crossed Is** correlator **nonzero** due to η-η' **mixing** at T=0

$$\chi_P^{ls}(T) = -\frac{2}{m_q m_s}\chi_{top}(T) = -\frac{m_q}{2m_s}\left(\chi_P^{\pi}(T) - \chi_P^{\eta_l}(T)\right) = -\frac{2m_q}{m_s}\chi_{5,disc}(T)$$

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- In the O(4) phase:

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- At exact O(4) restoration

$$\boxed{\chi_{5,disc} \overset{O(4)}{\sim} 0, \quad \chi_{top} \overset{O(4)}{\sim} 0}$$

$\hookrightarrow$ $O(4) \times U(1)_A$ **pattern**

[A. Gómez-Nicola and JRE (2017)]

Chiral patterns and partners from WIs in the physical case

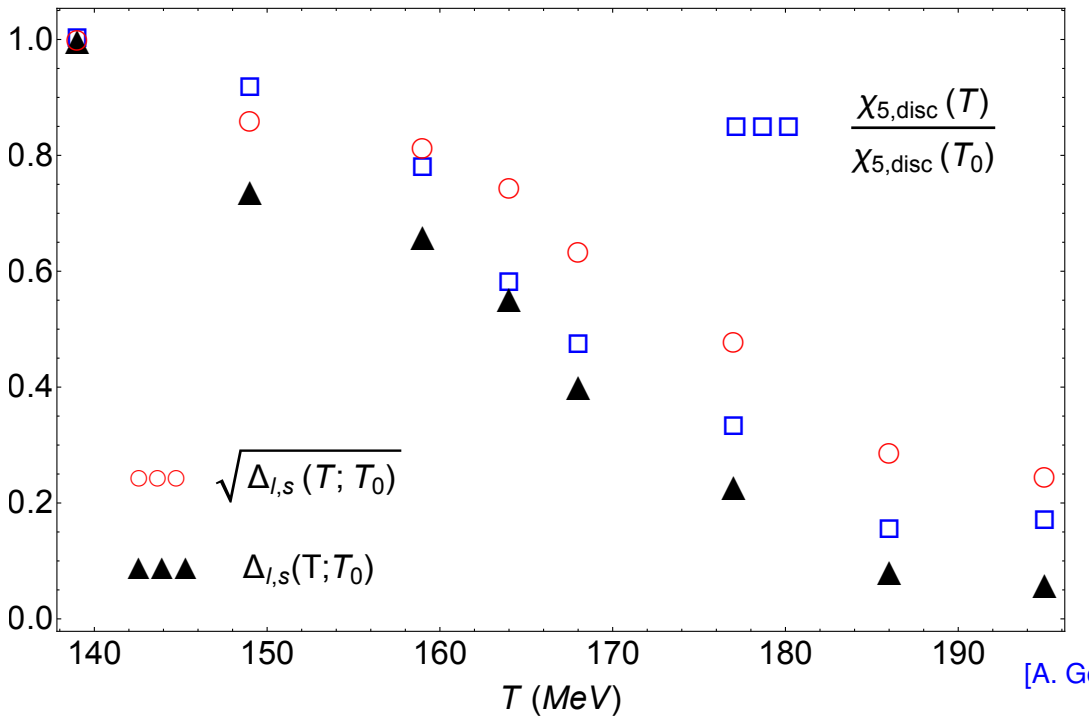
- **Physical case**: $N_f = 2 + 1$ and $m_q \neq 0$

  $\hookrightarrow$ no **exact** $\chi_P^{\eta_l} - \chi_S^{\delta}$ **degeneration**

- What is the effect of this breaking on $\chi_{5,disc}$?

  $\hookrightarrow$ let's compare with the condensate $\langle \bar{q}q \rangle_l$

data from Buchoff et al. (HotQCD Collaboration) (2014)

