FeynCalc

An Introduction to Mathematica Tools for HEP
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Carlos Quezada Calonge

Organization:

Javier Martínez Martín
Carlos Quezada Calonge
Alexandre Salas Bernárdez
Juan José Sanz Cillero

What is FeynCalc (FC) ?

FeynCalc is a Mathematica package for algebraic calculations in Quantum Field Theory and semi-automatic evaluation of Feynman Diagrams.

-One-loop integration

-Dirac algebra, contractions, color algebra, etc

Preface

We count with three main tools to understand FeynArts, FeynCalc, etc:

1. Command InputForm[X] in Mathematica. Shows how Mathematica understands the expression X.
2. Command ?X . Shows the definition of X.
3. User Guide provided by the creators. FC Guide: https://feyncalc.github.io/reference

Using InputForm

Typical amplitude for a Feynman diagram

```
hhtohhtreeamp2 = PickLevel[Particles][hhtohhtreeamp] // ToFA1Conventions
```

$$\mathrm{FAFeynAmpList}\left(\mathrm{Process} \to \begin{pmatrix} S(1) & p1 & \mathrm{FCGV}(\mathrm{MH}) & \{\} \\ S(1) & p2 & \mathrm{FCGV}(\mathrm{MH}) & \{\} \end{pmatrix} \to \begin{pmatrix} S(1) & k1 & \mathrm{FCGV}(\mathrm{MH}) & \{\} \\ S(1) & k2 & \mathrm{FCGV}(\mathrm{MH}) & \{\} \end{pmatrix}, \mathrm{Model} \to \{\mathrm{SM}\}, \mathrm{GenericModel} \to \{\mathrm{Lorentz}\}, \mathrm{AmplitudeLevel} \to \{\mathrm{Particles}\}, \mathrm{ExcludeParticles} \to \{\}, \mathrm{ExcludeFieldPoints} \to \{\}, \mathrm{LastSelections} \to \{\}\right)$$

$$\left(\mathrm{FAFeynAmp}\left(\mathrm{GraphName}(, \mathrm{T1}, \mathrm{G1C1P1}, \mathrm{N1}), -\frac{3\,\mathrm{FCGV}(\mathrm{EL})^2\,\mathrm{FCGV}(\mathrm{MH})^2}{4\,\mathrm{FCGV}(\mathrm{MW})^2\,\mathrm{FCGV}(\mathrm{SW})^2}\right), \mathrm{FAFeynAmp}\left(\mathrm{GraphName}(, \mathrm{T2}, \mathrm{G1C2P1}, \mathrm{N2}), -\frac{9\,\mathrm{FCGV}(\mathrm{EL})^2\,\mathrm{FCGV}(\mathrm{MH})^4\,\frac{1}{(k1+k2)^2-\mathrm{FCGV}(\mathrm{MH})^2}}{4\,\mathrm{FCGV}(\mathrm{MW})^2\,\mathrm{FCGV}(\mathrm{SW})^2}\right),\right.$$

