

A precise α_s determination from the R-improved QCD Static Energy

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Introduction

- α_s most important parameter in QCD $\rightarrow$ should be determined with high precision.
- Our strategy $\rightarrow$ comparing the QCD Static Energy obtained in lattice simulations with highly accurate perturbative results.
- $V_{\text{QCD}} \propto \alpha_s \rightarrow$ very sensitive.
- We improve this method building on previous analyses [PRD, 86 (2012) 114031, PRD, 90 (2020) 074038] in several ways:
 - Leading renormalon subtraction $\rightarrow$ short-distance scheme (MSR).
 - Resummation of associated large logs with R-evolution.
 - Profiles functions for the renormalization scales.
- We can fit (for the first time) lattice data up to $r \sim 1 \text{ fm} \rightarrow E \sim 200 \text{ MeV}$.

Static Energy and Static Potential

- The Static Energy is defined as the potential energy between an infinitely massive quark anti-quark pair at a distance r , corrected by ultra-soft effects. In pNRQCD

$$E_s(r) = V_s(r, \mu) + \delta_{\text{us}}(r, \mu).$$

- The Static Potential is the basic object to understand the behavior of non-relativistic QCD:

$$V_s(r, \mu) = V_s^{\text{soft}}(r, \mu) + V_s^{\text{us}}(r, \mu),$$

$$V_s^{\text{soft}}(r) = -C_F \frac{\alpha_s(\mu)}{r} \sum_{i=0}^3 \left[\frac{\alpha_s(\mu)}{4\pi} \right]^i \sum_{j=0}^i a_{ij} \log^j(r\mu e^{\gamma_E}).$$

- Coefficients a_{i0} are known to four loops. $a_{ij \geq 0}$ obtained with RGE. $\alpha_s(\mu) = \alpha_s^{(n_\ell=3)}(\mu)$.
- V_s known in perturbation theory up to $\mathcal{O}(\alpha_s^4) \rightarrow$ ultra-soft contributions show up for the first time

$$V_s^{\text{us}}(r, \mu) = -\frac{C_A^3 C_F}{12\pi} \frac{\alpha_s^4(\mu)}{r} \log(\mu r e^{\gamma_E}).$$

Ultra-soft term and resummation

- μ_{us} appears both in the ultrasoft static potential V_s^{us} and in the matrix element δ_{us}

$$\delta_{us}(\mu_s, \mu_{us}) = -\frac{C_A^3 C_F}{12\pi} \frac{\alpha_s^3(\mu_s) \alpha_s(\mu_{us})}{r} \log \left[\frac{C_A \alpha_s(\mu_s) e^{-5/6}}{\mu_{us} r} \right]$$

$$V_s^{\text{us}}(r, \mu_s, \mu_{us}) = -\frac{C_A^3 C_F}{12\pi} \frac{\alpha_s^3(\mu_s) \alpha_s(\mu_{us})}{r} \log(\mu_{us} r e^{\gamma_E})$$

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- One can see the necessity of resummation with these two (incompatible) choices that minimize logs.
 - $\mu_{us} \sim \alpha_s/r$ [use this one: no large logs in δ_{us}]
 - $\mu_{us} \sim 1/r$ [discarded: sum up logs in V_s^{us}]

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$$\mu \frac{dV_s(r, \mu_s, \mu)}{d\mu} = -\frac{2C_F C_A^3}{24r} \frac{\alpha_s(\mu)}{\pi} \left[1 + B \frac{\alpha_s(\mu)}{\pi} \right] \alpha_s^3(\mu_s) \left\{ 1 + 3 \frac{\alpha_s(\mu_s)}{4\pi} \left[a_{1,0} + 2\beta_0 \log(r\mu_s e^{\gamma_E}) \right] \right\}.$$

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- Integrating from $\mu = \mu_s \sim 1/r$ to $\mu = \mu_{us} \sim \alpha_s/r$

$$\begin{aligned} V_s(r, \mu_s, \mu_{us}) &= V_s(r, \mu_s) + U_{us}(r, \mu_s, \mu_{us}), \\ U_{us}(r, \mu_s, \mu_{us}) &= \frac{C_A^3 C_F}{6\beta_0 r} \alpha_s^3(\mu_s) \left\{ \left(1 + 3 \frac{\alpha_s(\mu_s)}{4\pi} [a_{1,0} + 2\beta_0 \log(r\mu_s e^{\gamma_E})] \right) \log \left[\frac{\alpha_s(\mu_{us})}{\alpha_s(\mu_s)} \right] \right. \\ &\quad \left. + \left(B - \frac{\beta_1}{4\beta_0} \right) \left[\frac{\alpha_s(\mu_{us})}{\pi} - \frac{\alpha_s(\mu_s)}{\pi} \right] \right\} \end{aligned}$$