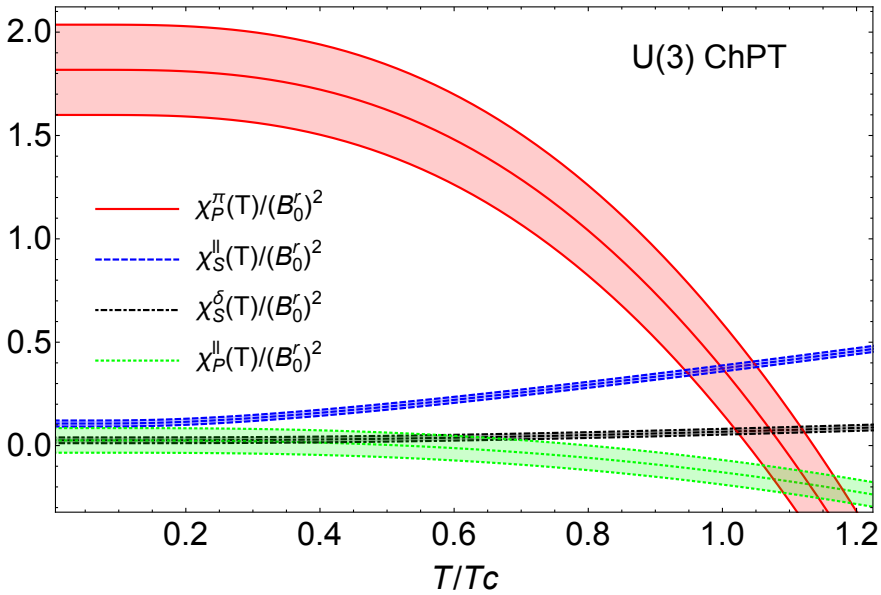


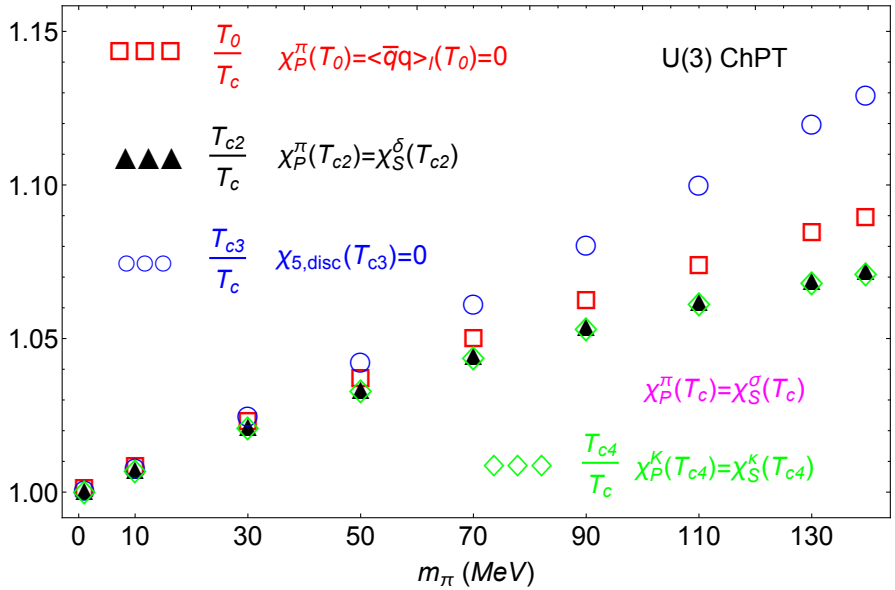
[A. Gómez-Nicola and JRE (2017)]

Chiral patterns and partners in ChPT

- How to study the **chiral limit**?
- **ChPT** provides a way to study the **dependence** on the **pion mass**

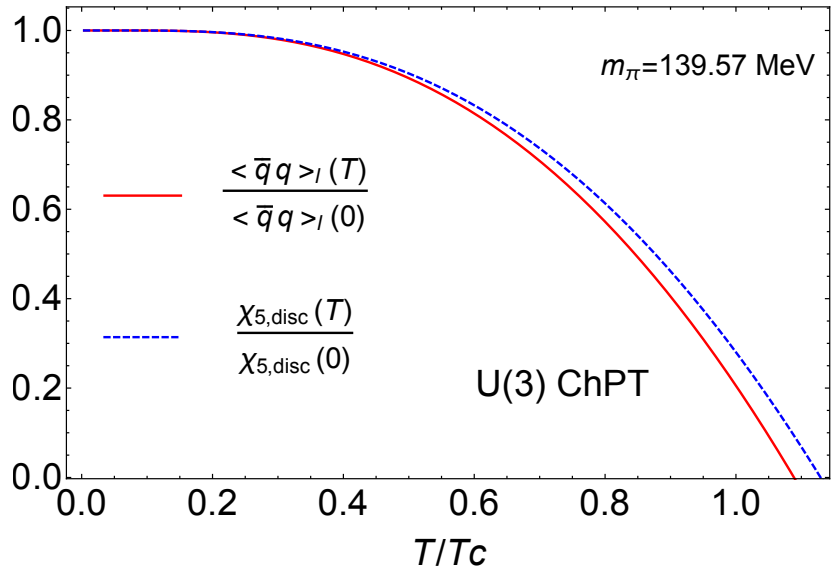
  ↪ use $U(3)$ **ChPT** to study χ_P^{ab} and χ_S^{ab}

