

2nd European School on the Physics of the EIC and Related Topics
Presentations by students, Wednesday 25 June

Energy-momentum tensor and spatial distribution

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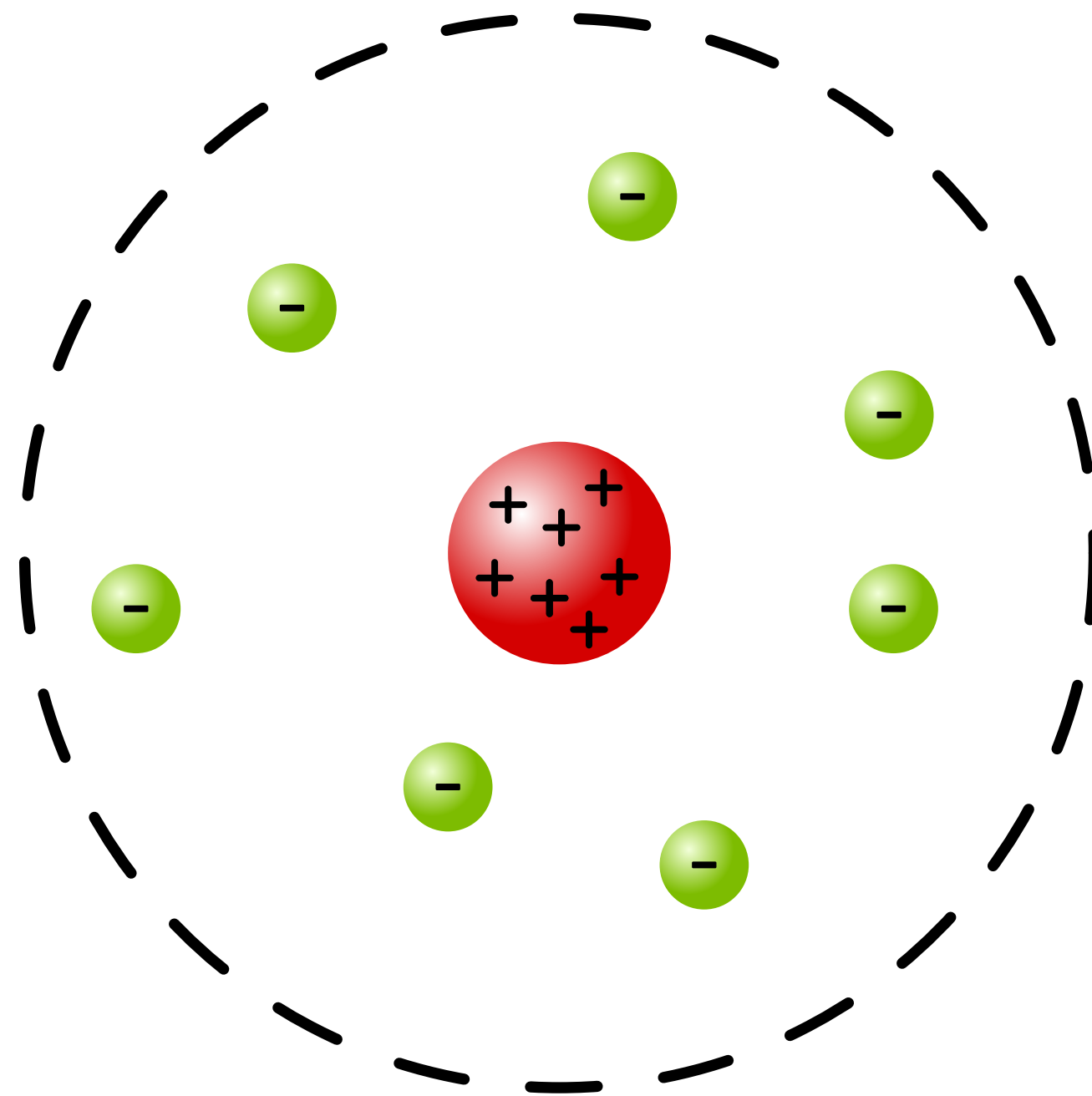
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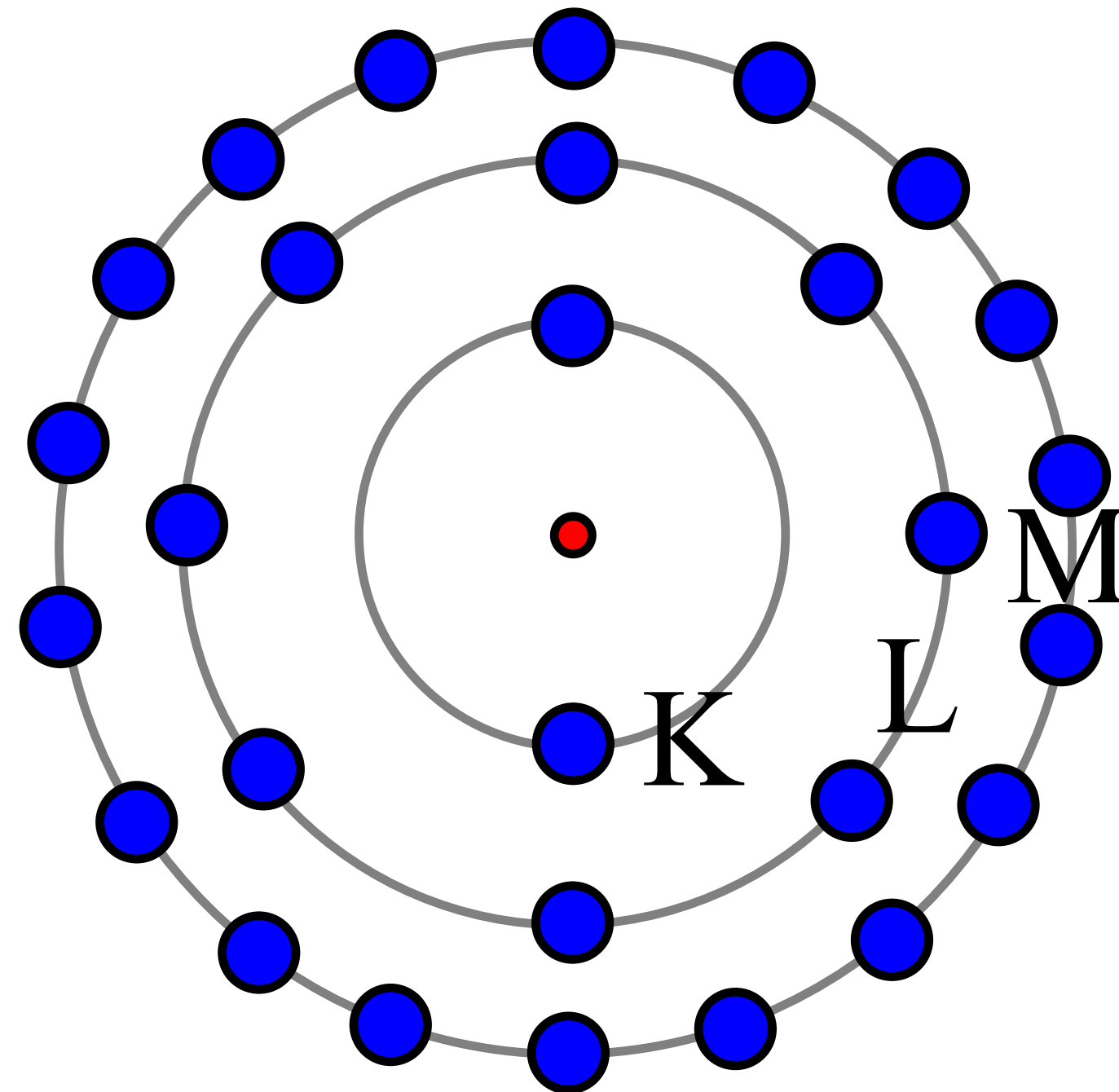
Introduction

Atom?