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- Replacing in the static energy $\rightarrow$ no large logs for $\mu_s \sim 1/r$ and $\mu_{us} \sim \alpha_s/r$

$$\begin{aligned} E_s(r) = & V_s^{\text{soft}}(r, \mu_s) + \frac{C_A^3 C_F}{12} \frac{\alpha_s^3(\mu_s)}{r} \left\{ \frac{2}{\beta_0} \left(B - \frac{\beta_1}{4\beta_0} \right) \left[\frac{\alpha_s(\mu_{us})}{\pi} - \frac{\alpha_s(\mu_s)}{\pi} \right] \right. \\ & + \frac{2}{\beta_0} \left(1 + 3 \frac{\alpha_s(\mu_s)}{4\pi} [a_{1,0} + 2\beta_0 \log(r\mu_s e^{\gamma_E})] \right) \log \left[\frac{\alpha_s(\mu_{us})}{\alpha_s(\mu_s)} \right] \\ & \left. - \frac{\alpha_s(\mu_{us})}{\pi} \log \left[\frac{C_A \alpha_s(\mu_s) e^{-5/6}}{r\mu_{us}} \right] - \frac{\alpha_s(\mu_s)}{\pi} \log(r\mu_s e^{\gamma_E}) \right\}. \end{aligned}$$

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- To cancel the renormalon we need a short-distance mass scheme $\rightarrow$ MSR mass [JHEP 04 (2018) 003]

$$\begin{aligned} \delta m_Q^{\text{MSR}}(\mu, R) &\equiv m_Q^{\text{pole}} - m_Q^{\text{MSR}}(R) = R \sum_{n=1}^{\infty} \delta_n^R \left[\frac{\alpha_s(R)}{4\pi} \right]^n \\ &= R \sum_{n=1}^{\infty} \left[\frac{\alpha_s(\mu)}{4\pi} \right]^n \sum_{j=0}^{n-1} \delta_{nj}^R \log^j \left(\frac{\mu}{R} \right). \end{aligned}$$

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- **New type of large logs** $\rightarrow$ resummation using R-evolution [PRL, 101 (2008) 151602].

Previous works

- To see the advantages of R-evolution we go back to the first work that used the static energy to obtain α_s [PRD, 86 (2012) 114031]

$$E_s(r) = V_s(r, \mu_s, \mu_{us}) + \delta_{US}(r, \mu_s, \mu_{us}) + RS(\rho),$$

- $V_s(r, \mu_s, \mu_{us}) \rightarrow \ln(r\mu_{us})$
- $RS(\rho) \rightarrow \ln(\rho/\mu_{us})$
- There is no right choice for $\mu_{us} \rightarrow$ fits only possible for small values of r .
- The force was used in [PRD, 100 (2019) 114511]

$$F_s(r) = \frac{dE_s(r)}{dr}$$

- Force avoids explicit subtraction.
- Using fully canonical scales $\sim 1/r \rightarrow$ limits fit-range to $r \sim 0.076$ fm.

R-improvement, renormalon subtractions

- The next goal is to obtain a renormalon-free potential.

$$E_{Q\bar{Q}}(r) = 2m_Q^{\text{MSR}}(R_0) + 2\delta m_Q^{\text{MSR}}(R_0, \mu) + E_s(r) \equiv 2m_Q^{\text{MSR}}(R_0) + V_s^{\text{MSR}}(r, \mu, R_0).$$

- We sum up large logs of μ/R_0 .

$$\begin{aligned}\delta m_Q^{\text{MSR}}(R_0) &= \delta m_Q^{\text{MSR}}(R_0) + \delta m_Q^{\text{MSR}}(R) - \delta m_Q^{\text{MSR}}(R) \\ &= \delta m_Q^{\text{MSR}}(R) + m_Q^{\text{MSR}}(R) - m_Q^{\text{MSR}}(R_0) \\ &= \delta m_Q^{\text{MSR}}(R) + \Delta^{\text{MSR}}(R, R_0).\end{aligned}$$