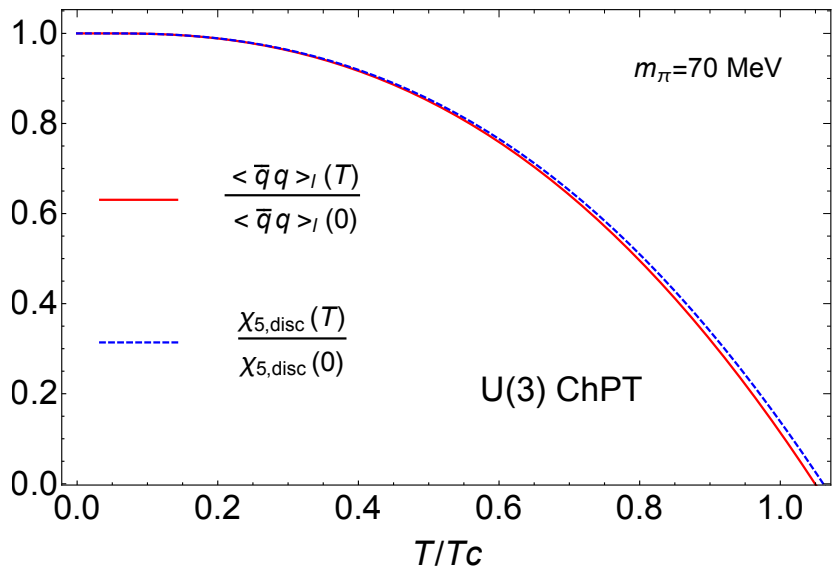


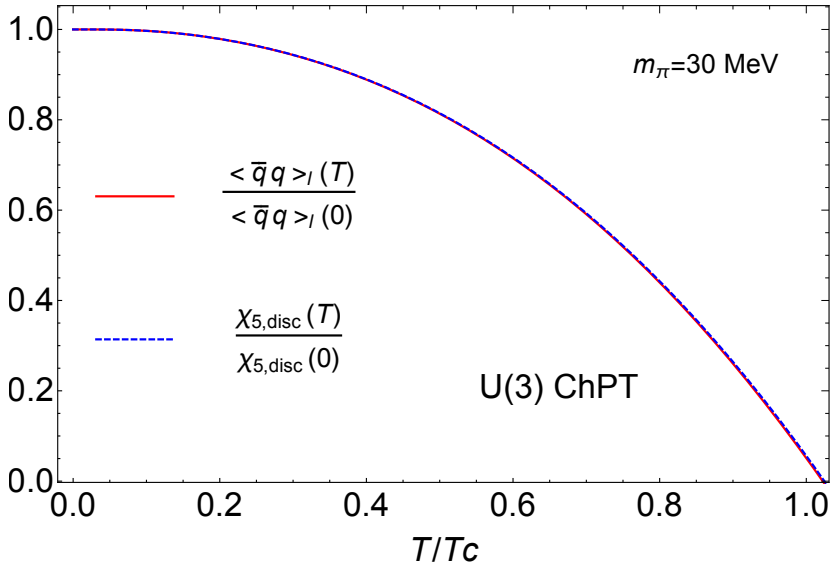


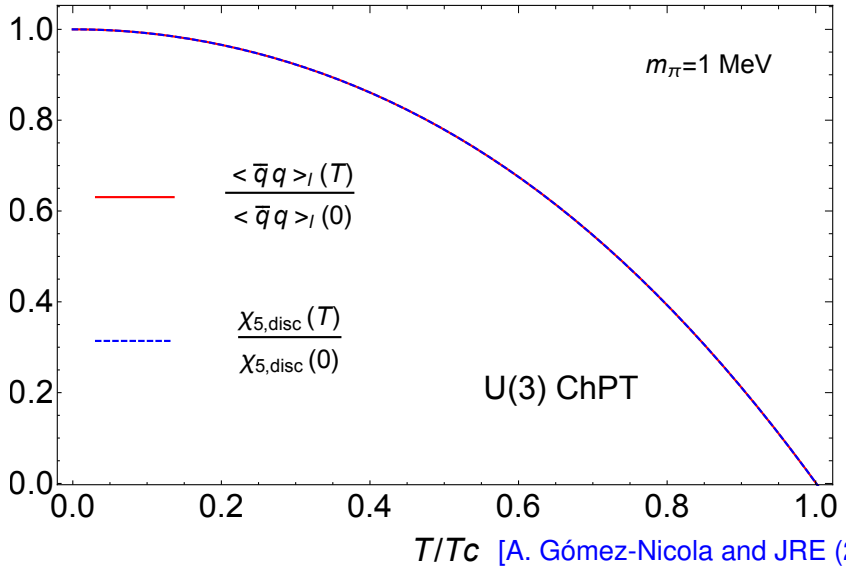
[A. Gómez-Nicola and JRE (2017)]

Chiral Patterns and Partners in ChPT









[A. Gómez-Nicola and JRE (2018)]

Chiral patterns and partners from WIs: $I = 1/2$

- I = 1/2 sector: K– κ degeneration $\rightarrow$ measures $O(4) \times U(1)_A$ **restoration**

$$\chi_P^K(T) = -\frac{\langle \bar{q}q \rangle_I(T) + 2\langle \bar{s}s \rangle(T)}{m_q + m_s}$$

- WIs for a vector transformations

$$\left\langle \frac{\delta \mathcal{O}_S(y)}{\delta \alpha_V^a(x)} \right\rangle = \left\langle \mathcal{O}_S(y)\bar{\psi}(x) \left[\frac{\lambda^a}{2}, \mathcal{M} \right] \psi(x) \right\rangle$$

- Non-trivial relation for $I = 1/2$ $\quad \chi_S^\kappa(T) = \dfrac{\langle \bar{q}q \rangle_I(T) - 2\langle \bar{s}s \rangle(T)}{m_s - m_q}$

- Combining

$$\chi_S^\kappa(T) - \chi_P^K(T) = \frac{2m_s}{m_s^2 - m_q^2} \left[\langle \bar{q}q \rangle_I(T) - 2\frac{m_q}{m_s} \langle \bar{s}s \rangle(T) \right] = \frac{2m_s}{m_s^2 - m_q^2} \Delta_{Is}(T)$$

- $\chi_S^\kappa(T) \overset{O(4)}{\sim} \chi_P^K(T)$ at exact chiral restoration

- $\chi_S^\kappa(T) \overset{O(4)\times U(1)_A}{\sim} \chi_P^K(T)$ in the physical case

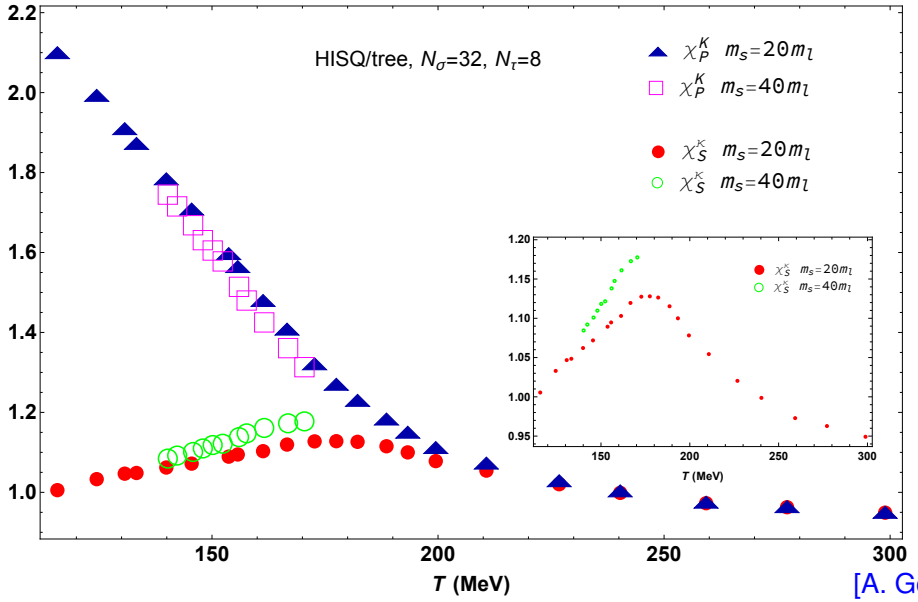
  $\hookrightarrow$ given by $\Delta_{Is}(T)$

[A. Gómez-Nicola and JRE (2017)]

Chiral Patterns and Partners from WIs: $I = 1/2$

- No $\chi_K - \chi_\kappa$ lattice data available

  $\hookrightarrow$ reconstruct them from WIs and lattice condensate data [Bazavov et al. (HotQCD) (2014)]



[A. Gómez-Nicola, JRE, A. Vioque (2021)]

- χ_κ shows a **peak** close to T_c, which is enhanced as m_l/m_s is reduced
- $\chi_K - \chi_\kappa|_{m_s \gg m_l} = 2\langle \bar{q}q \rangle_l / m_s + \mathcal{O}\left(m_s^{-2}\right)$