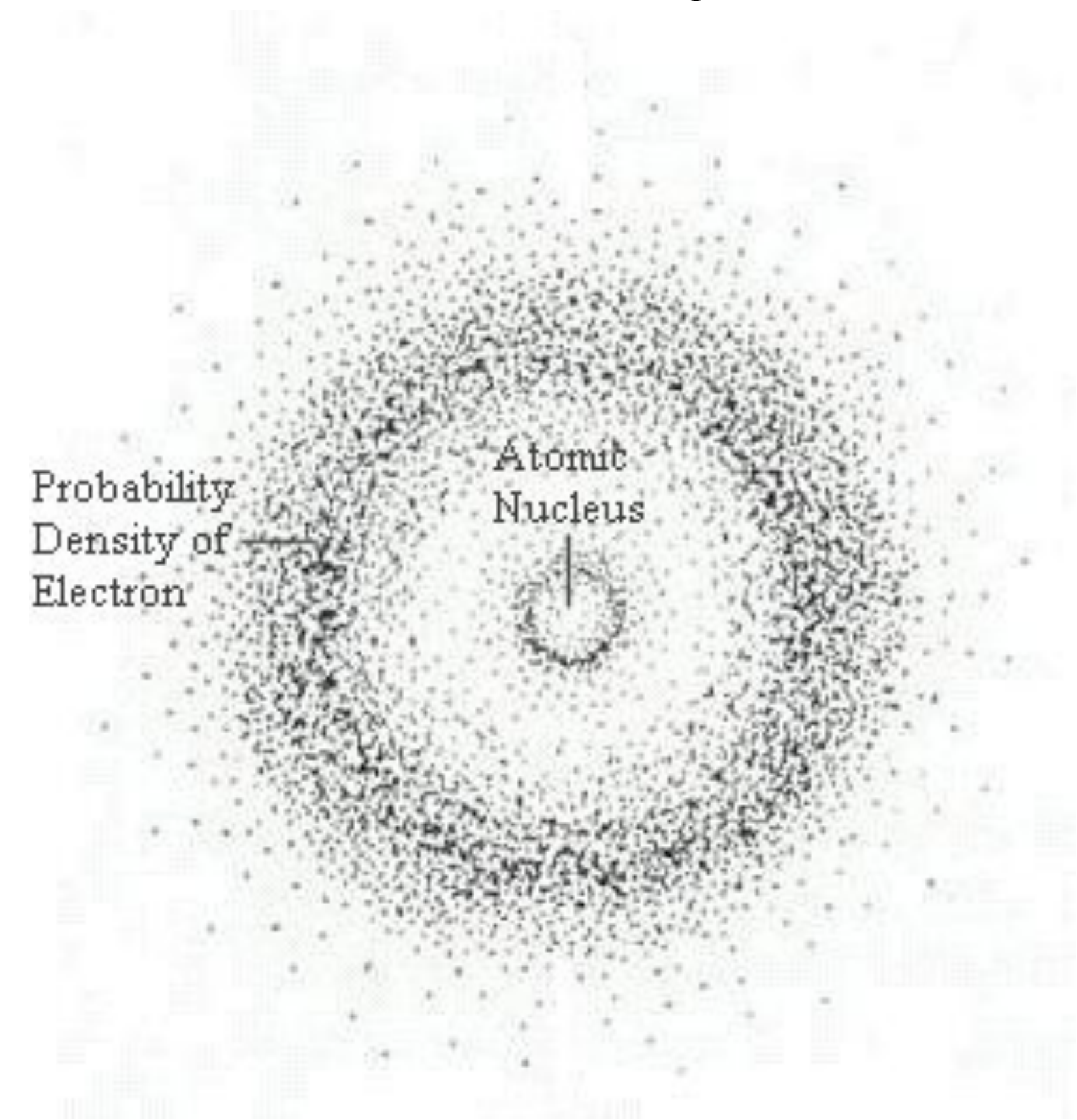
In 1909, E. Rutherford's model



In 1921, N. Bohr's model



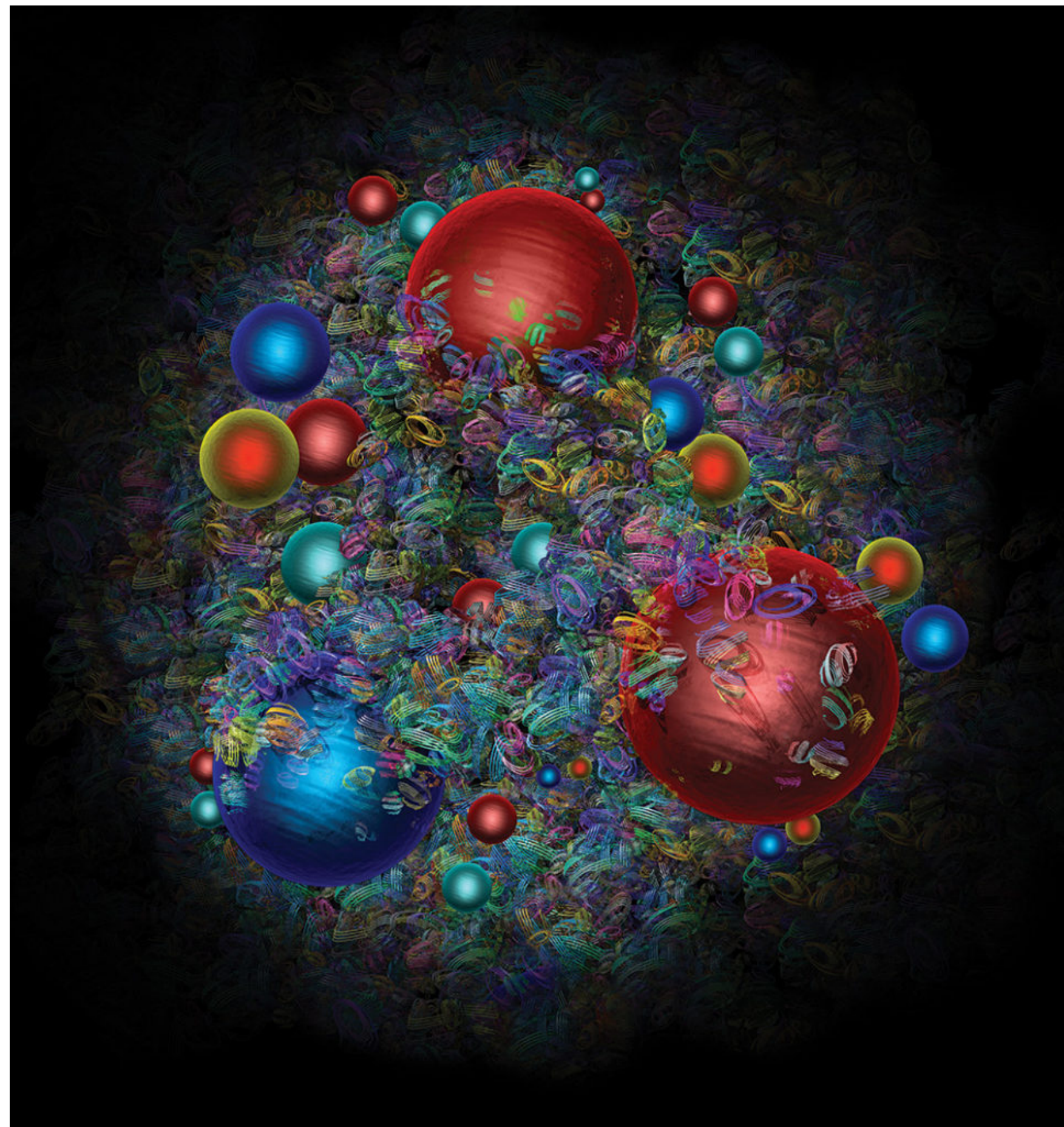
In 1932, E. Schrödinger model



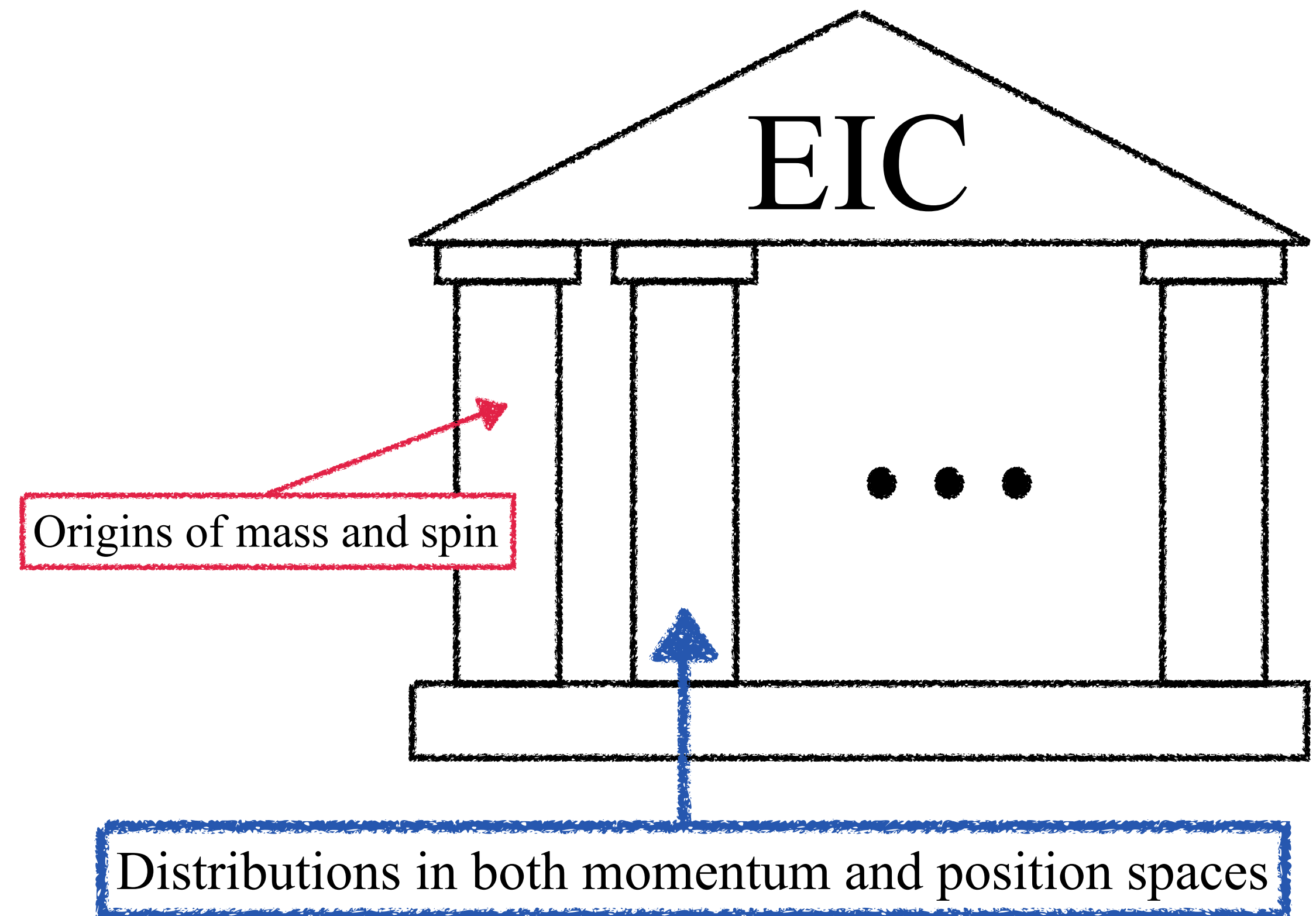
What is the nucleon?

Abdul Khalek et al., NPA 1026 (2022)

Modern understanding



Electron-Ion Collider (EIC) project



Energy-momentum tensor

Energy-momentum tensor

Energy-momentum tensor (EMT)

Conserved current under space-time translations

$$T^{\mu\nu} = \begin{pmatrix} \text{Energy} & \text{Momentum} \\ \text{Energy flux} & \text{Stress tensor} \end{pmatrix} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$$

