INSTITUTO GALEGO DE FÍSICA DE ALTAS ENERXÍAS
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Jet propagation in the Glasma phase in a Light-Front Hamiltonian formalism

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Motivation and introduction

Develop a **new formalism** allowing for a **rigorous quantum treatment** of particle evolution in the Glasma phase

Show that we can **reproduce the classical results** for momentum broadening and the equivalence between both approaches

Include purely quantum effects that can not be properly capture in the classical treatment of jets

NEXT GOAL!

Motivation and introduction

Studies using EKRT+hydro+BDMPS-Z suggest that energy **loss must be suppressed** in the initial stage of heavy-ion collisions [Andres, Armesto, Niemi, Paatelainen, Salgado (2020)]

Classical jet in Glasma studies show that $\hat{q}$ is **very large** [Ipp, Müller, Schuh (2020)] but it is **difficult to include energy loss** via quantum gluon radiation

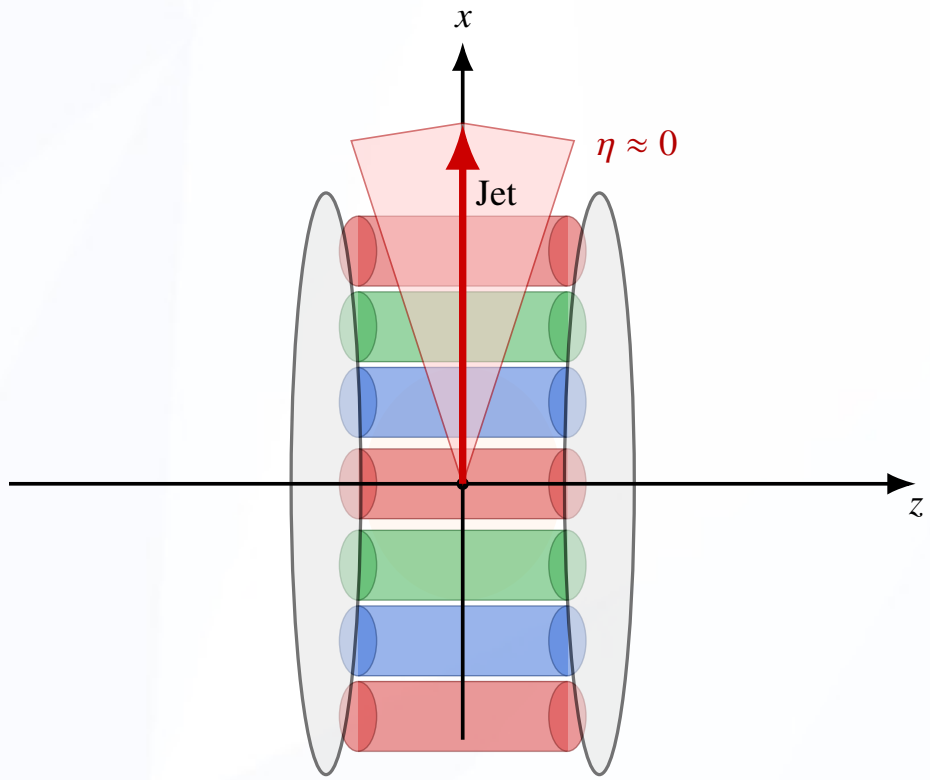
Our formalism will allow to **consistently include gluon radiation** and hopefully give fundamental information on this puzzle