$$\left.\mathrm{FAFeynAmp}\left(\mathrm{GraphName}(, \mathrm{T3}, \mathrm{G1C3P1}, \mathrm{N3}), -\frac{9\,\mathrm{FCGV}(\mathrm{EL})^2\,\mathrm{FCGV}(\mathrm{MH})^4\,\frac{1}{(k2-p2)^2-\mathrm{FCGV}(\mathrm{MH})^2}}{4\,\mathrm{FCGV}(\mathrm{MW})^2\,\mathrm{FCGV}(\mathrm{SW})^2}\right), \mathrm{FAFeynAmp}\left(\mathrm{GraphName}(, \mathrm{T4}, \mathrm{G1C4P1}, \mathrm{N4}), -\frac{9\,\mathrm{FCGV}(\mathrm{EL})^2\,\mathrm{FCGV}(\mathrm{MH})^4\,\frac{1}{(k1-p2)^2-\mathrm{FCGV}(\mathrm{MH})^2}}{4\,\mathrm{FCGV}(\mathrm{MW})^2\,\mathrm{FCGV}(\mathrm{SW})^2}\right)\right)$$

```
InputForm[hhtohhtreeamp2]
forma de entrada

//InputForm=
FAFeynAmpList[Process -> {{S[1], p1, FCGV["MH"], {}}, {S[1], p2, FCGV["MH"], {}}} -> {{S[1], k1, FCGV["MH"], {}}, {S[1], k2, FCGV["MH"], {}}}, Model -> {"SM"}, GenericModel -> {"Lorentz"},
  AmplitudeLevel -> {Particles}, ExcludeParticles -> {}, ExcludeFieldPoints -> {}, LastSelections -> {}][FAFeynAmp[GraphName["", "T1", "G1C1P1", "N1"],
  (-3*FCGV["EL"]^2*FCGV["MH"]^2)/(4*FCGV["MW"]^2*FCGV["SW"]^2)], FAFeynAmp[GraphName["", "T2", "G1C2P1", "N2"], (-9*FAPropagatorDenominator[k1 + k2, FCGV["MH"]]*FCGV["EL"]^2*FCGV["MH"]^4)/
   (4*FCGV["MW"]^2*FCGV["SW"]^2)], FAFeynAmp[GraphName["", "T3", "G1C3P1", "N3"], (-9*FAPropagatorDenominator[k2 - p2, FCGV["MH"]]*FCGV["EL"]^2*FCGV["MH"]^4)/(4*FCGV["MW"]^2*FCGV["SW"]^2)],
 FAFeynAmp[GraphName["", "T4", "G1C4P1", "N4"], (-9*FAPropagatorDenominator[k1 - p2, FCGV["MH"]]*FCGV["EL"]^2*FCGV["MH"]^4)/(4*FCGV["MW"]^2*FCGV["SW"]^2)]]
```

This allows us to write substitutions /. FCGV[“MH”] -> MH

How does it work?

Takes the output of FeynArts written in FeynCalc language created first by CreateFeynAmp and then translate to FeynCalc

```
omegaomegatoomegaomegaamp = CreateFeynAmp[omegaomegatoomegaomega, AmplitudeLevel → Particles, GaugeRules → {}]  /. M$FACouplingsHEFT
```

```
omegaomegatoomegaomegaampnew = PickLevel[Particles][omegaomegatoomegaomegaamp] // ToFA1Conventions
```

