JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

Centre of Excellence
in Quark Matter

Diffractive Structure Functions with JIMWLK

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2nd European School on the Physics of the EIC and Related Topics

June 28, 2025 @ Benicàssim

European Research Council
Established by the European Commission

Introduction

Maaninka

Population (2014-11-30)[2]	
• **Total**	3,747
• **Density**	6.5/km^2

Jyväskylä

• **Urban**	117,974
• **Urban density**	1,188.7/km^2

Benicàssim

Population (2018)[1]	
• **Total**	18,055
• **Density**	500/km^2

- Masters thesis 12/2024: Pure glue lattice QCD

Kvarkittoman QCD:n faasitransitio hilalla

by Runko, Pyry

Published 2024

Theses | Master thesis

- Summer student at Cern in JYU ALICE group in 2023, Flow analysis

Diffractive Structure Functions with JIMWLK

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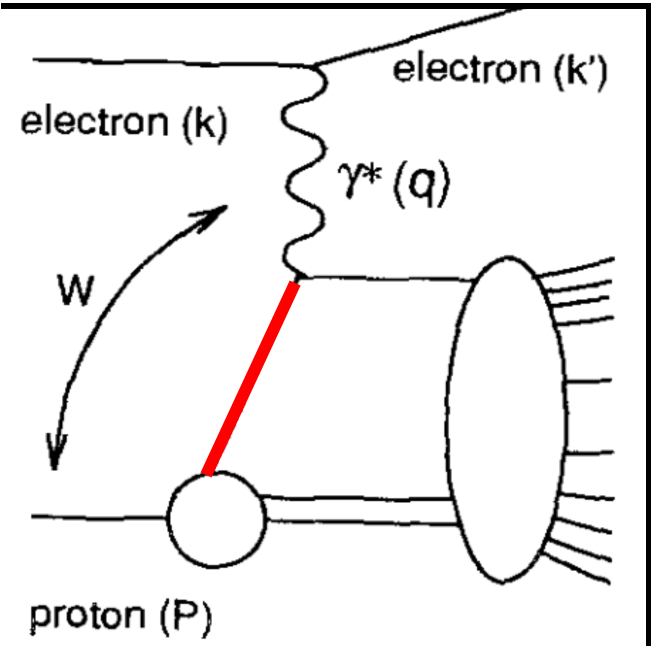
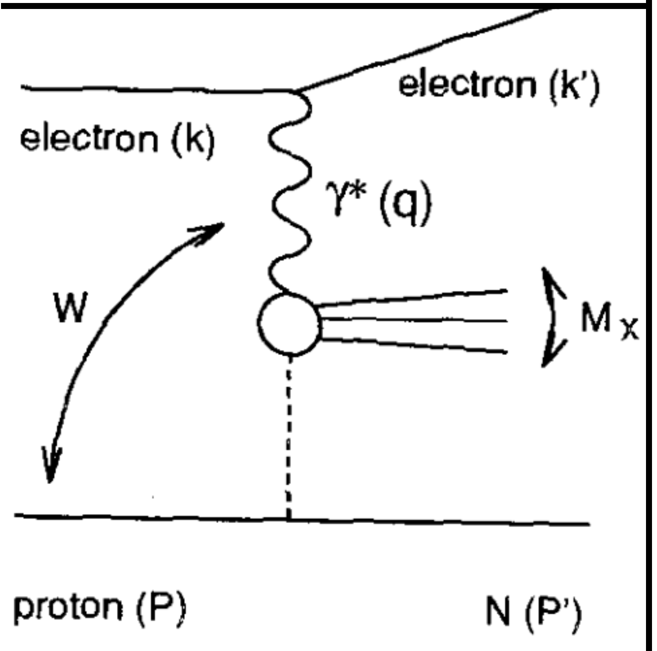
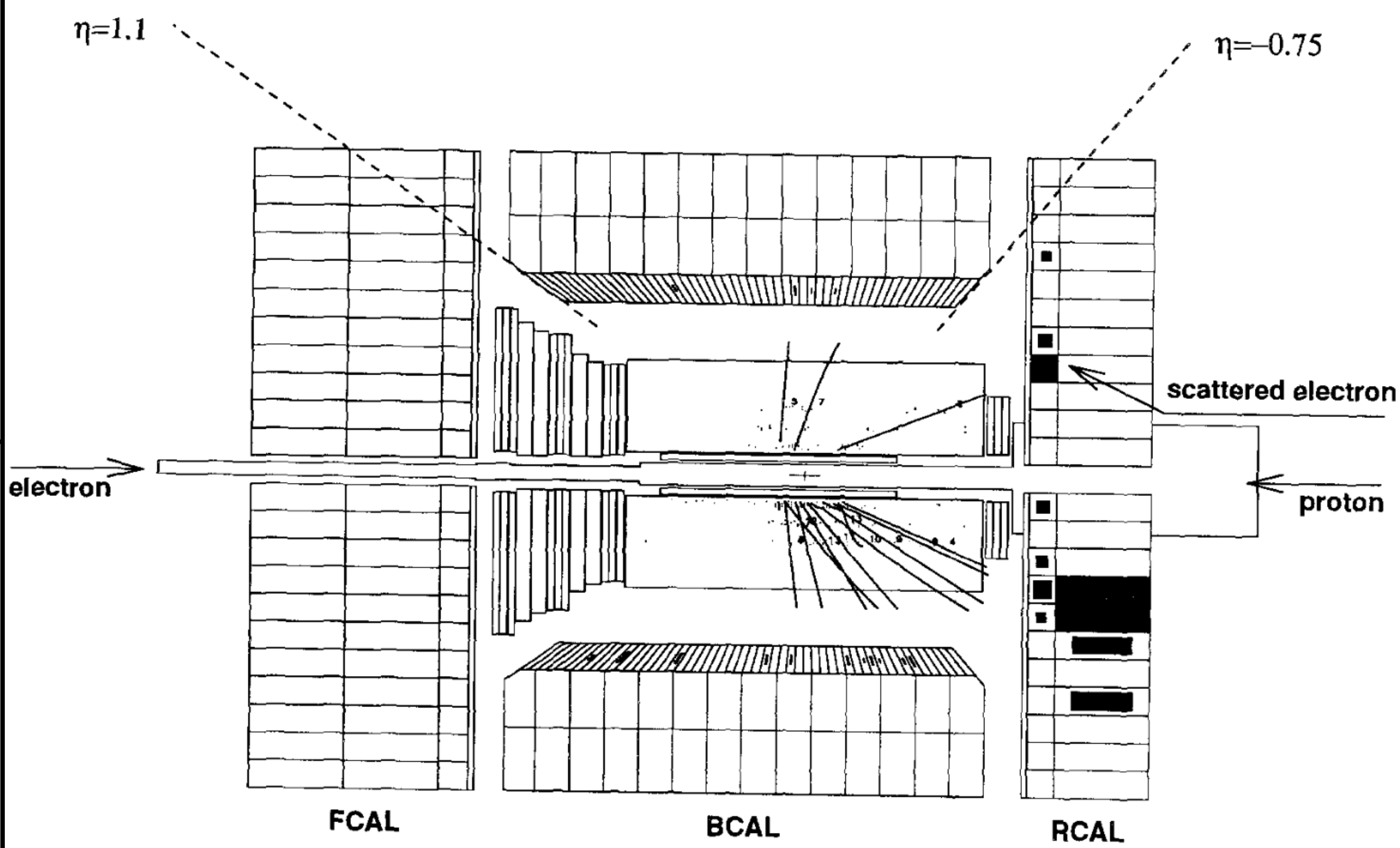






https://doi.org/10.1016/0370-2693(93)91645-4, ZEUS collaboration (1993)

Diffractive DIS

- Characteristic "Rapidity gap" in the detector between outgoing target and remnants of the virtual photon (X)
- Rapidity gap implies colorless interaction via the pomeron ($\mathbb{P}$)
- ~ 15% of DIS events were diffractive at HERA ('92 - '07)
- Note: in proton rest frame the virtual photon has energy ~ TeV!