The setup

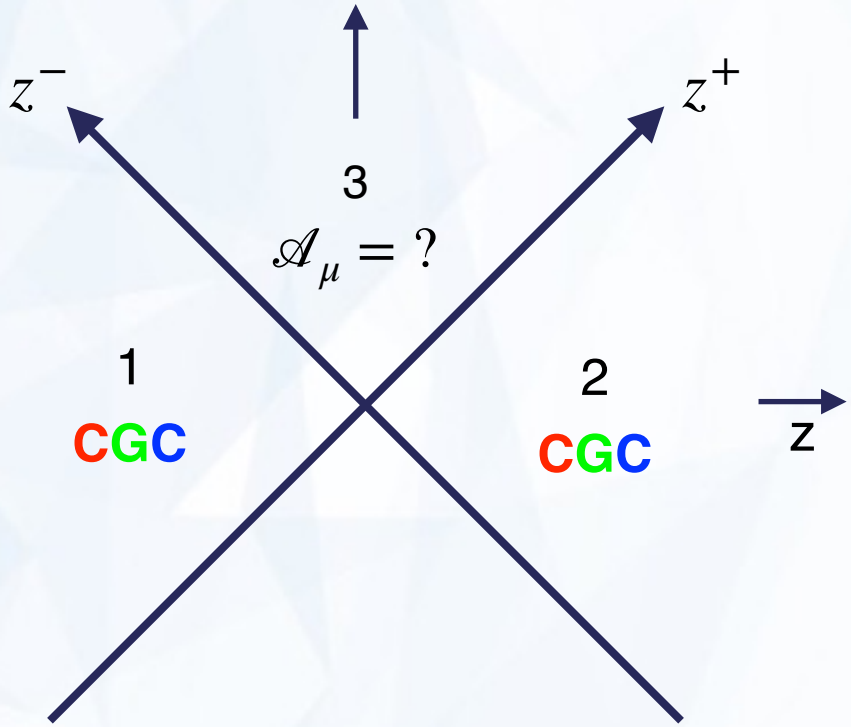
Heavy-ion collision in the z-direction

Mid-rapidity jet in the x-direction

The standard notation calls “transverse direction” to the y-direction and “longitudinal direction” to the z-direction, but both of them are perpendicular w.r.t the jet



The Glasma fields



We work on the **MV model**, where the relevant scale in the **saturation scale** Q_S

High-energy nuclei before the collision can be **modeled using Color Glass Condensate** (**CGC**)

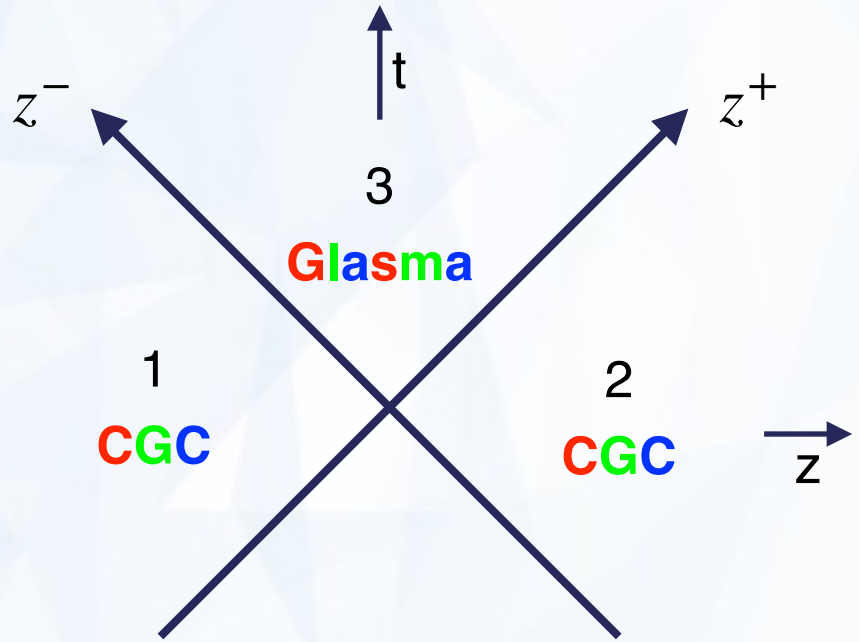
Based on scale separation

- Large x, static color charges
- Small x, classical color fields

Must follow **classical Yang-Mills** equations

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

The Glasma fields



We use **real-time lattice gauge theory** to numerically solve Yang-Mills equation [Kransnitz, Venugopalan (2019)]

Initial condition for the fields after the collision can be derived from the **CGC**

[Kovner, McLerran, Weigert (1995)]