$$\text{FAFeynAmpList}\left(\text{Process} \to \begin{pmatrix} S(3) & \text{p1} & \text{Mz} & \{\} \\ S(3) & \text{p2} & \text{Mz} & \{\} \end{pmatrix} \to \begin{pmatrix} S(3) & \text{k1} & \text{Mz} & \{\} \\ S(3) & \text{k2} & \text{Mz} & \{\} \end{pmatrix}, \text{Model} \to \{\text{HEFT3\_FA}\}, \text{GenericModel} \to \{\text{HEFT3\_FA}\},\right.$$
$$\text{AmplitudeLevel} \to \{\text{Particles}\}, \text{ExcludeParticles} \to \{\}, \text{ExcludeFieldPoints} \to \{\}, \text{LastSelections} \to \{\}\Big)\Big(\text{FAFeynAmp}\Big(\text{GraphName}(, \text{T1}, \text{G1C1P1}, \text{N1}),$$
$$-i\left(\frac{8\,i\,(\text{a4}+\text{a5})\,(-\text{k1}).(-\text{k2})\,\text{p1}.\text{p2}}{v^4} + \frac{8\,i\,(\text{a4}+\text{a5})\,(-\text{k1}).\text{p2}\,(-\text{k2}).\text{p1}}{v^4} + \frac{8\,i\,(\text{a4}+\text{a5})\,(-\text{k1}).\text{p1}\,(-\text{k2}).\text{p2}}{v^4} - \frac{2\,i\,(-\text{k1}).(-\text{k2})}{v^2} - \frac{2\,i\,(-\text{k1}).\text{p1}}{v^2} - \frac{2\,i\,(-\text{k1}).\text{p2}}{v^2} - \frac{2\,i\,(-\text{k2}).\text{p1}}{v^2} - \frac{2\,i\,(-\text{k2}).\text{p2}}{v^2} - \frac{2\,i\,\text{p1}.\text{p2}}{v^2}\right)\Big),$$
$$\text{FAFeynAmp}\left(\text{GraphName}(, \text{T2}, \text{G1C2P1}, \text{N2}), -\frac{4\,a^2\,(-\text{k1}).(-\text{k2})\,\text{p1}.\text{p2}\,\frac{1}{(\text{k1}+\text{k2})^2-\text{FCGV}(\text{MH})^2\,(\text{xiSubscript}[S(4)])}}{v^2}\right), \text{FAFeynAmp}\Big(\text{GraphName}(, \text{T3}, \text{G1C3P1}, \text{N3}),$$
$$-\frac{4\,a^2\,(-\text{k1}).\text{p1}\,(-\text{k2}).\text{p2}\,\frac{1}{(\text{k2}-\text{p2})^2-\text{FCGV}(\text{MH})^2\,(\text{xiSubscript}[S(4)])}}{v^2}\Big), \text{FAFeynAmp}\left(\text{GraphName}(, \text{T4}, \text{G1C4P1}, \text{N4}), -\frac{4\,a^2\,(-\text{k1}).\text{p2}\,(-\text{k2}).\text{p1}\,\frac{1}{(\text{k1}-\text{p2})^2-\text{FCGV}(\text{MH})^2\,(\text{xiSubscript}[S(4)])}}{v^2}\right)\Big)$$

Example 1: Goldstone scattering

```
omegaomegatoomegaomegaampnew[[2, 2]]
```

$$-\frac{4\,a^2\,(-\mathrm{k1}).(-\mathrm{k2})\ \mathrm{p1.p2}\ \frac{1}{(\mathrm{k1}+\mathrm{k2})^2-\mathrm{FCGV(MH)}^2\ \mathrm{xi}_{S(4)}}}{v^2}$$

```
omegaomegatoomegaomegaampnew[[2, 2]] // InputForm
                                       forma de entrada

InputForm=
(-4*a^2*FAPropagatorDenominator[k1 + k2, Sqrt[FAGaugeXi[S[4]]]*FCGV["MH"]]*FAScalarProduct[-k1, -k2]*
  FAScalarProduct[p1, p2])/v^2
```

```
omegaomegatoomegaamps =
 Table[omegaomegatoomegaomegaampnew[[i, 2]], {i, 1, 4}] /. {FCGV[x_] → ToExpression[x]} /. {p2 → k4, p1 → k3} /. FAPropagatorDenominator[x_, y_] → 1/(x^2 - y^2) /.
 tabla                                                      convierte en expresión
  {(k1 + k2)^2 → S, (k2 - k4)^2 → T, (k1 - k4)^2 → U}
```

$$=\Big\{-i\left(\frac{8\,i\,(a4+a5)\,(-\mathrm{k1}).(-\mathrm{k2})\ \mathrm{k3.k4}}{v^4}+\frac{8\,i\,(a4+a5)\,(-\mathrm{k1}).\mathrm{k4}\ (-\mathrm{k2}).\mathrm{k3}}{v^4}+\frac{8\,i\,(a4+a5)\,(-\mathrm{k1}).\mathrm{k3}\ (-\mathrm{k2}).\mathrm{k4}}{v^4}-\frac{2\,i\,(-\mathrm{k1}).(-\mathrm{k2})}{v^2}-\frac{2\,i\,(-\mathrm{k1}).\mathrm{k3}}{v^2}-\frac{2\,i\,(-\mathrm{k1}).\mathrm{k4}}{v^2}-\frac{2\,i\,(-\mathrm{k2}).\mathrm{k3}}{v^2}-\frac{2\,i\,(-\mathrm{k2}).\mathrm{k4}}{v^2}-\frac{2\,i\ \mathrm{k3.k4}}{v^2}\right),$$