***H**adron-**E**lektron-**R**ing**a**nlage*

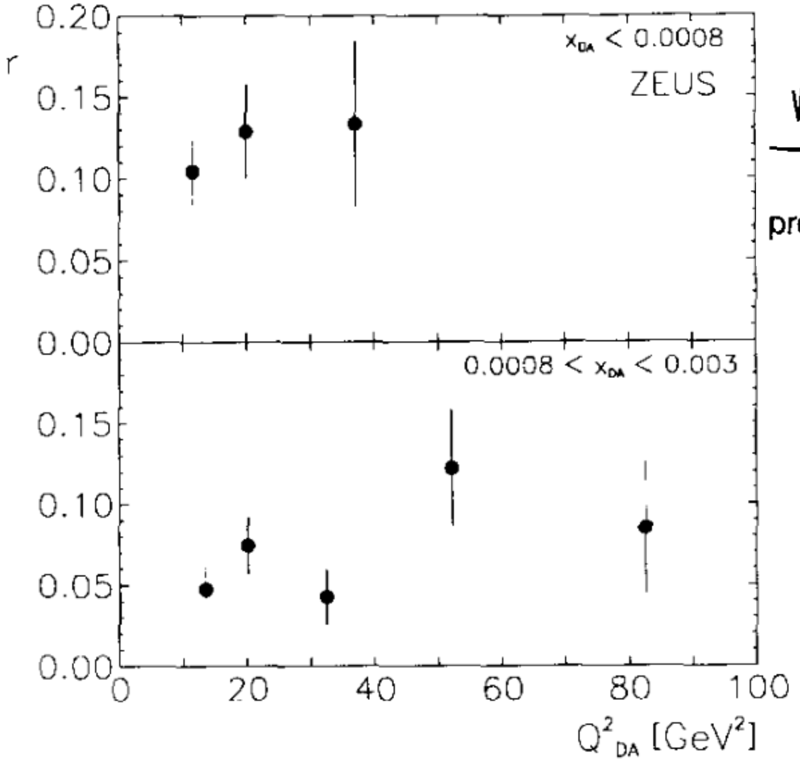
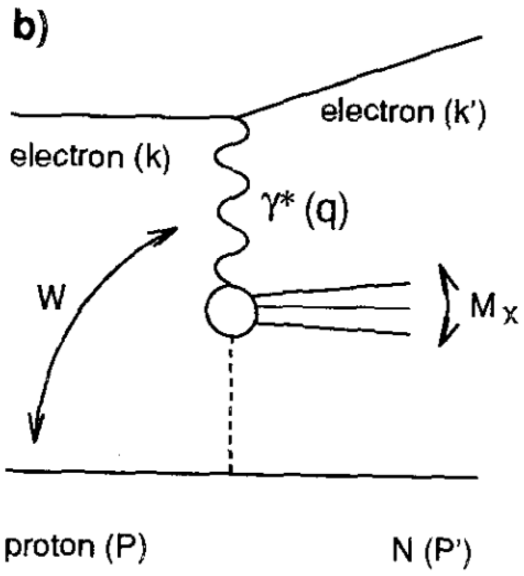


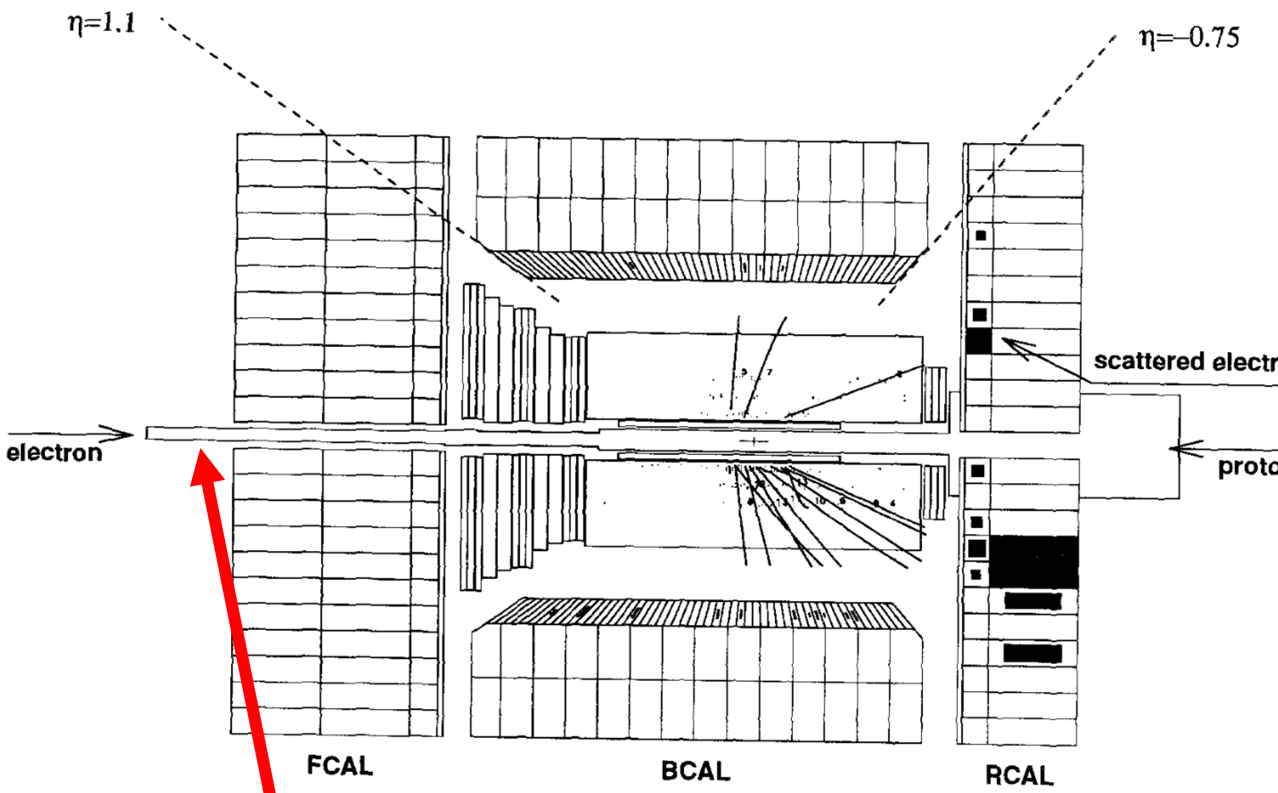
Fig. 4. Fraction r of events with a large rapidity gap, $\eta_{max} < 1.5$, as a function of Q^2_{DA} for two ranges of x_{DA}. No acceptance corrections have been applied.