Classical fields evolve with **sourceless Yang-Mills equation** in region (3)

$$[D_\mu, F^{\mu\nu}] = 0$$

↓

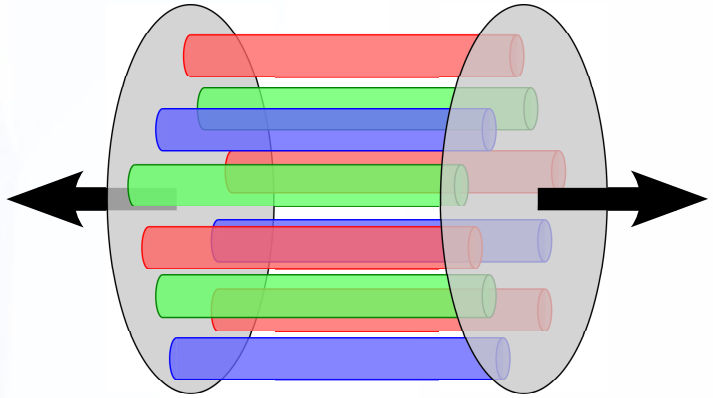
Boost-invariant classical **Glasma** fields

The Glasma fields

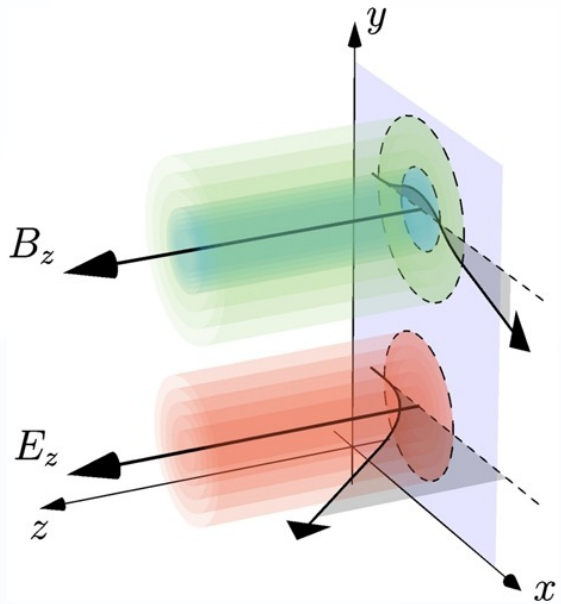
The Glasma potential $\mathcal{A}^{\mu}$ generates **chromo-electric and chromo-magnetic fields**

This fields are organized in **flux tubes of diameter** $\sim 1/Q_s$ that expand along the longitudinal direction

Initially the chromo-electric and chromo-magnetic fields **only have components along the longitudinal direction**



[Fukushima (2016)]



[Ipp, Müller, Schuh (2020)]

Classical momentum broadening

The dynamics of a classical particle in a color field are given by the **Wong's equations** [Wong (1970)]

$$\frac{dp^{\mu}_{kin}}{d\tau} = gQ^{a}(\tau)F^{\mu\nu,a}(\tau)\frac{dx_{\nu}}{d\tau} \qquad \frac{dQ^{a}}{d\tau} = -gf^{abc}Q^{c}(\tau)\mathcal{A}^{b}_{\mu}(\tau)\frac{dx^{\mu}}{d\tau}$$

The solution for the charge is a color rotation

$$Q(\tau) = \mathcal{U}(\tau,0)\,Q(0)\,\mathcal{U}^{\dagger}(\tau,0)$$

where

$$\mathcal{U}(\tau;0) = \mathcal{P}\exp\left(-ig\int_{0}^{\tau}d\tau'\frac{dx^{\mu}}{d\tau'}\mathcal{A}_{\mu}(\tau')\right)$$

Classical color charges satisfy

$$\int dQQ^{a}Q^{b} = T_{R}\delta^{ab}$$

Classical momentum broadening

Classical momentum broadening is given by the solution to the first Wong equation [Ipp, Müller, Schuh (2020)]

$$\langle (\delta p_i^{kin}(\tau))^2 \rangle_Q = g^2 \int_0^\tau d\tau' \int_0^\tau d\tau'' \, \mathrm{tr} \left[\tilde{f}_\mu(\tau') \tilde{f}_\mu(\tau'') \right]$$

where $\tilde{f}_\mu(\tau) = \mathcal{U}(\tau;0) f_\mu(\tau) \mathcal{U}^\dagger(\tau;0)$ is the parallel transported **Lorentz force**

$$f_y = E_y - B_z \qquad f_z = E_z + B_y$$

The quark propagation Hamiltonian

Minimal coupling Lagrangian

$$\mathscr{L} = \overline{\Psi}(x)(i\gamma^{\mu}D_{\mu} - m)\Psi(x)$$

Legendre transform

$D_{\mu} = \partial_{\mu} + ig\mathscr{A}_{\mu}$ where $\mathscr{A} = \mathscr{A}^{a}t^{a}$ is the **classical Glasma background field**