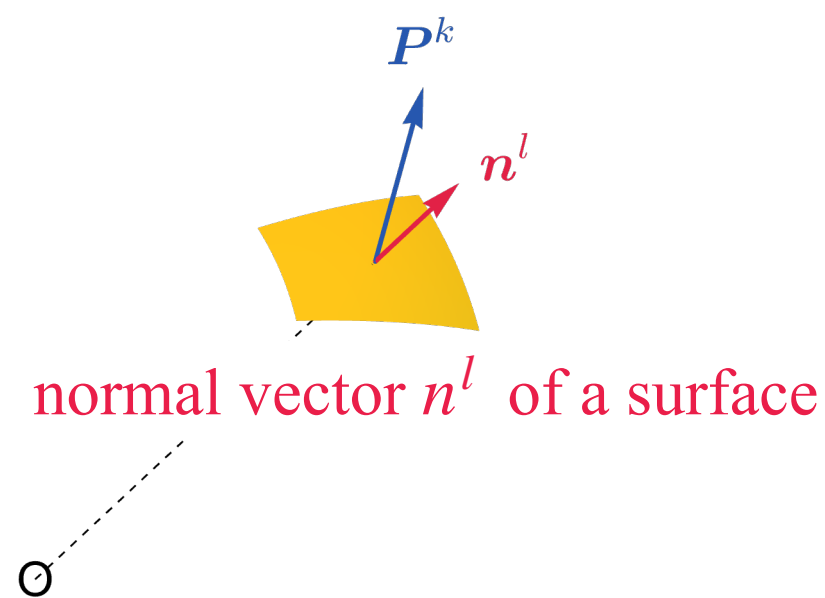
Conservation law

$$\partial_\mu T^{\mu\nu} = 0$$

S. Saha (Mon 30th)

Stress tensor T^{lk}

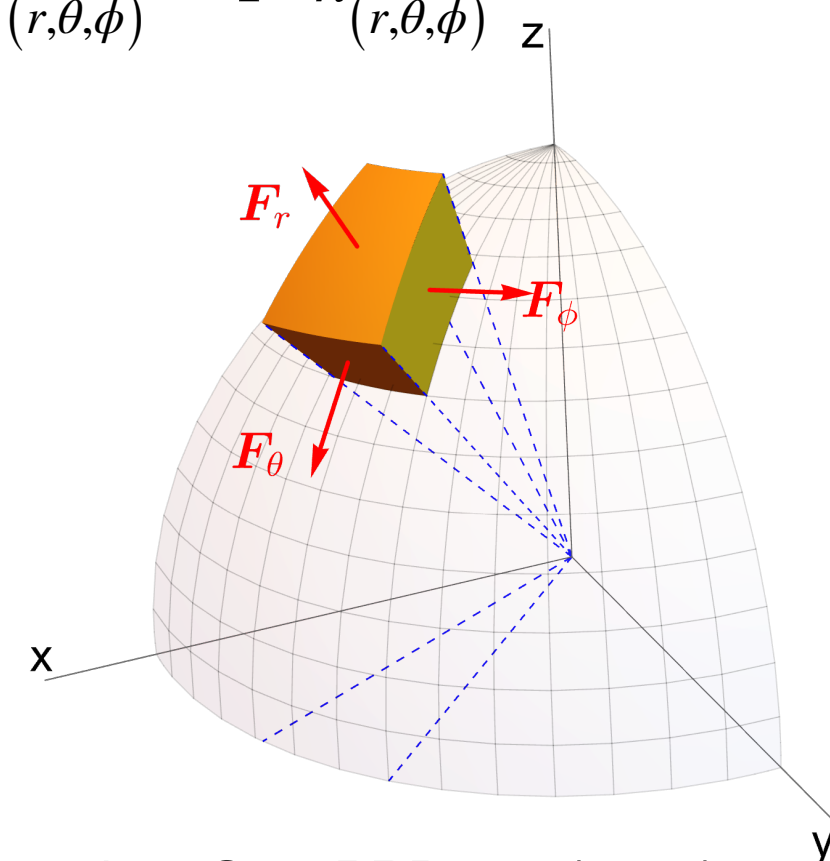
momentum flux



projection

Mechanical forces

$$F^k_{(r,\theta,\phi)} = T^{lk} n^l_{(r,\theta,\phi)}$$



Kim, Kim, Polyakov, Son, PRD 103 (2021)

Angular momentum

Orbital angular momentum

$$L^i = \int d^3x \epsilon^{ijk} x^j T^{0k}(x)$$

$$\longrightarrow J^i = L^i + S^i$$

Spin sum rule & individual contribution

c.f. intrinsic spin

$$S^i = \frac{1}{2} \int d^3x \bar{\psi}(x) \gamma^i \gamma_5 \psi(x)$$

Lorcé, Mantovani, Pasquini, PLB 776 (2018)