$$-\frac{4\,a^2\,(-\mathrm{k1}).(-\mathrm{k2})\ \mathrm{k3.k4}}{v^2\left(S-\mathrm{MH}^2\ (\mathrm{xiSubscript}[S(4)])\right)},\ -\frac{4\,a^2\,(-\mathrm{k1}).\mathrm{k3}\ (-\mathrm{k2}).\mathrm{k4}}{v^2\left(T-\mathrm{MH}^2\ (\mathrm{xiSubscript}[S(4)])\right)},\ -\frac{4\,a^2\,(-\mathrm{k1}).\mathrm{k4}\ (-\mathrm{k2}).\mathrm{k3}}{v^2\left(U-\mathrm{MH}^2\ (\mathrm{xiSubscript}[S(4)])\right)}\Big\}$$

```
omegaomegatoomegaamps[[2]] // InputForm
```
forma de entrada

InputForm=

```
(-4*a^2*FAScalarProduct[-k1, -k2]*FAScalarProduct[k3, k4])/(v^2*(S - "MH"^2*FAGaugeXi[S[4]]))
```

```
omegaomegatoomegaampsbis =
 Simplify[
  omegaomegatoomegaamps /. {a4 → 0, a5 → 0} //. {FAScalarProduct[-x_, y_] → -FAScalarProduct[x, y], FAScalarProduct[x_, -y_] → -FAScalarProduct[x, y]} /.
    {FAScalarProduct[k3, k4] → (S - FAScalarProduct[k3, k3] - FAScalarProduct[k4, k4])/2,
     FAScalarProduct[k1, k2] → (S - FAScalarProduct[k1, k1] - FAScalarProduct[k2, k2])/2,
     FAScalarProduct[k3, k4] → (S - FAScalarProduct[k3, k3] - FAScalarProduct[k4, k4])/2,
     FAScalarProduct[k1, k3] → (-T + FAScalarProduct[k1, k1] + FAScalarProduct[k3, k3])/2,
     FAScalarProduct[k2, k4] → (-T + FAScalarProduct[k2, k2] + FAScalarProduct[k4, k4])/2,
     FAScalarProduct[k1, k4] → (-U + FAScalarProduct[k1, k1] + FAScalarProduct[k4, k4])/2,
     FAScalarProduct[k2, k3] → (-U + FAScalarProduct[k2, k2] + FAScalarProduct[k3, k3])/2} /.
   {FAScalarProduct[k1, k1] → 0, FAScalarProduct[k2, k2] → 0, FAScalarProduct[k3, k3] → 0, FAScalarProduct[k4, k4] → 0}]
```
simplifica

$$\left\{-\frac{2(S+T+U)}{v^2}, -\frac{a^2 S^2}{v^2\left(S-\mathrm{MH}^2\,(\mathrm{xiSubscript}[S(4)])\right)}, -\frac{a^2 T^2}{v^2\left(T-\mathrm{MH}^2\,(\mathrm{xiSubscript}[S(4)])\right)}, -\frac{a^2 U^2}{v^2\left(U-\mathrm{MH}^2\,(\mathrm{xiSubscript}[S(4)])\right)}\right\}$$

