Quantum Computing Fragmentation Functions

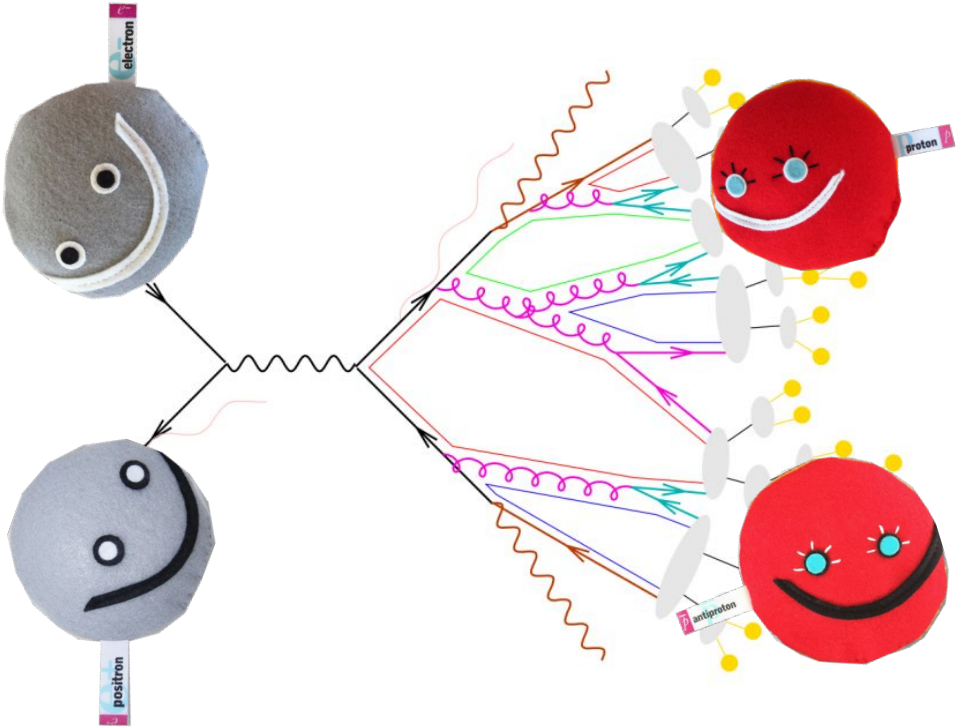
Universidad Complutense de Madrid & IPARCOS

Juan José Gálvez Viruet*

arxiv: 2510.18869

*juagalve@ucm.es

QCD is confined: Hard interactions reconstructed from universal **fragmentation functions (FF)** into hadrons
- the cleanest process is **e+ e– annihilation**



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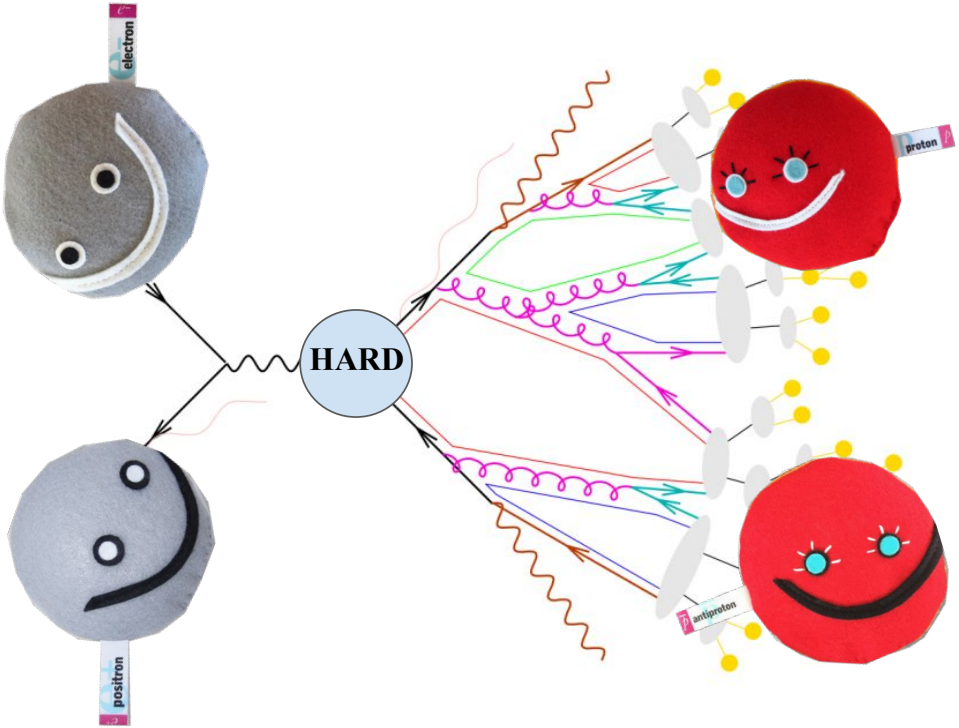

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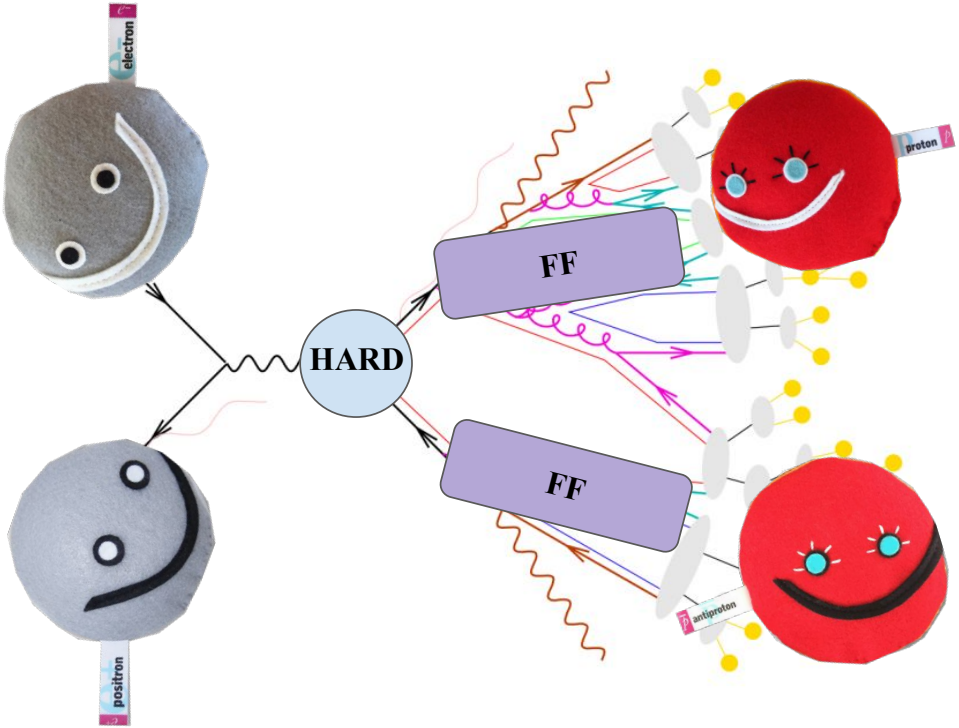

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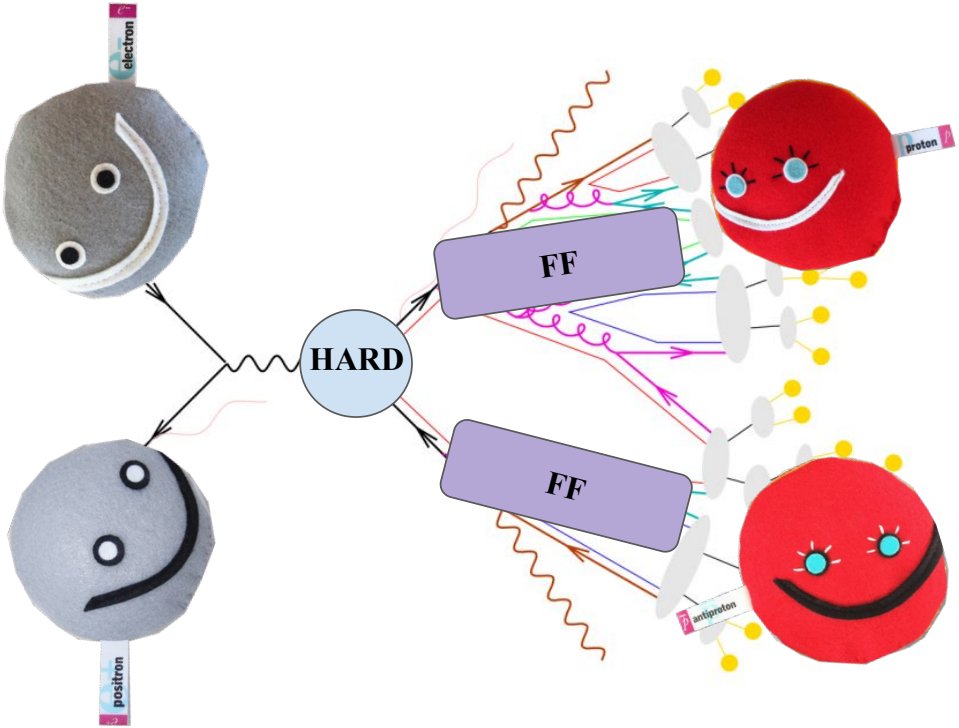
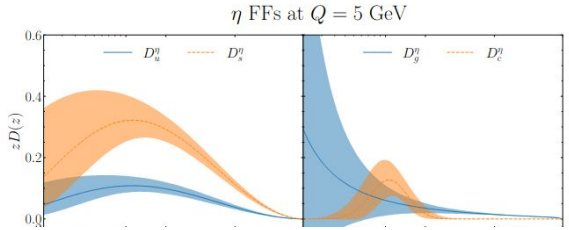


Obtained ...