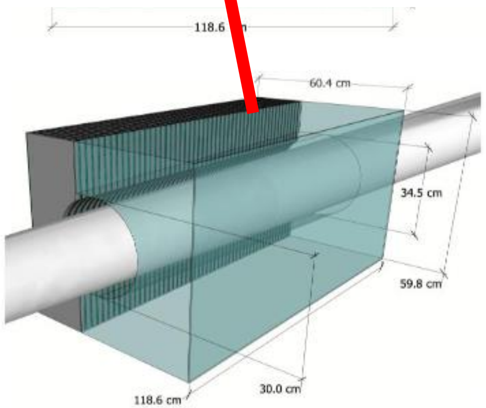


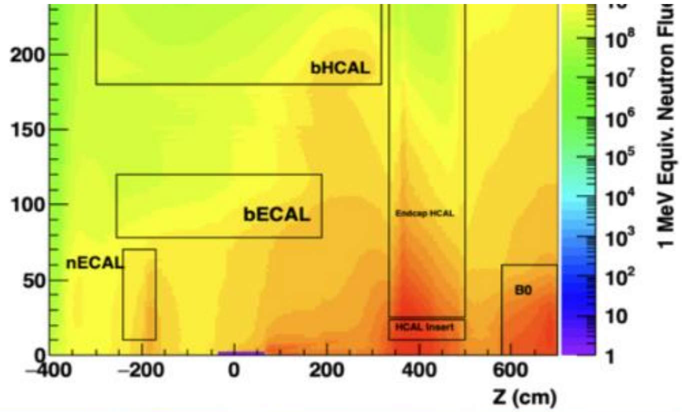
https://doi.org/10.1016/0370-2693(93)91645-4, ZEUS collaboration (1993)

Forward insert calorimeter

- Insert to extend central detector acceptance as much as possible in the forward direction
  - Crucial for containment of hadronic final state for inclusive kinematic reconstruction and rapidity gap diffractive measurements
  - 3.2 < η < 4
  - Needs to accommodate complex beam pipe geometry
- Small hexagonal scintillator tiles for better spatial uniformity & increased light yield
- High radiation load
  - SiPMs will be serviceable
  - ~10^{12} neq/cm^2/year







Diffractive Structure Functions with JIMWLK

https://arxiv.org/abs/1806.06783 Mäntysaari, Schenke (2018)

Structure functions

$$\frac{d^4\sigma_{ep}^{diff}}{d\beta dQ^2 dx_{IP} dt} = \frac{2\pi\alpha^2}{\beta Q^4}(1+(1-y)^2)F_2^{D(4)}(\beta, Q^2, x_{IP}, t)$$

- Most precise HERA data* is given as a reduced cross section (2012)
- Idea: Use JIMWLK to compute the energy ($x_{\mathbb{P}}$) dependence for diffractive structure functions (new!) for the proton and heavy nuclei → saturation
- Goals: Compare with HERA & predict for the EIC, constrain parameters with additional cross sections

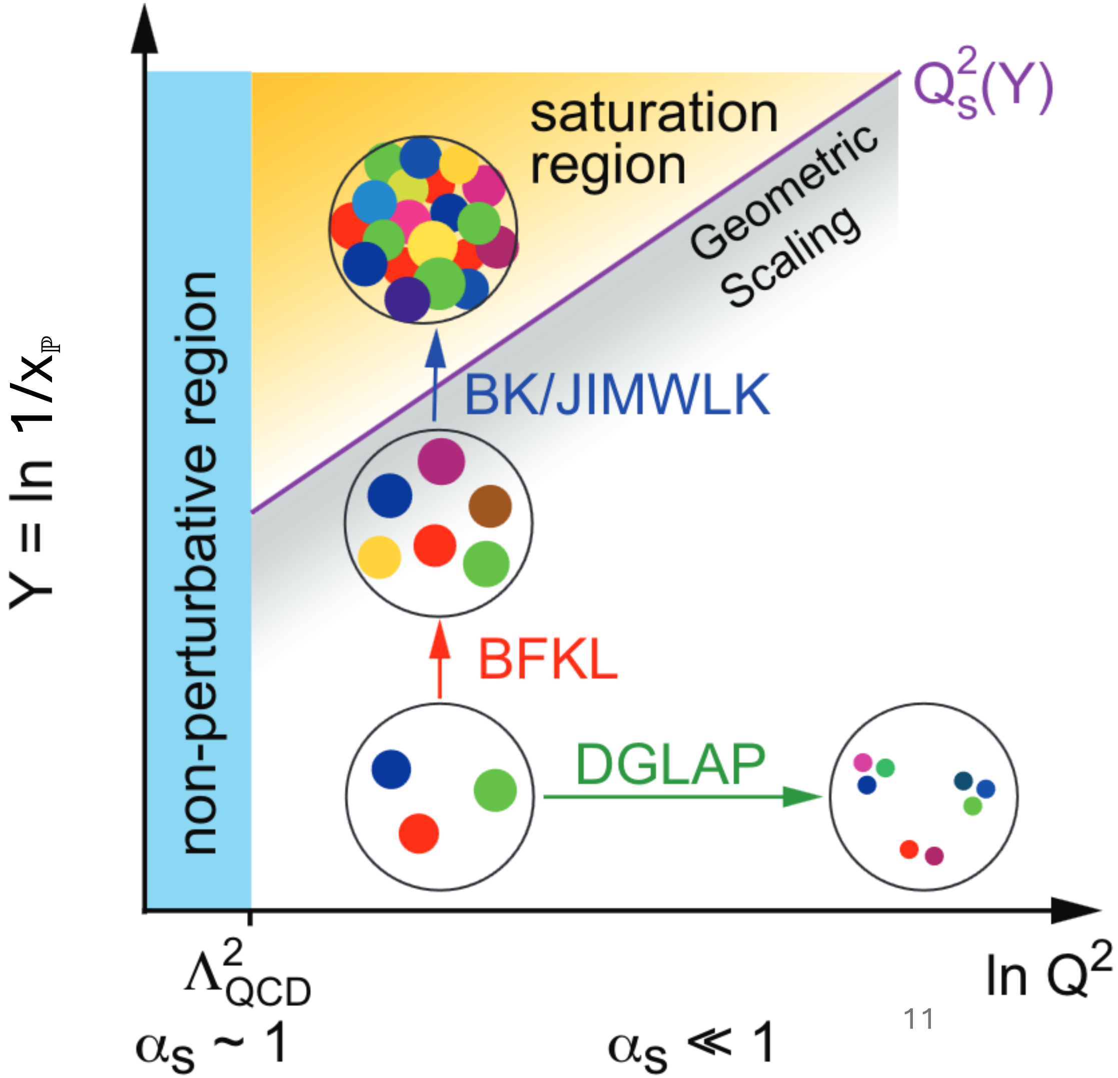
* https://arxiv.org/abs/1207.4864 H1, ZEUS (2012)