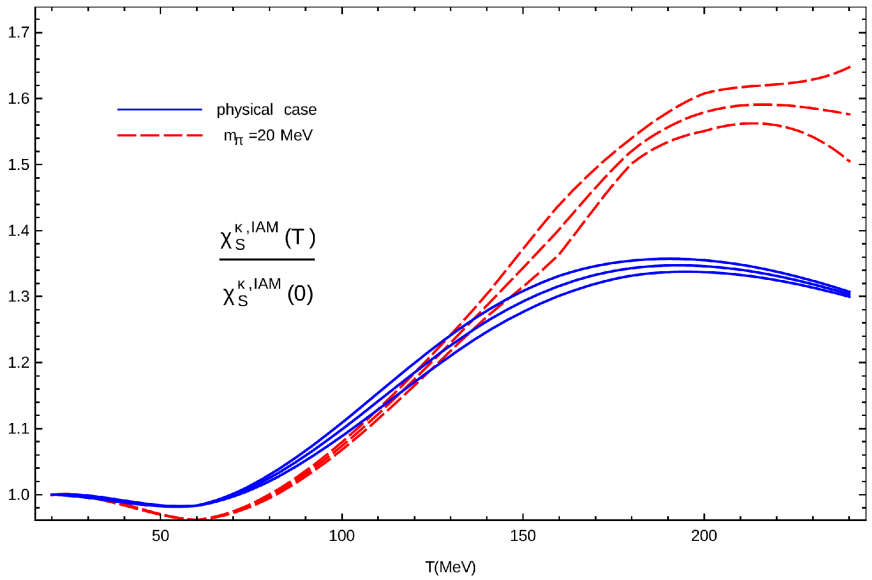
  $\hookrightarrow$ consistent with $N_f = 2$ simulations
- $O(4) \times U(1)_A$ **restoration** is driven by m_l/m_s and $\langle \bar{s}s \rangle$

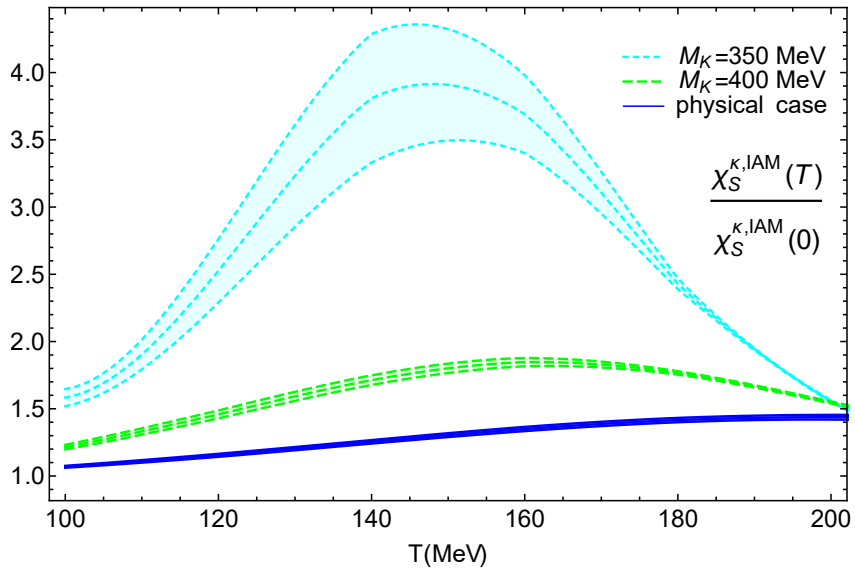
Chiral Patterns and Partners from WIs: $I = 1/2$

- More lattice data is needed
- Compute πK scattering in ChPT at finite temperature and unitarize using the IAM

[J. R. Peláez talk on Tuesday]

- χ_κ from $\kappa(700)$ resonance mass at finite T





[A. Gómez-Nicola, JRE, A. Vioque (2023)]

Conclusions

- WIs allow one to study **partner degeneration**

  $\hookrightarrow$ **benchmark** for lattice and model analyses

- WIs suggest a $O(4) \times U(1)_A$ pattern for **exact chiral restoration**

  $\hookrightarrow$ also suggested in ChPT

- In the physical case there is a **explicit** $O(4)$ **breaking**, distortion of $\delta - \eta_l$ degeneration

  WIs and ChPT suggest $\chi_{5,disc} \sim \Delta_{ls}$

- $I = 1/2$ channel provides additional information

  $\hookrightarrow$ $N_f = 2$ and $N_f = 2 + 1$ results reconciled in the $m_s \gg m_l$ limit.

  $\hookrightarrow$ m_l/m_s and $\langle \bar{s}s \rangle$ responsible of $O(4) \times U(1)_A$ **restoration**