Then the final amplitude is the known expression for HEFT

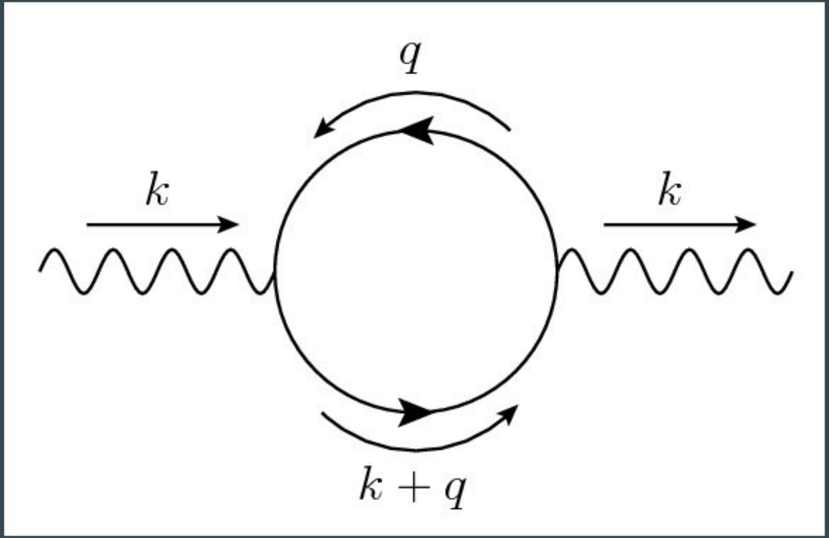
```
omegaomegatoomegaamptotal = Total[omegaomegatoomegaampsbis /. FAGaugeXi[S[4]] → 1, 1]
```

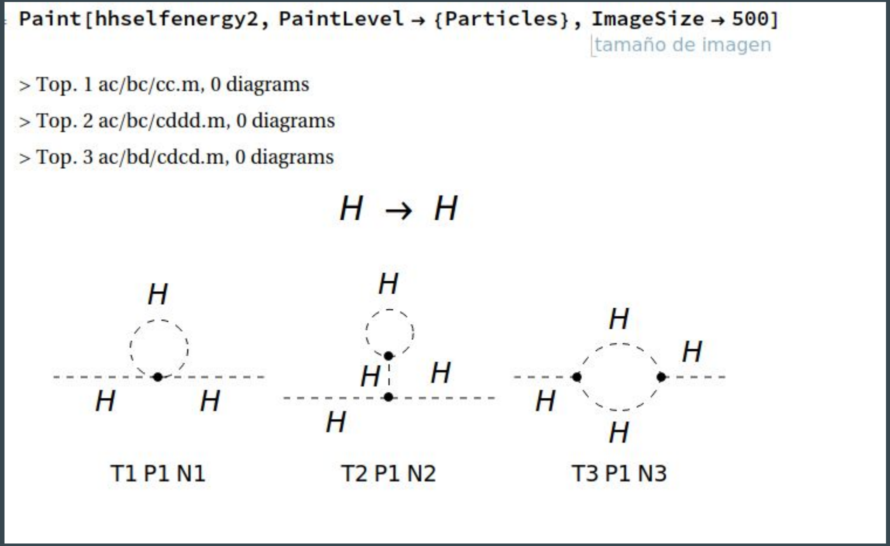
total

$$= -\frac{a^2 S^2}{v^2 (S - \mathrm{MH}^2)} - \frac{a^2 T^2}{v^2 (T - \mathrm{MH}^2)} - \frac{a^2 U^2}{v^2 (U - \mathrm{MH}^2)} - \frac{2 (S + T + U)}{v^2}$$

Example 2: Higgs self energy at one-loop

We have an amplitudes that depends on the momentum inside the loop q (q1 in FeynCalc by default)





Example 2: Higgs self energy at one-loop

The amplitude for looks like this

```
hhselfenergy2ampnew = PickLevel[Particles][hhselfenergy2amp] // ToFA1Conventions
```