Lorcé, Mukherjee, Singh, Won, arXiv 2606.20468

Mechanical torque

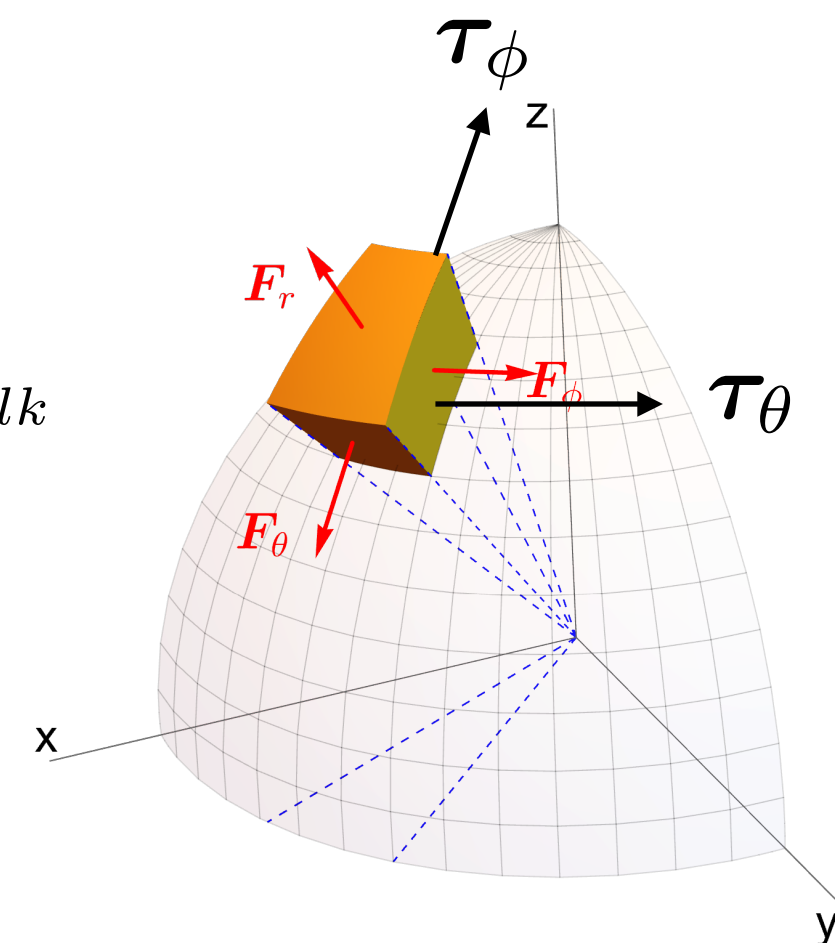
Generalized torque

$$C^{il} = \int d^3x \mathcal{C}^{il}(x)$$

operator density $\mathcal{C}^{il} = \epsilon^{ijk} r^j T^{lk}$

$$\begin{aligned} \longrightarrow \mathcal{C}^{il} n^l_{(\theta,\phi)} &= \epsilon^{ijk} r^j F^k_{(\theta,\phi)} \\ \text{projection} &= \tau^i_{(\theta,\phi)} \end{aligned}$$

Mechanical torque



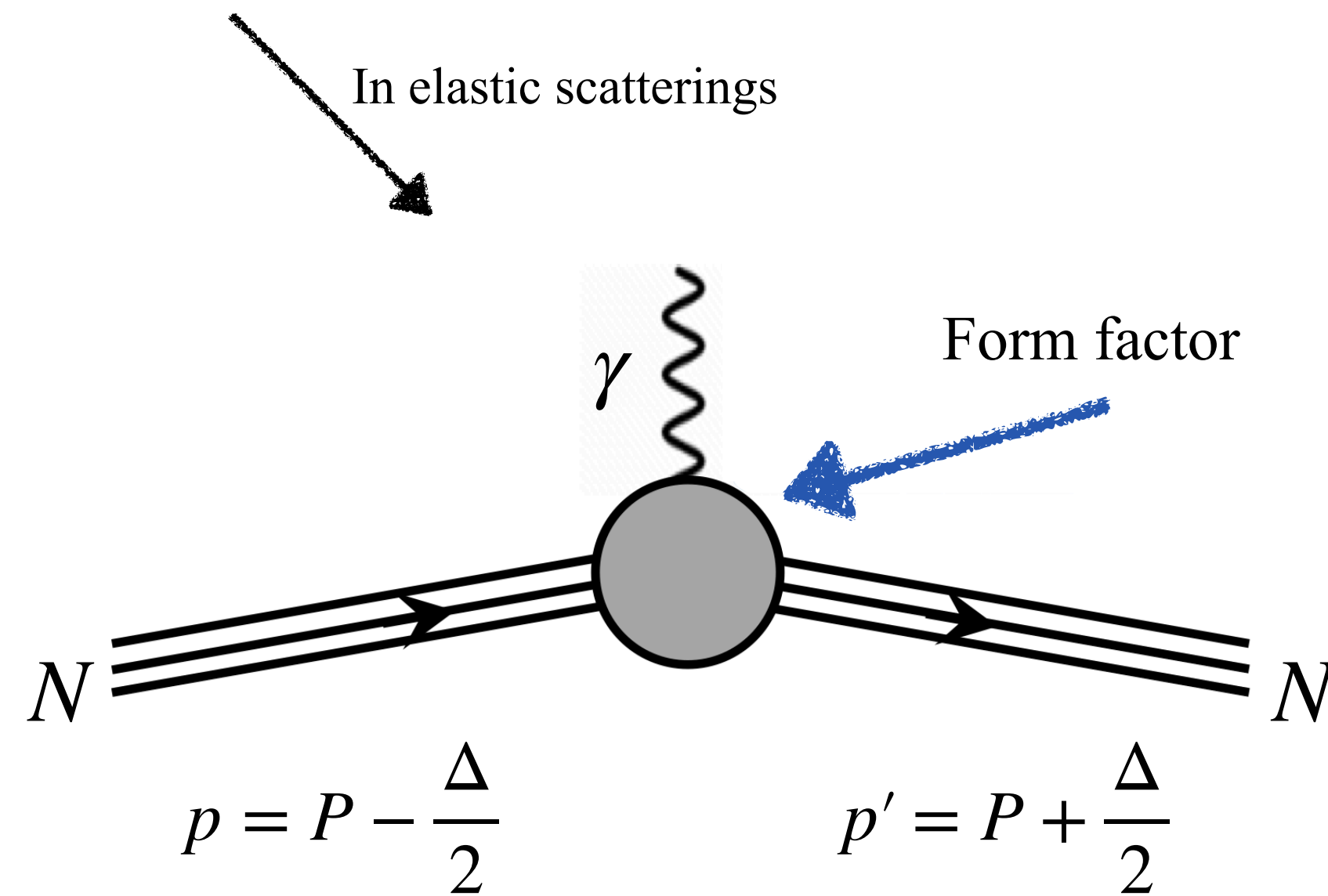
Lorcé, Song, PLB 864 (2025)

Relativistic spatial distribution

Form factor

Electric form factor

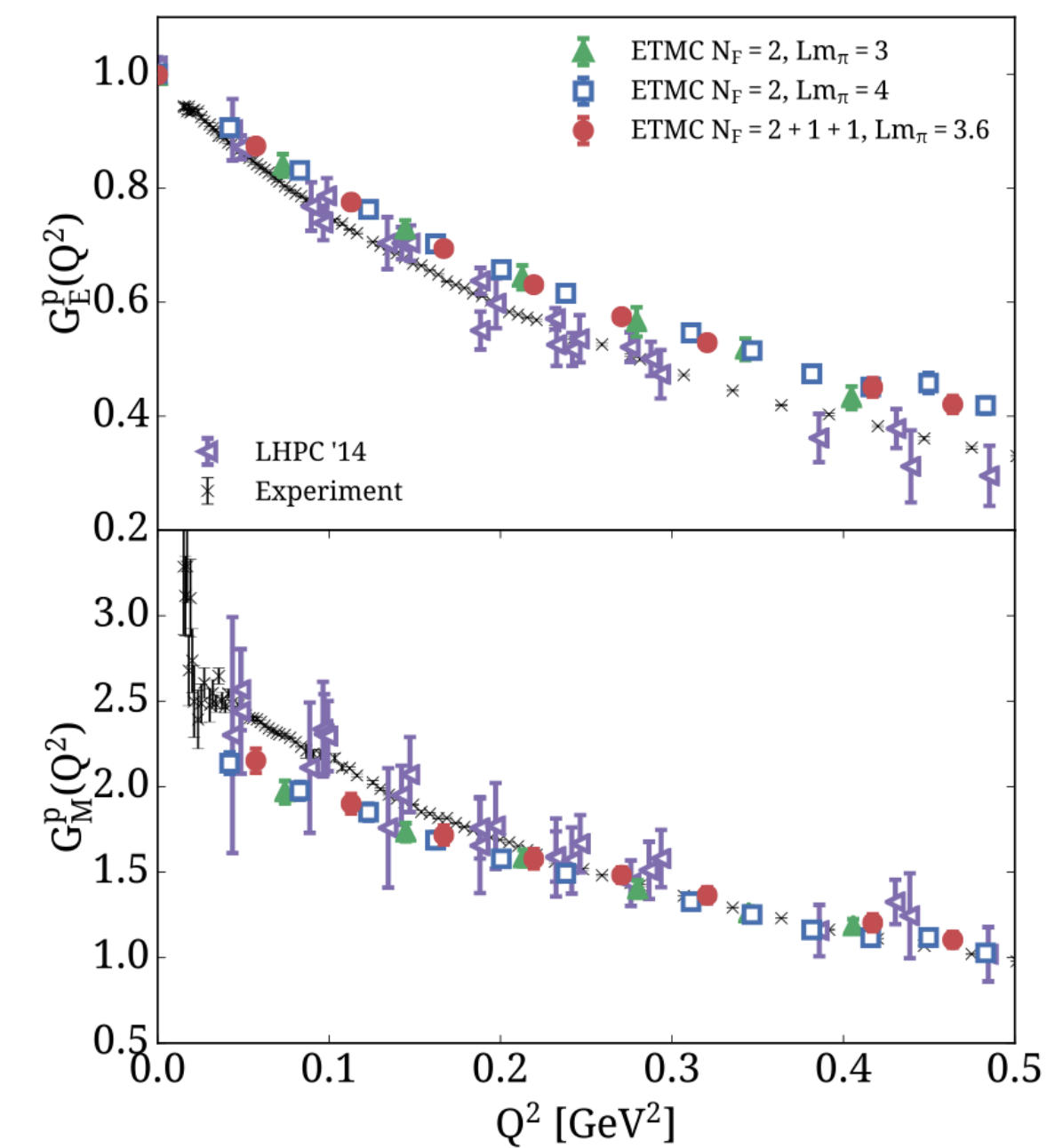
Internal structure, e . g . , $\mu_N \neq 1$