Diffractive Structure Functions with JIMWLK

JIMWLK equation (Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner)

- A renormalization group equation at low x (generalises BK)
  - Color Glass Condensate: quarks are static sources of dynamic color fields
  - Classical color fields and scattering off them is described using Wilson lines *V ~ P exp(-iA)*, **stochastic variables**
- JIMWLK: evolve the **probability distributions** of low x Wilson lines to even lower x
- Langevin formulation of LO JIMWLK:

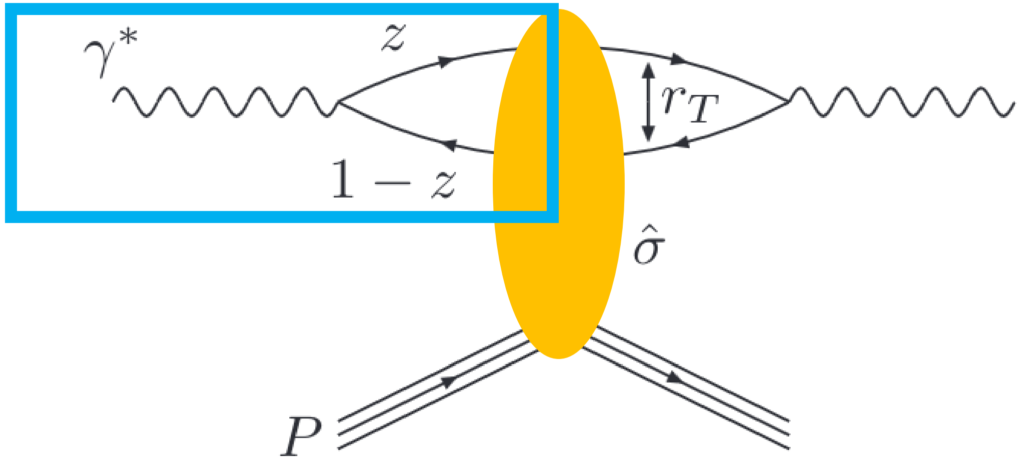
$$\frac{\mathrm{d}}{\mathrm{d}y} V_{\mathbf{x}} = V_{\mathbf{x}}(it^a) \left[\int \mathrm{d}^2\mathbf{z}\, \varepsilon^{ab,i}_{\mathbf{x},\mathbf{z}}\, \xi_{\mathbf{z}}(y)^b_i + \sigma^a_{\mathbf{x}} \right]$$



Longitudinal diffractive cross section

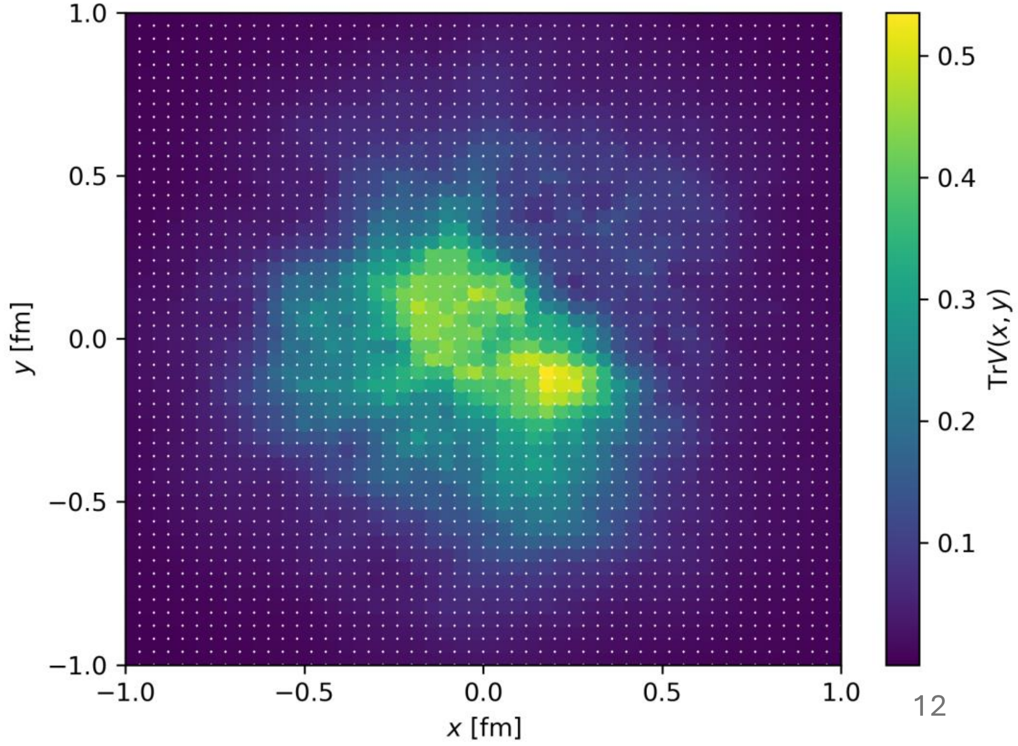
$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{L,q\bar{q}}}{\mathrm{d}M_X^2\mathrm{d}|t|} \propto \left|\left\langle \int \mathrm{d}^2x_0\mathrm{d}^2x_1 \psi_{\gamma_L^*\to q\bar{q}}(\mathbf{r}_T, z)\left(1-\frac{1}{N_c}\mathrm{Tr}\left[V(\mathbf{x}_0)V^\dagger(\mathbf{x}_1)\right]\right)\times \mathrm{phase}\right\rangle\right|^2$$

- Coherent cross section = proton stays intact
- Factorizes at high energy, schematically above
- Average over Wilson line configurations



Dipole picture @ LO: photon to quark-antiquark pair

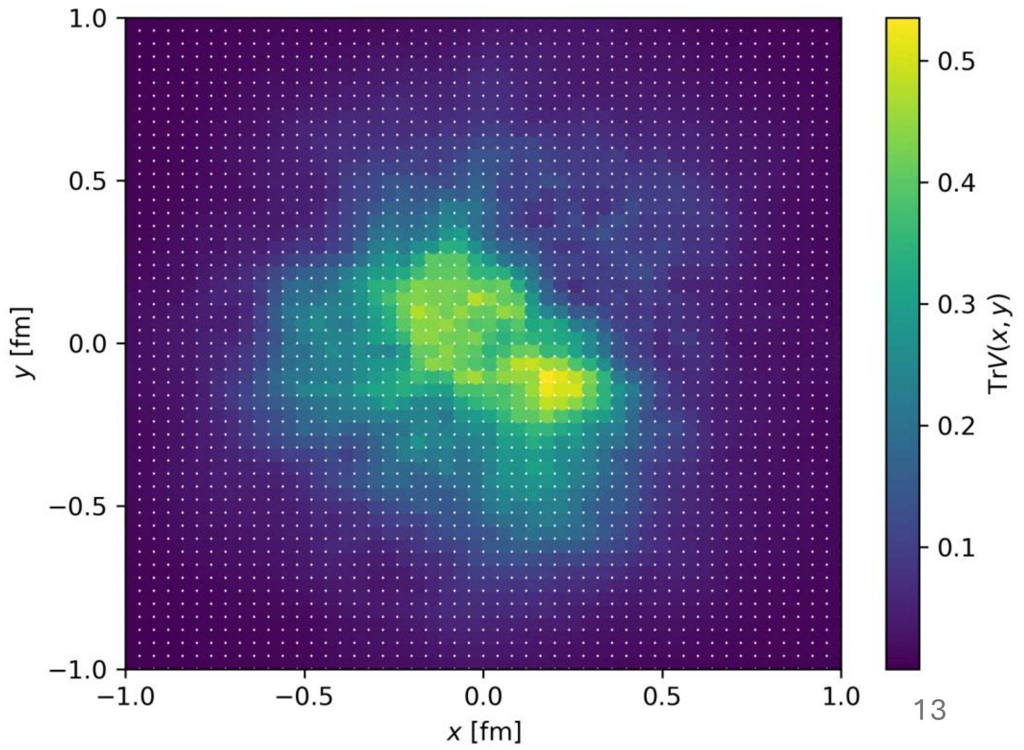
https://github.com/schenke/ipglasma



What's new?