By experimental fits



https://doi.org/10.1103/t5ds-vvc4

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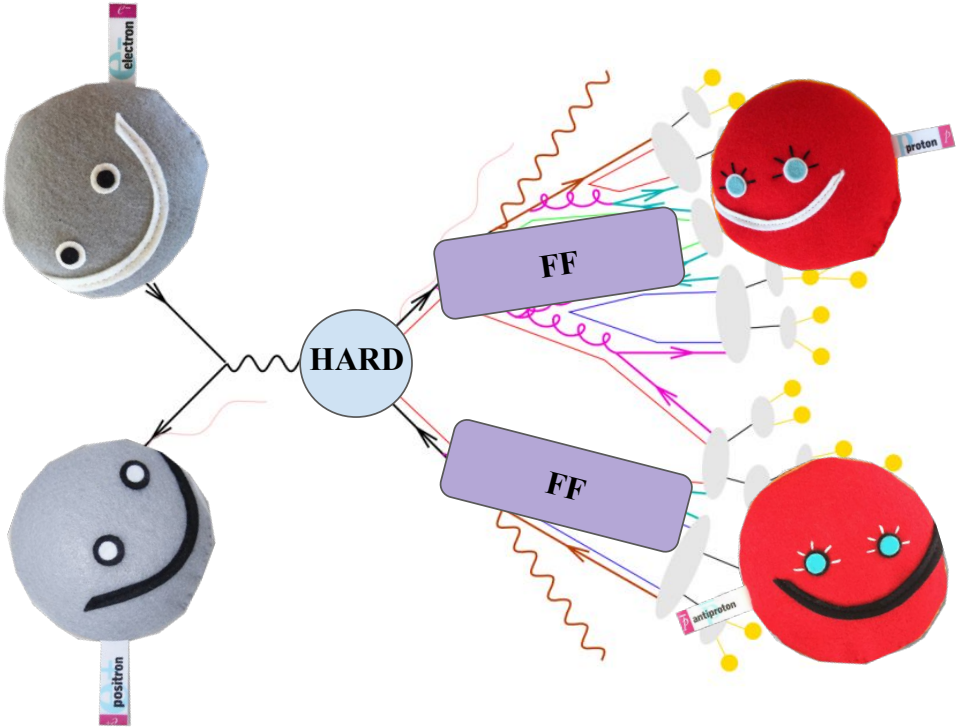

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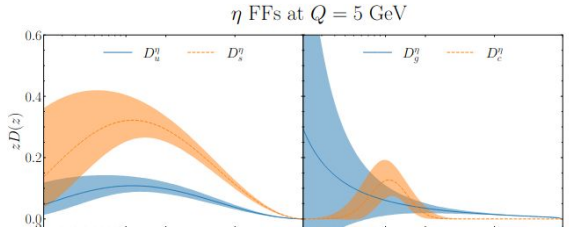

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On the Lattice?

$$D_j^h(z) \equiv \frac{\mathrm{Tr}_c}{N_{c,j}} \sum_X \langle j,p|h,X_{\mathrm{out}}\rangle\langle h,X_{\mathrm{out}}|j,p\rangle$$

doi:10.1103/PhysRevD.109.016006

- Exp. value of initial parton j traced over color

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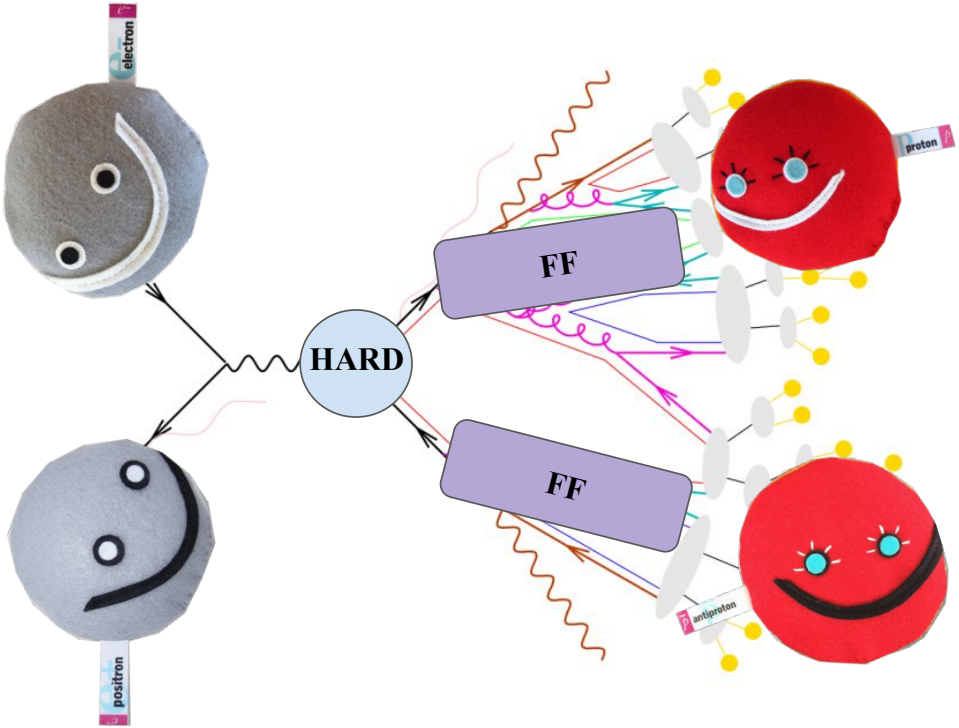

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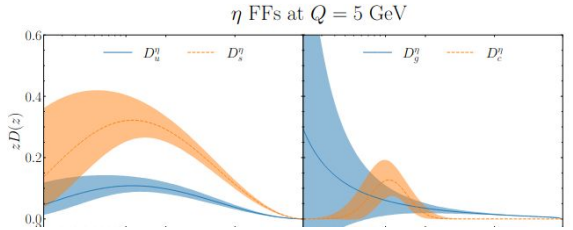

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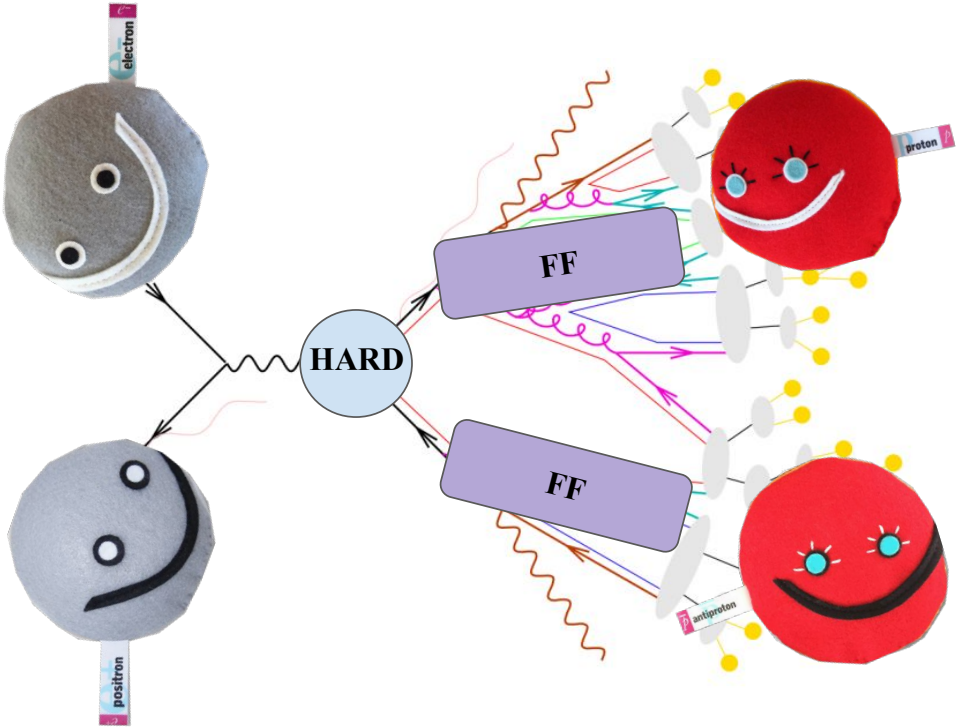

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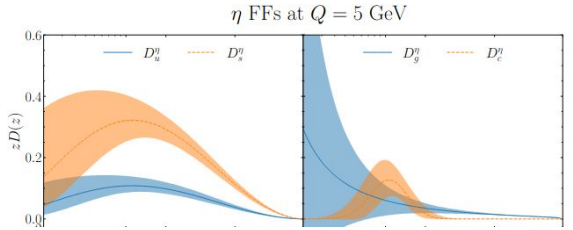

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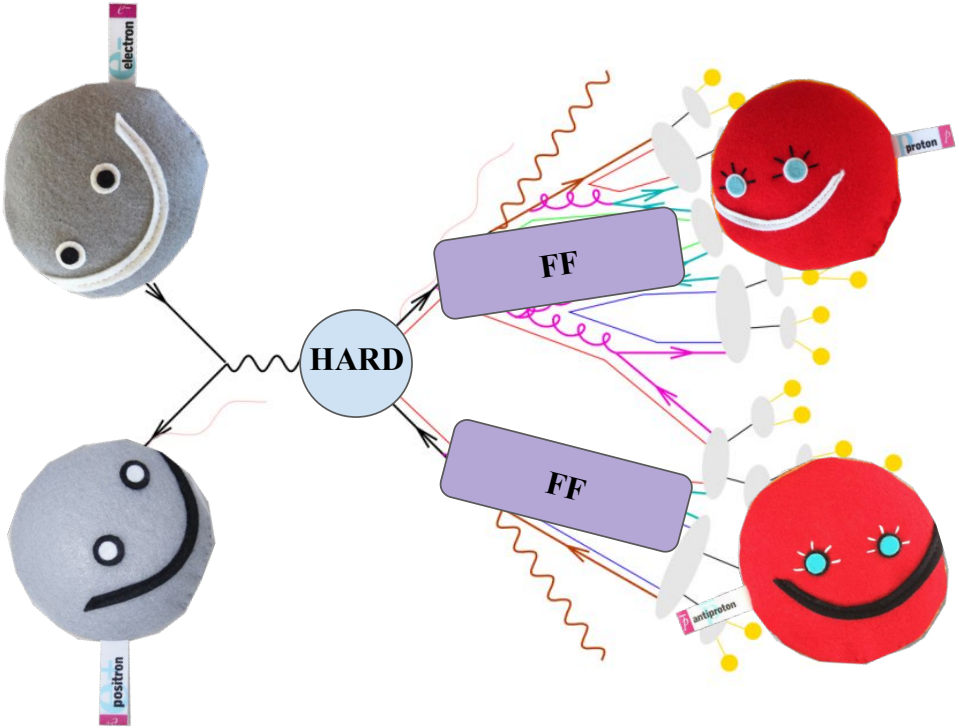

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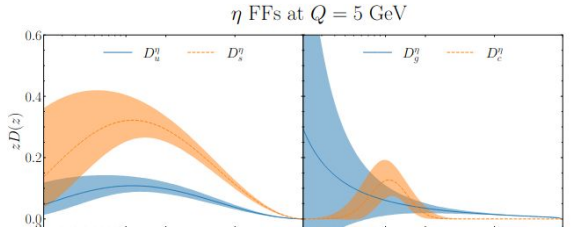

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Not a problem on a Quantum Computer!

