# Machine learning unveils the quark mass dependence of the pseudo-scalar meson decay constants in three-flavor NNLO $\chi$PT

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**In collaboration with R. Molina and Fernando G.**

**Project is still going**

# Decay constant of pion

- The pion decay constant

$$\left\langle 0 \left| A_\mu(0) \right| \pi^-(p) \right\rangle = ip_\mu f_\pi \qquad A_\mu = \bar{u}\gamma_\mu\gamma_5 d$$

- $f_\pi$ enters in the pion leptonic decay $\pi \to l\nu$

$$\Gamma^{(0)} = \frac{G_F^2 |V_{ud}|^2 f_\pi^2}{8\pi} m_\pi m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)$$

- In relation between the weak coupling $g_A$ and strong coupling constant $g_{\pi N}$:

$$f_\pi g_{\pi N} = \sqrt{2} m_p g_A$$

- The current value of $f_\pi$:

$$f_{\pi^+} = 130.2(1.2)\ \text{MeV}$$

PDG, PRD100, 030001 (2024)

# Decay constant in EFTs

- The lowest order chiral Lagrangian for the Goldstone boson sector

$$\mathscr{L}_{\phi\phi} = \frac{f^2}{4}\mathrm{Tr}\left[\partial_\mu U(\partial^\mu U)^\dagger + \chi U^\dagger + U\chi^\dagger\right] \qquad U = \exp\left(i\frac{\phi}{f}\right)$$

- The lowest order Lagrangian of Goldstone boson and Octet baryon interaction

$$\mathscr{L}_{\mathrm{WT}} = \frac{i}{4f^2}\mathrm{Tr}[\bar{B}\gamma^\mu B(\partial_\mu\phi\phi - \phi\partial_\mu\phi) + \bar{B}\gamma^\mu(\phi\partial_\mu\phi - \partial_\mu\phi\phi)B]$$

- Pseudo-scalar meson and Heavy meson interaction at lowest order

$$\mathscr{L}_{P\phi} = \frac{1}{4f^2}[\mathscr{P}(\partial_\mu\phi\phi - \phi\partial_\mu\phi)\partial_\mu\mathscr{P}^\dagger + \partial_\mu\mathscr{P}(\phi\partial_\mu\phi - \partial_\mu\phi\phi)\mathscr{P}^\dagger]$$

$$\mathscr{P} = (D_0, D^+, D_s^+)$$

$$\phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \qquad B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

# Pseudo-scalar meson mass and decay constant up to NNLO

- Pseudo-scalar meson mass

$$m_\phi^2 = \underbrace{m_{0\phi}^2 + m_\phi^{2(4)}(L_{ir})}_{\mathcal{O}(p^4)} + \underbrace{m_{\mathrm{ct},\phi}^{2(6)}(C_{ir}) + m_{\mathrm{loop},\phi}^{2(6)}(L_{ir})}_{\mathcal{O}(p^6)}$$

- Decay constant

$$f_\phi = f_0\left(1 + f_\phi^{(4)} + f_{\mathrm{ct},a}^{(6)} + f_{\mathrm{loop},\phi}^{(6)}\right)$$

$$m_{0\pi}^2 = 2B_0 m_{u(d)}$$

$$m_{0K}^2 = B_0(m_{u(d)} + m_s)$$

- Total 8 $L_{ir}$s in the experssion
- At NNLO sector, there are about 90 $C_{ir}$s **Johan Bigness, Status of Strong ChPT**
  - $C_{ir}$, $i = 12-21, 31-33$ appearing in masses of pion and kaon, $f_\pi$ and $f_K$
  - $C_{18}$ and $C_{33}$ only contributing to mass of eta and $f_\eta$
  - Up to 13 of $C_{ir}$ in pseudo-scalar meson mass and decay constant
  - Total 8+13=21 parameters

# NNLO

Gabriel A. et al, NPB568 (2000), 319-363

- Pion mass up to NLO

$$m_\pi^2 = M_{0\,\pi}^2 \left[1 + \mu_\pi - \frac{\mu_\eta}{3} + \frac{16 M_{0\,K}^2}{f_0^2}\left(2L_6^r - L_4^r\right)\right] + \frac{8 M_{0\,\pi}^2}{f_0^2}\left(2L_6^r + 2L_8^r - L_4^r - L_5^r\right)$$