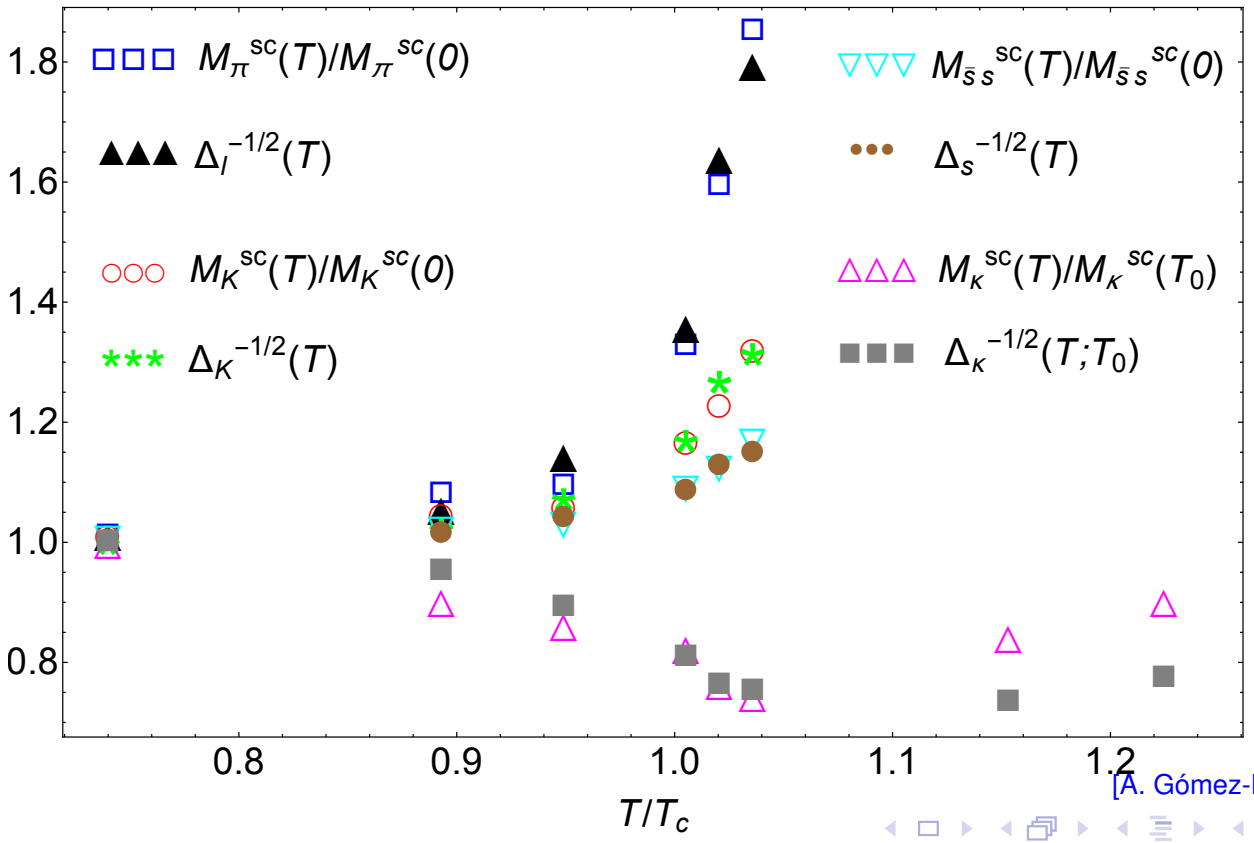
Spare slides

Ward identities and Screening masses

- $\chi_{P,S}(T) = K_{P,S}(p=0,T) \sim \dfrac{g_{P,S}(T)^2}{M^{pole}_{P,S}(T)^2}$

- Assuming soft T behavior for residues and M^{pole}/M^{sc}

  $\hookrightarrow$ WIs explain the T behavior of screening masses



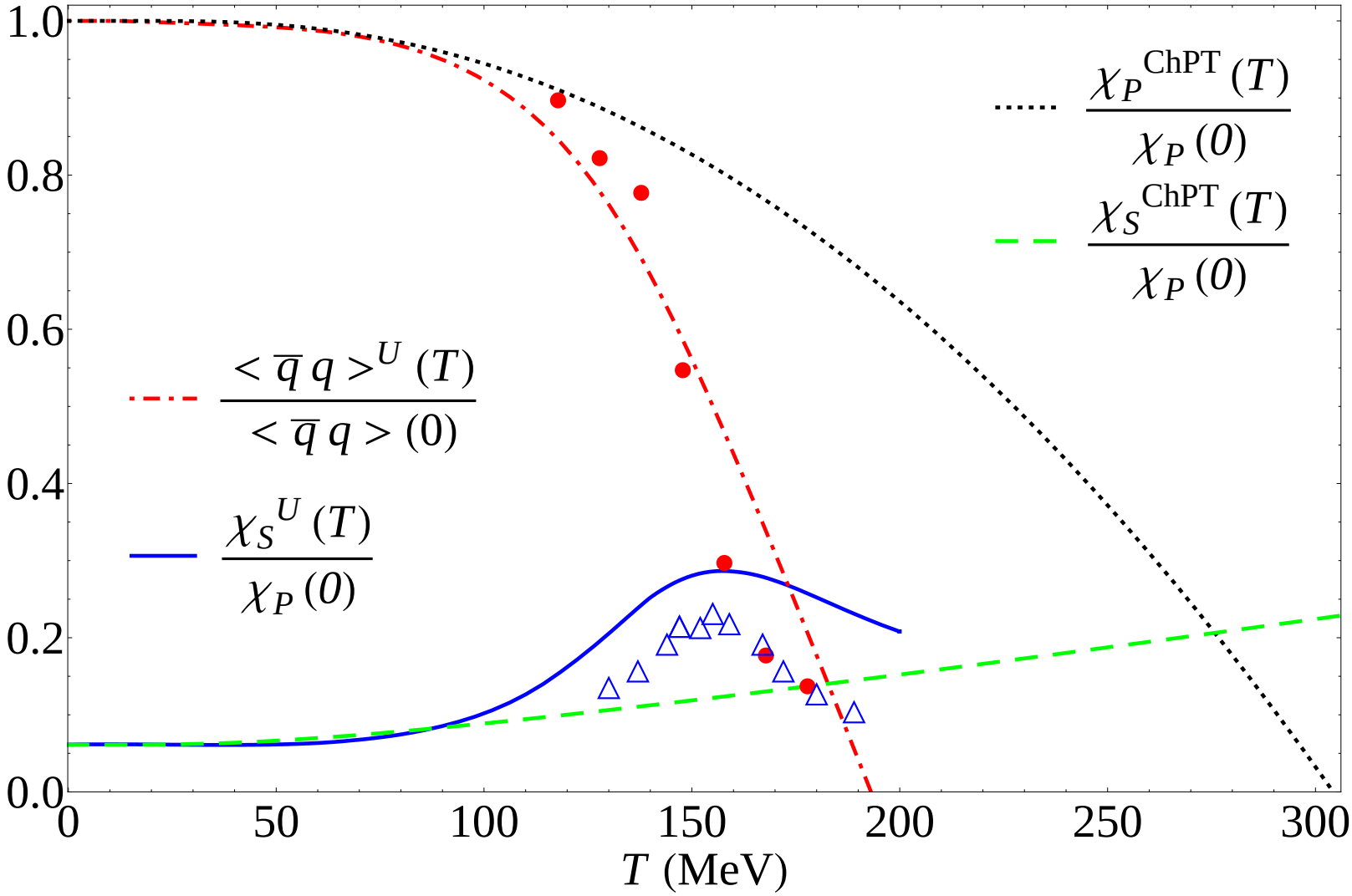
[A. Gómez-Nicola and JRE (2018)]