$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{L,q\bar{q}}}{\mathrm{d}M_X^2\,\mathrm{d}|t|} \propto \left|\left\langle \int \mathrm{d}^2x_0\mathrm{d}^2x_1 \psi_{\gamma_L^*\to q\bar{q}}(\mathbf{r}_T, z)\left(1 - \frac{1}{N_c}\mathrm{Tr}\left[V(\mathbf{x}_0)V^\dagger(\mathbf{x}_1)\right]\right)\times \mathrm{phase}\right\rangle\right|^2$$

- Energy dependence is solved using JIMWLK *
  - Wilson lines already used to describe other HERA data ^
  - Evolution of the proton geometry is naturally accounted for



* https://github.com/hejajama/jimwlk
Schenke, Mäntysaari

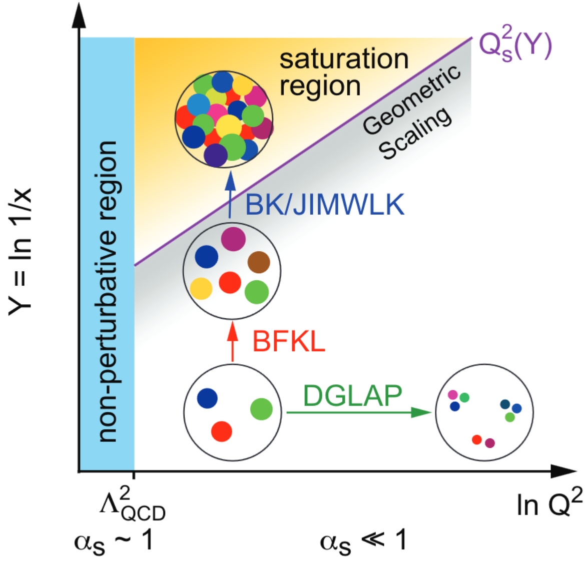
^ https://arxiv.org/abs/1806.06783
Mäntysaari, Schenke

Plans for the future

- Investigate the Gaussian approximation for JIMWLK with TMDs
  - Often used to reduce the computational load, but validity has been checked only for (two) specific coordinate configurations [3]

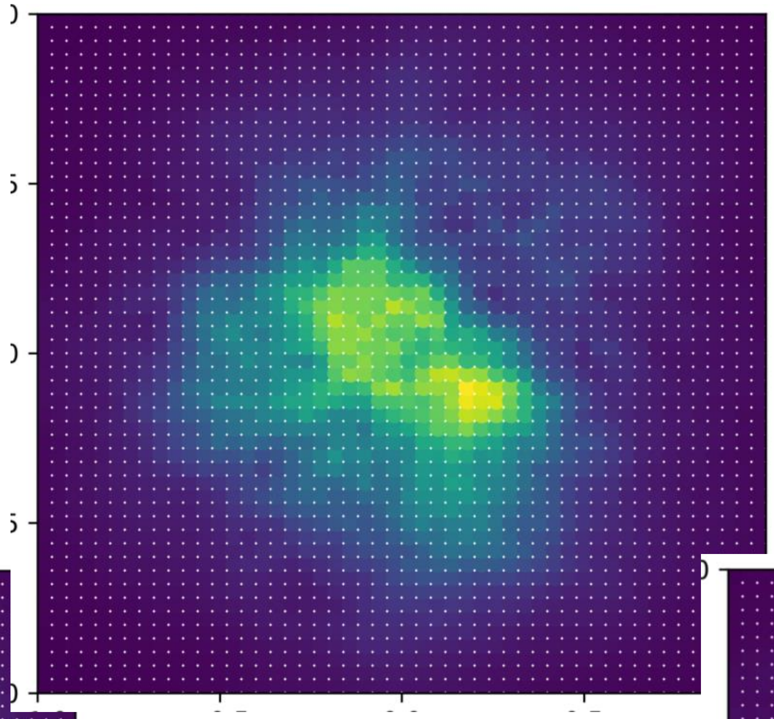
- Work towards a Langevin formulation of NLO JIMWLK

[3] https://arxiv.org/abs/1108.4764 Dumitru, Jalilian-Marian, Lappi, Schenke, Venugopalan (2011)