- The loop part of $m_\pi$

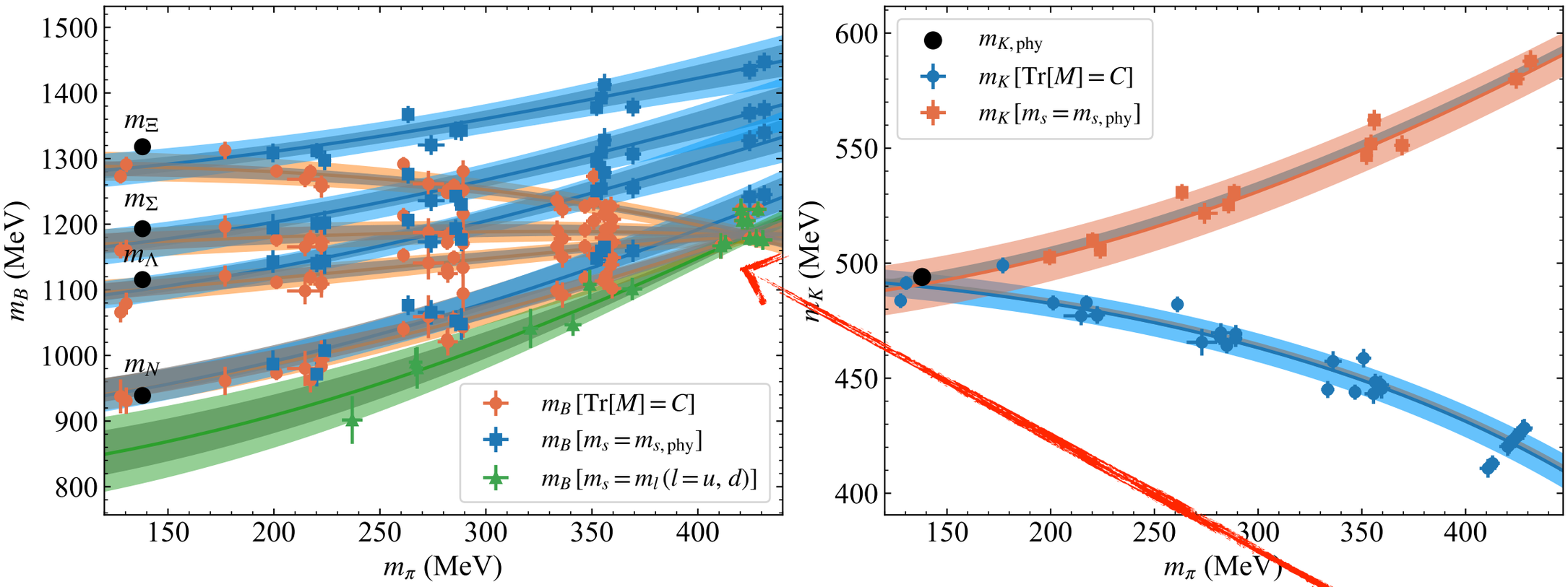
Cross term of the LECs

$$\begin{aligned} m_{\mathrm{Loop},\pi}^{2(6)}(L_{ir}) = \cdots + & -128 m_\pi^2 m_K^4\big) L_4^r L_5^r + \big(512 m_\pi^6 + 2048 m_\pi^4 m_K^2 + 2048 m_\pi^2 m_K^4\big) L_4^r L_6^r \\ & + \big(512 m_\pi^6 + 768 m_\pi^4 m_K^2 + 256 m_\pi^2 m_K^4\big) L_4^r L_8^r - 128 m_\pi^6 L_5^{r\,2} \\ & + \big(512 m_\pi^6 + 768 m_\pi^4 m_K^2 + 256 m_\pi^2 m_K^4\big) L_5^r L_6^r + 512 m_\pi^6 L_5^r L_8^r \\ & + \big(-512 m_\pi^6 - 2048 m_\pi^4 m_K^2 - 2048 m_\pi^2 m_K^4\big) L_6^{r\,2} \\ & + \big(-1024 m_\pi^6 - 1536 m_\pi^4 m_K^2 - 512 m_\pi^2 m_K^4\big) L_6^r L_8^r - 512 m_\pi^6 L_8^{r\,2} \quad + \cdots \end{aligned}$$

- The counter term

$$\begin{aligned} m_{\mathrm{ct},\pi}^{2(6)} = & -32 m_\pi^6 C_{12}^r - 32 m_\pi^6 C_{13}^r - 16 m_\pi^6 C_{14}^r - 16 m_\pi^6 C_{15}^r - 48 m_\pi^6 C_{16}^r \\ & - 16 m_\pi^6 C_{17}^r + 48 m_\pi^6 C_{19}^r + 80 m_\pi^6 C_{20}^r + 48 m_\pi^6 C_{21}^r + 32 m_\pi^6 C_{31}^r \\ & + 32 m_\pi^6 C_{32}^r - 64 m_\pi^4 m_K^2 C_{13}^r - 32 m_\pi^4 m_K^2 C_{15}^r + 64 m_\pi^4 m_K^2 C_{16}^r \\ & + 192 m_\pi^4 m_K^2 C_{21}^r + 64 m_\pi^4 m_K^2 C_{32}^r - 64 m_\pi^2 m_K^4 C_{16}^r \\ & + 64 m_\pi^2 m_K^4 C_{20}^r + 192 m_\pi^2 m_K^4 C_{21}^r \,, \end{aligned}$$

# Three types of trajectories in LQCD




**Z. Zhuang et al, Sci.Bull. 70 (2025), 1953-1961**

SU3 symmetry

- Three settings of $m_\pi$ lying in a chiral trajectory in LQCD simulation
  - $\mathrm{Tr}[M] = C$, $m_s = m_{s,\mathrm{phy}}$, and $m_s = m_{u(d)}$