Ward identities for two-point functions

- Considering in the WI master formula a $\mathcal{O}^{ab}(y_1, y_2) = P^a(y_1)S^b(y_2)$

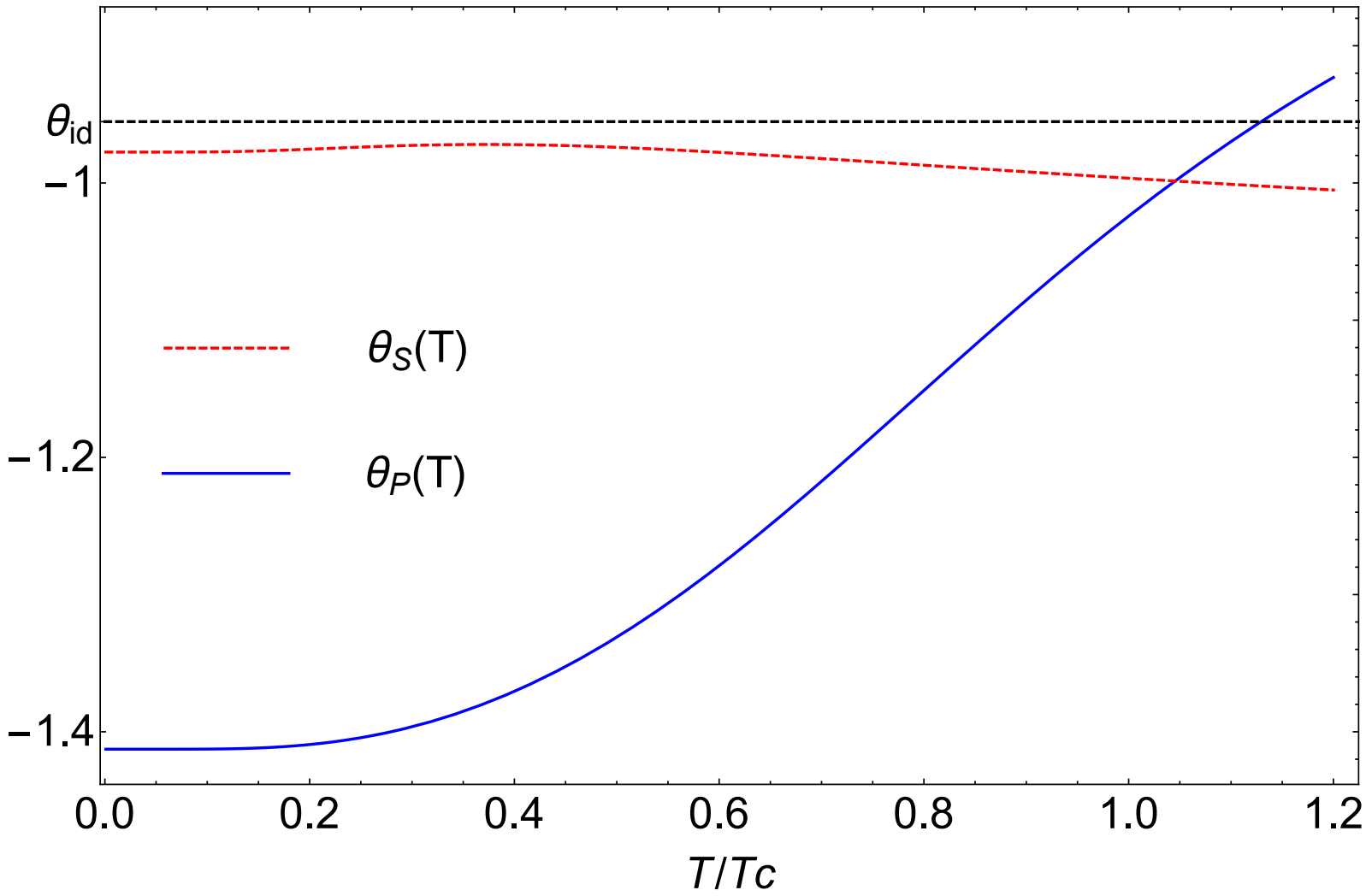
$$
\begin{aligned}
P_\pi(y) - S_\sigma(y) &= m_q \int_T dx\, \langle \mathcal{T}\sigma_l(y)\pi(x)\pi(0)\rangle \\
P_{\eta_l}(y) - S_\delta(y) &= m_q \int_T dx\, \langle \mathcal{T}\delta(y)\pi(x)\eta_l(0)\rangle \\
P_{ls}(y) &= \frac{1}{3}\hat{m} \int_T dx\, \langle \mathcal{T}\eta_s(y)\pi(x)\delta(0)\rangle \\
S_{ls}(y) &= -\frac{1}{3}\hat{m} \int_T dx\, \langle \mathcal{T}\sigma_s(y)\pi(x)\pi(0)\rangle \\
P_\pi(y) - S_\delta(y) &= \int_T dx\, \langle \mathcal{T}\pi(y)\delta(0)\tilde{\eta}(x)\rangle \\
P_{\eta_l}(y) - S_\sigma(y) &= \int_T dx\, \langle \mathcal{T}\eta_l(y)\sigma_l(0)\tilde{\eta}(x)\rangle \\
P_{ls}(y) - S_{ls}(y) &= \int_T dx\, \langle \mathcal{T}\eta_l(y)\sigma_s(0)\tilde{\eta}(x)\rangle \\
P_{ss}(y) - S_{ss}(y) &= \int_T dx\, \langle \mathcal{T}\eta_s(y)\sigma_s(0)\tilde{\eta}(x)\rangle
\end{aligned}
$$