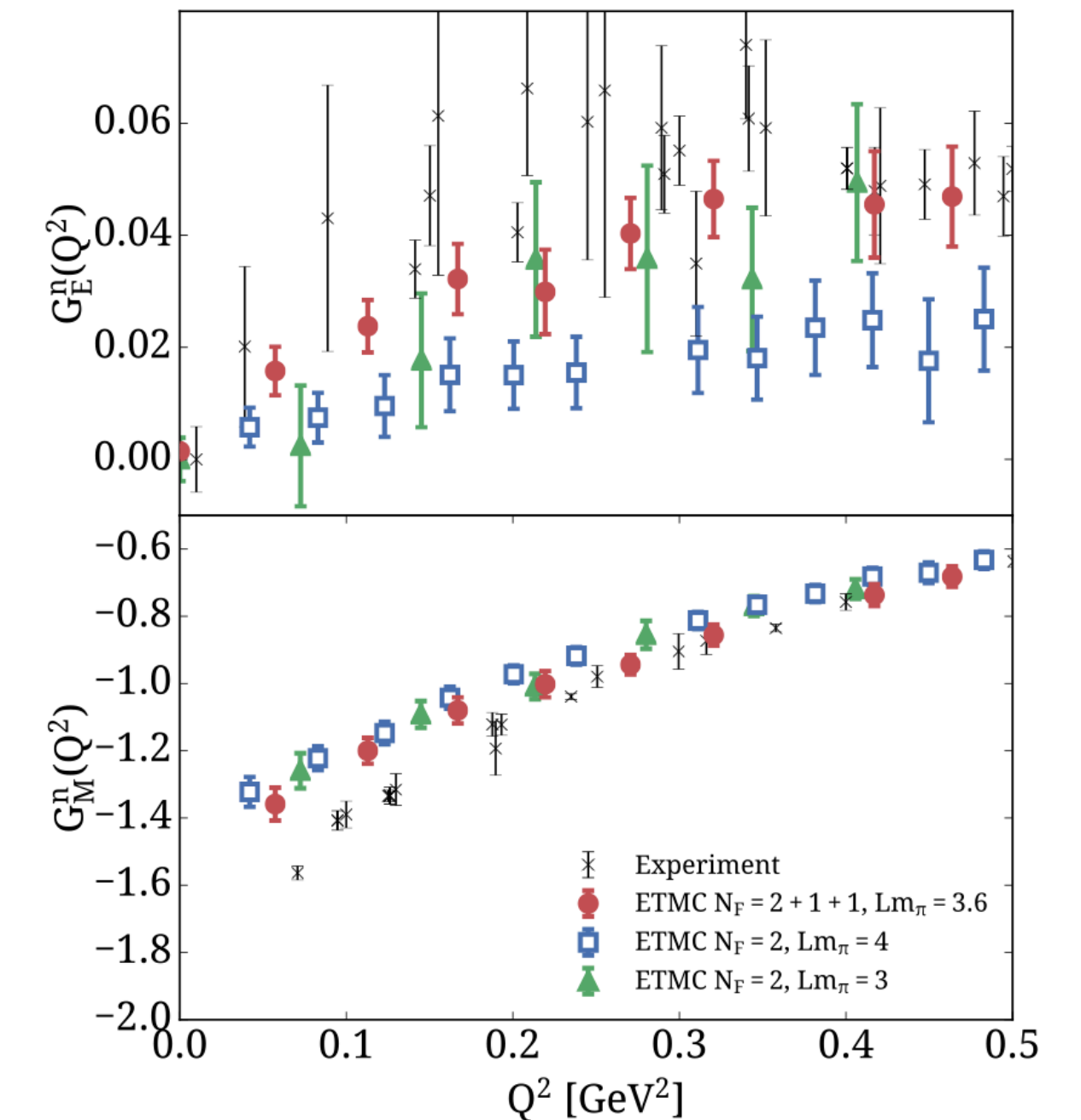
$$t = -\Delta^2 = -Q^2$$

$$\tau = \frac{Q^2}{4M^2}$$

Lorentz invariant quantities



Alexandrou, et. al., PRD 100 (2019)



Spatial distribution depending on Lorentz frame

Different spatial distributions

Classical charge distribution

Sachs, PR 126 (1962)

Jaffe, RRD 103 (2021)

$$\rho|_{\text{Sachs}}(\mathbf{r}) := \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} G_E(t)$$

Particle size
Validity: $R \gg \bar{\lambda}$
Reduced Compton wavelength

I. Charge radius

$$\sqrt{\langle r^2 \rangle_{\text{exp}}^Q} = 0.84 \text{ fm v.s. } \bar{\lambda} = \frac{\hbar}{M_p c} \sim 0.21 \text{ fm}$$

PDG, PTEP 2022 (2022)

