III CONGRESO IPARCOS 2024

Transverse diffeomorphisms

basic ideas & cosmological consequences

11 . 12 . 24

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OUTLINE

DARÍO

1

Refs.

[2307.14861]

[2402.17422]

TDiff : Basic ideas

Motivation | Building the theory | Symmetry breaking | New interactions | EMT conservation | Constraint

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DIEGO

2

Cosmological consequences

Refs.
[2409.11991]

Multi-field model | Effective interactions | Dark sector | Analysis | Conclusions

①

TDiff : Basic ideas

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  - ➢ The Universe presents an ***accelerated expansion***.
  - ➢ To form cosmic structures, we need ***more matter*** than we see.

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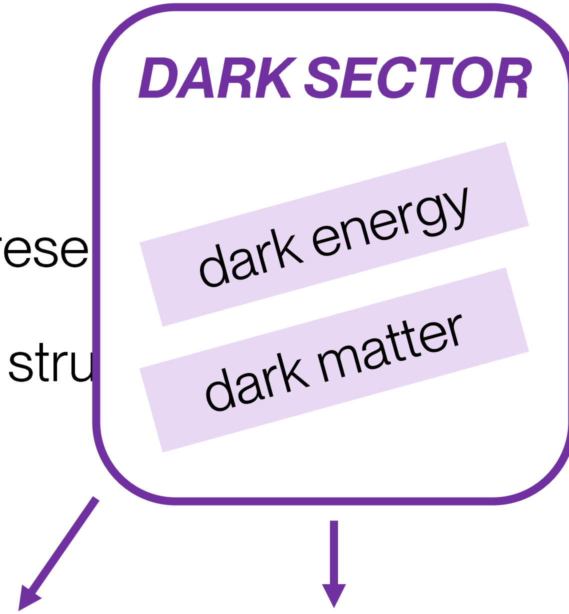

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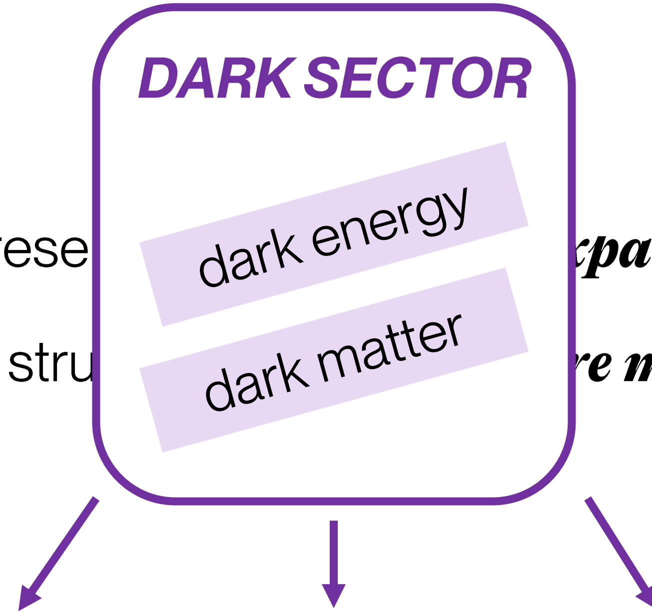

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- What are the symmetries of the dark sector of the Universe?

We will consider the possibility of ***broken symmetries*** in the dark sector.

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(… at least in the simplest case of “minimal coupling”)

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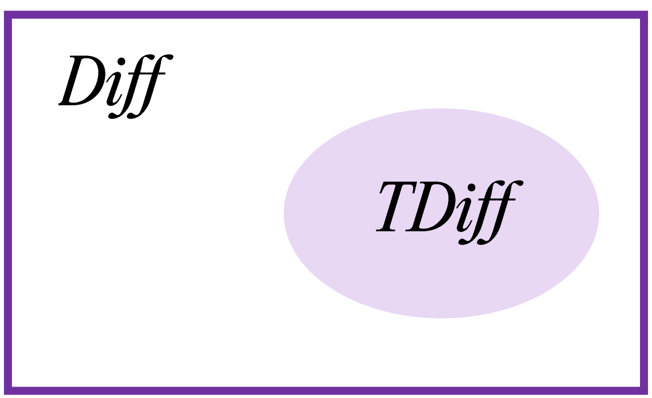