- $\Delta^{\text{MSR}}(R, R_0)$: solution to MSR mass R-RGE $\rightarrow$ sums up logs of R/R_0 .
- We have $\delta m_Q^{\text{MSR}}(R) \sim \log(\mu/R)$. By choosing $\mu = R \rightarrow$ no large logs.
- We define the R-improved static potential:

$$V_s^{\text{MSR}}(r, \mu, R_0) = V_s(r, \mu) + 2\delta^{\text{MSR}}(R, \mu) + 2\Delta^{\text{MSR}}(R, R_0).$$

Subtraction schemes: RS scheme

- It is defined from the pole mass by subtracting its leading asymptotic behavior [JHEP 06 (2001) 022].

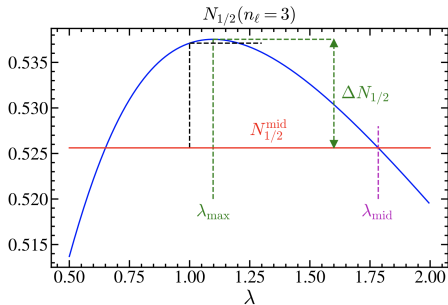
$$m_Q^{\text{pole}} - m_Q^{\text{RS}}(R) = \frac{2\pi}{\beta_0} R N_{1/2} \sum_{n=1}^{\infty} \left[\frac{\beta_0 \alpha_s(R)}{2\pi} \right]^n \sum_{\ell=0}^{\infty} g_{\ell} \left(1 + \hat{b}_1 \right)_{n-1-\ell},$$

- $N_{1/2}$ is the normalization of the leading renormalon

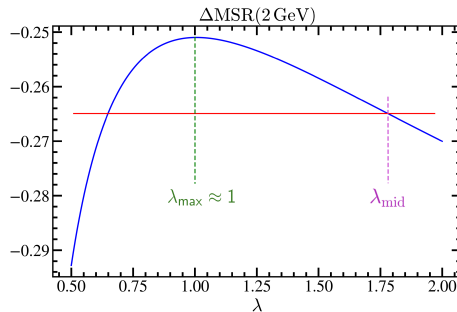
$$N_{1/2}^{(n)} = \frac{\beta_0}{2\pi} \sum_{k=0}^n \frac{S_k}{\left(1 + \hat{b}_1 \right)_k} \quad S_k = \sum_{k=0}^j \tilde{\gamma}_k^R \sum_{i=0}^{j-k} (-1)^i \tilde{b}_i^N \tilde{g}_{j-i-k}^N,$$

- $N_{1/2}$ depends on λ , it reshuffles higher perturbative orders. We vary it to estimate $N_{1/2}$ (similar to scale variation).
- λ is also used to estimate R-evolution uncertainty.

Dependence on λ



[JHEP 04 (2018) 003]



- The canonical value of $\lambda = 1$ is clearly biased.
- It is necessary to vary it to estimate the uncertainty coming from higher order missing terms.
- We pick $\lambda_{\text{mid}} = 1.784$ and vary it from 1.5 to 2.1.

Profiles

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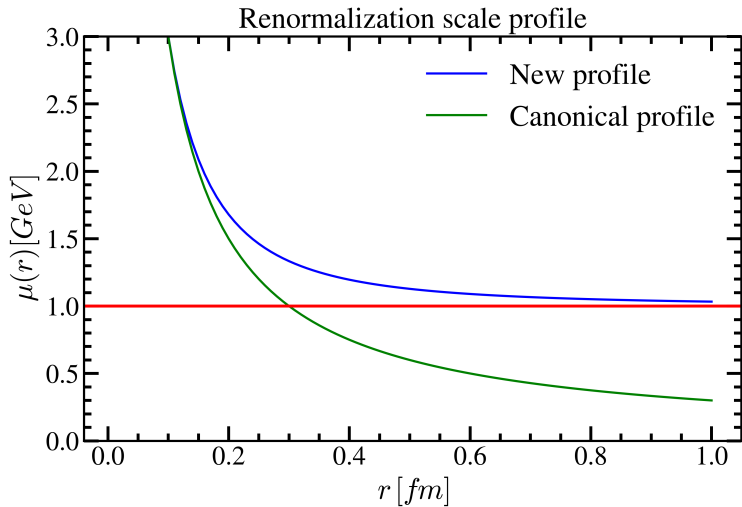
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- Choosing $\mu_s \sim 1/r$ doesn't always work $\rightarrow \mu_s < \Lambda_{\text{QCD}}$ for $r \gtrsim 0.2 \text{ fm} \sim 1 \text{ GeV}$.
- Solution: use profile functions that ensure series convergence:

$$\mu_s = \sqrt{\left(\frac{\xi}{r}\right)^2 + \frac{b}{r} + a^2} + \mu_0 - a = \begin{cases} \frac{\xi}{r} & \text{for } r \rightarrow 0 \\ \mu_0 & \text{for } r \rightarrow \infty \end{cases}$$

with $\xi = \mathcal{O}(1)$ and $\mu_0 \sim 1 \text{ GeV}$. This makes sure the series is stable and convergent over the entire spectrum.

Profiles



Fits

- Lattice QCD data from HotQCD. $(2 + 1)$ flavor simulations. We have 9 lattices sets with 2512 points between all of them.
- The static potential is defined up to an additive constant $\rightarrow$ each experiment n has a different offset A_n .
- Minimization of χ^2 function.

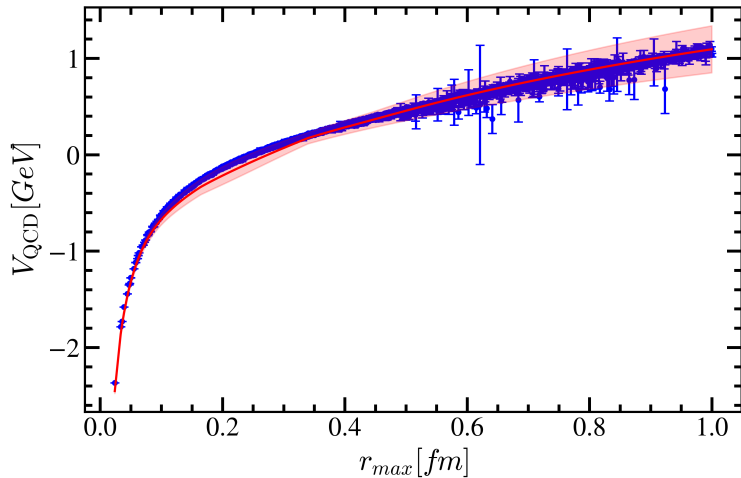
$$\chi^2 = \sum_{n=1}^{N_{\text{exp}}} \sum_{i=1}^{N_i} \left(\frac{V_{n,i}^{\text{pert}} + A_n - V_{n,i}^{\text{exp}}}{\sigma_{n,i}^{\text{exp}}} \right)^2.$$

- We can marginalize analytically first with respect to the offsets.

$$\frac{\partial \chi^2}{\partial A_n} = 0 \implies \tilde{A}_n = - \frac{\sum_i \frac{(V_{n,i} - V_{n,i}^{\text{exp}})}{\sigma_{n,i}^2}}{\sum_i \frac{1}{\sigma_{n,i}^2}} \implies \tilde{\chi}^2 = \sum_{n,i} \frac{(V_i - V_i^{\text{exp}})^2}{\sigma_i^2} - \frac{\left[\sum_{n,i} \frac{(V_i - V_i^{\text{exp}})}{\sigma_i^2} \right]^2}{\sum_{n,i} \frac{1}{\sigma_i^2}}$$

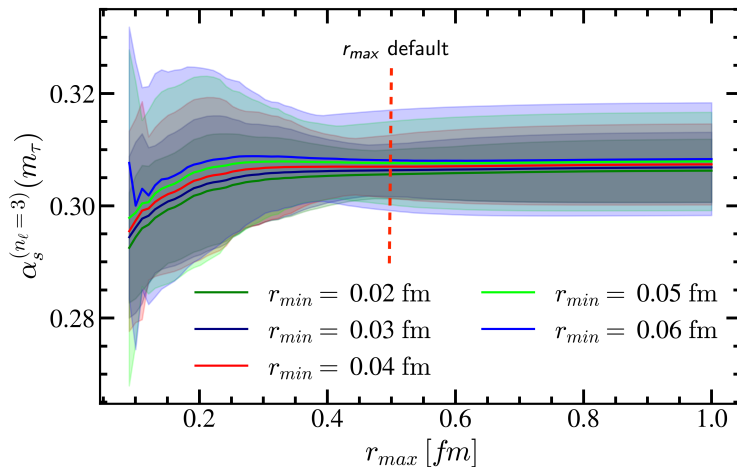
- We perform the fit with 500 random profiles.