II. Recoil correction

$$P_{\text{BF}}^0 \approx M, Q \ll 2M$$

$$P_{\text{BF}}^0 = M\sqrt{1+\tau}, Q \gtrsim 2M$$

Modified distribution in rest frame

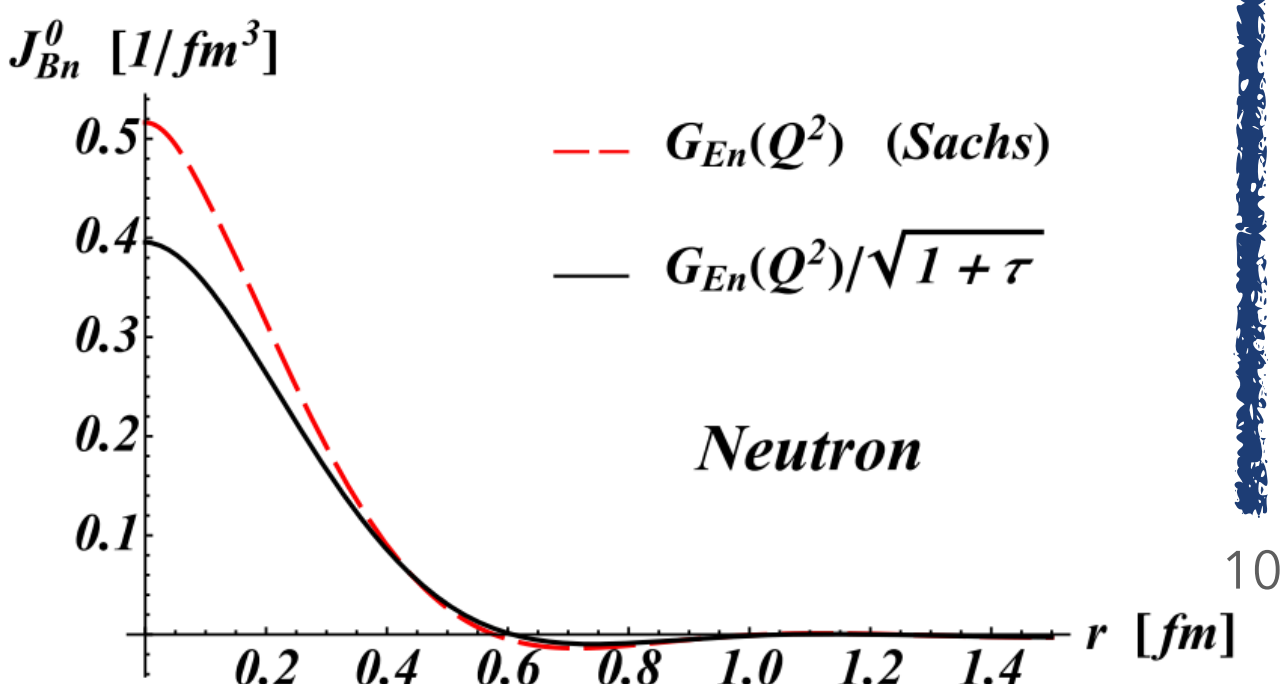
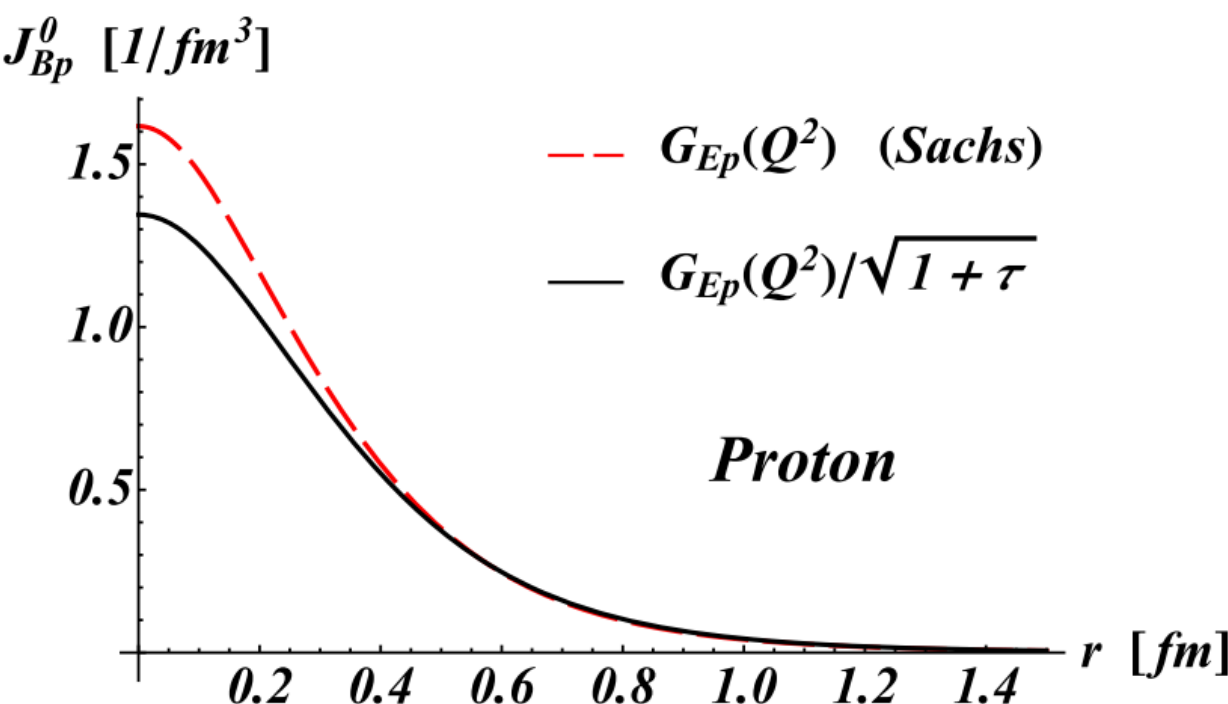
Friar, Negele, ANP 8 (1975)

Lorcé, PRL 125 (2020)

$$\rho_{\text{BF}}(\mathbf{r}) := \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} \frac{\langle p' | \hat{j}^0(0) | p \rangle |_{\text{BF}}}{2P_{\text{BF}}^0}$$

$$= \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} \frac{M}{P_{\text{BF}}^0} G_E(t)$$

Non-relativistic limit
 $\frac{M}{P_{\text{BF}}^0} \approx 1 \rightarrow \rho_{\text{BF}} \rightarrow \rho|_{\text{Sachs}}$



Distribution in infinite momentum frame

$\mathbf{b}_\perp$: impact parameter vector

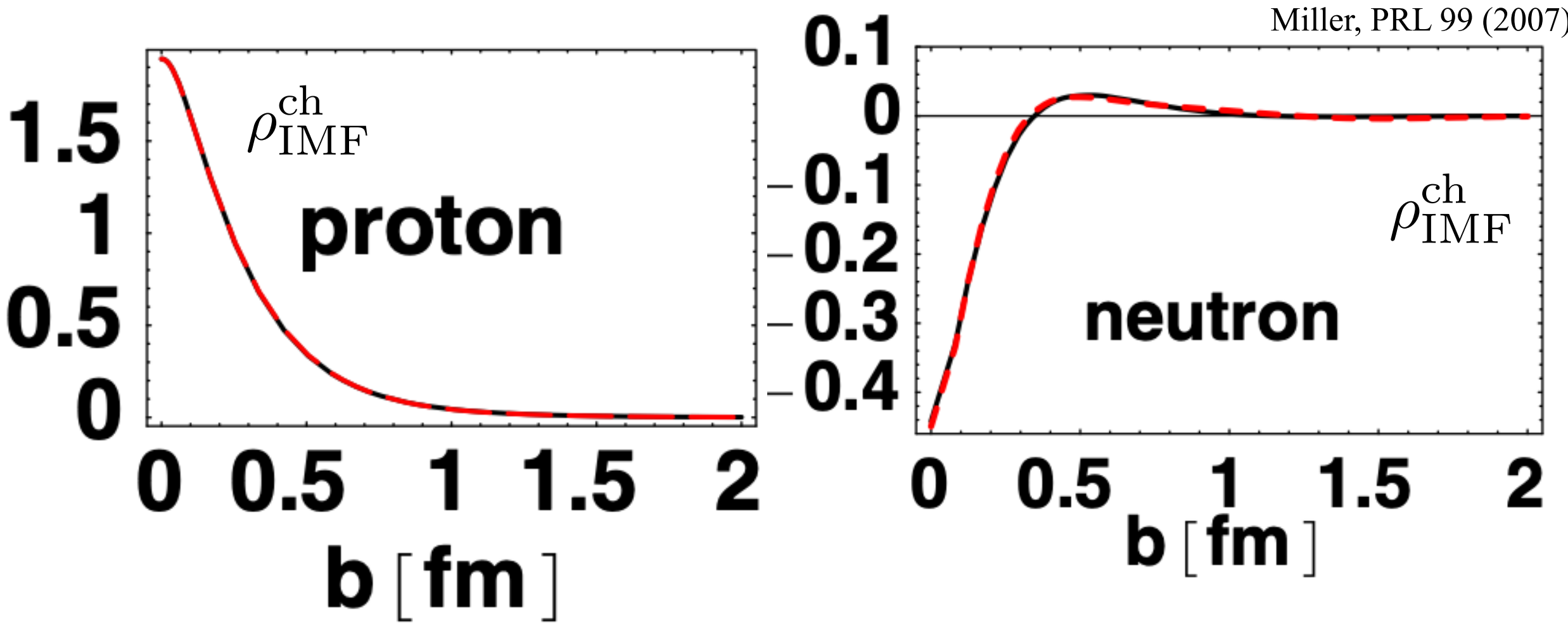
$$\rho_{\text{IMF}}^{\text{ch}}(\mathbf{b}_\perp) := \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{\langle p' | \hat{j}^0(0) | p \rangle |_{\text{IMF}}}{2P^+}$$

$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\underbrace{F_1(t)}_{\text{Dirac form factor}} + \frac{(\boldsymbol{\sigma} \times i\boldsymbol{\Delta}_\perp)_z}{2M} \underbrace{F_2(t)}_{\text{Pauli form factor}} \right]$$

Lorcé, Wang, PRD 105 (2022)

Chen, Lorcé, PRD 106 (2022)

No recoil correction! $\because$ Galilean symmetry in the transverse plane



Miller, PRL 99 (2007)

Interpolation between two different distributions!