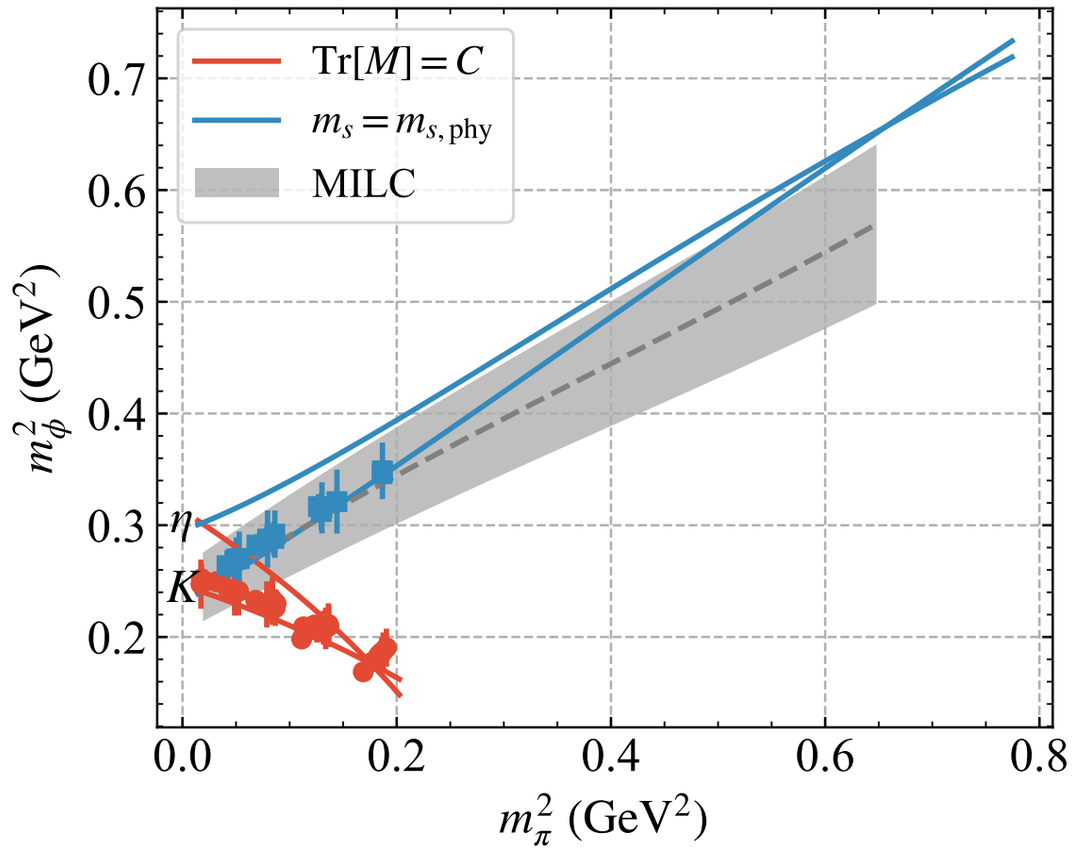
$$M = \mathrm{diag}\{m_u, m_d, m_s\}$$

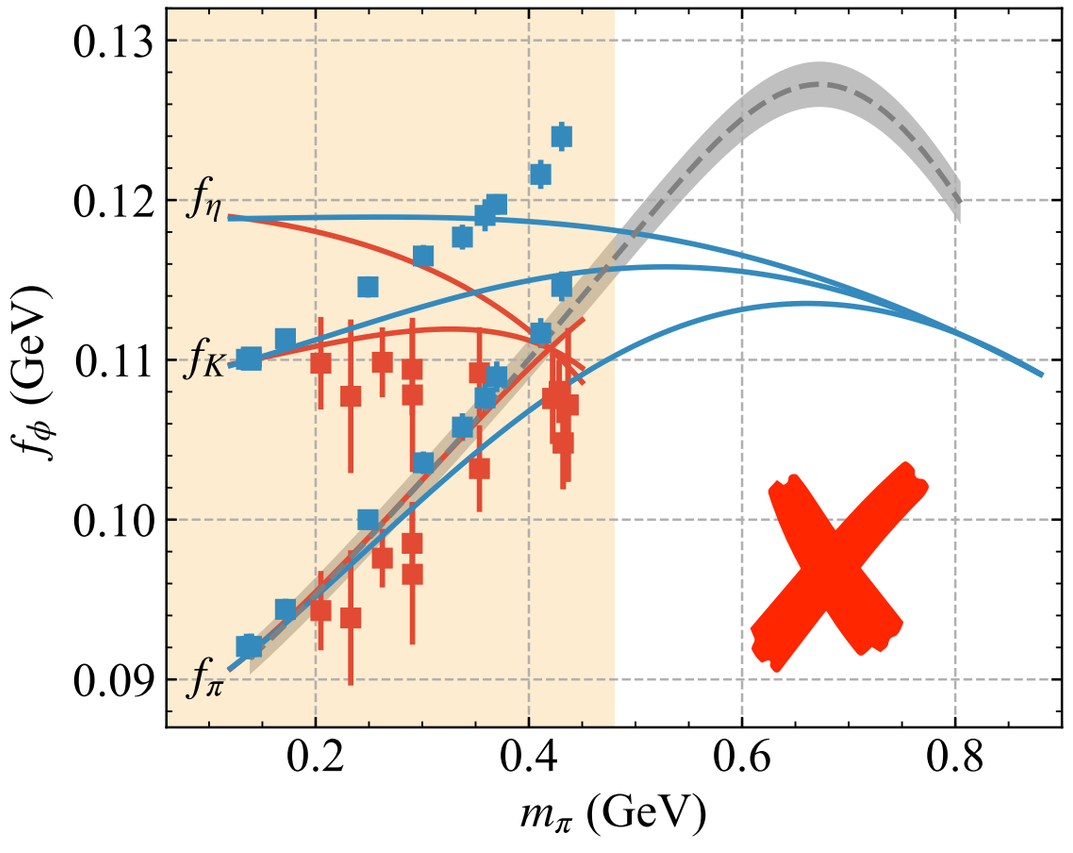
$$m_{u(d)} = m_{s,\mathrm{phy}} \rightarrow m_\pi \sim 800\ \mathrm{MeV}$$

$$m_{u,d,s} = \frac{C}{3} \rightarrow m_\pi \sim 420\ \mathrm{MeV}$$

# NLO ChPT works up to 450 MeV

$L_{ir}$ **provided by R. Molina et al, JHEP11, 017 (2020)**




**The bands from MILC, RMP82, 1349 (2010)**

- NLO ChPT is insufficient to predict decay constants higher than 450 MeV
- Considering the higher order corrections is necessary ***Our motivation!!!***

# Least Absolute Shrinkage and Selection Operator (LASSO)

**Lasso Regression** is a linear regression technique that combines prediction with feature selection (model selection). It does this by adding a penalty term to the cost function shrinking less relevant feature's coefficient to zero.

- The cost function of Linear Regression

$$J = \sum_{i=1}^{m} \left( y^{(i)} - h(x^{(i)}) \right)^2$$

- The cost function of Lasso Regression by adding the L1 penalty term

$$J = \sum_{i=1}^{m} \left( y^{(i)} - h(x^{(i)}) \right)^2 + \frac{\lambda}{10} \sum_{j=1}^{n} |w_j|$$

$w_j$: the weight for the $j^{th}$ feature

$n$: the number of the feature in the dataset

$\lambda$: the regularization strength

# How does Lasso work?

$$J = \sum_{i=1}^{m} \left( y^{(i)} - h(x^{(i)}) \right)^2 + \lambda \sum_{j=1}^{n} |w_j|$$

- If $\lambda = 0$, Lasso likes a Linear Regression
- If $\lambda \sim \infty$, all features are shrunk to zero
- increasing $\lambda$ increases bias but reduces variance

How Lasso Regression operates:

1. Set the intercept and coefficients (features) to zero
2. Update the coefficient by algorithms, example: coordinate descent
3. Enforce sparsity with the L1 penalty term, causing some coefficients to 0
4. The model simplifies as the most significant features are selected

# Application on ChPT up to NNLO

- The cost function modified as follows

$$\chi^2\left(\vec{L},\vec{C}\right)=\sum_i\frac{\left(y_i-f_i(\vec{L},\vec{C})\right)^2}{\sigma_i^2}+\frac{\lambda}{10}\sum_j|\mathrm{LECs}_j|$$

- Rewriting the counter term of $m_\phi^2$ and $f_\phi$ $(\phi=\pi,K)$ in a linear model