Algorithms for **quantum simulations** of hadron physics:

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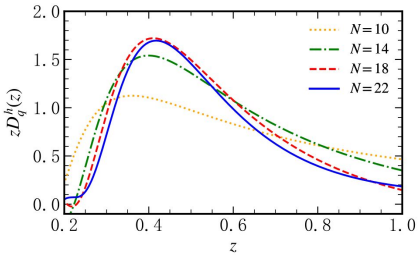

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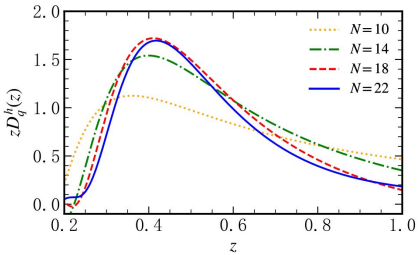

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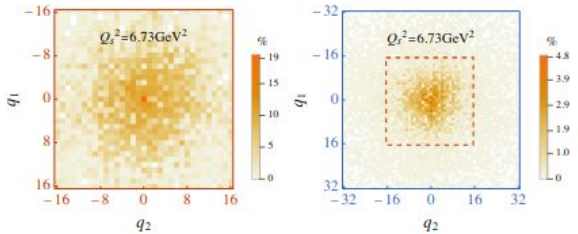

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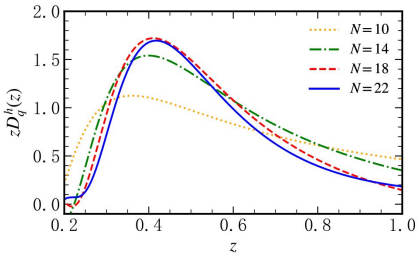

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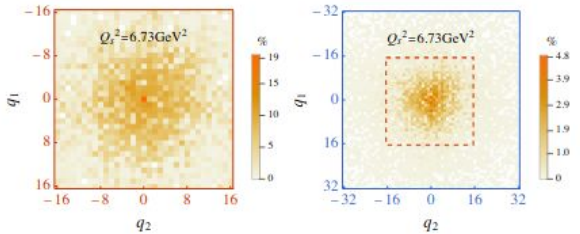

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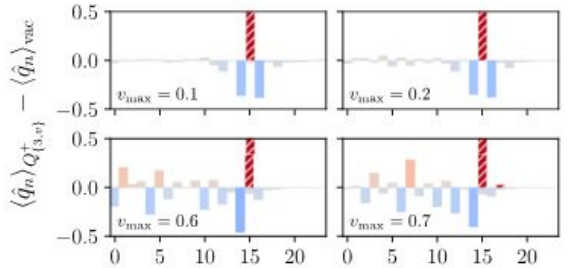

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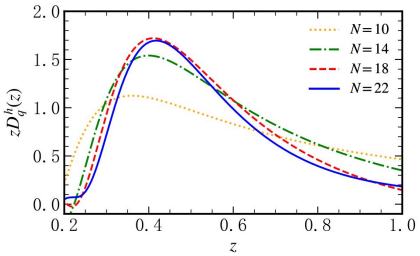

- See https://arxiv.org/abs/2510.26293 for (our) review

Preparations for Quantum Computing in Hadron Physics

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María Gómez-Rocha
Dept. de Física Atómica, Molecular y Nuclear and Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, 18071 Granada, Spain

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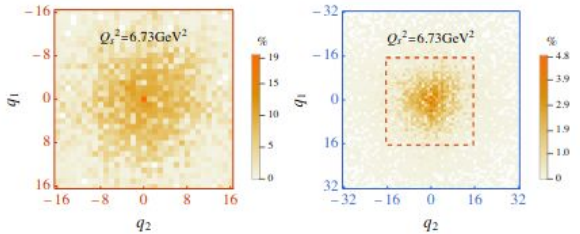

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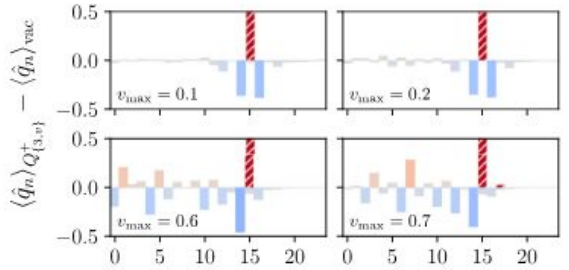
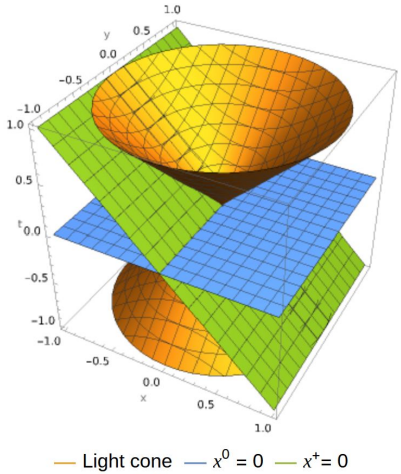


Ab initio QCD on the Light-Front



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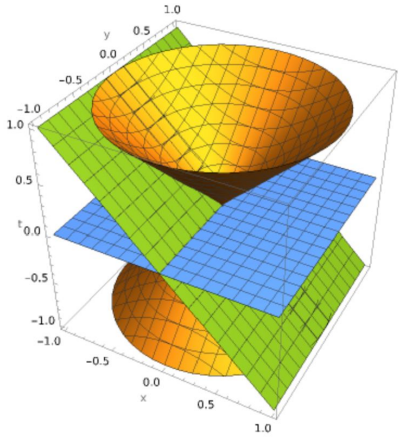

Coordinates

Instant Form (usual quantisation)

$$(x^0, x^1, x^2, x^3)$$

Light Front

$$(x^+, x^-, x^1, x^2)$$

$$x^+ = x^3 + x^0$$

$$x^- = x^3 - x^0$$

$$x^1,\ x^2 \qquad x^\perp$$

Collision axis

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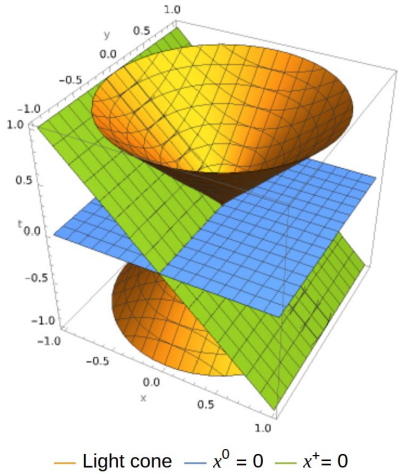

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$$p^- = \frac{m^2 + |p^\perp|^2}{p^+}$$

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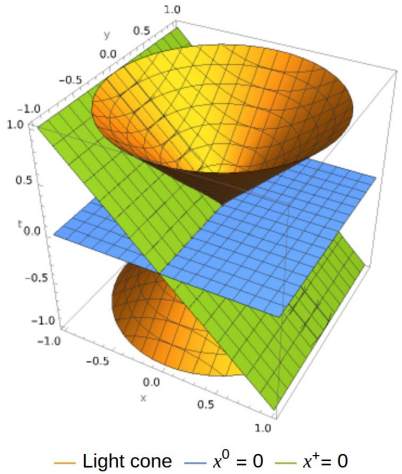

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$$p^0 = \sqrt{m^2 + |p|^2} \qquad p^- = \frac{m^2 + |p^\perp|^2}{p^+}$$

Gauges:

$$A^0 = 0 \qquad A^+ = 0$$

From

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi + g\, A^a_\mu J^{\mu,a}_D$$ define LF Hamiltonian by $$P^- = \sum \Pi\, \partial_+ A - \mathcal{L}$$

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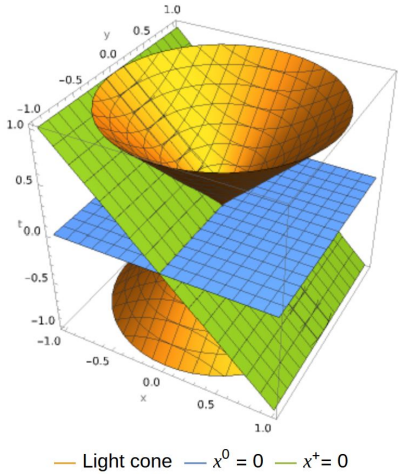

