Matching the dipole picture with collinear factorization

2nd European School on the Physics of the EIC

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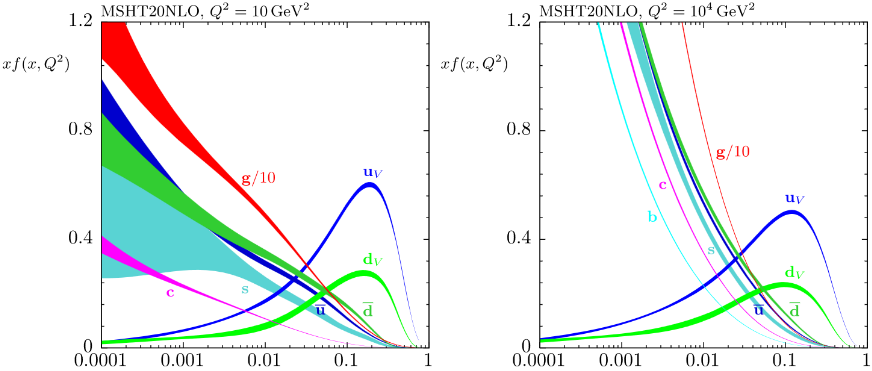
Parton distribution functions and gluon saturation

- At small momentum fraction, x, the gluon density skyrockets.

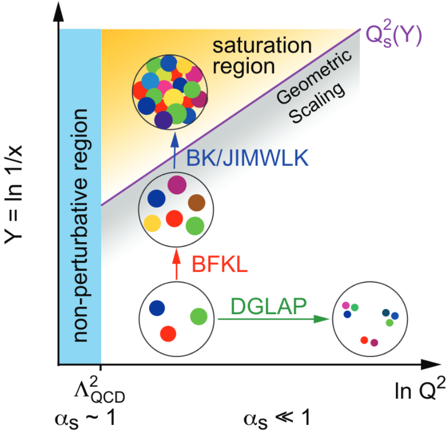


Gluon saturation

- How do we understand hadron structure as x becomes small? $\rightarrow$ Color Glass Condensate

- Things becomes separated at large Q^2 $\rightarrow$ Collinear Factorization (CF).

- One can both decrease x and increase Q^2. Can one match these frameworks with each other?

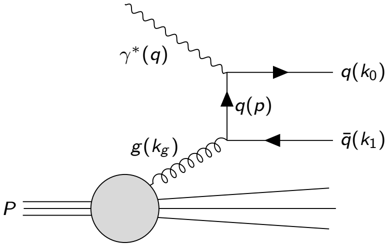


Outline

- Introduce the CF structure function result for total cross section as our aim for the matching
- Discuss the dipole picture
- Idea for matching: Kinematical Constraint and large Q^2 limit in CGC

Collinear Factorization

- The parton model and CF is calculated in the infinite momentum frame (IMF)



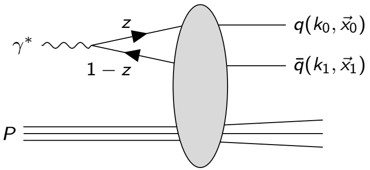
- Structure function F_2^g for massless quarks where $p = \xi k_g$

$$F_2^g \propto g(\mu^2) \otimes \frac{\alpha_s}{2\pi} \left\{ \log\left(\frac{Q^2}{\mu^2}\right) P_{qg} + \left(C_g - f_g^{scheme}\right) \right\}$$

$$C_g = P_{qg} \log\left(\frac{1-\xi}{\xi}\right) + 1 - 8\xi + 8\xi^2$$

The dipole picture

- The dipole picture is applied in the target rest frame (TRF).



- It is common to work in the mixed space representation

$$\sigma_{\mathrm{L,T}}^{\gamma^* P} \propto \int \frac{d^2k_0}{(2\pi)^2} \frac{d^2k_1}{(2\pi)^2} \frac{dz}{z(1-z)} d^2x_0\, d^2x_1\, d^2x_0'\, d^2x_1'\, e^{i\vec{k}_0\cdot(\vec{x}_0'-\vec{x}_0)} e^{i\vec{k}_1\cdot(\vec{x}_1'-\vec{x}_1)}$$

$$\times\, \tilde{\Psi}_{L,T}^{\gamma^*\to q\bar{q}}(z, \vec{x}_0, \vec{x}_1) \left(\tilde{\Psi}_{L,T}^{\gamma^*\to q\bar{q}}(z, \vec{x}_0', \vec{x}_1')\right)^\dagger \frac{1}{N_c} \left\langle \mathrm{Tr}\left[\left[U(\vec{x}_0)U^\dagger(\vec{x}_1) - \mathbb{I}\right]\left(\left[U(\vec{x}_0')U^\dagger(\vec{x}_1') - \mathbb{I}\right]\right)^\dagger\right]\right\rangle$$