FAFeynAmpList(Process → (S(1) p1 FCGV(MH) {}) → (S(1) k1 FCGV(MH) {}), Model → {SM}, GenericModel → {Lorentz}, AmplitudeLevel → {Particles}, ExcludeParticles → {F, U, V, −F(1), F(1), −F(2), F(2), −F(3), F(3), −F(4), F(4), −F(1, {1}), F(1, {1}), −F(1, {2}), F(1, {2}), −F(1, {3}), F(1, {3}), −F(2, {1}), F(2, {1}), −F(2, {2}), F(2, {2}), −F(2, {3}), F(2, {3}), −F(3, {1, _}), F(3, {1, _}), −F(3, {2, _}), F(3, {2, _}), −F(3, {3, _}), F(3, {3, _}), −F(4, {1, _}), F(4, {1, _}), −F(4, {2, _}), F(4, {2, _}), −F(4, {3, _}), F(4, {3, _}), S(2), −S(3), S(3), −U(1), U(1), −U(2), U(2), −U(3), U(3), −U(4), U(4), V(1), V(2), −V(3), V(3)}, ExcludeFieldPoints → {}, LastSelections → {S(1)})

$$\Bigg(\text{FAFeynAmp}\left(\text{GraphName}(,\ \text{T1},\ \text{G1C1P1},\ \text{N1}),\ \text{q1},\ -\frac{3\,i\ \text{FCGV(EL)}^2\ \text{FCGV(MH)}^2\ \text{FAFeynAmpDenominator}\left(\frac{1}{\text{q1}^2-\text{FCGV(MH)}^2}\right)}{128\,\pi^4\ \text{FCGV(MW)}^2\ \text{FCGV(SW)}^2}\right),$$

$$\text{FAFeynAmp}\left(\text{GraphName}(,\ \text{T2},\ \text{G1C2P1},\ \text{N2}),\ \text{q1},\ -\frac{9\,i\ \text{FCGV(EL)}^2\ \text{FCGV(MH)}^4\ \text{FAFeynAmpDenominator}\left(\frac{1}{\text{q1}^2-\text{FCGV(MH)}^2}\right)\ \frac{1}{(\text{p1}-\text{k1})^2-\text{FCGV(MH)}^2}}{128\,\pi^4\ \text{FCGV(MW)}^2\ \text{FCGV(SW)}^2}\right),$$

$$\text{FAFeynAmp}\left(\text{GraphName}(,\ \text{T3},\ \text{G1C3P1},\ \text{N3}),\ \text{q1},\ -\frac{9\,i\ \text{FCGV(EL)}^2\ \text{FCGV(MH)}^4\ \text{FAFeynAmpDenominator}\left(\frac{1}{\text{q1}^2-\text{FCGV(MH)}^2},\ \frac{1}{(\text{q1}-\text{k1})^2-\text{FCGV(MH)}^2}\right)}{128\,\pi^4\ \text{FCGV(MW)}^2\ \text{FCGV(SW)}^2}\right)\Bigg)$$

Example 2: Higgs self energy at one-loop

Let us inspect the amplitude

```
Dimensions[hhselfenergy2ampnew]
dimensiones

{3}
```

```
hhselfenergy2ampnew[[3, 3]] // InputForm
                               forma de entrada

putForm=
(((-9*I)/128)*FAFeynAmpDenominator[FAPropagatorDenominator[q1, FCGV["MH"]], FAPropagatorDenominator[-k1 + q1, FCGV["MH"]]]*FCGV["EL"]^2*FCGV["MH"]^4)/(Pi^4*FC
```