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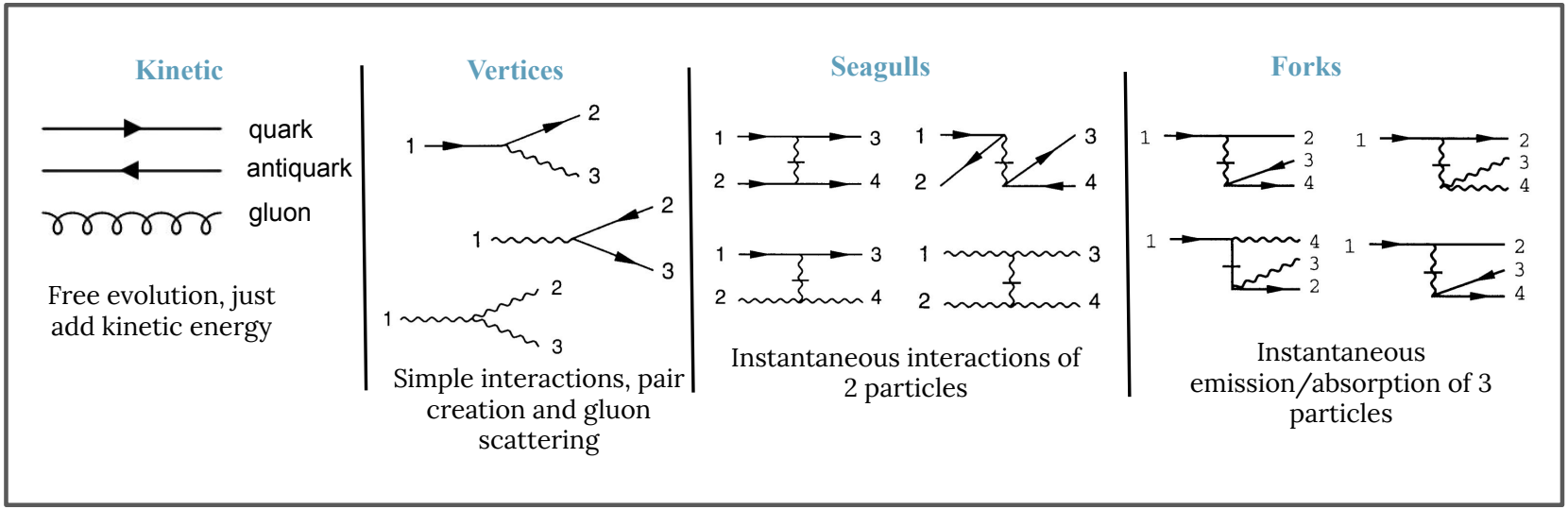
$$P_+ = \frac{1}{2}\int \mathrm{d}x_+ \mathrm{d}^2 x_\perp \left(\bar{\tilde{\Psi}} \gamma^+ \frac{m^2 + (i\nabla_\perp)^2}{i\partial^+} \tilde{\Psi} + \tilde{A}^\mu_a (i\nabla_\perp)^2 \tilde{A}^a_\mu \right) + g \int \mathrm{d}x_+ \mathrm{d}^2 x_\perp\, \tilde{J}^\mu_a \tilde{A}^a_\mu$$

$$+ \frac{g^2}{4}\int \mathrm{d}x_+ \mathrm{d}^2 x_\perp\, \tilde{B}^{\mu\nu}_a \tilde{B}^a_{\mu\nu} + \frac{g^2}{2}\int \mathrm{d}x_+ \mathrm{d}^2 x_\perp \tilde{J}^+_a \frac{1}{(i\partial^+)^2} \tilde{J}^+_a + \frac{g^2}{2}\int \mathrm{d}x_+ \mathrm{d}^2 x_\perp\, \bar{\tilde{\Psi}} \gamma^\mu T^a \tilde{A}^a_\mu \frac{\gamma^+}{i\partial^+} (\gamma^\nu T^b \tilde{A}^b_\nu \tilde{\Psi}) \,.$$

Hamiltonian as a dictionary of interactions among **on shell** partons:

doi.org/10.1016/S0370-1573(97)00089-6



V terms

T terms

Encoding I - Goal

Bosons

$$a_\rho^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{S}_i \cdot \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_\rho^\dagger\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

$$a_\rho^{(n)} = \sum_{i=1}^{n} \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{01} \otimes \mathfrak{s}_\rho\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{S}_i$$

Commutation relations up to boundary term

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} - \mathcal{S}_n \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{S}_n$$

Fermions

$$b_\rho^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{A}_i \cdot \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_\rho^\dagger\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

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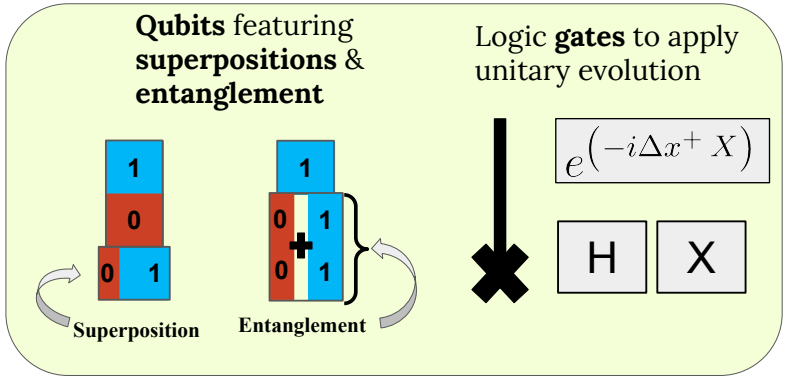
Anticommutation relations up to boundary term

$$\left\{b_\rho^{(n)}, b_\eta^{(n)\dagger}\right\} = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} + \mathcal{A}_n \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{A}_n$$

Details in PhysRevD.110.116018

Quantum Computers

Quantum computers have two main components



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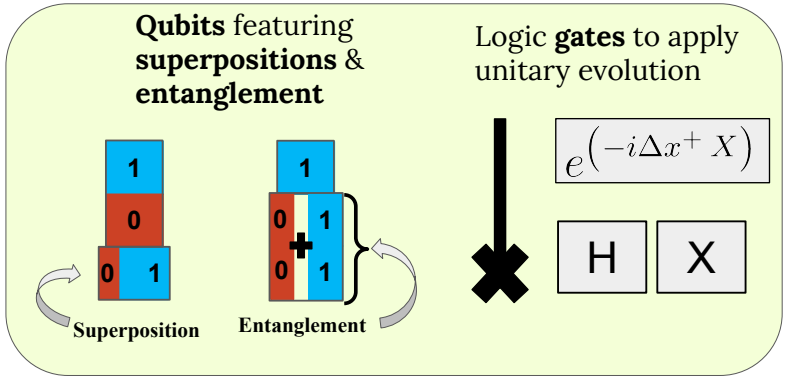

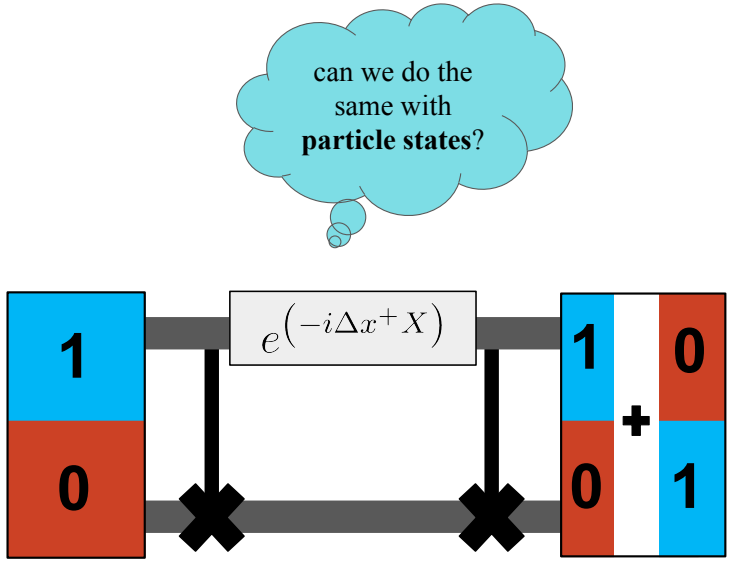

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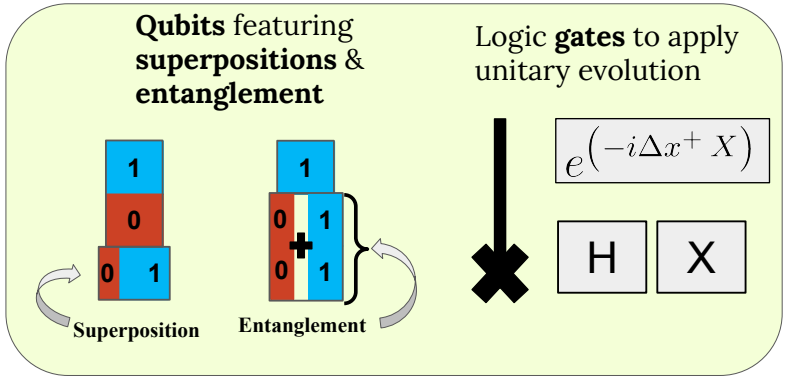

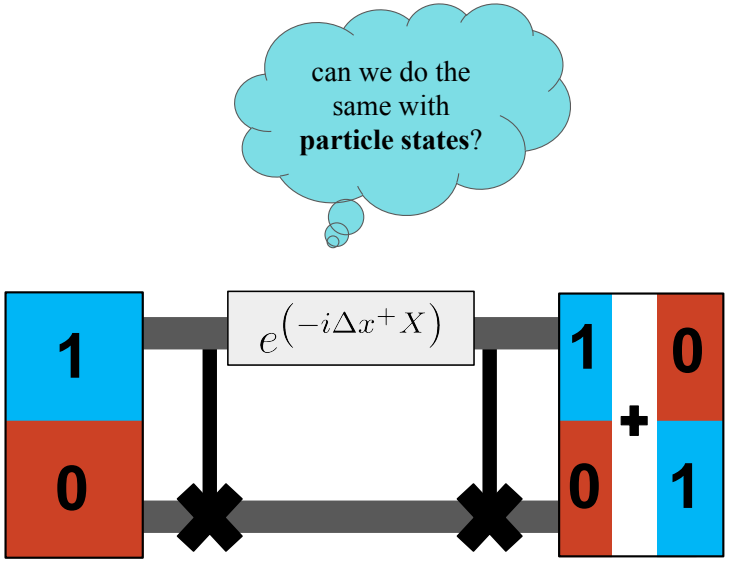

Four basic operations

Turn on
$$\mathfrak{C}_{10} = |1\rangle\langle 0|$$

Control
$$\mathfrak{C}_{11} = |1\rangle\langle 1|$$

Turn off
$$\mathfrak{C}_{01} = |0\rangle\langle 1|$$

Anticontrol
$$\mathfrak{C}_{00} = |0\rangle\langle 0|$$

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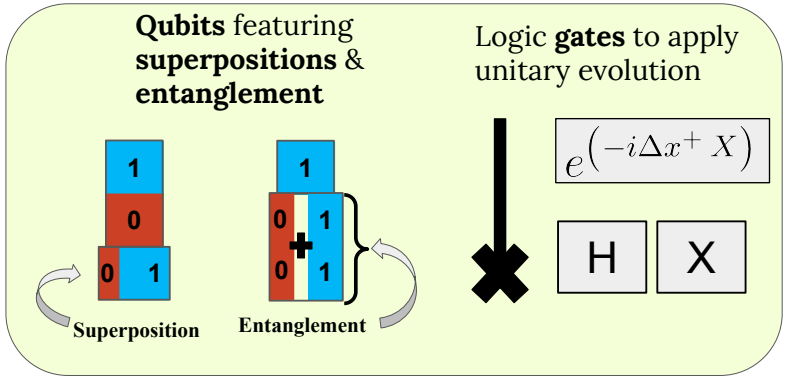