Quantum Phase-space formalism

Hadronic matrix element

In quantum phase-space formalism

$$\langle \Psi | \hat{O}(\mathbf{r}) | \Psi \rangle = \int \frac{d^3 P}{(2\pi)^3} \int d^3 R \rho_{\Psi}(\mathbf{R}, \mathbf{P}) \langle \hat{O}(\mathbf{r}) \rangle_{\mathbf{R}, \mathbf{P}}$$

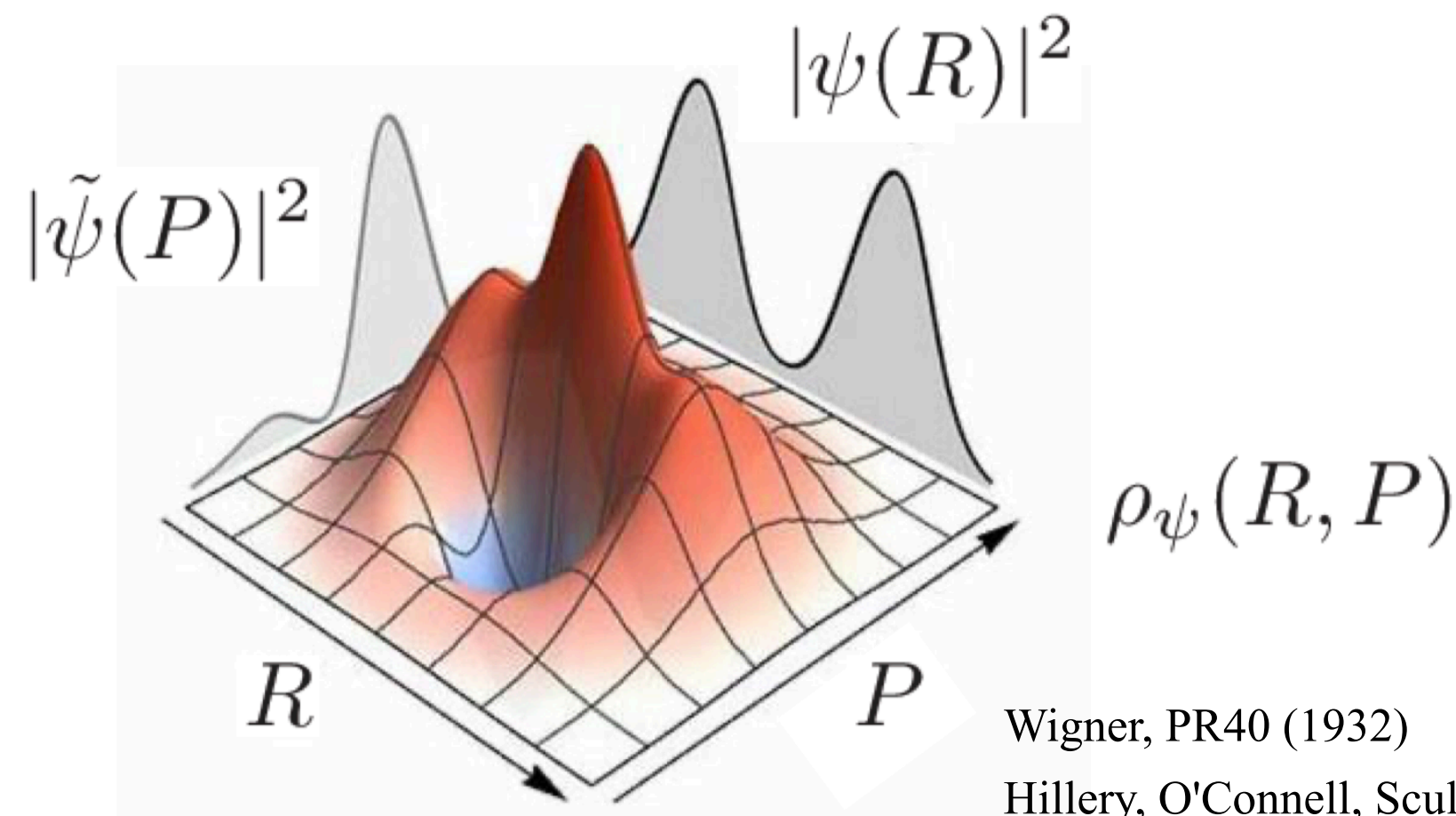
I. Nucleon Wigner distribution (hadronic wavepacket)

$$\begin{aligned} \rho_{\Psi}(\mathbf{R}, \mathbf{P}) &= \int \frac{d^3 q}{(2\pi)^3} e^{-i\mathbf{q} \cdot \mathbf{R}} \tilde{\psi}^{\dagger}\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) \tilde{\psi}\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \\ &= \int d^3 Y e^{i\mathbf{P} \cdot \mathbf{Y}} \psi^{\dagger}\left(\mathbf{R} + \frac{\mathbf{Y}}{2}\right) \psi\left(\mathbf{R} - \frac{\mathbf{Y}}{2}\right) \end{aligned}$$

Wavepacket

$$\psi(\mathbf{r}) = \langle \mathbf{r} | \Psi \rangle$$

$$\tilde{\psi}(\mathbf{p}) = \frac{1}{\sqrt{2p^0}} \langle \mathbf{p} | \Psi \rangle$$



Wigner, PR40 (1932)

Hillery, O'Connell, Scully, Wigner, PR106 (1984)

Bialynicki-Birula, Gornicki, Rafelski, PRD 44 (1991)

Quasi-probabilistic interpretation

$$\int d^3 R \rho_{\Psi}(\mathbf{R}, \mathbf{P}) = |\tilde{\psi}(\mathbf{P})|^2$$

$$\int \frac{d^3 P}{(2\pi)^3} \rho_{\Psi}(\mathbf{R}, \mathbf{P}) = |\psi(\mathbf{R})|^2$$

II. Matrix element (internal distribution)

$$\langle \hat{O}(\mathbf{r}) \rangle_{\mathbf{R}, \mathbf{P}} = \int \frac{d^3 \Delta}{(2\pi)^3} e^{i\Delta \cdot \mathbf{R}} \frac{1}{\sqrt{2p_i^0} \sqrt{2p_f^0}} \left\langle P + \frac{\Delta}{2} \left| \hat{O}(\mathbf{r}) \right| P - \frac{\Delta}{2} \right\rangle$$

Projection onto the 2D space

$$\int dr_z \langle \hat{O}(\mathbf{r}) \rangle_{\mathbf{R}, \mathbf{P}} = \int \frac{d^2 \Delta}{(2\pi)^2} e^{i\Delta_{\perp} \cdot \mathbf{R}_{\perp}} \frac{1}{\sqrt{2p_i^0} \sqrt{2p_f^0}} \left\langle P + \frac{\Delta}{2} \left| \hat{O}(\mathbf{r}_{\perp}) \right| P - \frac{\Delta}{2} \right\rangle \Big|_{\Delta_z=0}$$

Transverse Lorentz frame with $\Delta_z = 0$

Lorcé, Moutarde, Trawiński, EPJC 79 (2019)

Lorcé, EPJC 78 (2018)

Lorcé, PRL 125 (2020)

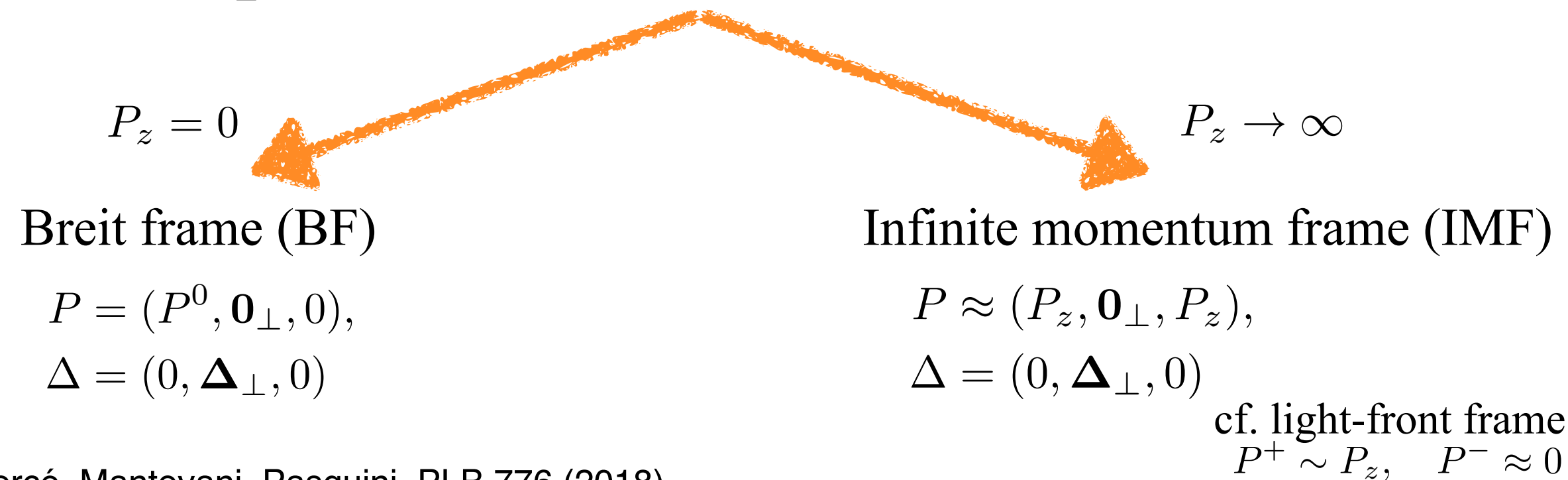
Elastic frame and relativistic spatial distribution