$$x \longrightarrow \hat{x}$$

$$J = \det\left(\frac{\partial \hat{x}}{\partial x}\right) = 1$$

$$\hat{x}^\mu = x^\mu + \xi^\mu, \;\; \partial_\mu \xi^\mu = 0$$

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Einstein: $$G_{\mu\nu} = 8\pi G \, T_{\mu\nu} \qquad T_{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta S_{\mathrm{M}}}{\delta g^{\mu\nu}}$$

Klein-Gordon: $$\partial_\mu \left[f_k(g) \partial^\mu \psi \right] + f_v(g) V'(\psi) = 0$$

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! *This is true for any TDiff theory where the gravitational action is the Einstein-Hilbert one*

Constraint & phenomenology

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new phenomenology!

②

Cosmological consequences

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This will provide the geometrical constraint that will be responsible for the interactions

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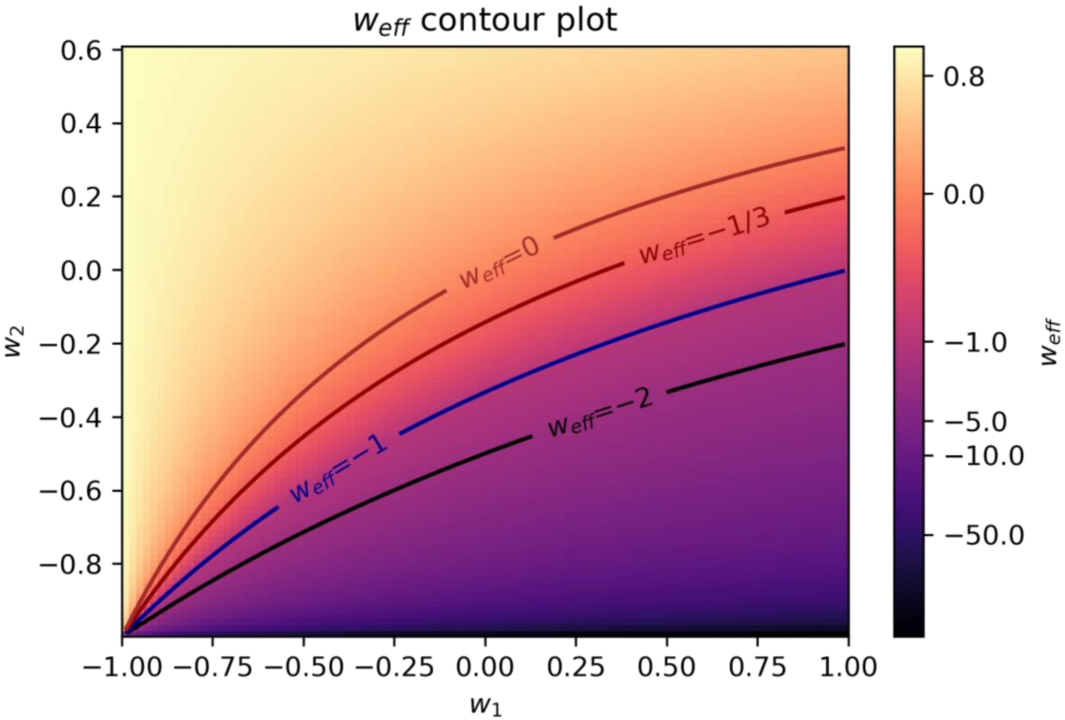

Fig.1. Effective equation of state parameter for the subdominant field in terms of w_1 and w_2 .