Potential

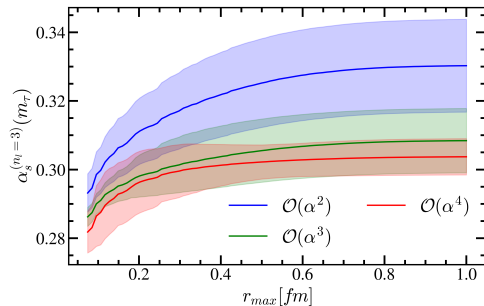
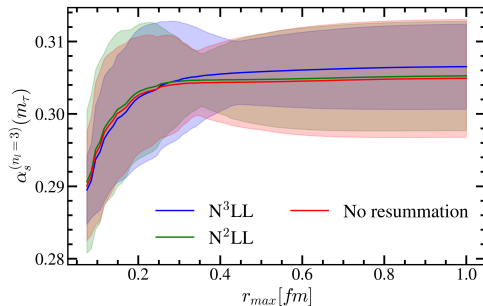


Study of the dataset

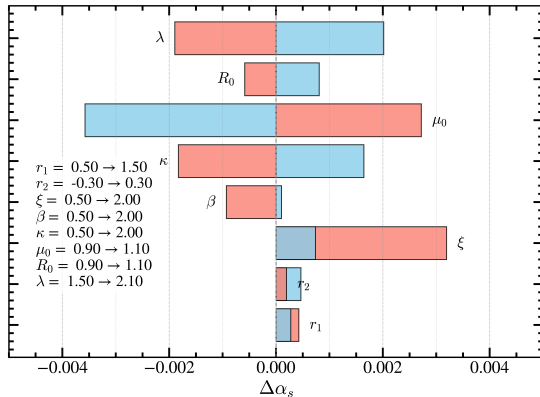
Fit for $r \in [r_{\min}, r_{\max}]$. $r_{\text{data}} \geq 0.023$ fm.



Order-by-order agreement



Variation of profiles parameters one at a time



$$\mu_s = \sqrt{\left(\frac{\xi}{r}\right)^2 + \frac{b}{r} + a^2} + \mu_0 - a$$

$$R = \sqrt{\left(\frac{\beta}{r}\right)^2 + \frac{b}{r} + a^2} + R_0 - a$$

$$\mu_{us} = \kappa \frac{C_A}{2} \{ \mu_s \alpha_s(\mu) - \mu_0 \alpha_s(\mu_0) \} + \mu_0.$$

Preliminary results

- We perform a fit with $r \in [0.03, 0.5]$ fm. Error includes: perturbative (dominant), lattice (negligible) and dataset dependence (also negligible).

$$\text{All data} \rightarrow \alpha_s^{(n_f=3)}(m_\tau) = 0.3107 \pm 0.00001_{\text{lattice}} \pm 0.053_{\text{pert}} \pm 0.0001_{r_{\text{max}}} \pm 0.0007_{r_{\text{min}}}$$

$\downarrow$

$$\alpha_s^{(n_f=3)}(m_\tau) = 0.311 \pm 0.053$$

$$\alpha_s^{(n_f=5)}(m_Z) = 0.1175 \pm 0.0007$$

Competitive with the w.a. $\alpha_s^{(5)}(m_Z) = 0.1180 \pm 0.0009$ and compatible at $0.5\text{-}\sigma$ level. Comparing with previous analyses:

$$\alpha_s^{(n_f=5)}(m_Z) = 0.11660^{+0.00110}_{-0.00056} \text{ [PRD 100 (2019) 11]}$$

$$\alpha_s^{(n_f=5)}(m_Z) = 0.1181 \pm 0.0009 \text{ [JHEP 09 (2020) 016]}$$

Conclusions

- Used the static energy to determine α_s , building (and improving) on previous analyses.
- Performed ultra-soft large-log resummation up to N³LL.
- Employed the MSR mass and R-evolution to improve the static potential.
- Designed profile functions to increase the validity of the potential up to $r \sim 1 \text{ fm} \sim 200 \text{ MeV} \rightarrow$ lowest value reached.
- Carried out fits to lattice data to obtain a very competitive result for α_s .