There are no quantum gluons because we are **truncating the quark Fock space to the $|q>$ sector**, not considering radiation

$$P^{-} = \int dx^{-}d^{2}\vec{x}_{\perp}\left[\frac{1}{2}\bar{\Psi}\gamma^{+}\frac{m^{2}-\nabla^{\perp 2}}{i\partial^{+}-g\mathscr{A}^{+}}\Psi + g\bar{\Psi}(\gamma^{+}\mathscr{A}_{+}+\gamma^{i}\mathscr{A}_{i})\Psi + \frac{g^{2}}{2}\bar{\Psi}\gamma^{i}\mathscr{A}_{i}\frac{\gamma^{+}}{i\partial^{+}-g\mathscr{A}^{+}}\gamma^{j}\mathscr{A}_{j}\Psi\right]$$

We work on the **light-front from**, where $x^{+} = t + x$ plays the role of the time-evolution variable

The quark propagation Hamiltonian

Quantum many-body representation

$$P^- = \frac{m^2 + (\vec{p}_\perp - g\vec{\mathcal{A}}_\perp)^2}{p^+ - g\mathcal{A}^+} + 2g\mathcal{A}^- - g\frac{B^x S^x}{p^+ - g\mathcal{A}^+}$$

Quark **kinetic energy**

Interaction potential

Quark spin coupling to the chromomagnetic field, **Zeeman effect**

Formally reduced to the propagation of a non-relativistic particle of mass $p^+ - g\mathcal{A}^+$ in the 2-dimensional transverse plane

Basis representation

Employ a **basis of eigenstates of the free-Hamiltonian** to construct the Hilbert space

$$\frac{m^2+\vec{p}_\perp^{\,2}}{p^+}|\beta\rangle = P_\beta^-|\beta\rangle \quad \Rightarrow \quad |\psi; x^+> = \sum_\beta c_\beta(x^+)|\beta>$$

where $|\beta\rangle = |p^+, \vec{p}_\perp, c, \lambda\rangle$

Discretize the transverse space in a lattice of size $2L_\perp$ and $2N_\perp$ sites, so the spacing is $a_\perp = L_\perp/N_\perp$

Momentum modes are then discretized $p_i = k_i d_p$ with $d_p = \pi/L_\perp$ and $k_i = -N_\perp, \ldots, N_\perp$

Time evolution

$$i\frac{\partial}{\partial x^+}|\psi, x^+\rangle = \frac{P^-}{2}|\psi; x^+\rangle \qquad \lim_{p^+\to\infty} P^- = 2g\mathscr{A}^-$$

In terms of the basis coefficients and matrix elements $\mathscr{M}_{\beta\beta'}(x^+) \equiv \langle\beta|P^-|\beta'\rangle$

$$\mathbf{c}(x^+) = \mathscr{T}_+ \exp\left[-\frac{i}{2}\int_0^{x^+} ds\mathscr{M}(s)\right]\mathbf{c}(0)$$

The **expectation value** of any quantum operator is then

$$\langle\mathcal{O}(x^+)\rangle = \langle\psi; x^+|\mathcal{O}(x^+)|\psi; x^+\rangle = \sum_{\beta\beta'} c_\beta(x^+)c_{\beta'}(x^+)\mathcal{O}_{\beta\beta'}(x^+)$$

Time evolution

In some cases it will be useful to work in the **Heisenberg picture**

$$\mathcal{O}_H(x^+) = U^\dagger(x^+;0)\mathcal{O}(x^+)U(x^+;0) \qquad |\psi\rangle_H = |\psi;0\rangle$$

Time evolution is then contained in the operators

$$\frac{d\mathcal{O}_H}{dx^+} = i\left[\frac{1}{2}P^-, \mathcal{O}_H\right] + \left(\frac{\partial\mathcal{O}}{\partial x^+}\right)_H$$

The **expectation value** of operators is obtained in the same way

$$\langle\mathcal{O}(x^+)\rangle = \langle\psi;0|\,\mathcal{O}_H(x^+)\,|\psi;0\rangle$$

Quantum momentum broadening

The quantum operator representing the momentum of the particle can be easily obtained in the Heisenberg picture

$$\vec{p}^{\,kin}_{\perp,H} = (p^+ - g\mathscr{A}^+)\frac{d\vec{x}_\perp}{dx^+} = i(p^+ - g\mathscr{A}^+)\left[\frac{P^-_H}{2}, \vec{x}_{\perp,H}\right] = \vec{p}_\perp - g\vec{\mathscr{A}}_\perp(x^+, \vec{x}_\perp)$$

The physical kinetic momentum of the particle inside the medium is different to the canonic momentum conjugated to the position in the Lagrangian!!!