- The structure function for the gluon in this picture is given by

$$F_2 = \frac{Q^2}{4\pi^2\alpha_{EM}}(F_L + F_T), \qquad F_L = \frac{Q^2}{4\pi^2\alpha_{EM}}\sigma_L^{\gamma^* P}, \qquad F_T = \frac{Q^2}{4\pi^2\alpha_{EM}}\sigma_T^{\gamma^* P}$$

The dipole picture

- If one integrates over momenta one gets the optical theorem result.

$$\sigma_{\mathrm{LO}}^{\gamma^* P} \propto \int \frac{dz}{z(1-z)} d^2x_0\, d^2x_1 \left|\tilde{\Psi}^{\gamma^* \to q\bar{q}}(z, \vec{x}_0, \vec{x}_1)\right|^2 \left\langle \mathrm{Tr}\left[U(\vec{x}_0)U^\dagger(\vec{x}_1) - \mathbb{I}\right]\right\rangle$$

- As Q^2 becomes large, one can show

$$\sigma_{\mathrm{LO}}^{\gamma^* P} \propto \log\left(Q^2\right)$$

- We want to capture more than the $\log\left(Q^2\right)$ behavior

Game, set, match

- Use the invariant mass of the final state $q\bar{q}$-pair to match the DP cross section to the CF cross section
- Dipole picture kinematics, TRF

$$M_{q\bar{q}}^2 = (k_0 + k_1)^2 = \ldots = \frac{m_f^2 + \vec{p}^{\,2}}{z(1-z)}$$

- Parton model kinematics, IMF

$$M_{q\bar{q}}^2 = (k_0 + k_1)^2 = \ldots = Q^2 \left(\frac{1-\xi}{\xi} \right)$$

$$\frac{m_f^2 + \vec{p}^{\,2}}{z(1-z)} = Q^2 \left(\frac{1-\xi}{\xi} \right)$$

Kinematic Constraint

- It is suitable to use the coordinates

$$\vec{u} = \vec{x}_0 - \vec{x}_1 , \qquad \vec{p} = (1-z)\vec{k}_0 - z\vec{k}_1$$
$$\vec{v} = z\vec{x}_0 + (1-z)\vec{x}_1 , \qquad \vec{k}_g = \vec{k}_0 + \vec{k}_1$$

$$\sigma_{L,T}^{\gamma^* P} \propto \int \frac{d^2 k_g}{(2\pi)^2} \frac{d^2 p}{(2\pi)^2} d^2u d^2v d^2u' d^2v' dz\, e^{i\vec{k}_g\cdot(\vec{v}'-\vec{v})} e^{i\vec{p}\cdot(\vec{u}'-\vec{u})} \tilde{\Psi}_{L,T}^{\gamma^*\to q\bar{q}}(z,\vec{u}) \left(\tilde{\Psi}_{L,T}^{\gamma^*\to q\bar{q}}(z,\vec{u}\,')\right)^\dagger$$
$$\times \frac{1}{N_c} \left\langle \mathrm{Tr}\left[\left\{ U(\vec{u}(1-z)+\vec{v}) U^\dagger(-\vec{u}z+\vec{v}) - \mathbb{I} \right\} \left\{ U(-\vec{u}'z+\vec{v}') U^\dagger(\vec{u}'(1-z)+\vec{v}') - \mathbb{I} \right\} \right] \right\rangle$$

- The light cone wavefunctions are defined as

$$\tilde{\Psi}_L \tilde{\Psi}_L^* \propto Q^2 z^2 (1-z)^2 K_0(\varepsilon_f|\vec{u}|) K_0^*(\varepsilon_f|\vec{u}'|)$$
$$\tilde{\Psi}_T \tilde{\Psi}_T^* \propto (z^2 + (1-z)^2)\varepsilon_f^2 \frac{\vec{u}\cdot\vec{u}'}{|\vec{u}||\vec{u}'|} K_1(\varepsilon_f|\vec{u}|) K_1^*(\varepsilon_f|\vec{u}'|) + m_f^2 K_0(\varepsilon_f|\vec{u}|) K_0^*(\varepsilon_f|\vec{u}'|)$$

The Wilson line correlator at small $\vec{u}$ and $\vec{u}'$

- As Q^2 becomes large, $\vec{u}$ and $\vec{u}\,'$ becomes small which allows us to expand the Wilson line correlator.

$$\langle \mathrm{Tr}\left[U(\vec{u}(1-z)+\vec{v})\ldots\right]\rangle \xrightarrow{\vec{u},\,\vec{u}\,'\to 0} -u_i u_j' \left\langle \mathrm{Tr}\left[\left(\partial_i U(\vec{v})\right) U^\dagger(\vec{v}) \left(\partial_j U(\vec{v}')\right) U^\dagger(\vec{v}')\right]\right\rangle$$