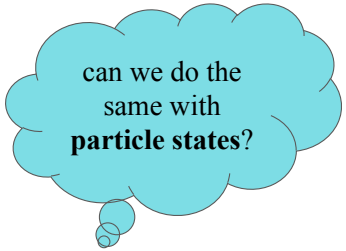

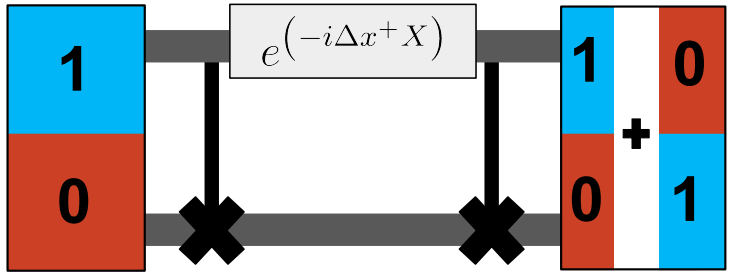

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Pauli matrices

$$I = \mathfrak{C}_{00} + \mathfrak{C}_{11} \qquad X = \mathfrak{C}_{10} + \mathfrak{C}_{01}$$

$$Y = i\mathfrak{C}_{10} - i\mathfrak{C}_{01} \qquad Z = \mathfrak{C}_{00} - \mathfrak{C}_{11}$$

$$H = \frac{1}{\sqrt{2}}(\mathfrak{C}_{00} + \mathfrak{C}_{01} + \mathfrak{C}_{10} - \mathfrak{C}_{11})$$

Encoding II - Particle Registers

1. Encode each **single-particle** state as a binary number on **qubit registers**

N states $\rightarrow \log_2 N$ qubits

Gluons have 8 colors (Gell-Mann basis):

$$\lambda_1 : (r\bar{b} + b\bar{r})/\sqrt{2} \quad \rightleftharpoons \quad \boxed{0}\boxed{0}\boxed{0} \quad \mathfrak{s}_1^\dagger = |000\rangle\langle 000|$$

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... ...

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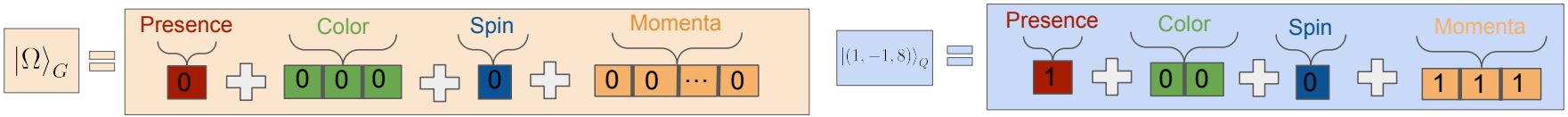
Add spin, momenta (discretized) and **presence**



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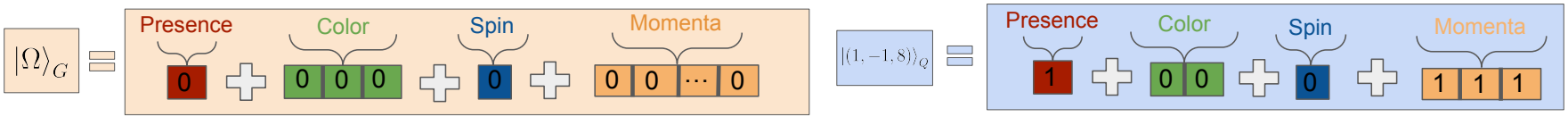

To initiate a register, change modes and presence

$$|(1,\uparrow,(1,0,0))\rangle_G = \left(\mathfrak{C}_{10} \otimes \mathfrak{s}^\dagger_{(1,\uparrow,(1,0,0))}\right)_1 |\Omega\rangle_G$$

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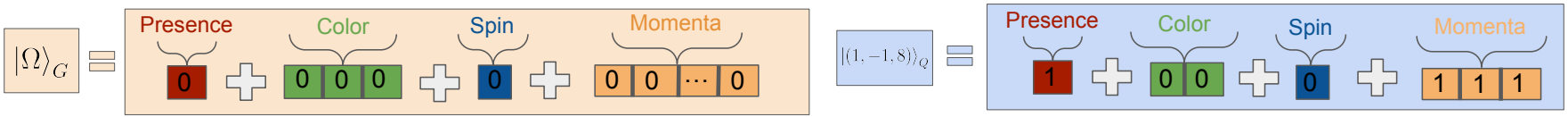

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Acts on presence

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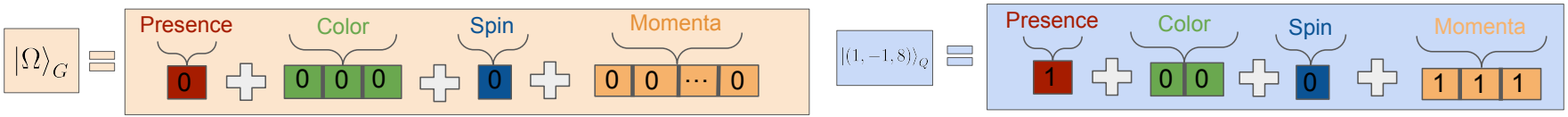

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Acts on quantum numbers

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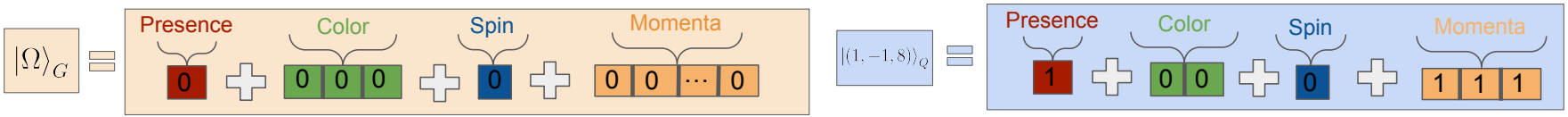

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$$|(1,\uparrow,(1,0,0))\rangle_G = \left(\mathfrak{C}_{10} \otimes \mathfrak{s}^\dagger_{(1,\uparrow,(1,0,0))}\right)_1 |\Omega\rangle_G = \mathfrak{C}_{10}\,|0\rangle \otimes \mathfrak{s}_1^\dagger\,|000\rangle \otimes \mathfrak{s}_\uparrow^\dagger\,|0\rangle \otimes \mathfrak{s}^\dagger_{(1,0,0)}\,|00\dots0\rangle$$

Encoding III - Multi-particle memories

2. Define multi-particle states combining registers

No particles

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No particles

1 particle

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How to distinguish? Use presence!

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Introduce projectors to select registers:

$$\mathbb{P}_i^{(n)} = \overbrace{(\mathfrak{C}_{00} \otimes \mathfrak{i})_n \otimes \cdots \otimes (\mathfrak{C}_{00} \otimes \mathfrak{i})_{i+1} \otimes \underbrace{(\mathfrak{C}_{11} \otimes \mathfrak{i})_i \otimes \cdots \otimes (\mathfrak{C}_{11} \otimes \mathfrak{i})_1}_{i \text{ particles}}}^{n \text{ registers}}$$

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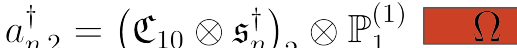
Combine projectors and set operators to create particles on specific registers

$$a^\dagger_{\eta,2} = \left(\mathfrak{C}_{10}\otimes\mathfrak{s}^\dagger_\eta\right)_2\otimes\mathbb{P}_1^{(1)}$$



$a^\dagger_{\eta,2}$

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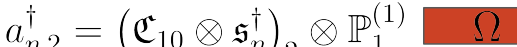

$a^\dagger_{\eta,2}$

3. Add (anti)symmetrizers for (fermion)bosons

$\mathcal{S}_2/\mathcal{A}_2$

$$\mathcal{S}_j = \left(\mathcal{I}+\mathcal{P}_{(j)(j-1)}+\cdots+\mathcal{P}_{(j)(1)}\right)/\sqrt{j} \qquad \mathcal{A}_j = \left(\mathcal{I}-\mathcal{P}_{(j)(j-1)}-\cdots-\mathcal{P}_{(j)(1)}\right)/\sqrt{j}$$

Encoding IV - Commutation relations revisited

Bosons

$$a_\rho^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{S}_i \cdot \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_\rho^\dagger\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

$$a_\rho^{(n)} = \sum_{i=1}^{n} \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{01} \otimes \mathfrak{s}_\rho\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)} \cdot \mathcal{S}_i$$

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Commutation relations

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)}$$

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Commutation relations up to boundary term

$$\left[a_\rho^{(n)}, a_\eta^{(n)\dagger}\right] = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} - \mathcal{S}_n \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{S}_n$$

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Fermions

$$b_\rho^{(n)\dagger} = \sum_{i=1}^{n} \mathcal{A}_i \cdot \mathbb{P}_0^{(n-i)} \otimes \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_\rho^\dagger\right)_i \otimes \mathbb{P}_{i-1}^{(i-1)}$$

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Anticommutation relations up to boundary term

$$\left\{b_\rho^{(n)}, b_\eta^{(n)\dagger}\right\} = \delta_{\rho\eta}\, \mathbb{P}_0^{(1)} \otimes \mathbb{I}^{(n-1)} + \mathcal{A}_n \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_\rho^\dagger \mathfrak{s}_\eta\right)_n \otimes \mathbb{P}_{n-1}^{(n-1)} \cdot \mathcal{A}_n$$