$$\alpha_s^{(n_f=5)}(m_Z) = 0.1175 \pm 0.0007 \quad [\text{preliminary}]$$

BACKUP

Renormalon subtractions

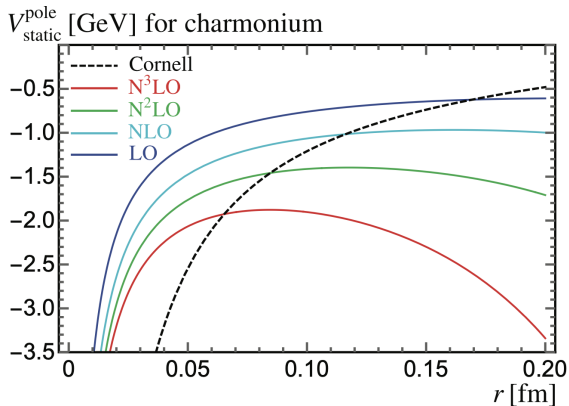


Figure: [Eur. Phys. J. C 79 (2019) 4, 323]

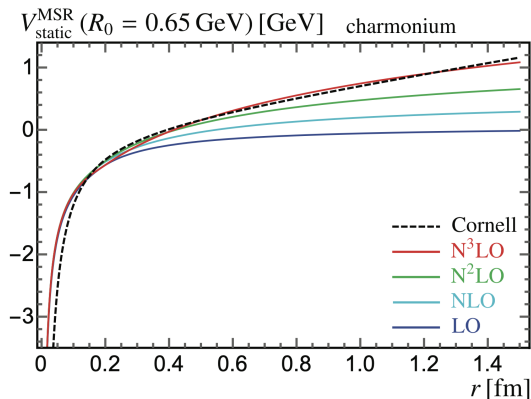


Figure: [Eur. Phys. J. C 79 (2019) 4, 323]

Force-type subtractions

- Integrating the Force is equal to perform R-evolution.
- The renormalon doesn't depend on $r \rightarrow$ we can subtract the potential at r_0 .

$$\begin{aligned} E_s^F(r, r_0) &\equiv E_s(r) - E_s(r_0) = E_s(r) - E_s(r_1) + [E_s(r_1) - E_s(r_0)] \\ &= E_s(r) - E_s(r_1) + \int_{r_0}^{r_1} dr' F_s(r') \equiv E_s(r) - E_s(r_1) + \Delta_F(r_0, r_1) \end{aligned}$$

- [PRD, 90(7), 074038] chooses $r_1 = r \rightarrow$ only Δ_F left.
- Connecting with R-evolution, the subtraction term is (choosing $R = 1/r_1$ and $\mu = R$):

$$E_s(r_1) \equiv \delta_{\text{soft}}^F(R) = \frac{1}{2} V_s^{\text{soft}}\left(\frac{1}{R}\right) = -2\pi C_F R \sum_{i=1} \left[\frac{\alpha_s(R)}{4\pi} \right]^i \sum_{j=0}^{i-1} a_{i-1,j} \gamma_E^j \equiv R \sum_{i=1} \left[\frac{\alpha_s(R)}{4\pi} \right]^i \delta_i^F.$$

- We can express Δ_F as an R-evolution integral:

$$\begin{aligned} \gamma_{\text{soft}}^F(R) &= -\frac{1}{2} \left[r^2 F_s^{\text{soft}}(r) \right]_{r=1/R}, \\ \Delta_F^{\text{soft}}(r_0, r_1) &= \int_{r_0}^{r_1} \frac{dr'}{(r')^2} \left[(r')^2 F_s^{\text{soft}}(r') \right] = \frac{1}{2} \int_{1/r_0}^{1/r_1} dR' \gamma_{\text{soft}}^F(R'). \end{aligned}$$

- It inherits the infrared sensitivity of the Static Potential.

α_s dependence on ξ