Create a table containing the amplitude of each diagram

```
hhselfenergy2amps = Table[hhselfenergy2ampnew[[i, 3]], {i, 1, 3}] /. {FCGV[x_] → ToExpression[x]} /. {p1 → k1}
                    tabla                                                convierte en expresión
```

$$\left\{-\frac{3\, i\, EL^2\, MH^2\, \text{FAFeynAmpDenominator}\left(\frac{1}{q1^2-MH^2}\right)}{128\, \pi^4\, MW^2\, SW^2},\ -\frac{9\, i\, EL^2\, MH^4 \left(-\frac{1}{MH^2}\right) \text{FAFeynAmpDenominator}\left(\frac{1}{q1^2-MH^2}\right)}{128\, \pi^4\, MW^2\, SW^2},\ -\frac{9\, i\, EL^2\, MH^4\, \text{FAFeynAmpDenominator}\left(\frac{1}{q1^2-MH^2}, \frac{1}{(q1-k1)^2-MH^2}\right)}{128\, \pi^4\, MW^2\, SW^2}\right\}$$

Example 2: Higgs self energy at one-loop

To integrate the momenta in the loop we use the function OneLoop

```
OneLoop[q1, hhselfenergy2amps]
```

In order for OneLoop to work the Propagators and Momenta need to be defined correctly (right dimension, syntax and might change among versions of FeynCalc)

```
hhselfenergy2amps // InputForm
                     forma de entrada

//InputForm=
{(((-3*I)/128)*"EL"^2*"MH"^2*FAFeynAmpDenominator[FAPropagatorDenominator[q1, "MH"]])/("MW"^2*"SW"^2*Pi^4),
 (((-9*I)/128)*"EL"^2*"MH"^4*FAFeynAmpDenominator[FAPropagatorDenominator[q1, "MH"]*FAPropagatorDenominator[0, "MH"])/
  ("MW"^2*"SW"^2*Pi^4), (((-9*I)/128)*"EL"^2*"MH"^4*FAFeynAmpDenominator[FAPropagatorDenominator[q1, "MH"],
   FAPropagatorDenominator[-k1 + q1, "MH"]])/("MW"^2*"SW"^2*Pi^4)}
```

Example 2: Higgs self energy at one-loop

Write the Propagators and Momenta in the correct way

```
hhselfenergy2ampsmod =
 hhselfenergy2amps /. FAFeynAmpDenominator[FAPropagatorDenominator[x_, y_], FAPropagatorDenominator[z_, w_]] →
    FAFeynAmpDenominator[FAPropagatorDenominator[x, y]] * FAFeynAmpDenominator[FAPropagatorDenominator[z, w]] /.
  FAFeynAmpDenominator[FAPropagatorDenominator[x_, y_]] → FeynAmpDenominator[PropagatorDenominator[Momentum[x, D], y]]
                                                                                                              deriva
```

$$\left\{-\frac{3\, i\, \mathrm{EL}^2\, \mathrm{MH}^2}{128\, \pi^4\, \mathrm{MW}^2\, \mathrm{SW}^2\, (\mathrm{q1}^2-\mathrm{MH}^2)}, -\frac{9\, i\, \mathrm{EL}^2\, \mathrm{MH}^4 \left(-\frac{1}{\mathrm{MH}^2}\right)}{128\, \pi^4\, \mathrm{MW}^2\, \mathrm{SW}^2\, (\mathrm{q1}^2-\mathrm{MH}^2)}, -\frac{9\, i\, \mathrm{EL}^2\, \mathrm{MH}^4}{128\, \pi^4\, \mathrm{MW}^2\, \mathrm{SW}^2\, (\mathrm{q1}^2-\mathrm{MH}^2)\, ((\mathrm{q1}-\mathrm{k1})^2-\mathrm{MH}^2)}\right\}$$

Now we can apply OneLoop to obtain the result in terms of the PaVe functions

```
hhselfenergy2ampsPaVe = Table[OneLoop[q1, hhselfenergy2ampsmod[[i]]], {i, 1, 3}]
                        tabla
```

$$\left\{\frac{3\, \mathrm{EL}^2\, \mathrm{MH}^2\, A_0(\mathrm{MH}^2)}{128\, \pi^2\, \mathrm{MW}^2\, \mathrm{SW}^2}, \frac{9\, \mathrm{EL}^2\, \mathrm{MH}^4\, (-\mathrm{MH}^2)\, \frac{1}{\mathrm{MH}^4}\, A_0(\mathrm{MH}^2)}{128\, \pi^2\, \mathrm{MW}^2\, \mathrm{SW}^2}, \frac{9\, \mathrm{EL}^2\, \mathrm{MH}^4\, B_0(\overline{\mathrm{k1}}^2, \mathrm{MH}^2, \mathrm{MH}^2)}{128\, \pi^2\, \mathrm{MW}^2\, \mathrm{SW}^2}\right\}$$