- Feature: $C_{ir}$

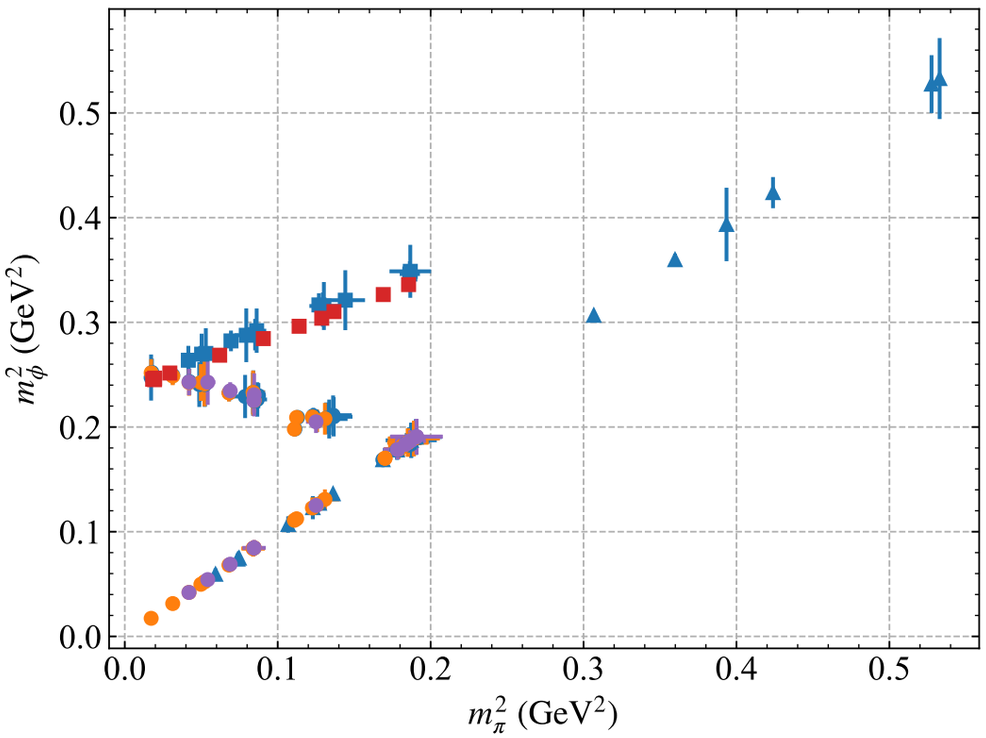
$$\begin{array}{c} \\ m_\pi^2 \\ m_K^2 \\ f_\pi \\ f_K \end{array}\begin{array}{c}\text{Feature}\\ \begin{pmatrix} -32m_{0\pi}^6 & -64m_{0K}^2m_{0\pi}^4-32m_{0\pi}^6 & -16m_{0\pi}^6 & -32m_{0K}^2m_{0\pi}^4-16m_{0\pi}^6 & \cdots \\ -32m_{0K}^6 & -64m_{0K}^2-32m_{0K}^4m_{0\pi}^2 & -32m_{0K}^6+32m_{0K}^4m_{0\pi}^2-16m_{0K}^2m_{0\pi}^4 & -32m_{0K}^6-16m_{0K}^4m_{0\pi}^2 & \cdots \\ 0 & 0 & 8m_{0\pi^4}f_0^2 & (16m_{0K}^2m_{0\pi}^2+8m_{0\pi}^4)f_0^2 & \cdots \\ 0 & 0 & (16m_{0K}^4-16m_{0K}^2m_{0\pi}^2+8m_{0\pi}^4)f_0^2 & (16m_{0K}^4+8m_{0K}^2m_{0\pi}^2)f_0^2 & \cdots \end{pmatrix}\end{array}\begin{pmatrix} C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \\ \vdots \end{pmatrix}$$

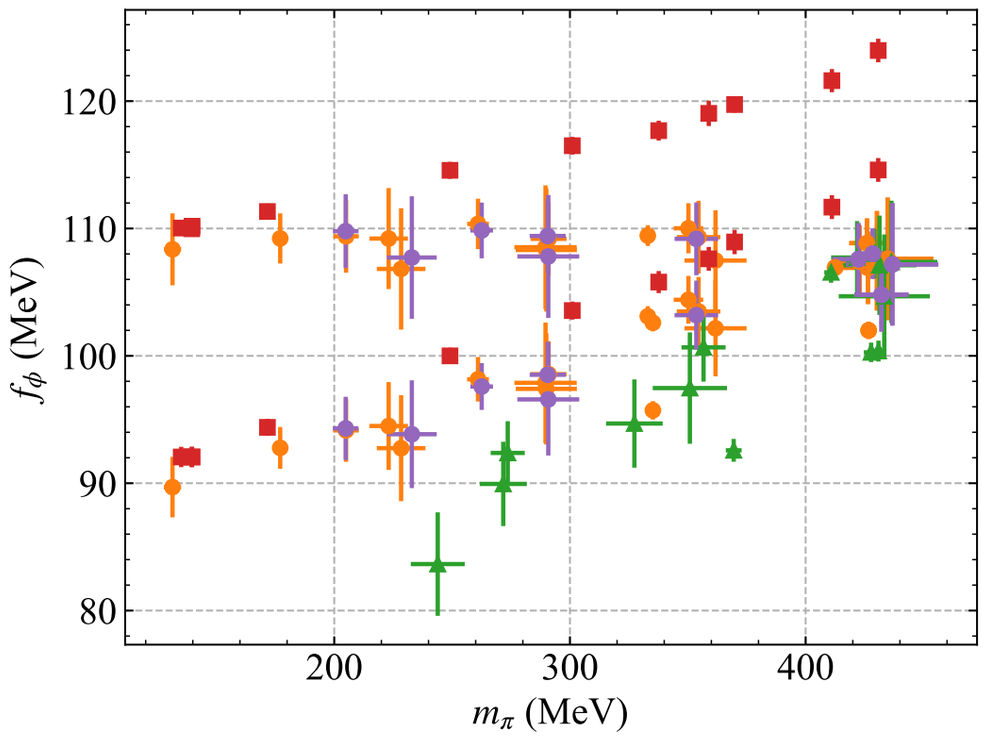
# Fitting strategy

- Fit 1: We preform a global fit of data with NLO ($L_{ir}$) and NNLO ($C_{ir}$) LECs as free parameters (As comparison)
- The Lasso method is applied to the NNLO LECs