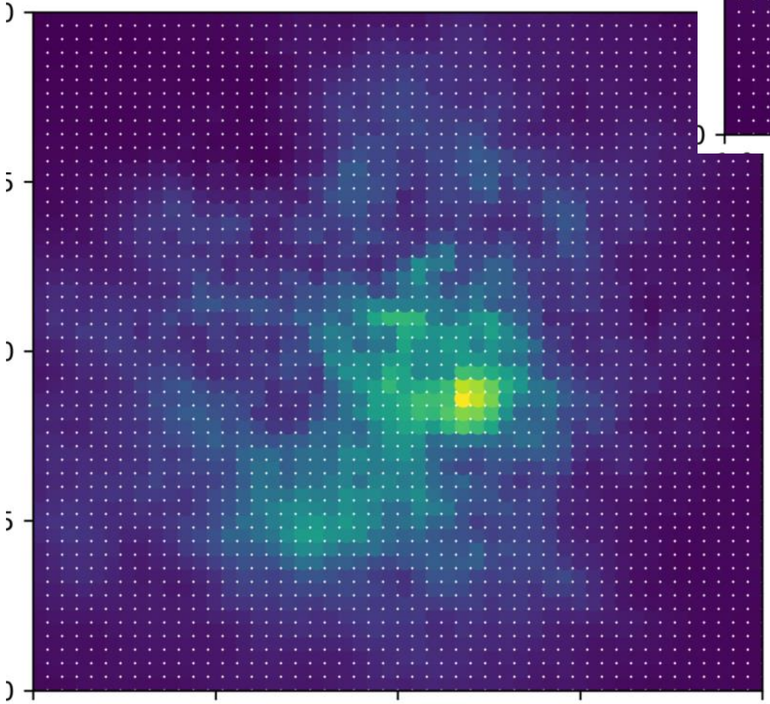


Y = 0

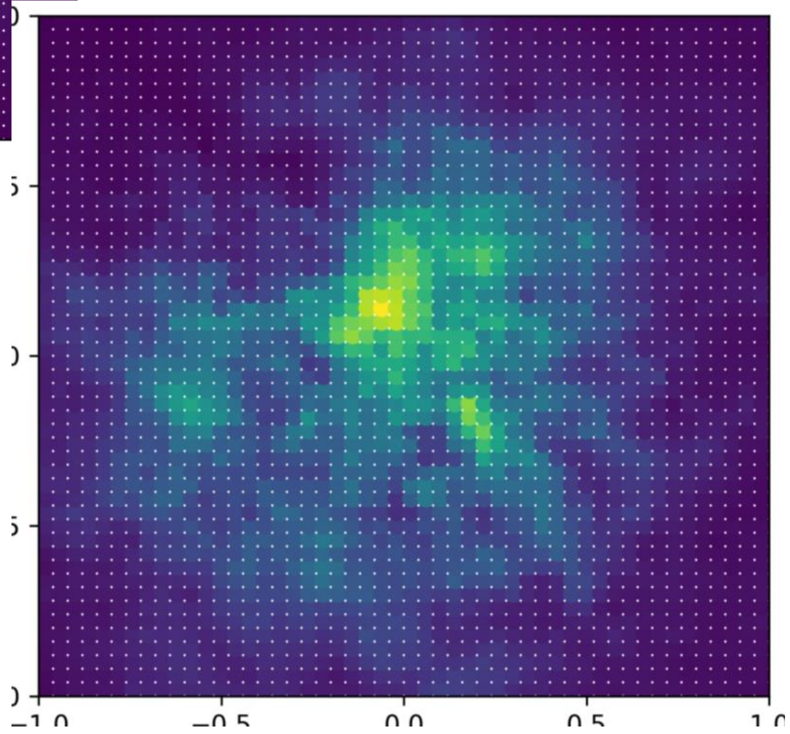
X_0=0.01



Y = 4



Y = 8



$$\frac{\mathrm{d}}{\mathrm{d}y} V_{\mathbf{x}} = V_{\mathbf{x}}(it^a) \left[\int \mathrm{d}^2\mathbf{z}\, \varepsilon^{ab,i}_{\mathbf{x},\mathbf{z}}\ \xi_{\mathbf{z}}(y)^b_i + \sigma^a_{\mathbf{x}} \right]$$

https://arxiv.org/abs/1806.06783
Mäntysaari, Schenke (2018)

The random noise is Gaussian and local in coordinates, color, and rapidity with expectation value zero and

$$\langle \xi^a_{\mathbf{x},i}(y) \xi^b_{\mathbf{y},j}(y') \rangle = \delta^{ab} \delta^{ij} \delta^{(2)}_{\mathbf{xy}} \delta(y - y'). \qquad (4)$$

The deterministic drift term is

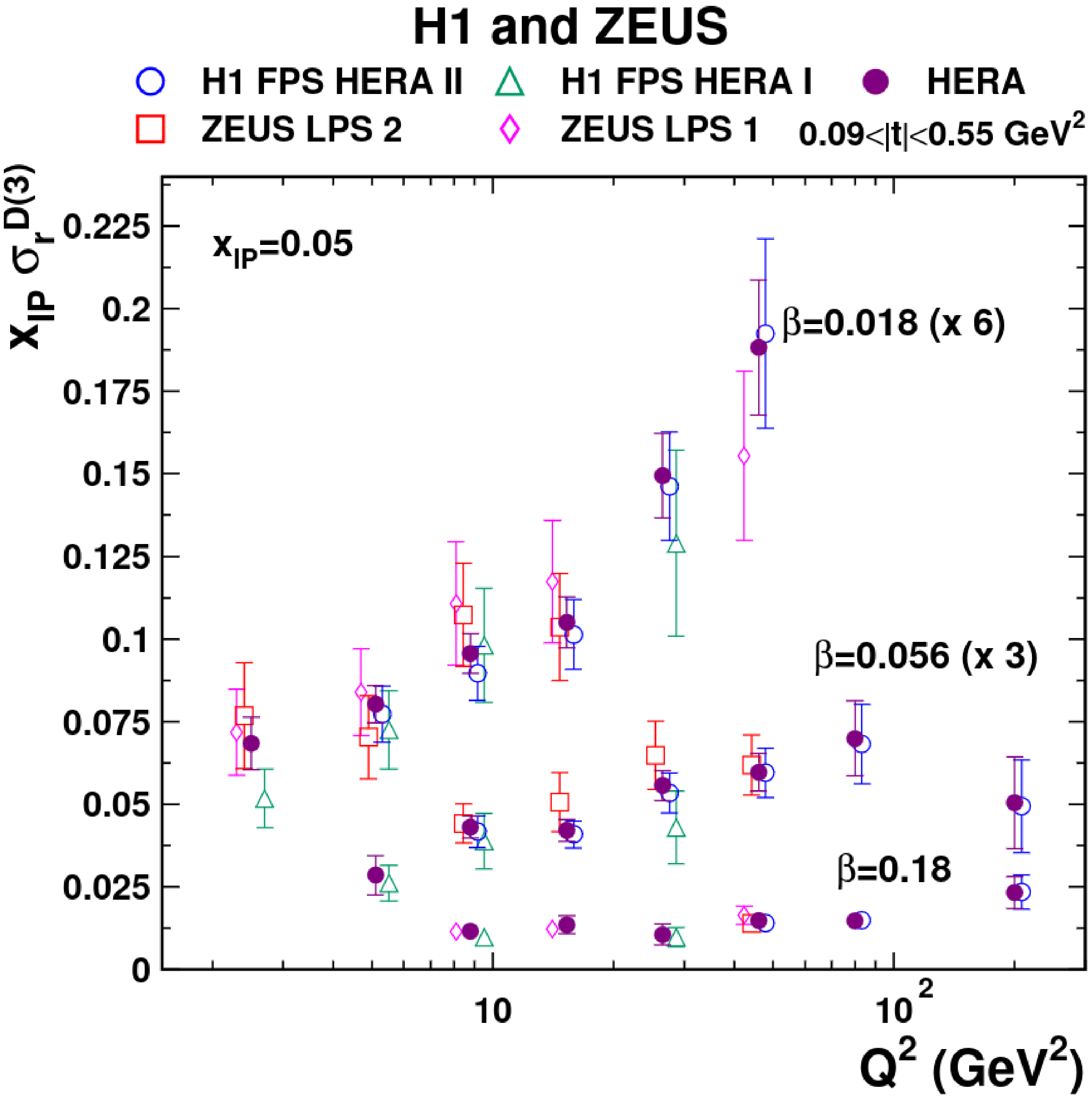
$$\sigma^a_{\mathbf{x}} = -i \frac{\alpha_{\mathrm{s}}}{2\pi^2} \int \mathrm{d}^2\mathbf{z}\, S_{\mathbf{x}-\mathbf{z}}\, \mathrm{Tr}\left[T^a U^\dagger_{\mathbf{x}} U_{\mathbf{z}} \right], \qquad (3)$$

with $S_{\mathbf{x}} = 1/\mathbf{x}^2$ and T^a the generators of the adjoint representation. $U_{\mathbf{x}}$ are Wilson lines in the adjoint representation.