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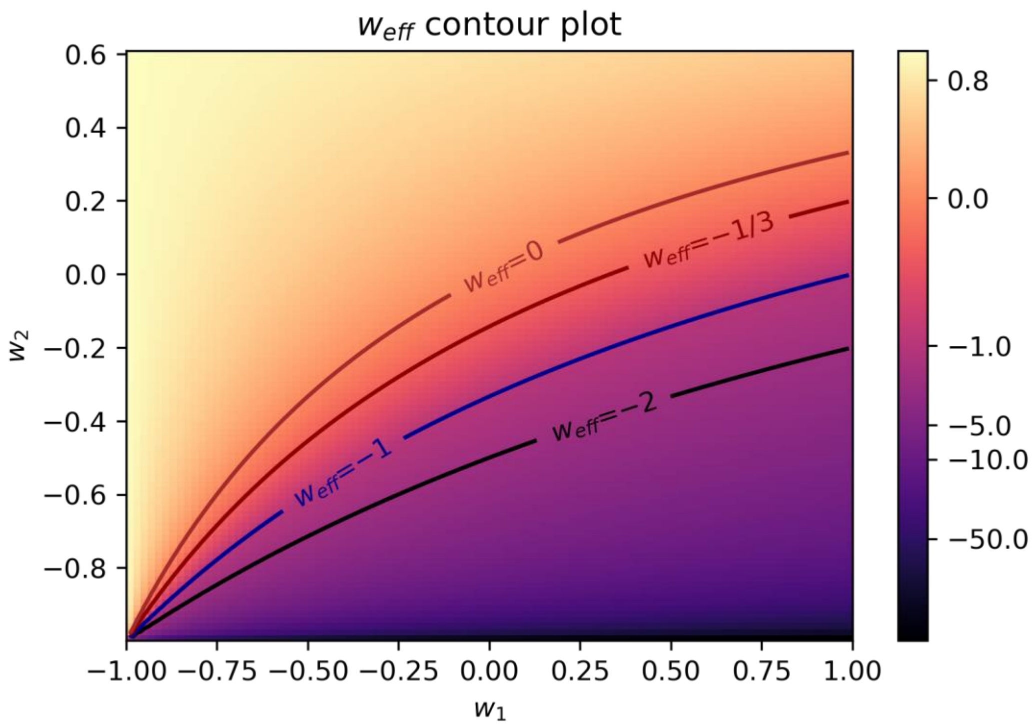

Fig.1. Effective equation of state parameter for the subdominant field in terms of w_1 and w_2 .

- Wide range of possible phenomenology.
- Possible to have a cosmological constant in subdominant regimes.
- Possibility of phantom-crossing

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! *We fixed $\omega_B = \Omega_b h^2$ to the value obtained from the abundance of light elements and marginalized H_0.*

$$\Omega_B h^2 = 0.02240 \pm 0.00069$$

Analysis

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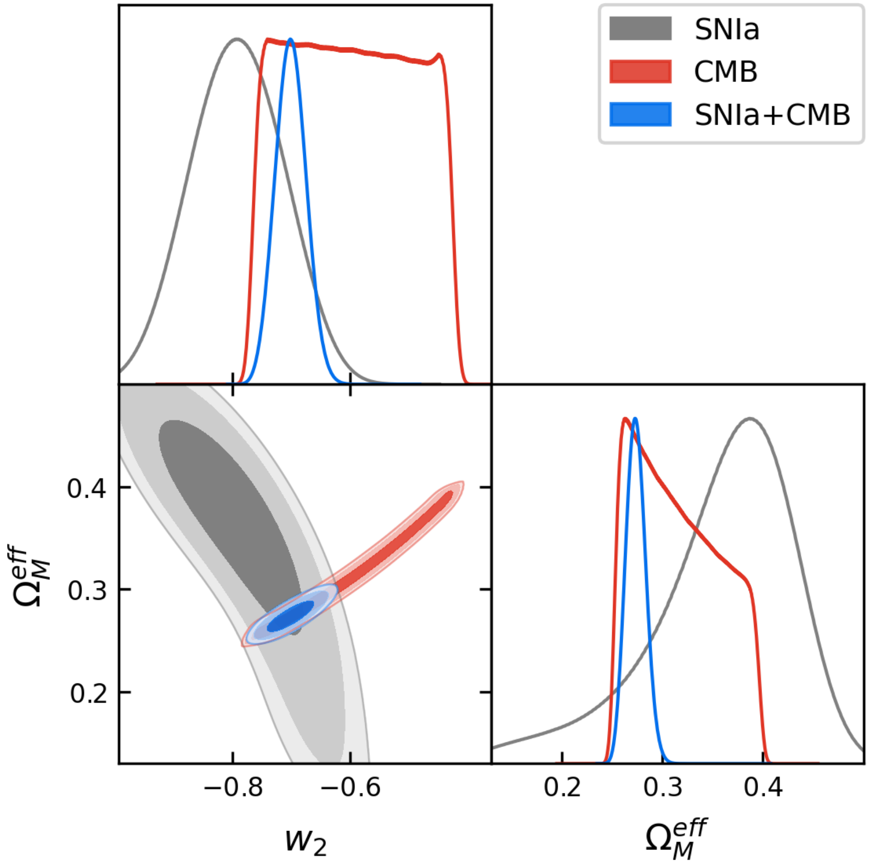