Details in PhysRevD.110.116018

Encoding IV – Commutation relations revisited

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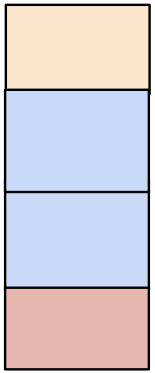
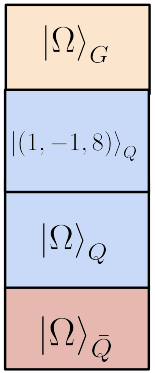
Algorithm I - Set up and exponentiation

1. Minimal: memory with **up to** 2 quarks, 1 antiquark and 1 gluon



Algorithm I - Set up and exponentiation

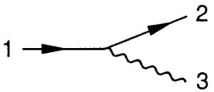
1. Minimal: memory with **up to** 2 quarks, 1 antiquark and 1 gluon
2. Start with quark of max momenta



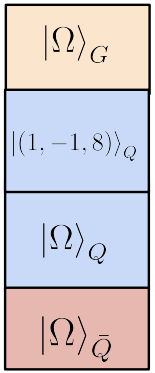
Algorithm I - Set up and exponentiation

1. Minimal: memory with **up to** 2 quarks, 1 antiquark and 1 gluon
2. Start with quark of max momenta
3. Apply codification

$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$



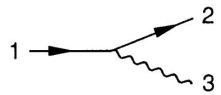
$$= \sum_{123} V(1;2,3) \underbrace{\left(\mathcal{S}_{j_b} \cdot \mathbb{P}_0^{(n_b - j_b)} \cdot \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_3^\dagger \right)_{j_b} \otimes \mathbb{P}_{j_b-1}^{(j_b-1)} \right.}_{\text{gluons}} \bar{\otimes} \underbrace{\left. \mathcal{A}_{j_f} \cdot \mathbb{P}_0^{(n_f - j_f)} \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_2^\dagger \mathfrak{s}_1 \right)_{j_f} \otimes \mathbb{P}_{j_f-1}^{(j_f-1)} \cdot \mathcal{A}_{j_f} + h.c. \right)}_{\text{fermions}}$$



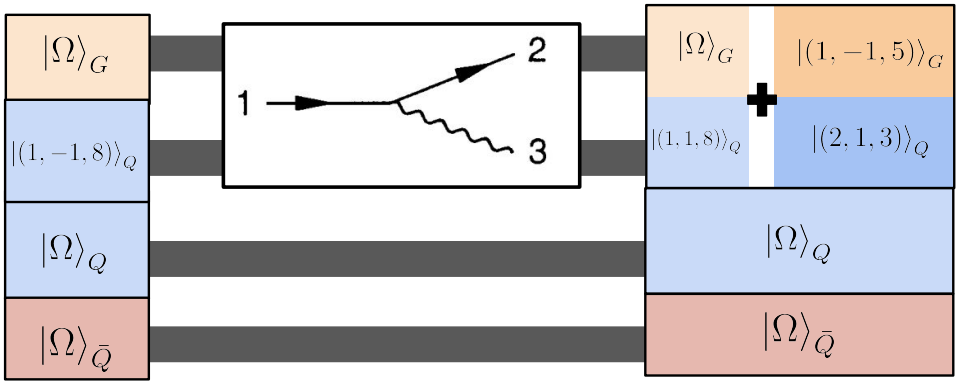
Algorithm I - Set up and exponentiation

1. Minimal: memory with **up to** 2 quarks, 1 antiquark and 1 gluon
2. Start with quark of max momenta
3. Apply codification and exponentiate using Trotter decompositions

$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$



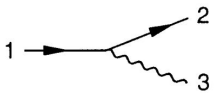
$$= \sum_{123} V(1;2,3) \underbrace{\left(\mathcal{S}_{j_b} \cdot \mathbb{P}_0^{(n_b - j_b)} \cdot \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_3^\dagger \right)_{j_b} \otimes \mathbb{P}_{j_b-1}^{(j_b-1)} \right.}_{\text{gluons}} \bar{\otimes} \underbrace{\left. \mathcal{A}_{j_f} \cdot \mathbb{P}_0^{(n_f - j_f)} \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_2^\dagger \mathfrak{s}_1 \right)_{j_f} \otimes \mathbb{P}_{j_f-1}^{(j_f-1)} \cdot \mathcal{A}_{j_f} + h.c. \right)}_{\text{fermions}}$$



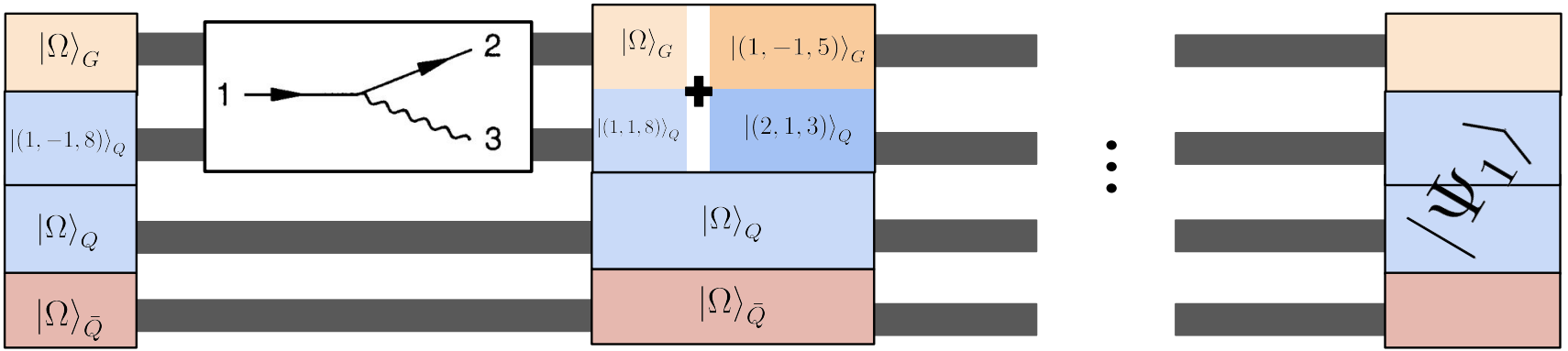
Algorithm I - Set up and exponentiation

1. Minimal: memory with **up to** 2 quarks, 1 antiquark and 1 gluon
2. Start with quark of max momenta
3. Apply codification and exponentiate using Trotter decompositions
4. Repeat for all Hamiltonian terms to evolve **1 Trotter step**

$$V_1 = \int V(1\,;\,2,\,3)\, a_3^\dagger\, b_2^\dagger\, b_1 + h.c.$$

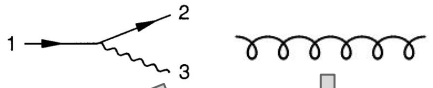


$$= \sum_{123} V(1;2,3) \underbrace{\left(\mathcal{S}_{j_b} \cdot \mathbb{P}_0^{(n_b - j_b)} \cdot \left(\mathfrak{C}_{10} \otimes \mathfrak{s}_3^\dagger \right)_{j_b} \otimes \mathbb{P}_{j_b-1}^{(j_b-1)} \right.}_{\text{gluons}} \underbrace{\left. \otimes \mathcal{A}_{j_f} \cdot \mathbb{P}_0^{(n_f - j_f)} \cdot \left(\mathfrak{C}_{11} \otimes \mathfrak{s}_2^\dagger \mathfrak{s}_1 \right)_{j_f} \otimes \mathbb{P}_{j_f-1}^{(j_f-1)} \cdot \mathcal{A}_{j_f} + h.c. \right)}_{\text{fermions}}$$



Algorithm II - Halting protocol

Errors expected when trying to create two gluons, focus on gluon scattering:



$$\exp\{-i\Delta t(V_1 + \lambda E_c)\}$$

Algorithm II – Halting protocol

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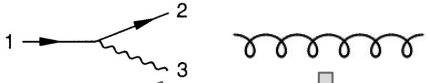
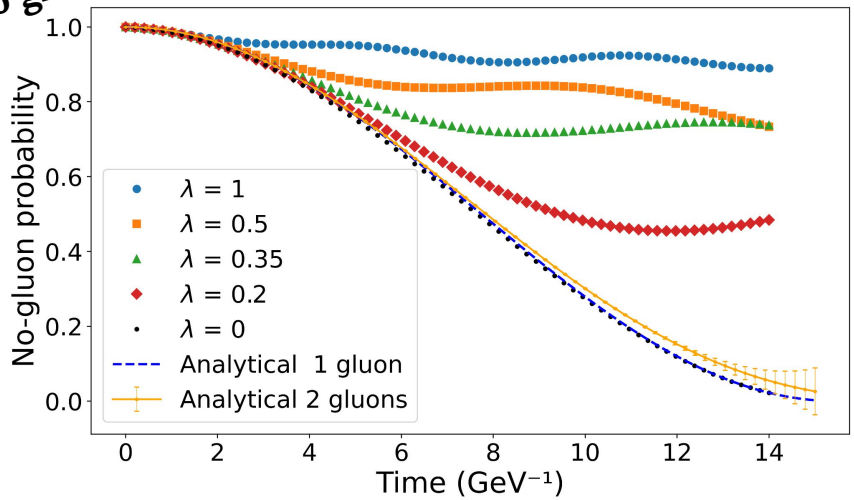


$$\exp\{-i\Delta t(V_1 + \lambda E_c)\}$$

No gluons



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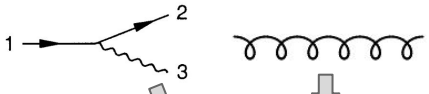

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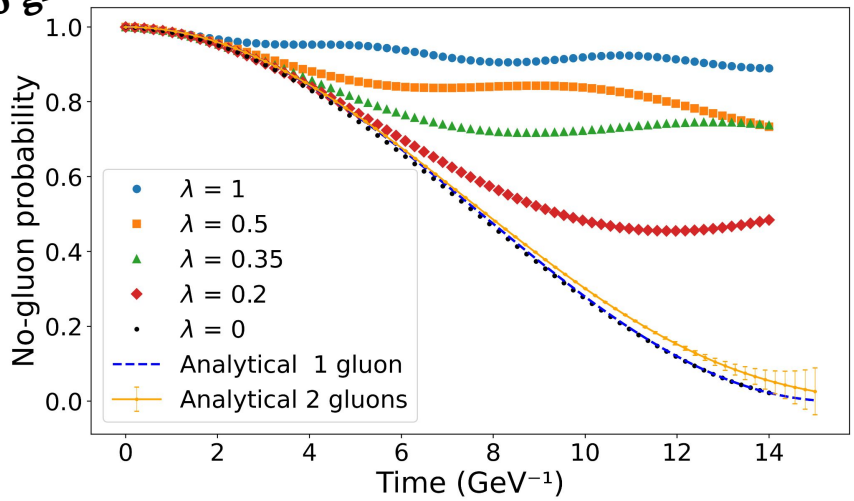