1. Fit 2A: Set $C_{ir} = 0$, and fit the pseudo-scalar meson masses and decay constants to obtain the $L_{ir}$ with the NNLO formulas including up to two loops. Then fix the $L_{ir}$ and and perform a Lasso method
2. Fit 2B: We leave the $C_{ir}$ as free parameters ($C_{ir} \neq 0$) and apply the Lasso method directly

# Dataset



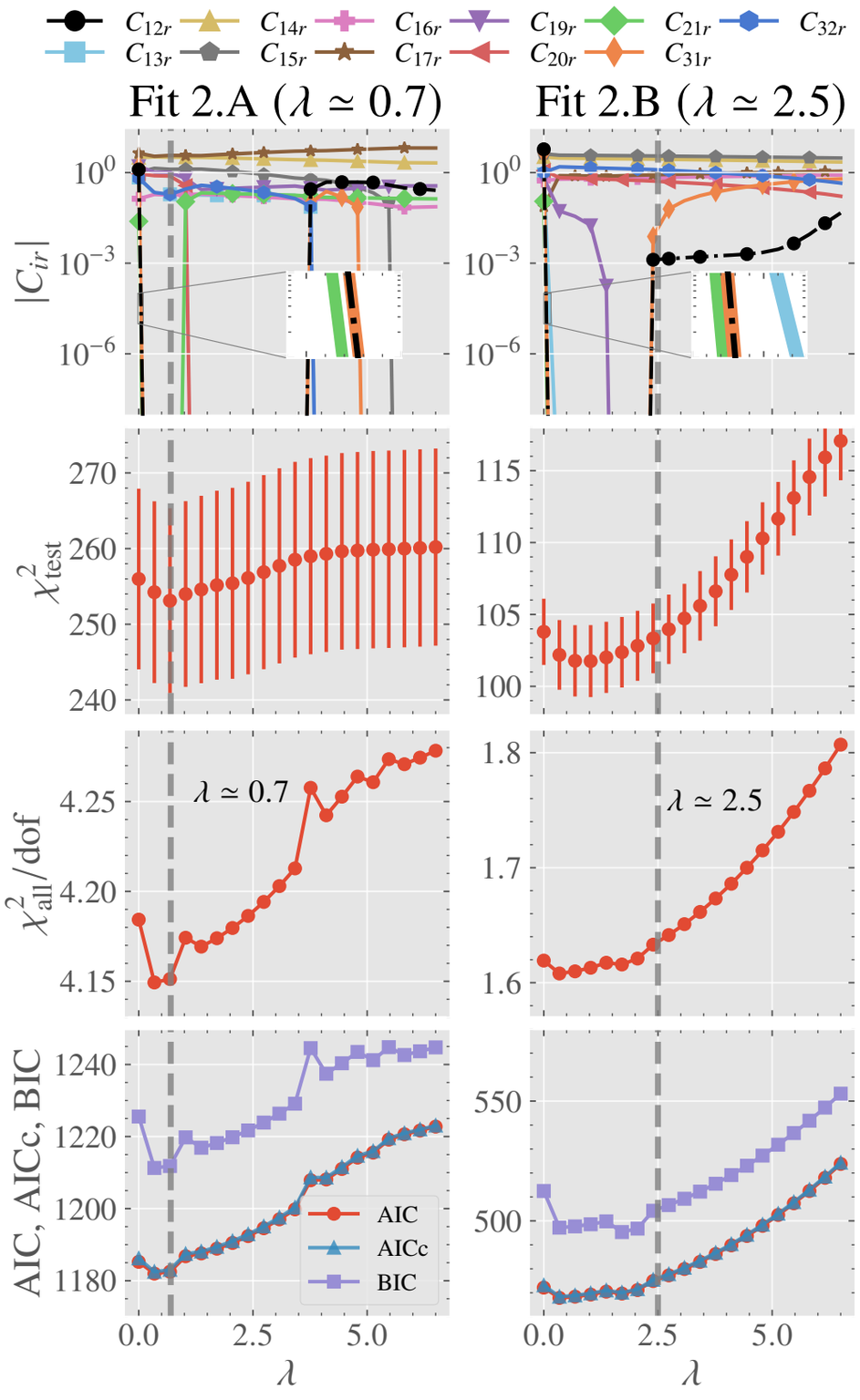




**UKQCD, PRD93, 074505 (2016), M. B. et al, PRD95, 074504 (2017)**

**M. Ce et al, PRD106, 114502 (2022), RQCD, PRD105, 054516 (2022), RQCD, JHEP05, 035 (2023)**

- These data points are separated to training set and validation set

# Results of the lasso up to NNLO



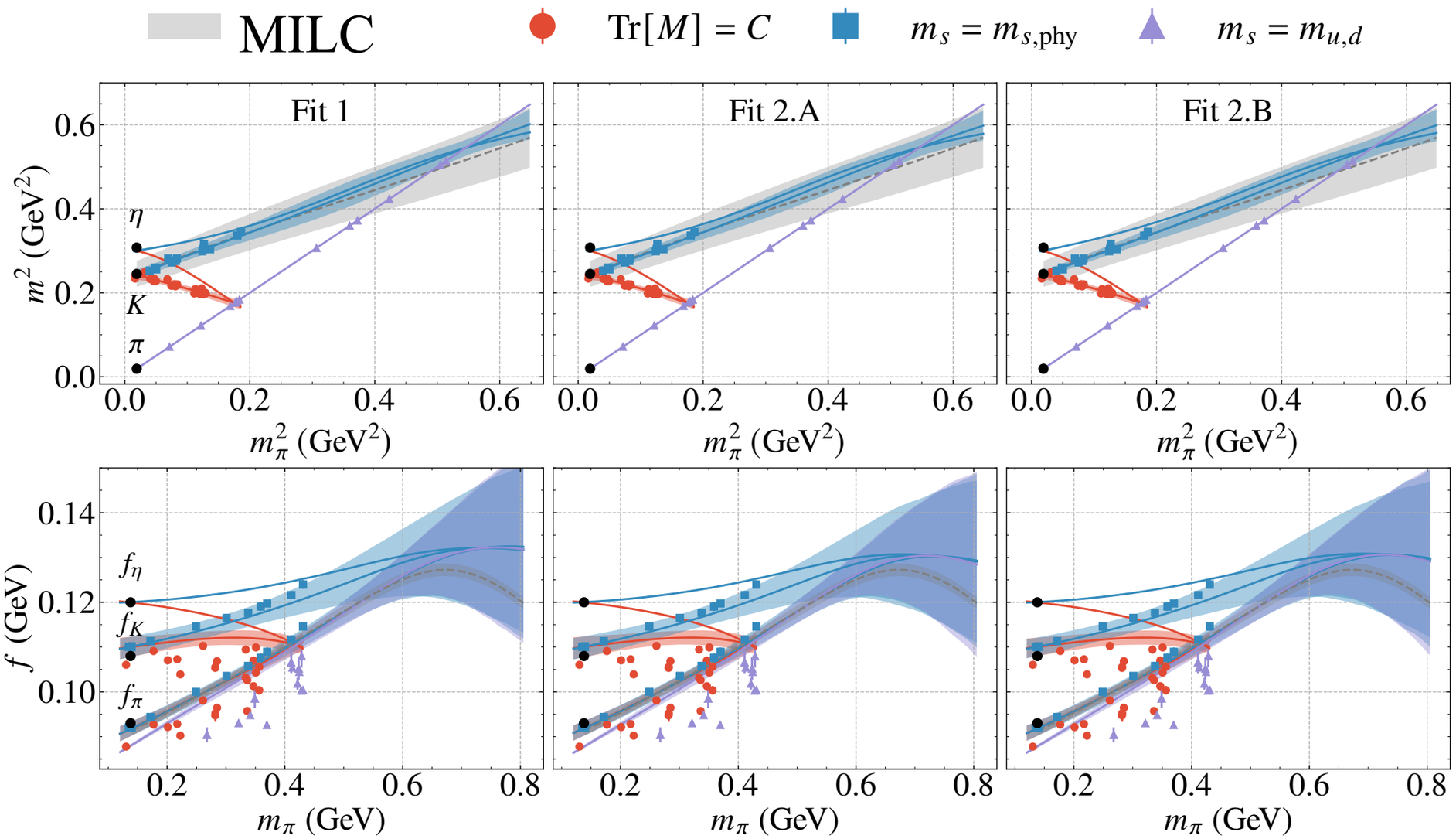

- To select a optimal $\lambda$, we use information criteria
  - Akaike information criterion (AIC) and AICc
  - Bayesian information Criterion (BIC)
- Algorithm: coordinate descent
- The redundant LECs from the Fit 2.A

  $C_{12r}$, $C_{21r}$, and $C_{31r}$
- The redundant LECs from the Fit 2.B

  $C_{13r}$, $C_{19r}$, and $C_{21r}$

Notice: Lasso does not tell you the values of $C_{ir}$

# Fitting results




- ChPT up to NNLO can describe the pseudo-scalar meson masses and decay constants up to 800 MeV