Ward identities and Screening masses



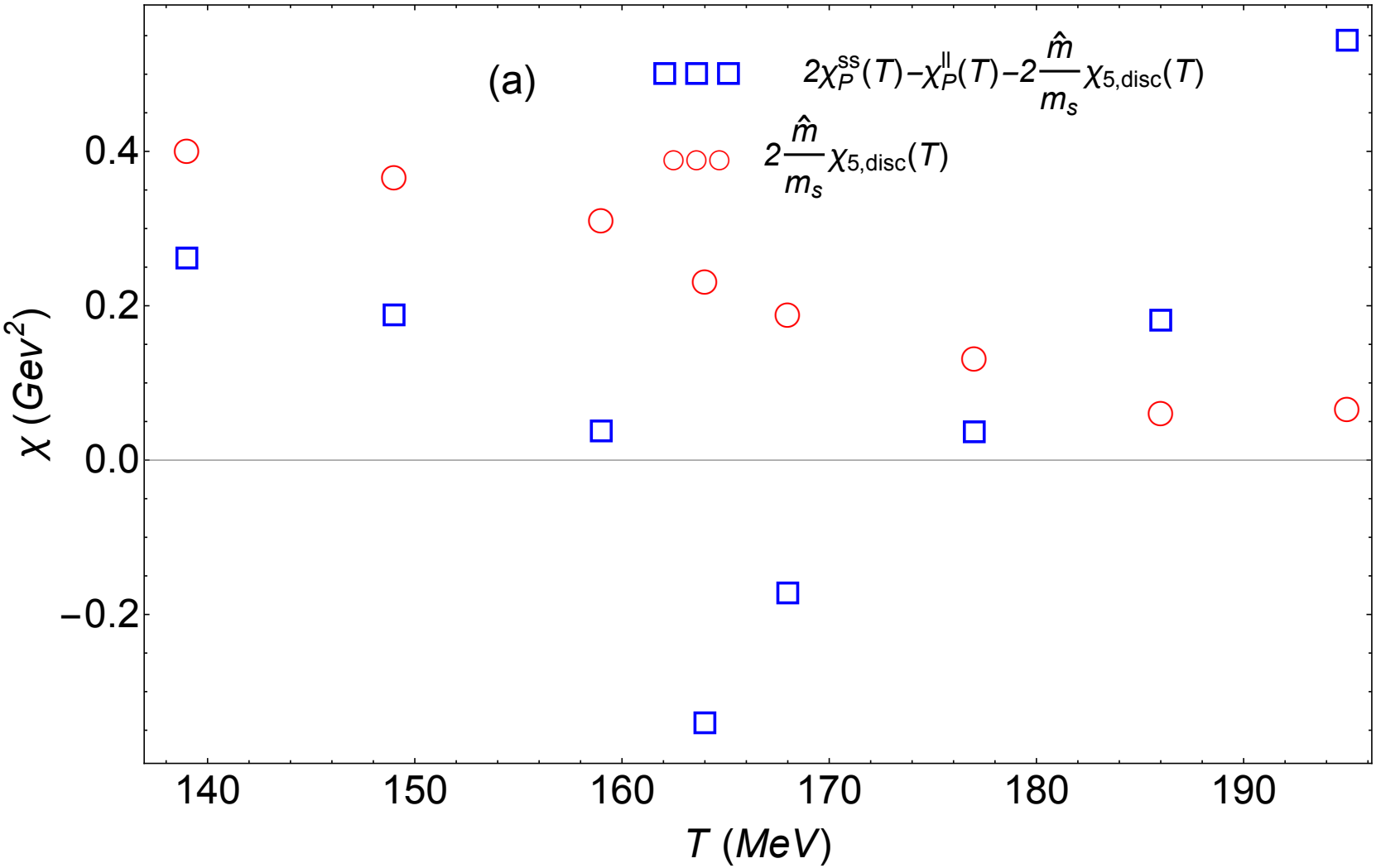
[A. Gómez-Nicola, JRE and R. Torres 2013]

Ward identities and Screening masses



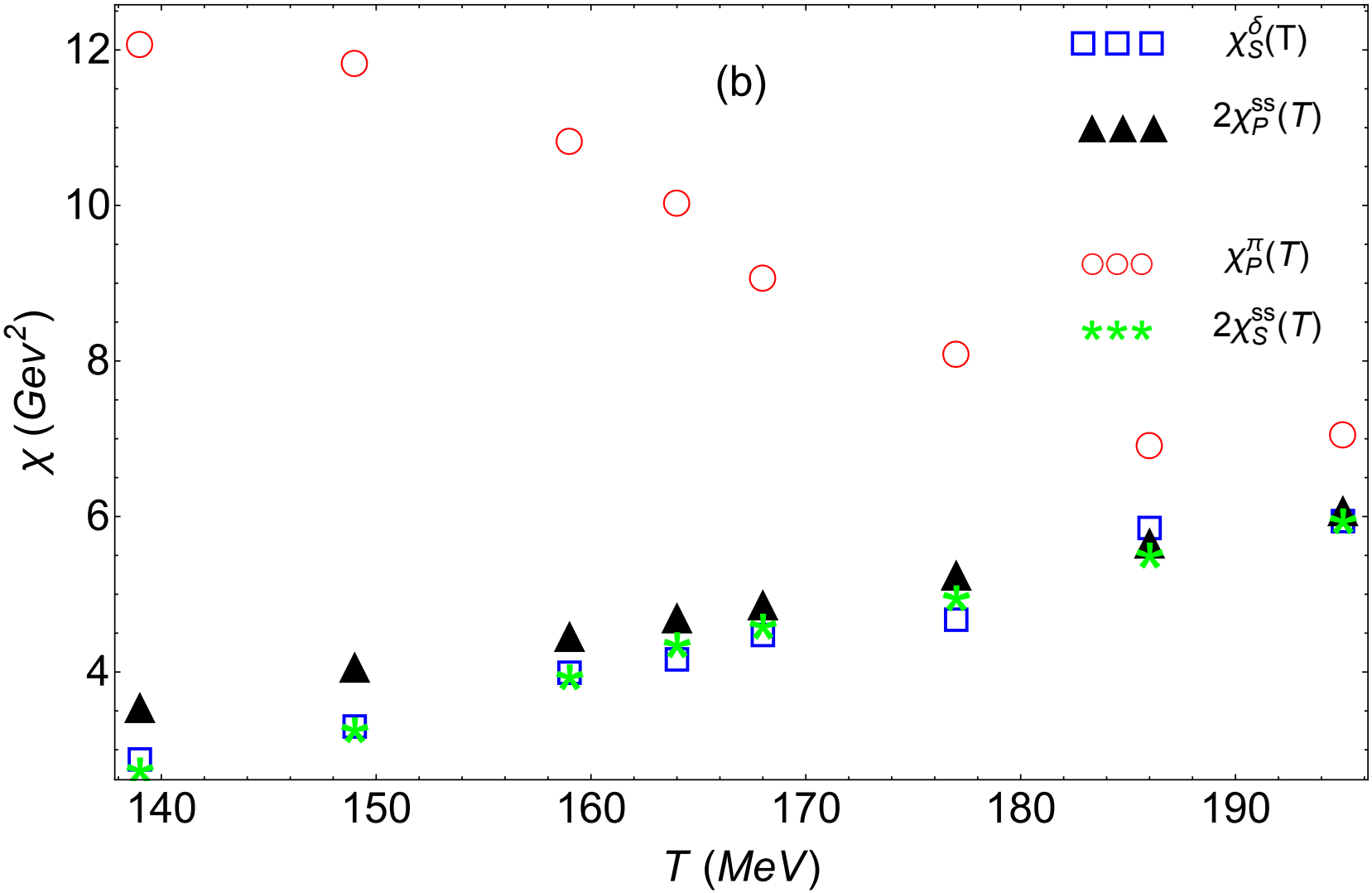
[A. Gómez-Nicola, JRE 2017]