We expect the entropy to increase monotonically:

$$S = \sum_i -p_i \log(p_i)$$

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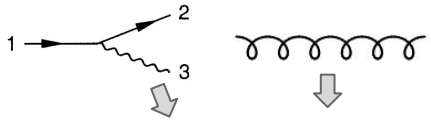
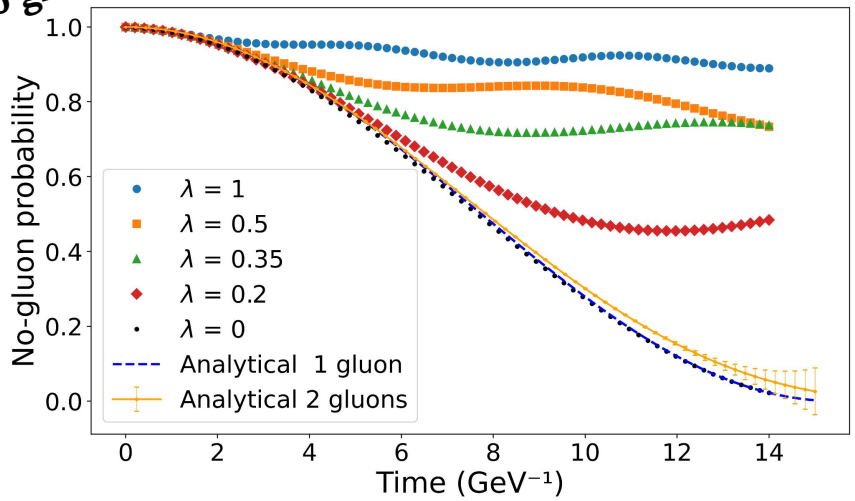


$$\exp\{-i\Delta t(V_1 + \lambda E_c)\}$$

No gluons



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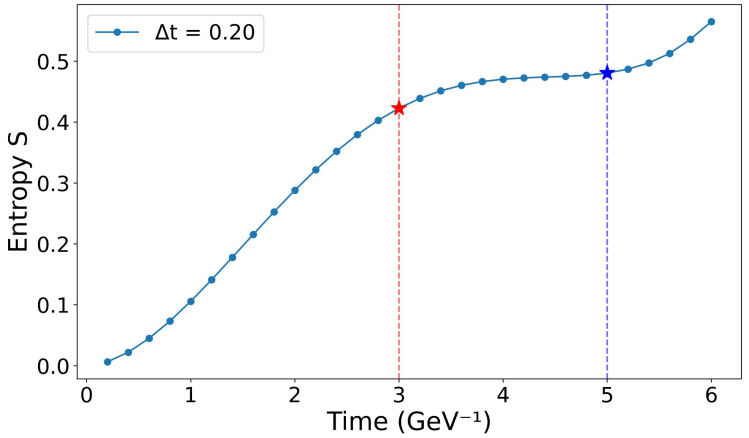

30 Trotter Steps for a total evolution time of 6 GeV^{-1}

Algorithm III - Measurement

5. Measure J/Ψ wavefunction

An ansatz

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An ansatz $$|J/\Psi\rangle = \sum \frac{\chi_0(x)}{\sqrt{x\,(z-x)}}\,\delta_{c_q c_{\bar{q}}}\;\sigma_{ij}\;|x\;i\,c_q,(z-x)\;j\,c_{\bar{q}}\rangle$$

doi.org/10.1140/epjc/s10052-022-10988-5

Algorithm III - Measurement

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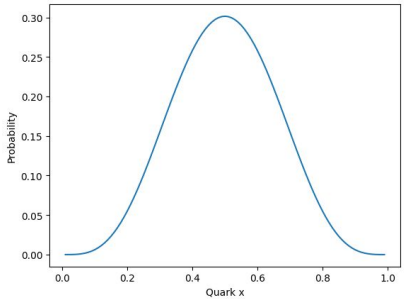
- Simple momentum fraction dependence



Algorithm III - Measurement

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An ansatz $$|J/\Psi\rangle = \sum \frac{\chi_0(x)}{\sqrt{x\,(z-x)}} \delta_{c_q c_{\bar{q}}} \sigma_{ij}\ |x\ i\ c_q, (z-x)\ j\ c_{\bar{q}}\rangle$$

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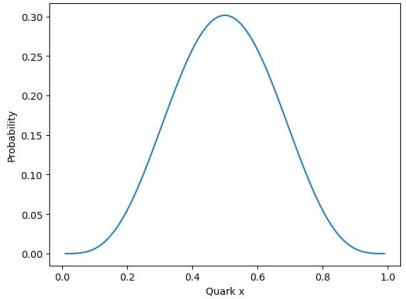
- Simple momentum fraction dependence



- Color singlet

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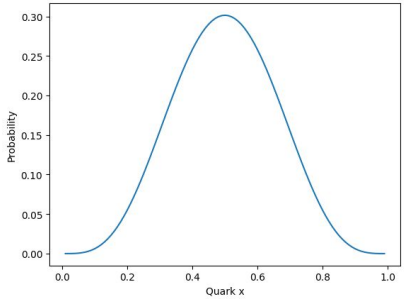

- Color singlet
- Spin singlet

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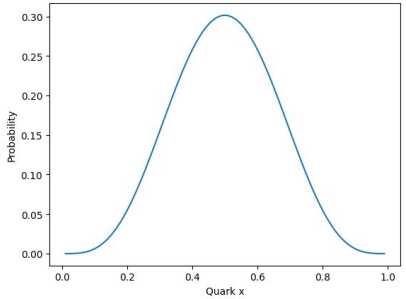

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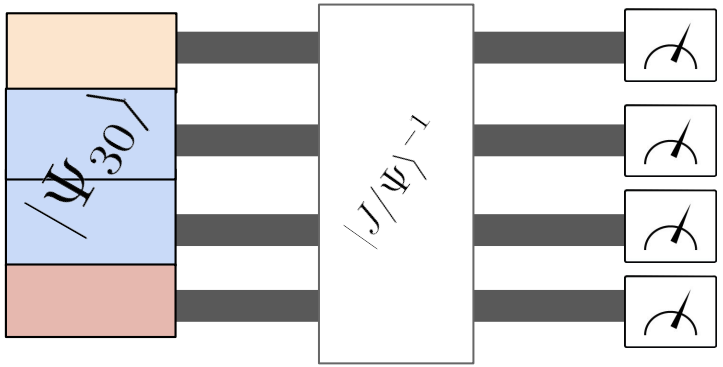

Algorithm III - Measurement

5. Measure J/Ψ wavefunction

An ansatz $|J/\Psi\rangle$ Should be obtained from Hamiltonian! $x)\ j\ c_{\bar{q}}\rangle$

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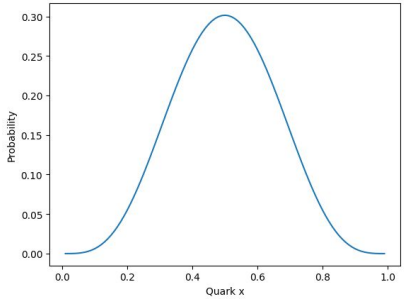
- Simple momentum fraction dependence



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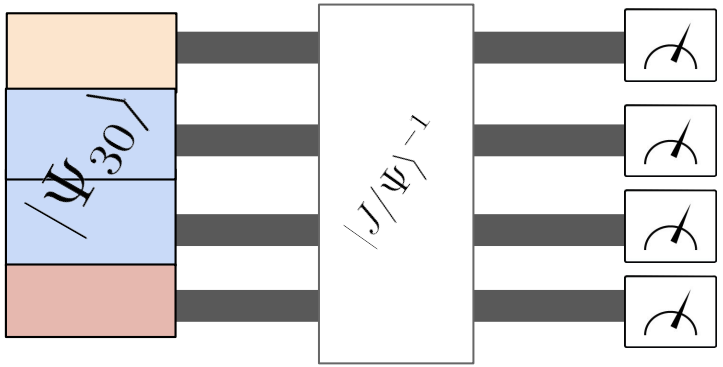
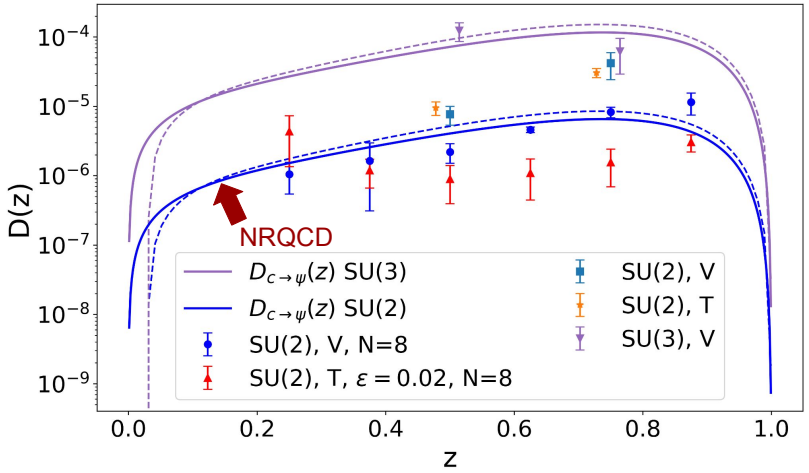