The free parameters to be determined by the HERA data are the following:

- $g^4\mu^2$: controls the saturation scale of the proton at $x = x_0$
- B_p: controls the proton size at the initial condition
- m: models confinement effects in the JIMWLK evolution limiting the growth of the proton at long distances
- constant α_s or Λ_{QCD} which sets the scale for the running of α_{s}

$$K^i_{\mathbf{x}} \to m|\mathbf{x}| K_1(m|\mathbf{x}|) \frac{x^i}{\mathbf{x}^2}.$$



H1 and ZEUS
H1 FPS HERA II
H1 FPS HERA I
HERA
ZEUS LPS 2
ZEUS LPS 1
0.09<|t|<0.55 GeV^2
$x_{IP}\,\sigma_r^{D(3)}$
x_{IP}=0.05
β=0.018 (x 6)
β=0.056 (x 3)
β=0.18
0
0.025
0.05
0.075
0.1
0.125
0.15
0.175
0.2
0.225
10
10^2
Q^2 (GeV^2)

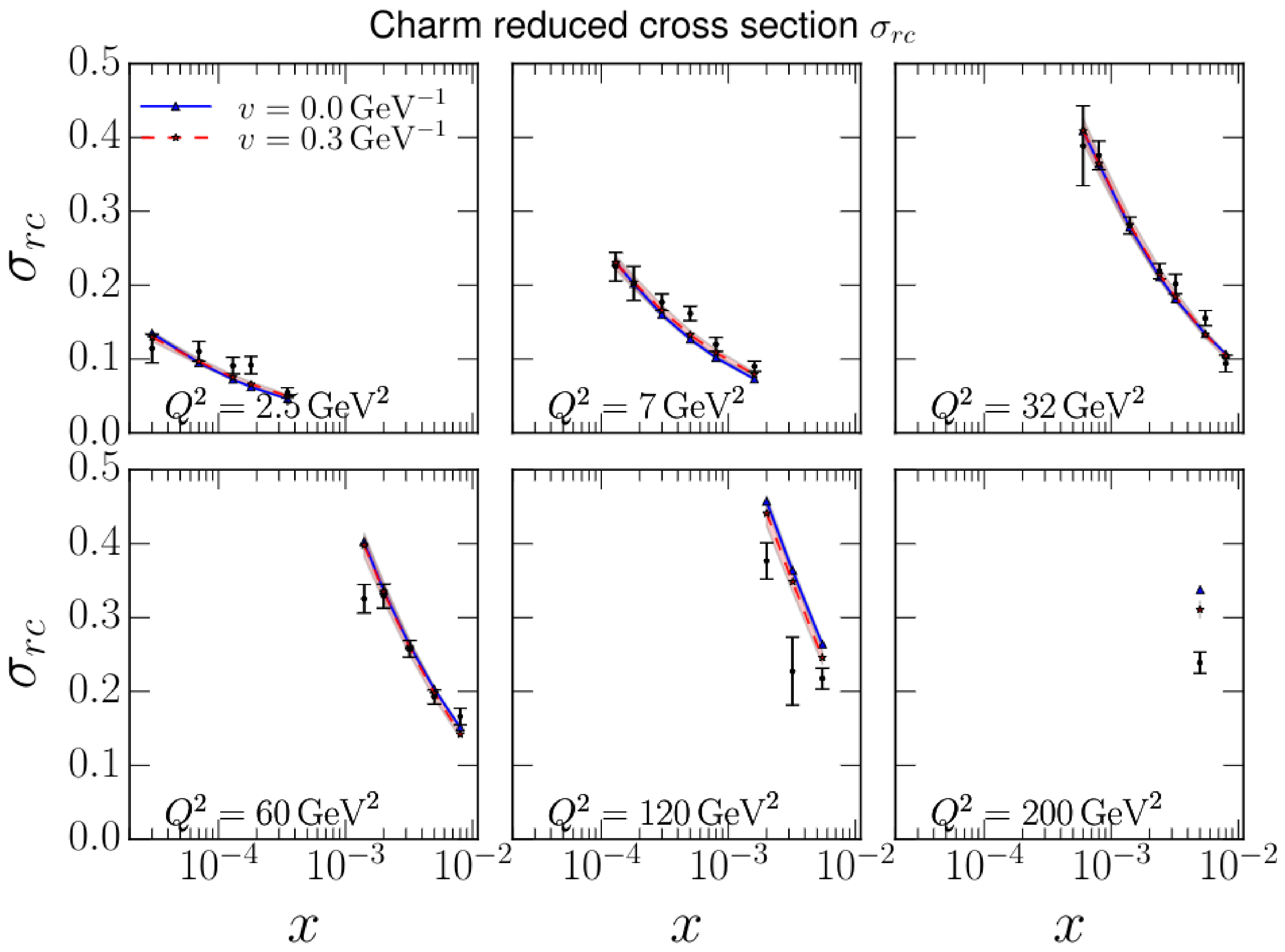
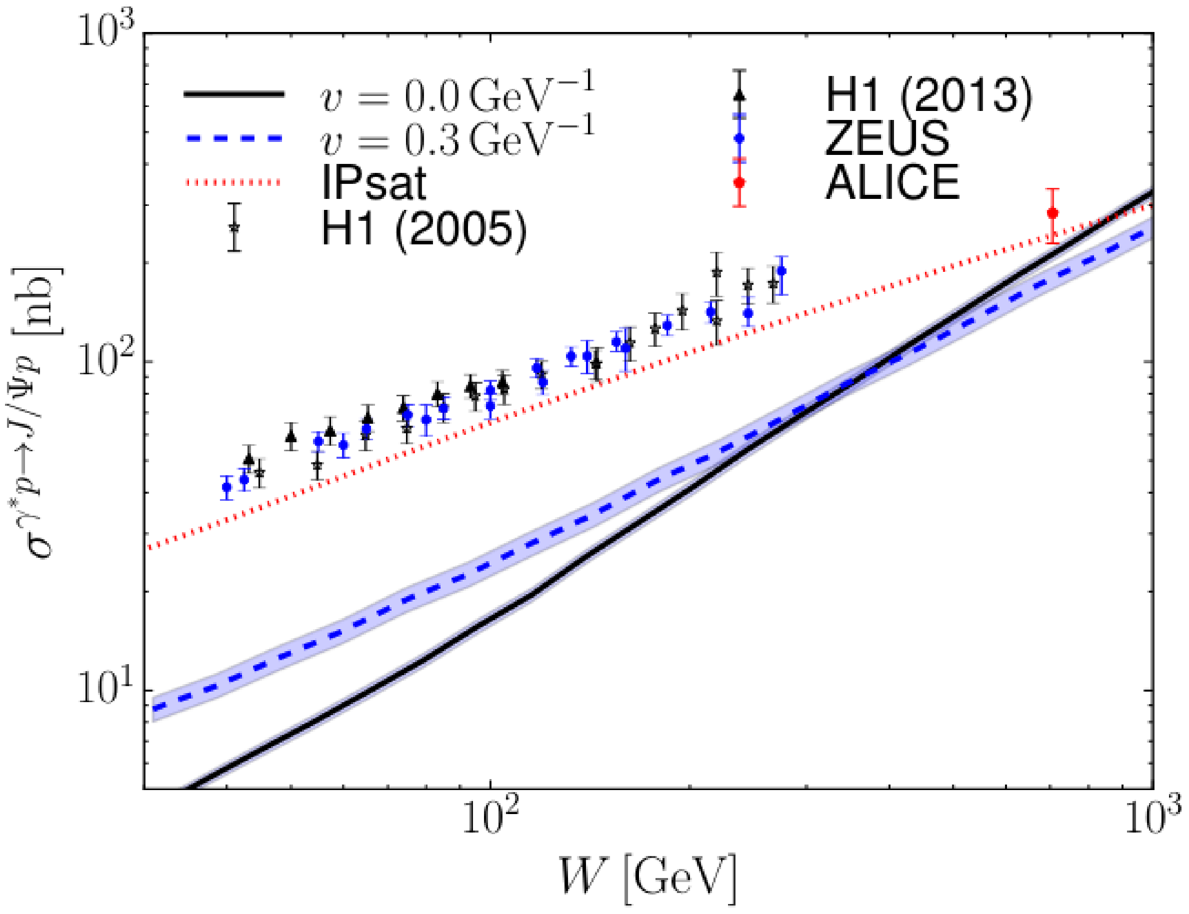
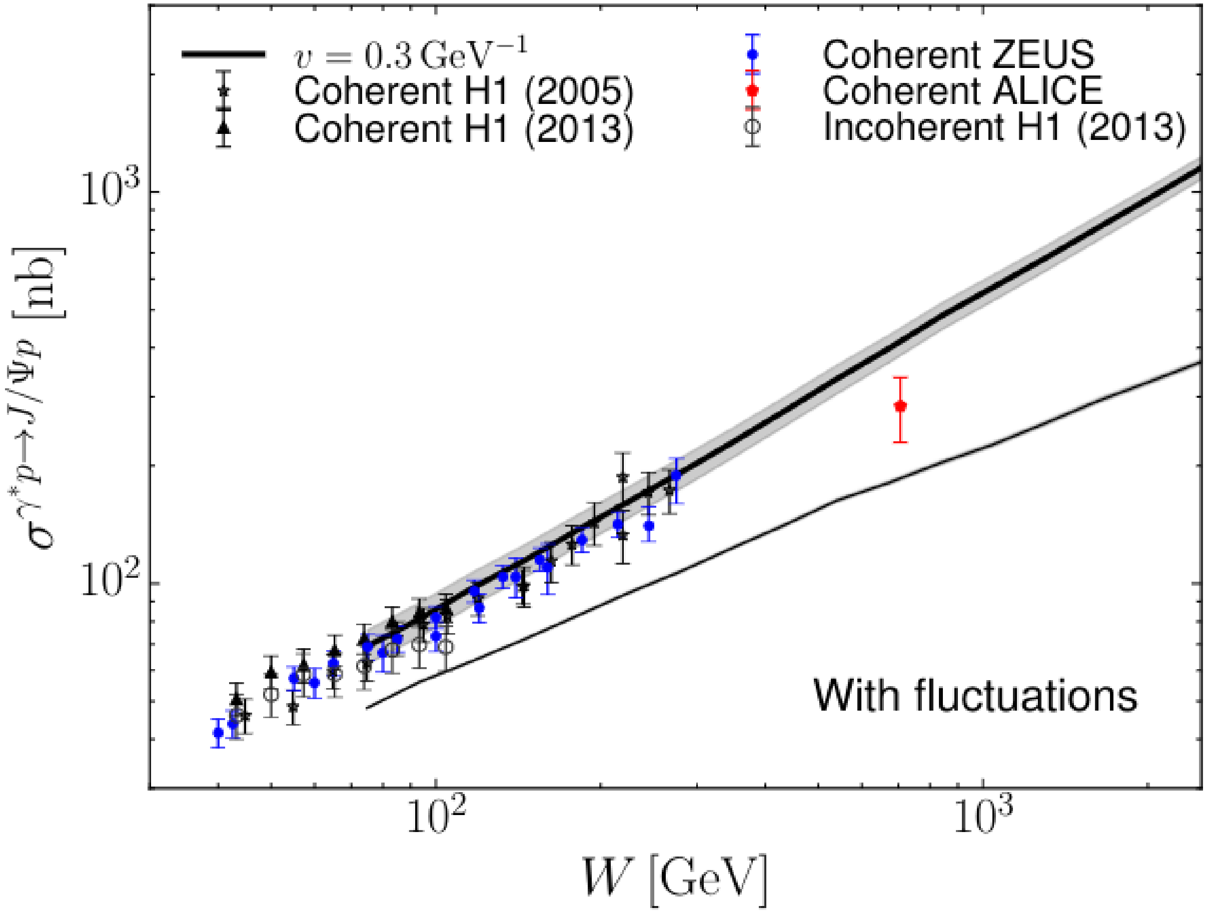