- A SU3 symmetry point for $m_s = m_{s,\mathrm{phy}}$ is about 739 MeV
- Only 8 of the 11 LECs $C_{ir}$ are necessary to reproduce the pseudo-scalar meson masses and decay constants

# The LECs from the fits

$L_{ir} \times 10^3$

| | $L_{2r}$ | $L_{4r}$ | $L_{5r}$ | $L_{6r}$ | $L_{7r}$ | $L_{8r}$ |
|---|---|---|---|---|---|---|
| Fit 1 | 0.727(2)(3) | 0.152(1)(3) | 0.289(4)(10) | 0.403(1)(2) | 0.423(12)(27) | −0.529(2)(6) |
| Fit 2A | 0.629(2)(4) | 0.094(1)(3) | 0.171(4)(10) | 0.333(1)(2) | 0.298(11)(23) | −0.421(2)(6) |
| Fit 2B | 0.645(7)(4) | 0.106(10)(3) | 0.144(25)(15) | 0.375(11)(7) | 0.470(85)(51) | −0.528(29)(17) |

$C_{ir} \times 10^6$

| $C_{12r}$ | $C_{13r}$ | $C_{14r}$ | $C_{15r}$ | $C_{16r}$ | $C_{17r}$ | $C_{19r}$ | $C_{20r}$ | $C_{21r}$ | $C_{31r}$ | $C_{32r}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.64(4)(1) | −1.49(2)(5) | −3.70(4)(8) | 4.93(3)(6) | 0.36(2)(3) | −0.72(8)(15) | 0.42(2)(5) | −0.83(1)(3) | −0.05(0)(1) | −1.64(4)(13) | 1.49(2)(5) |
| ··· | −0.92(2)(5) | −3.13(4)(9) | 4.42(3)(6) | 0.37(2)(3) | 0.25(8)(16) | −0.20(2)(4) | −0.59(1)(3) | ··· | ··· | 0.64(2)(5) |
| 0.03(4)(1) | ··· | −3.47(4)(9) | 4.48(3)(6) | 0.40(2)(3) | 0.08(8)(1) | ··· | −0.71(1)(3) | ··· | −0.92(4)(13) | 1.95(2)(4) |