Ward identities and Screening masses



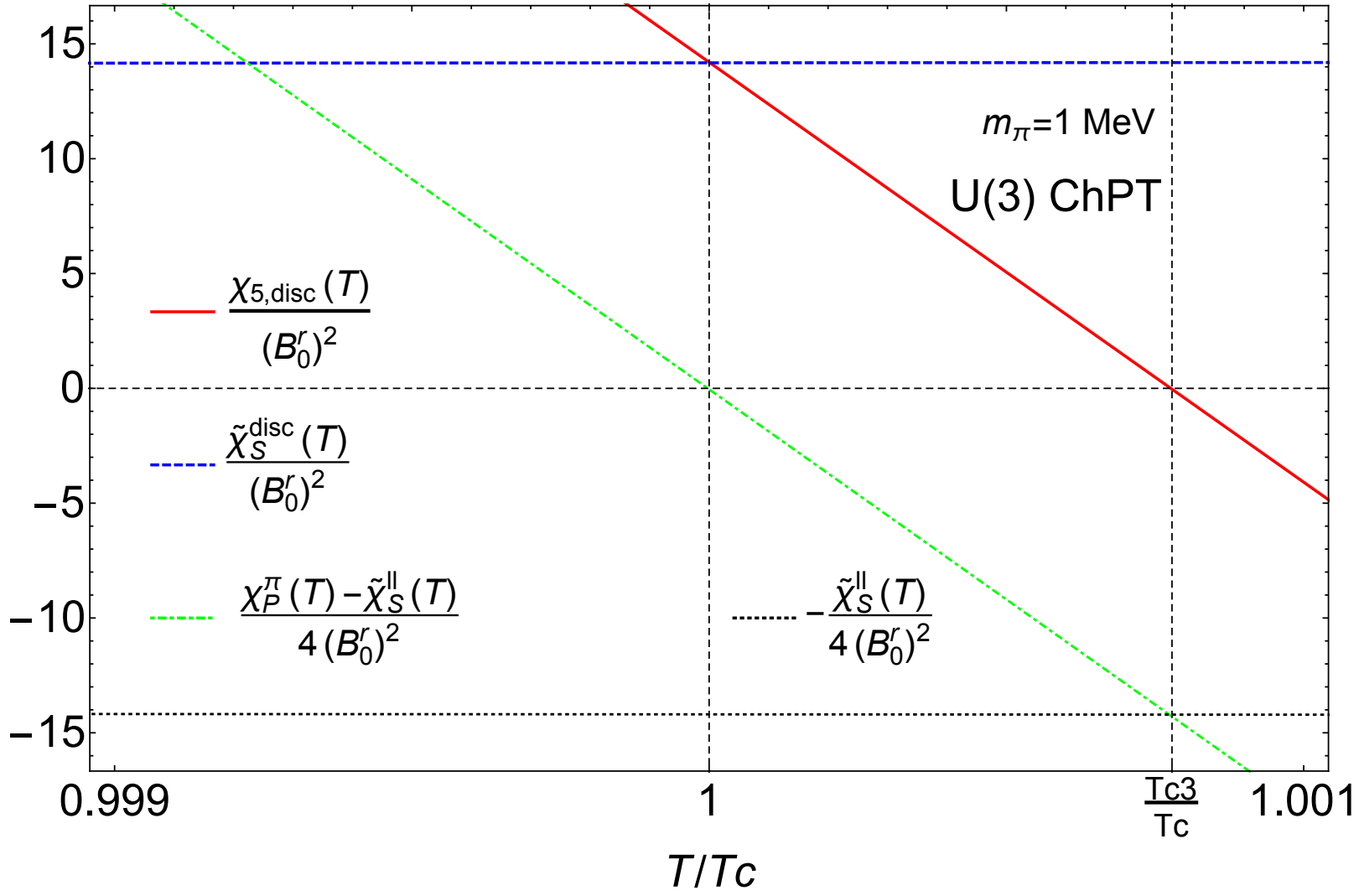
[A. Gómez-Nicola, JRE 2017]

Ward identities and Screening masses



[A. Gómez-Nicola, JRE 2017]

Ward identities and Screening masses



[A. Gómez-Nicola, JRE 2017]