Results & conclusions



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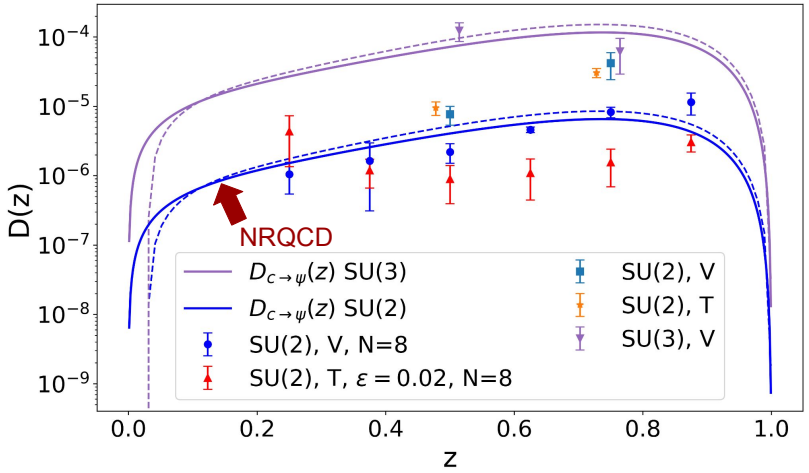
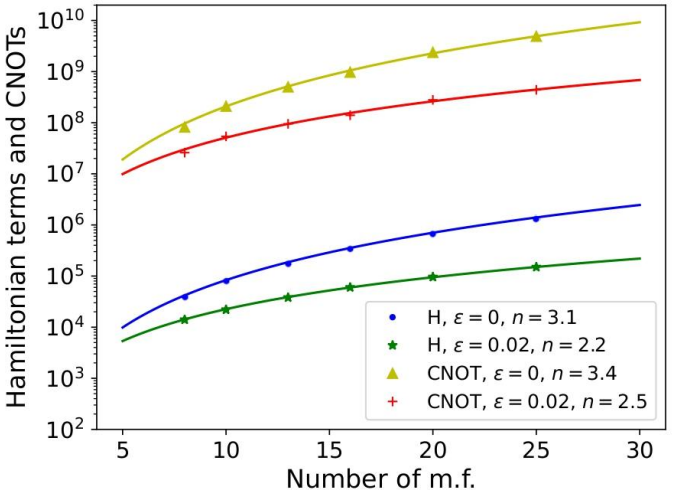


$$C(N;a,n) = a \cdot N^n$$



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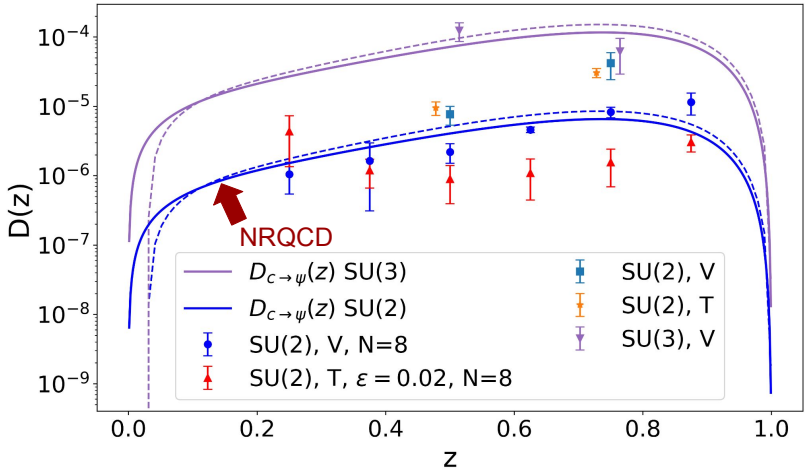

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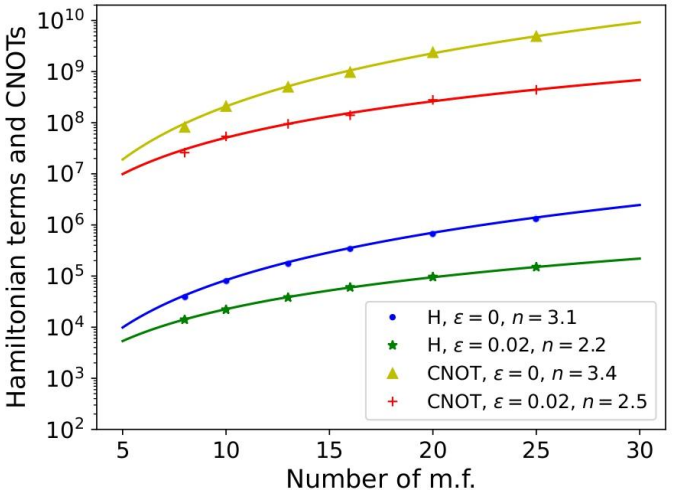
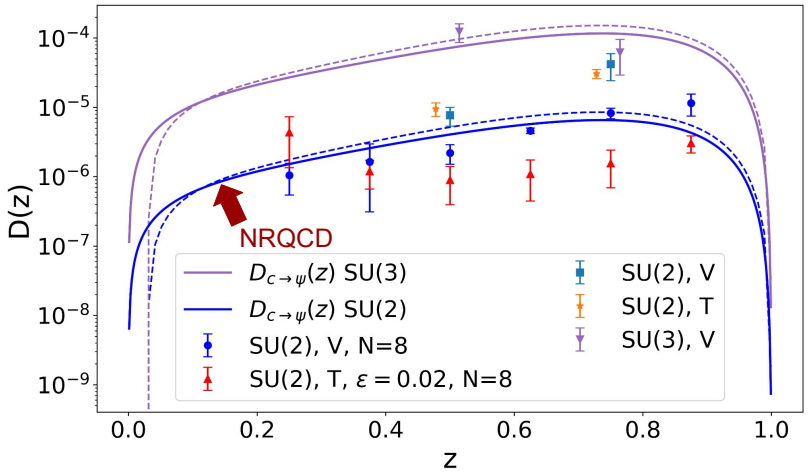


- **End to end** simulation of fragmentation in LF QCD with 2 quarks, 1 antiquark, and 1 gluon.

Results & conclusions



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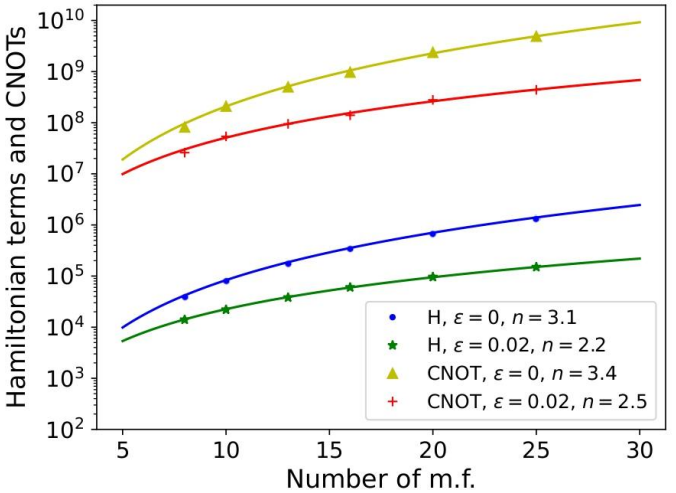

- **End to end** simulation of fragmentation in LF QCD with 2 quarks, 1 antiquark, and 1 gluon.
- 29 qubits ⟹ 17 GB of wavefunction

Run configuration	V/T	N_c	N grid	N_{qubits}	Δt (GeV^{-1})	ϵ	Runtime (min)
1	V	2	4	25	0.2	0	1.5
2	T	2	4	25	0.1	0	22
3	T	2	4	25	0.2	0.02	15
4	V	2	8	29	0.2	0	110
5	T	2	8	29	0.2	0.02	1800
6	V	3	4	29	0.25	0	120

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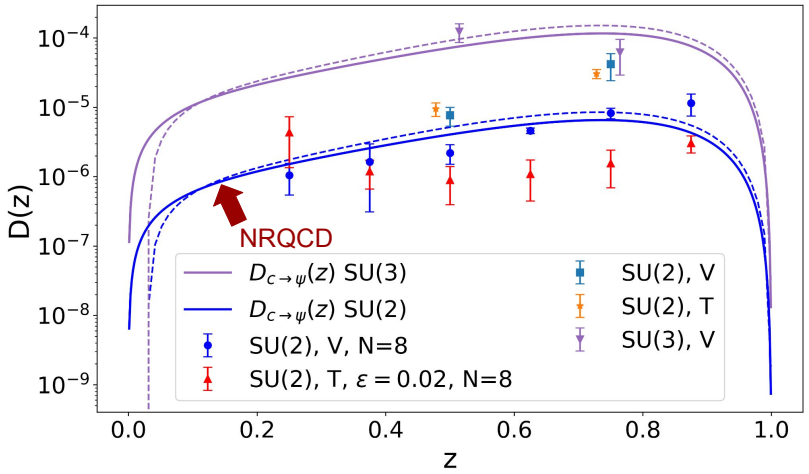
Results & conclusions



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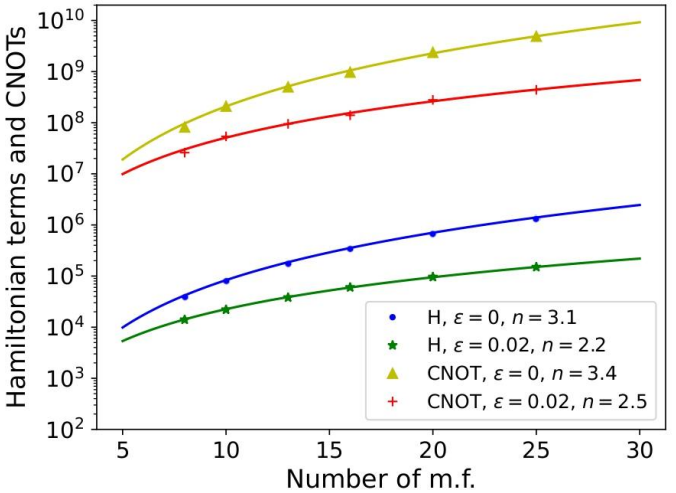

- **End to end** simulation of fragmentation in LF QCD with 2 quarks, 1 antiquark, and 1 gluon.
- 29 qubits $\Longrightarrow$ 17 GB of wavefunction
- Total number of gates $\sim 10^8$ far from today$\sim 10^3$ gates, but improving fast!

Run configuration	V/T	N_c	N grid	N_{qubits}	Δt (GeV^{-1})	ϵ	Runtime (min)
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Quantum Computing Fragmentation Functions

Universidad Complutense de Madrid & IPARCOS

Juan José Gálvez Viruet*

arxiv: 2510.18869

*juagalve@ucm.es

Appendix: More about scaling