# Correlation Fit 2A

| | $L_{2r}$ | $L_{4r}$ | $L_{5r}$ | $L_{6r}$ | $L_{7r}$ | $L_{8r}$ | $C_{13r}$ | $C_{14r}$ | $C_{15r}$ | $C_{16r}$ | $C_{17r}$ | $C_{19r}$ | $C_{20r}$ | $C_{32r}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $L_{2r}$ | 1.00 | 0.54 | 0.38 | 0.22 | -0.12 | 0.00 | -0.05 | -0.18 | 0.04 | 0.35 | 0.06 | -0.07 | 0.08 | 0.05 |
| $L_{4r}$ | 0.54 | 1.00 | -0.18 | 0.16 | -0.26 | 0.16 | -0.02 | 0.11 | 0.05 | -0.46 | -0.00 | -0.07 | -0.00 | 0.02 |
| $L_{5r}$ | 0.38 | -0.18 | 1.00 | 0.23 | 0.04 | 0.20 | 0.04 | -0.52 | -0.17 | 0.35 | -0.05 | -0.11 | -0.01 | -0.04 |
| $L_{6r}$ | 0.22 | 0.16 | 0.23 | 1.00 | 0.50 | -0.68 | -0.01 | 0.10 | 0.00 | -0.09 | -0.10 | -0.06 | -0.09 | 0.01 |
| $L_{7r}$ | -0.12 | -0.26 | 0.04 | 0.50 | 1.00 | -0.43 | 0.09 | 0.46 | 0.01 | -0.02 | -0.11 | -0.48 | 0.42 | -0.09 |
| $L_{8r}$ | 0.00 | 0.16 | 0.20 | -0.68 | -0.43 | 1.00 | 0.03 | -0.23 | -0.09 | -0.12 | -0.04 | -0.24 | -0.02 | -0.03 |
| $C_{13r}$ | -0.05 | -0.02 | 0.04 | -0.01 | 0.09 | 0.03 | 1.00 | 0.06 | -0.00 | -0.01 | -0.19 | 0.05 | 0.35 | 0.54 |
| $C_{14r}$ | -0.18 | 0.11 | -0.52 | 0.10 | 0.46 | -0.23 | 0.06 | 1.00 | -0.34 | -0.22 | 0.20 | -0.03 | 0.20 | -0.06 |
| $C_{15r}$ | 0.04 | 0.05 | -0.17 | 0.00 | 0.01 | -0.09 | -0.00 | -0.34 | 1.00 | -0.24 | -0.75 | 0.01 | 0.02 | 0.00 |
| $C_{16r}$ | 0.35 | -0.46 | 0.35 | -0.09 | -0.02 | -0.12 | -0.01 | -0.22 | -0.24 | 1.00 | 0.24 | 0.08 | 0.11 | 0.01 |
| $C_{17r}$ | 0.06 | -0.00 | -0.05 | -0.10 | -0.11 | -0.04 | -0.19 | 0.20 | -0.75 | 0.24 | 1.00 | -0.10 | -0.09 | 0.19 |
| $C_{19r}$ | -0.07 | -0.07 | -0.11 | -0.06 | -0.48 | -0.24 | 0.05 | -0.03 | 0.01 | 0.08 | -0.10 | 1.00 | -0.37 | -0.05 |
| $C_{20r}$ | 0.08 | -0.00 | -0.01 | -0.09 | 0.42 | -0.02 | 0.35 | 0.20 | 0.02 | 0.11 | -0.09 | -0.37 | 1.00 | -0.35 |
| $C_{32r}$ | 0.05 | 0.02 | -0.04 | 0.01 | -0.09 | -0.03 | 0.54 | -0.06 | 0.00 | 0.01 | 0.19 | -0.05 | -0.35 | 1.00 |