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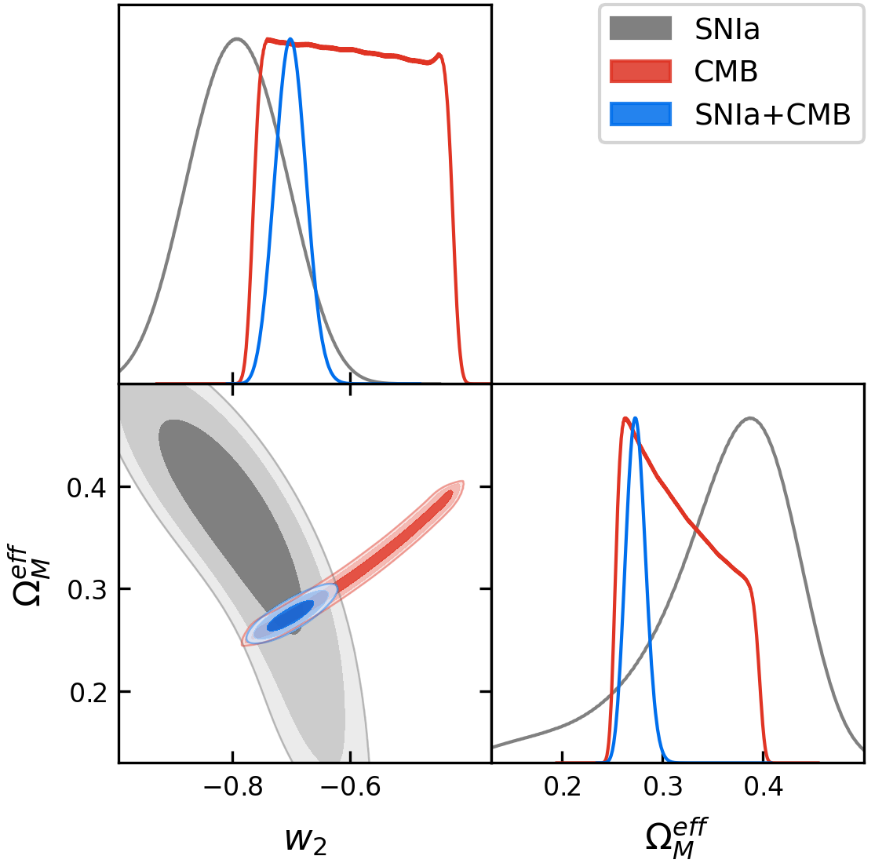