To quantify momentum broadening we define the operator

$$(\delta p^{kin}_{i,H}(x^+))^2 = \left(p^{kin}_{i,H}(x^+) - p^{kin}_{i,H}(0)\right)^2 = \left(p^{kin}_{i,H}(x^+)\right)^2 + \left(p^{kin}_{i,H}(0)\right)^2 - 2\Re p^{kin}_{i,H}(x^+)p^{kin}_{i,H}(0)$$

Quantum momentum broadening, direct calculation

We can directly compute

$$(\delta p_{i,H}^{kin}(x^+))^2 = \left(p_{i,H}^{kin}(x^+)\right)^2 + \left(p_{i,H}^{kin}(0)\right)^2 - 2\Re p_{i,H}^{kin}(x^+)p_{i,H}^{kin}(0)$$

First two terms are trivial to compute in the **Schrödinger picture**

$$\left\langle \left(p_{i,H}^{kin}(x^+)\right)^2 \right\rangle = \langle \psi; x^+ | p_i^{kin}(x^+)p_i^{kin}(x^+) | \psi; x^+ \rangle \qquad \left\langle \left(p_{i,H}^{kin}(0)\right)^2 \right\rangle = \langle \psi; 0 | p_i^{kin}(x^+)p_i^{kin}(x^+) | \psi; 0 \rangle$$

The cross term requires some extra work. Defining $|\tilde{\psi}_i; 0\rangle \equiv p_i^{kin}(0)|\psi; 0\rangle$

$$\langle p_{i,H}^{kin}(x^+)p_{i,H}(0)\rangle = \langle \psi; x^+ | p_i^{kin}(x^+) | \tilde{\psi}_i; x^+ \rangle$$

Quantum momentum broadening, Lorentz force calculation

To observe explicitly the similarities with the classical calculation, one can compute the **quantum observable associated to the Lorentz Force**

$$\vec{f}_{\perp,H}(x^+) = \frac{d\vec{p}^{\,kin}_{\perp,H}}{dx^+} = i\left[\frac{1}{2}P^-, \vec{p}^{\,kin}_{\perp,H}\right] + \left(\frac{\partial \vec{p}^{\,kin}_{\perp}}{\partial x^+}\right)_H = gU^\dagger(x^+,0)F_{i+}(x^+)U(x^+,0)$$