- This Wilson line correlator integrated over $\vec{v}$ and $\vec{v}\,'$ is known as the Weiszäcker-Williams gluon distribution

$$xg(x,\ \vec{k}_g) = \frac{-2}{\alpha_S}\int \frac{d^2v}{(2\pi)^2}\frac{d^2v'}{(2\pi)^2} e^{i\vec{k}_g\cdot(\vec{v}'-\vec{v})} \left\langle \mathrm{Tr}\left[\left(\partial_i U(\vec{v})\right) U^\dagger(\vec{v}) \left(\partial_i U(\vec{v}')\right) U^\dagger(\vec{v}')\right]\right\rangle$$

$$x\tilde{g}(x,\ \vec{k}_g) = \frac{S_\perp}{\pi^2\alpha_S}\frac{N_c^2-1}{N_c}\int \frac{d^2r}{(2\pi)^2}\frac{e^{i\vec{k}_g\cdot\vec{r}}}{\vec{r}^{\,2}}\left(1-e^{-\frac{\vec{r}^{\,2}Q_s^2}{4}}\right)$$

Cross sections at small dipole size

- After performing integrations over u, u' and angles one gets

$$\sigma_L^{\gamma^* P} \propto \alpha_S \int d^2k_g \, xg(x, \vec{k}_g) \int d^2p \, dz \, Q^2 z^2 (1-z)^2 \frac{p^2}{(p^2+\varepsilon_f^2)^4}$$

$$\sigma_T^{\gamma^* P} \propto \alpha_S \int d^2k_g \, xg(x, \vec{k}_g) \int d^2p \, dz \left(\frac{2m_f^2 p^2}{(p^2+\varepsilon_f^2)^4} + (z^2 + (1-z)^2) \frac{p^4+\varepsilon_f^4}{(p^2+\varepsilon_f^2)^4} \right)$$

- At this point it is suitable to perform the matching to the collinear factorization

$$1 = \int_{4m_f^2}^{W^2} dM_{q\bar{q}}^2 \, \delta \left(M_{q\bar{q}}^2 - \frac{\vec{p}^{\,2} + m_f^2}{z(1-z)} \right)$$

Cross section and F_2^g

- Multiplying the integrand by 1, integrating over z and the momentum $\vec{p}$, and performing the substitution $M_{q\bar{q}}^2 \to \xi$ gives us the cross section

$$\sigma_{L,\mathrm{LO}}^{\gamma^* P} \propto \alpha_S \int d^2k_g xg(x,\vec{k}_g) \int_{x_B}^{1} d\xi\xi\Big[Q^2(Q^2-2m_f^2)(1-\xi)\sqrt{1-4m_f^2\xi/Q^2(1-\xi)}$$
$$-4m_f^2(m_f^2\xi - Q^2(1-2\xi))\coth^{-1}\left(1/\sqrt{1-4m_f^2\xi/Q^2(1-\xi)}\right)\Big]$$
$$\sigma_{T,\mathrm{LO}}^{\gamma^* P} \propto \alpha_S \int d^2k_g\, xg(x,\vec{k}_g) \int_{x_B}^{1} d\xi\Big[-Q^2\sqrt{1-4m_f^2\xi/Q^2(1-\xi)}(Q^2(1-2\xi)^2+4m_f^2\xi(1-\xi))$$
$$+4(4m_f^2Q^2\xi(1-\xi)-8m_f^4\xi^2+Q^4(\xi^2+(1-\xi)^2)\coth^{-1}\left(1/\sqrt{1-4m_f^2\xi/Q^2(1-\xi)}\right)\Big]$$

- Taking the limit $m_f \to 0$ gives us the same result as for CF

$$F_{T,\mathrm{LO}} \propto \alpha_S \int d^2k_g\, xg(x,\vec{k}_g) \int_{x_B}^{1} d\xi \left[P_{qg}(\xi)\left(\log\left(\frac{Q^2}{m_f^2}\right)+\log\left(\frac{1-\xi}{\xi}\right)\right)-(1-2\xi)^2\right],$$
$$F_{L,\mathrm{LO}} \propto \alpha_S \int d^2k_g xg(x,\vec{k}_g) \int_{x_B}^{1} d\xi\, -\xi(1-\xi).$$

Summary and Outlook

- S: At small x, one uses the dipole picture and the color glass effective theory.
- S: We match the Dipole Picture Framework to the Collinear Factorization
- O: We want to improve on the precision of cross section calculations.
- O: Additionally, we want to compare calculations to existing HERA data and make predictions for the EIC experiments.
- O: Apply this approach to get corrections for the BK evolution equation.

Thank you