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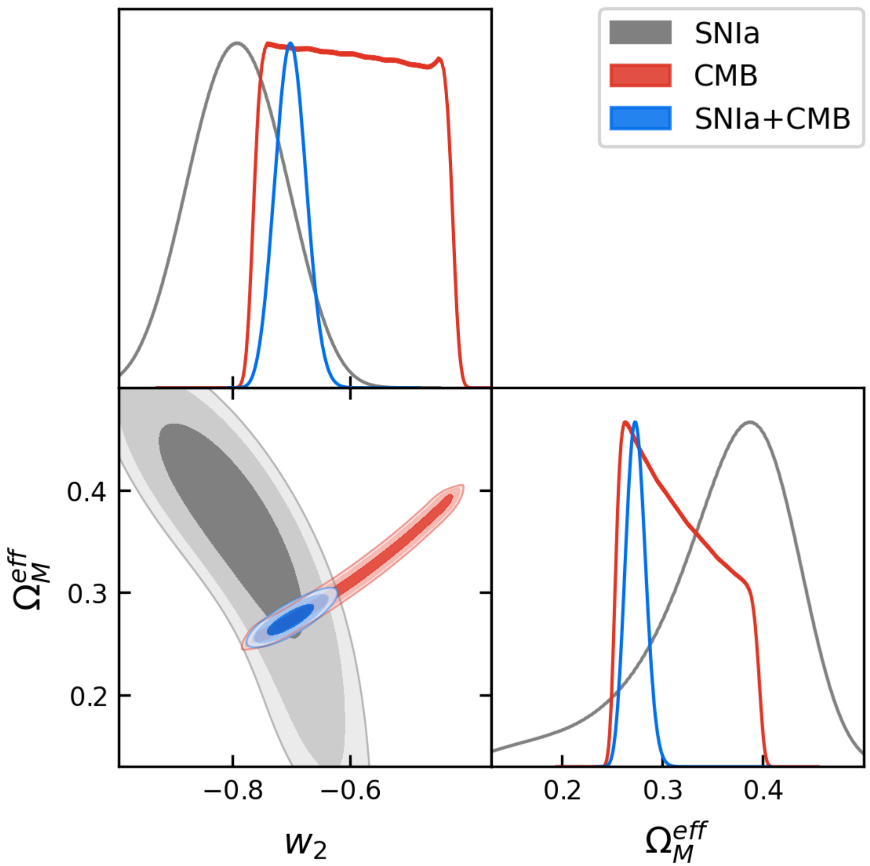

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SNIa	Best fit	$\chi^2_{\rm min}$
TDiff	$w_2 = -0.813^{+0.102}_{-0.060}$, $\Omega_{\rm M}^{\rm eff} = 0.387^{+0.056}_{-0.078}$	542.16
wCDM	$w = -1.142^{+0.145}_{-0.184}$, $\Omega_{\rm M} = 0.346^{+0.082}_{-0.083}$	542.64

CMB	Best fit	$\chi^2_{\rm min}$
TDiff	$w_2 = -0.722^{+0.191}_{-0.058}$, $\Omega_{\rm M}^{\rm eff} = 0.263^{+0.077}_{-0.016}$	-
wCDM	$w = -1.278^{+0.378}_{-0.042}$, $\Omega_{\rm M} = 0.236^{+0.097}_{-0.002}$	-

SNIa+CMB	Best fit	$\chi^2_{\rm min}$
TDiff	$w_2 = -0.703^{+0.026}_{-0.026}$, $\Omega_{\rm M}^{\rm eff} = 0.273^{+0.010}_{-0.010}$	557.97
wCDM	$w = -1.092^{+0.034}_{-0.034}$, $\Omega_{\rm M} = 0.292^{+0.010}_{-0.010}$	556.63