Integrating and squaring we get momentum broadening

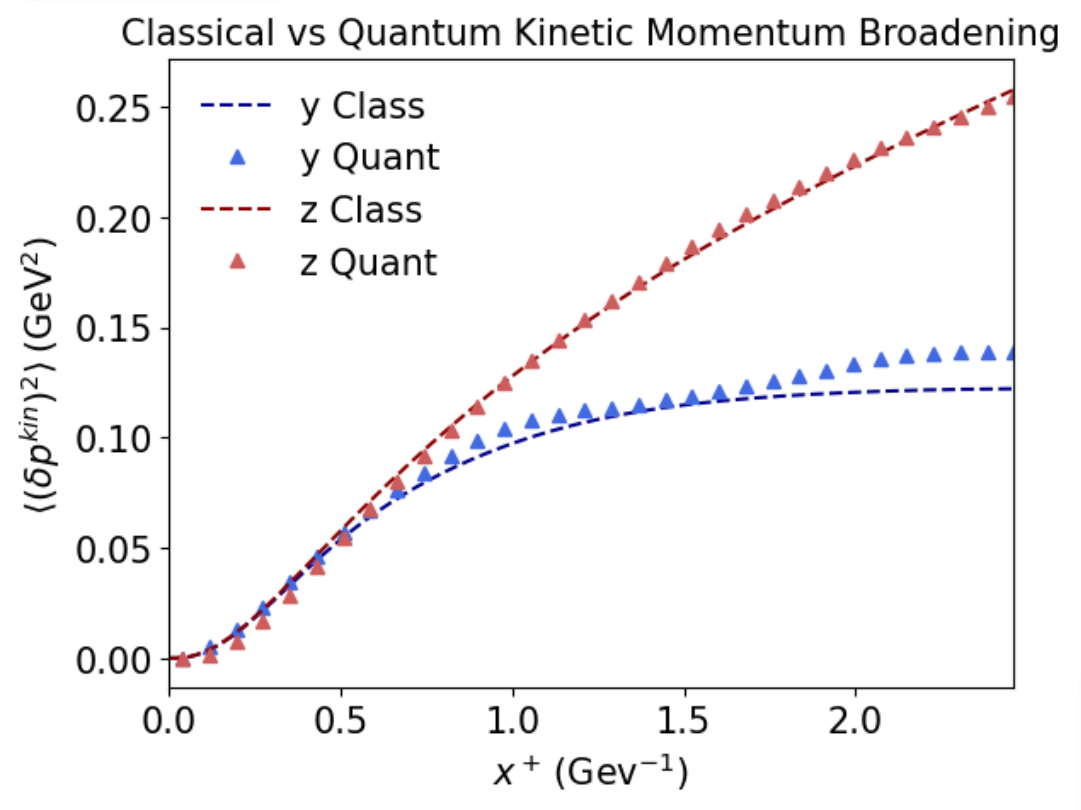
$$\langle (\delta p_i^{kin})^2 \rangle_{x^+} = \frac{g^2}{4}\int_0^{x^+} d\lambda \int_0^{x^+} d\lambda' \langle\langle \psi;0 | \tilde{f}_{i,H}(\lambda)\tilde{f}_{i,H}(\lambda') | \psi;0\rangle\rangle$$

Formally identical to classical momentum broadening but derived purely in quantum mechanics!!!

Results, momentum broadening

Compare our quantum direct calculation with the classical particle calculation

Good agreement between both



We observe the characteristic **anisotropic momentum broadening** from the Glasma larger in the beam direction than in the transverse one

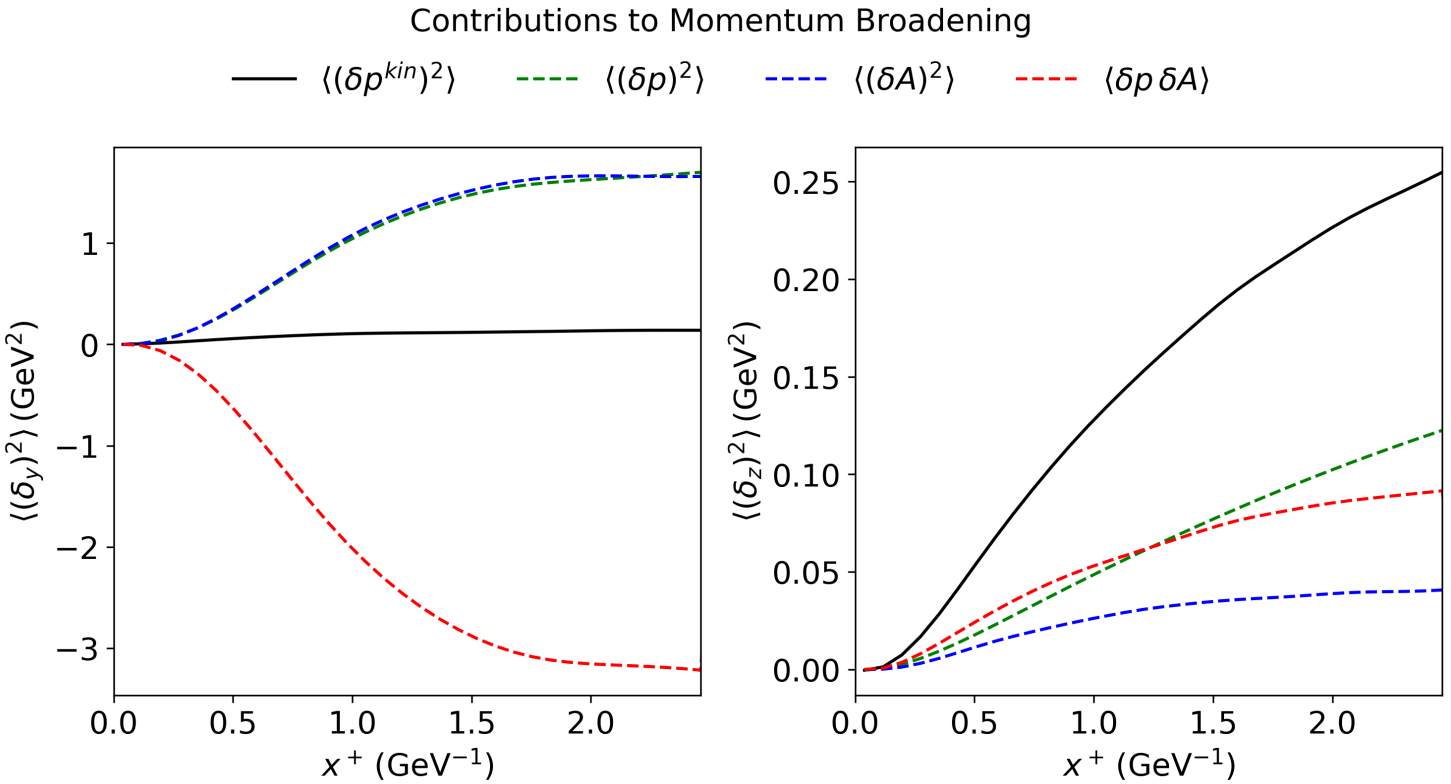
Our formalism is ready to search for new quantum effects in the initial stages!!!