FIG. 2: Description of the charm reduced cross section using the unmodified MV model ($v = 0, \chi^2/N = 4.3$) and with an ultraviolet damping in the initial condition ($v = 0.3$ GeV^{-1}, $\chi^2/N = 2.7$).

https://arxiv.org/abs/1806.06783

Mäntysaari, Schenke





https://arxiv.org/abs/1806.06783

Mäntysaari, Schenke

$$\frac{\mathrm{d}\sigma^{\mathrm{D}}_{L,q\bar{q}}}{\mathrm{d}M_X^2\mathrm{d}|t|} = \frac{N_c}{4\pi}\frac{1}{4(2\pi)^3}\sum_{z_i} dz \sum_{\theta_{l\Delta,i}} d\theta_{l\Delta}\left|\frac{1}{N_V}\sum_{V}^{N_V}\sum_{\substack{m,n,\\ v,w\in V}}^{N} a^4\tilde{\psi}_L^{\gamma^*\to u\bar{u}}\right.$$

$$\left.\times\left[S_{m,n,v,w}-1\right]e^{-i(|\mathbf{l}|\hat{\mathbf{i}}\cdot(\mathbf{x}_{m,n}-\mathbf{x}_{v,w})+|\boldsymbol{\Delta}|\cos\theta_{l\Delta,i}\cdot(z_i\mathbf{x}_{m,n}+(1-z_i)\mathbf{x}_{v,w})}\right|^2\Bigg|_{\substack{|\mathbf{l}|=\sqrt{z_i(1-z_i)}M_X\\ |\boldsymbol{\Delta}|=\sqrt{|t|}}}$$

$$\frac{d^4\sigma_{ep}^{diff}}{d\beta dQ^2 dx_{IP} dt} = \frac{2\pi\alpha^2}{\beta Q^4}(1+(1-y)^2)F_2^{D(4)}(\beta, Q^2, x_{IP}, t)$$

In addition to the usual DIS variables, $Q^2 = -q^2 = -(k-k')^2$, $y = (P\cdot q)/(P\cdot k)$, $x = Q^2/(2P\cdot q)$, other quantities are used to describe the kinematics of a diffractive DIS event:

$$x_{IP} = \frac{q(P-P')}{qp} \simeq \frac{Q^2+M_X^2}{Q^2+W^2}, \quad t = (P-P')^2, \quad \beta = \frac{Q^2}{2q\cdot(P-P')} \simeq \frac{Q^2}{Q^2+M_X^2}; \tag{1}$$