Analysis

- Equation of state evolution for the best fit

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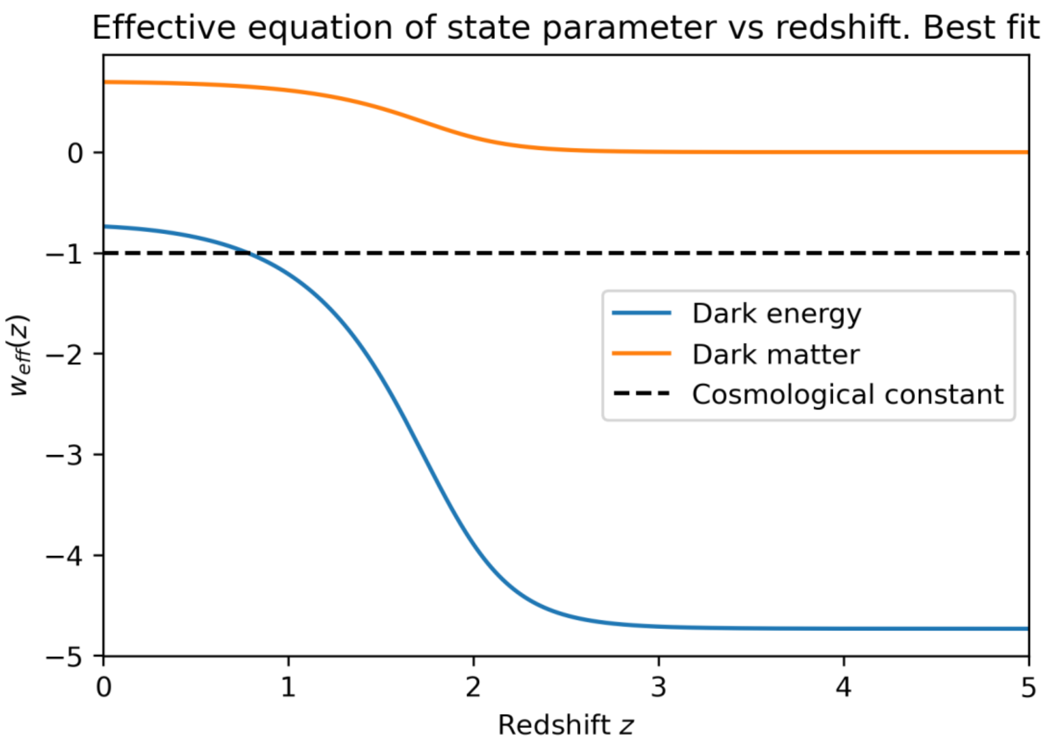

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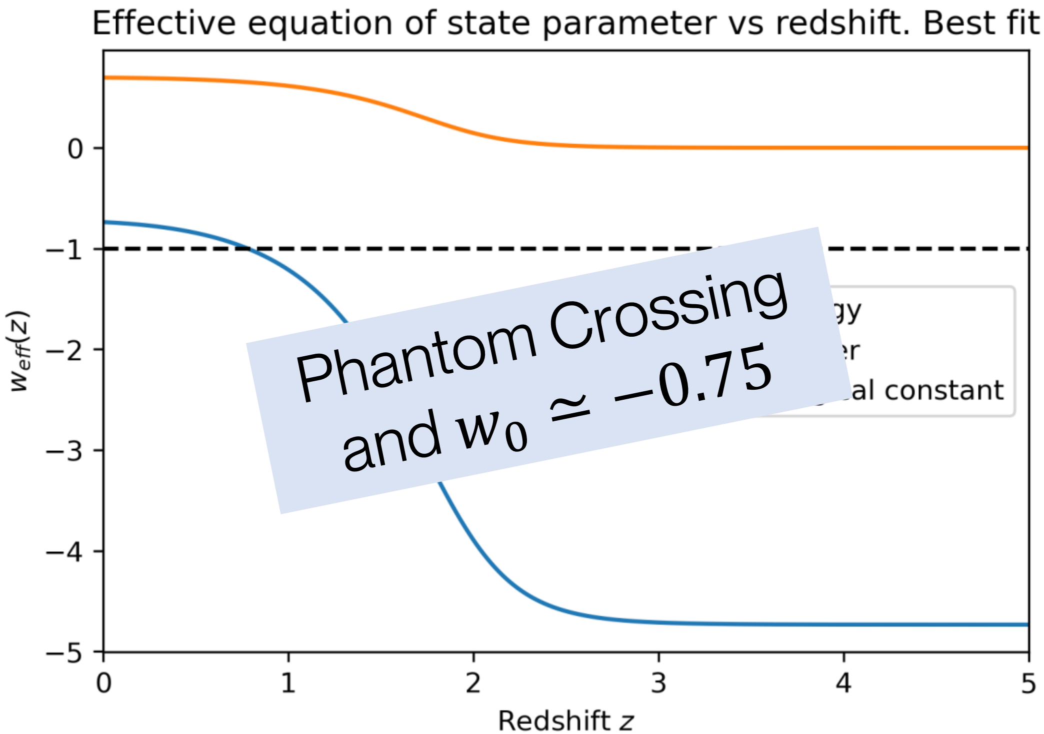

Phantom Crossing and $w_0 \simeq -0.75$

the end

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  Phantom crossing → H_0 tension?