Results, momentum decomposition

Decompose kinetic momentum on it's different contributions

In the transverse direction there are **large cancellations between gauge dependent components** to give a small physical broadening

In the longitudinal direction this components **add up** to give a larger broadening



When studying the evolution of a particle inside a medium it is fundamental to pay attention to the **difference between kinetic and canonic momentum**

Conclusions…

We presented a formalism to study the **quantum real-time evolution** of particles in the Glasma phase of a heavy-ion collision

We can **reproduce the previous classical particle results** if we truncate the quark Fock space to the $|q\rangle$ sector and therefore neglect quantum corrections

… and outlook

Include quantum effects non-present in the classical calculation as **subeikonal corrections** and **quantum gluon radiation**

Back-up slides…

Milne and LC coordinates

$$\tau = \sqrt{t^2 - z^2}\,, \quad x = x\,, \quad y = y\,, \quad \eta = \tanh^{-1}(z/t)$$

$$g^{\mu\nu}_{(M)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1/\tau^2 \end{pmatrix}$$

$$x^+ = t + x\,, \quad x^- = t - x\,, \quad y = y\,, \quad z = z$$

$$g^{\mu\nu}_{LC} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Approximations in the coordinate transformation

We consider a jet at approximately mid-rapidity, propagating along the x -direction at lightspeed so $x \sim t$ and $x^+ \sim 2t$

In the transformation between Milne and LC coordinates, as the jet is well localized around $z = 0$ we expand to first order

$$A_+(x^+, y, z) = \frac{1}{2}\left[-\frac{z}{t}A_\eta(t, x = t, y) + A_x(t, x = t, y)\right]$$

$$A_y(x^+, y, z) = A_y(t, x = t, y)$$

$$A_z(x^+, y, z) = A_\eta(t, x = t, y)/t$$

Real time lattice gauge theory

The gauge transformation of the gauge fields $\mathcal{A}_\mu(x)$ is **non-local** as it involves spatial derivatives

Gauge invariant is then **only exact up to same order in** a^μ

The solution to have exact gauge invariance is to use as degrees of freedom the **Wilson lines between neighbor points, the gauge links**

Gauge fields → **Wilson lines** → **Gauge links**

$$\mathcal{A}^\mu(x) \qquad W[c] = \mathcal{P}\exp\left(-ig\int_c dx^\mu \mathcal{A}_\mu(x)\right) \qquad U_{x,\mu} = \exp\left(iga^\mu \mathcal{A}_\mu(x + a^\mu/2)\right)$$

[Müller (2019)]

Simulation parameters

$$L_\perp = 0.5\,\mathrm{fm}$$
$$N_\perp = 128$$
$$Q_S = 1.5\,\mathrm{GeV}$$
$$m_g = 0.2\,\mathrm{GeV}$$
$$\tau_{sim} = 0.25\,\mathrm{fm}$$

Estimate numerical errors