# Correlation Fit 2B

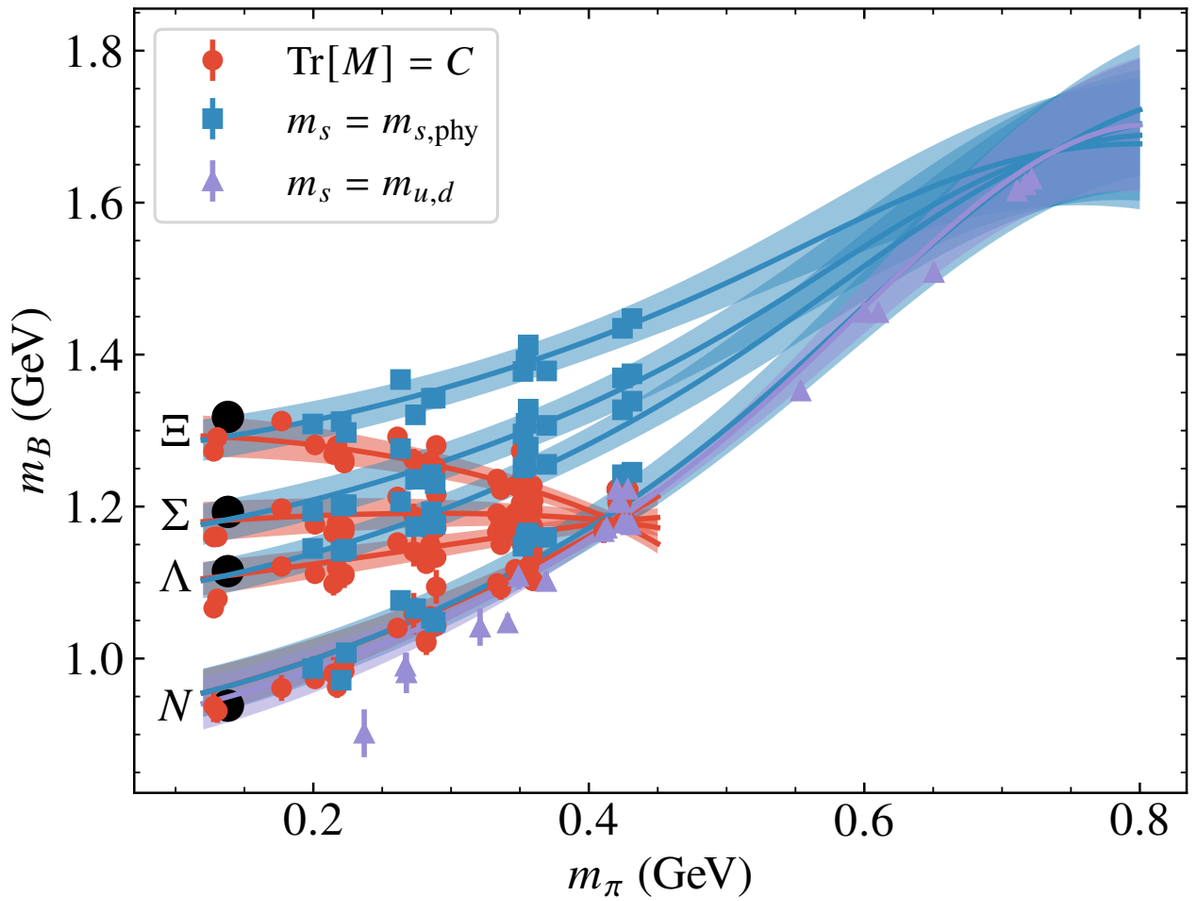
| | $L_{2r}$ | $L_{4r}$ | $L_{5r}$ | $L_{6r}$ | $L_{7r}$ | $L_{8r}$ | $C_{12r}$ | $C_{14r}$ | $C_{15r}$ | $C_{16r}$ | $C_{17r}$ | $C_{20r}$ | $C_{31r}$ | $C_{32r}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $L_{2r}$ | 1.00 | 0.54 | 0.38 | 0.22 | -0.14 | -0.01 | 0.01 | -0.18 | 0.04 | 0.36 | 0.05 | 0.08 | -0.01 | 0.08 |
| $L_{4r}$ | 0.54 | 1.00 | -0.19 | 0.16 | -0.28 | 0.15 | 0.02 | 0.11 | 0.06 | -0.46 | -0.02 | -0.01 | -0.02 | 0.04 |
| $L_{5r}$ | 0.38 | -0.19 | 1.00 | 0.23 | 0.05 | 0.20 | 0.08 | -0.51 | -0.17 | 0.36 | -0.08 | -0.00 | -0.08 | -0.01 |
| $L_{6r}$ | 0.22 | 0.16 | 0.23 | 1.00 | 0.48 | -0.70 | 0.01 | 0.11 | 0.00 | -0.09 | -0.11 | -0.10 | -0.01 | 0.02 |
| $L_{7r}$ | -0.14 | -0.28 | 0.05 | 0.48 | 1.00 | -0.46 | 0.27 | 0.48 | 0.04 | 0.00 | -0.24 | 0.45 | -0.27 | 0.05 |
| $L_{8r}$ | -0.01 | 0.15 | 0.20 | -0.70 | -0.46 | 1.00 | 0.11 | -0.22 | -0.07 | -0.11 | -0.09 | -0.03 | -0.11 | 0.02 |
| $C_{12r}$ | 0.01 | 0.02 | 0.08 | 0.01 | 0.27 | 0.11 | 1.00 | 0.05 | 0.03 | -0.02 | -0.11 | 0.27 | 0.41 | 0.26 |
| $C_{14r}$ | -0.18 | 0.11 | -0.51 | 0.11 | 0.48 | -0.22 | 0.05 | 1.00 | -0.33 | -0.22 | 0.19 | 0.21 | -0.05 | -0.06 |
| $C_{15r}$ | 0.04 | 0.06 | -0.17 | 0.00 | 0.04 | -0.07 | 0.03 | -0.33 | 1.00 | -0.25 | -0.76 | 0.04 | -0.03 | 0.03 |
| $C_{16r}$ | 0.36 | -0.46 | 0.36 | -0.09 | 0.00 | -0.11 | -0.02 | -0.22 | -0.25 | 1.00 | 0.25 | 0.12 | 0.02 | -0.00 |
| $C_{17r}$ | 0.05 | -0.02 | -0.08 | -0.11 | -0.24 | -0.09 | -0.11 | 0.19 | -0.76 | 0.25 | 1.00 | -0.16 | 0.11 | 0.20 |
| $C_{20r}$ | 0.08 | -0.01 | -0.00 | -0.10 | 0.45 | -0.03 | 0.27 | 0.21 | 0.04 | 0.12 | -0.16 | 1.00 | -0.27 | -0.38 |
| $C_{31r}$ | -0.01 | -0.02 | -0.08 | -0.01 | -0.27 | -0.11 | 0.41 | -0.05 | -0.03 | 0.02 | 0.11 | -0.27 | 1.00 | -0.26 |
| $C_{32r}$ | 0.08 | 0.04 | -0.01 | 0.02 | 0.05 | 0.02 | 0.26 | -0.06 | 0.03 | -0.00 | 0.20 | -0.38 | -0.26 | 1.00 |

# Pseudo-scalar meson masses and decay constants

| | $m_K$ | $m_\eta$ | $f_\pi$ | $f_K$ | $f_\eta$ | $m_\phi^{(SU3)}$ | $f_\phi^{(SU3)}$ |
|---|---|---|---|---|---|---|---|
| PDG [1] | 496 | 548 | 93 | 108 | 120 | ... | ... |
| $\mathrm{Tr}[M] = C$ | 492(7) | 548(7) | 92(2) | 110(2) | 120(12) | 420(7) | 111(2) |
| $m_s = m_{s,\mathrm{phy}}$ | 493(7) | 548(7) | 92(1) | 110(2) | 120(12) | 739(12) | 130(13) |

- The symmetry point for $\mathrm{Tr}[M] = C$ is 420 MeV
- The symmetry point for $m_s = m_{s,phy}$ is 739 MeV

# Octet baryon mass




| | $m_N$ | $m_\Lambda$ | $m_\Sigma$ | $m_\Xi$ | $m_\phi^{(SU3)}$ | $m_B^{(SU3)}$ |
|---|---|---|---|---|---|---|
| PDG | 938 | 1116 | 1193 | 1318 | ... | ... |
| $\mathrm{Tr}[M] = C$ | 964(17) | 1109(12) | 1182(15) | 1291(15) | 420(7) | 1181(7) |
| $m_s = m_{s,phy}$ | 964(16) | 1110(13) | 1183(15) | 1292(14) | 739(12) | 1669(39) |

# Conclusions

- We perform a global fit by including different chiral trajectories $\mathrm{Tr}[M] = C$, $m_s = m_{s,phy}$, and $m_s = m_{u(d)}$ lattice data
- Pseudo-scalar meson masses and decay constants have been determined till $m_\pi \sim 800$ MeV for the first time
- The Lasso algorithm is able to reduce the number of the LECs at NNLO
- Only 8 of the 11 LECs $C_{ir}$ are necessary to reproduce the pseudo-scalar meson masses and decay constants
- The quark mass dependence of the observables can be used as input in chiral based EFT theory, For example: pion mass dependence of $D_0^*(2300)$ and $\Lambda(1405)$

**Z. Zhuang et al, Sci.Bull. 70 (2025), 1953-1961**

***Thank you for your attention!!!***