Example 2: Higgs self energy at one-loop

I can set the momentum on-shell

```
hhselfenergy2ampsPaVe2 = hhselfenergy2ampsPaVe /. Pair[Momentum[k1], Momentum[k1]] → MH^2
```

$$\left\{\frac{3\,\mathrm{EL}^2\,\mathrm{MH}^2\,A_0(\mathrm{MH}^2)}{128\,\pi^2\,\mathrm{MW}^2\,\mathrm{SW}^2}, \frac{9\,\mathrm{EL}^2\,\mathrm{MH}^4\,(-\mathrm{MH}^2)\,\frac{1}{\mathrm{MH}^4}\,A_0(\mathrm{MH}^2)}{128\,\pi^2\,\mathrm{MW}^2\,\mathrm{SW}^2}, \frac{9\,\mathrm{EL}^2\,\mathrm{MH}^4\,B_0(\mathrm{MH}^2,\mathrm{MH}^2,\mathrm{MH}^2)}{128\,\pi^2\,\mathrm{MW}^2\,\mathrm{SW}^2}\right\}$$

```
hhselfenergy2ampsPaVetotal = Total[hhselfenergy2ampsPaVe2, 1]
                             total
```

$$\frac{9\,\mathrm{EL}^2\,\mathrm{MH}^4\,B_0(\mathrm{MH}^2,\mathrm{MH}^2,\mathrm{MH}^2)}{128\,\pi^2\,\mathrm{MW}^2\,\mathrm{SW}^2} + \frac{3\,\mathrm{EL}^2\,\mathrm{MH}^2\,A_0(\mathrm{MH}^2)}{128\,\pi^2\,\mathrm{MW}^2\,\mathrm{SW}^2} + \frac{9\,\mathrm{EL}^2\,(-\mathrm{MH}^2)\,\frac{1}{\mathrm{MH}^4}\,\mathrm{MH}^4\,A_0(\mathrm{MH}^2)}{128\,\pi^2\,\mathrm{MW}^2\,\mathrm{SW}^2}$$

The final amplitude is the sum of the three terms


```
hhselfenergy2ampsPaVetotal = Total[hhselfenergy2ampsPaVe2, 1]
                             total
```

$$\frac{9\,\mathrm{EL}^2\,\mathrm{MH}^4\,B_0(\mathrm{MH}^2,\mathrm{MH}^2,\mathrm{MH}^2)}{128\,\pi^2\,\mathrm{MW}^2\,\mathrm{SW}^2} + \frac{3\,\mathrm{EL}^2\,\mathrm{MH}^2\,A_0(\mathrm{MH}^2)}{128\,\pi^2\,\mathrm{MW}^2\,\mathrm{SW}^2} + \frac{9\,\mathrm{EL}^2\,(-\mathrm{MH}^2)\,\frac{1}{\mathrm{MH}^4}\,\mathrm{MH}^4\,A_0(\mathrm{MH}^2)}{128\,\pi^2\,\mathrm{MW}^2\,\mathrm{SW}^2}$$

Hands on!

Exercise 1:

- Use the amplitude from the previous session to calculate the process hh-> hh

Exercise 2:

- Use the amplitude from the previous session to calculate the one-loop self-energy of the Higgs boson with only Higgs bosons in the loop

Hands on!

Exercise 3 (extra):

- Use the amplitude from the previous session to calculate the process G0G0-> G0G0