Elastic frame

I. A generic frame in the 2D space

Connection between the rest (Breit) frame and the moving frame.

$$P = \frac{p' + p}{2} = (P^0, \mathbf{0}_\perp, P_z) \quad \Delta = p' - p = (0, \Delta_\perp, 0)$$



Lorcé, Mantovani, Pasquini, PLB 776 (2018)

II. Relativistic spin dynamics

Spin- j particle

Non-trivial structure $\because [K^i, K^j] = -i\varepsilon^{ijk} J^k$

$$\langle p', s' | \hat{O}^{\mu_1 \dots \mu_n} | p, s \rangle = \sum_{s'_{\text{BF}}, s_{\text{BF}}} D_{s_{\text{BF}} s}^{(j)}(p_{\text{BF}}, \Lambda) D_{s'_{\text{BF}} s'}^{*(j')}(p'_{\text{BF}}, \Lambda) \times \Lambda^{\mu_1}_{\alpha_1} \dots \Lambda^{\mu_n}_{\alpha_n} \langle p'_{\text{BF}}, s'_{\text{BF}} | \hat{O}^{\mu_1 \dots \mu_n} | p_{\text{BF}}, s_{\text{BF}} \rangle$$

Wigner rotation

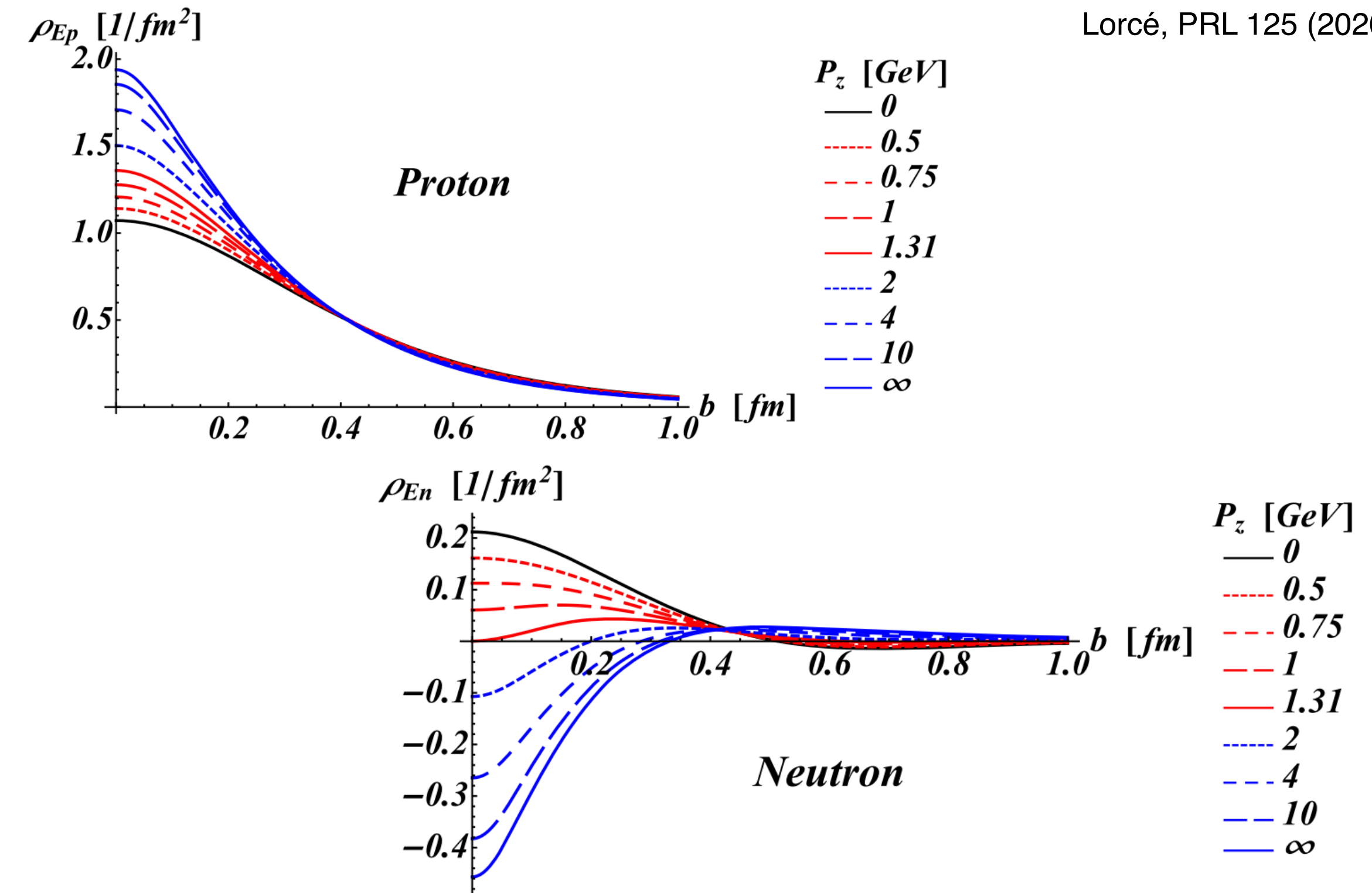
Lorentz transformation of BF amplitudes

Durand, DeCelles, Marr, PR 126 (1962)

Polyzou, Glöckle, Witala, FBS 54 (2013)

III. Relativistic charge distribution

Lorcé, PRL 125 (2020)



What about the EMT?

Numerical results - longitudinal EMT

Transversely polarized nucleon in the x -direction

The spatial distributions of the EMT

In the EF, the relativistic spatial distributions of the longitudinal components of the EMT

$$g_a(\mathbf{b}_\perp, P_z; s', s) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \left[\boxed{\delta_{s's}} \tilde{g}_a^U(Q^2, P_z) + \boxed{\frac{(\boldsymbol{\sigma}_{s's} \times i\mathbf{\Delta}_\perp)_z}{2M}} \tilde{g}_a^T(Q^2, P_z) \right], \quad \text{spin=position in multipole order}$$

Monopole; spin-independent

Dipole; spin-dependent

$$g = \rho, \mathcal{P}, \mathcal{I}, \Pi^{zz}$$

Energy, longitudinal momentum, longitudinal energy flux, axial momentum flux

For example, the amplitudes in the energy distribution

$$\tilde{\rho}_a^U(Q^2, P_z) = M \frac{\gamma}{\gamma_P} \left[\frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}} \{E_a(Q^2) + \beta^2 F_a(Q^2)\} + \frac{\sqrt{\tau} P_z}{(P^0 + M)\sqrt{1 + \tau}} 2\sqrt{\tau} \beta J_a(Q^2) \right],$$

$$\tilde{\rho}_a^T(Q^2, P_z) = M \frac{\gamma}{\gamma_P} \left[\frac{\sqrt{\tau} P_z}{(P^0 + M)\sqrt{1 + \tau}} \frac{1}{\sqrt{\tau}} \{-E_a(Q^2) - \beta^2 F_a(Q^2)\} + \frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}} \frac{1}{\sqrt{\tau}} 2\sqrt{\tau} \beta J_a(Q^2) \right],$$

Normalized by γ_P

→ Wigner rotation × Lorentz transformation

$$\hat{T}^{00} \rightarrow \gamma^2 \hat{T}^{00} + \gamma^2 \beta \hat{T}^{03} + \gamma^2 \beta \hat{T}^{30} + \gamma^2 \beta^2 \hat{T}^{33}$$

Normalized EMT distribution

$N \bullet \longrightarrow +x$

$P_z \rightarrow \infty$

Won, Lorcé, PRD 111 (2025)

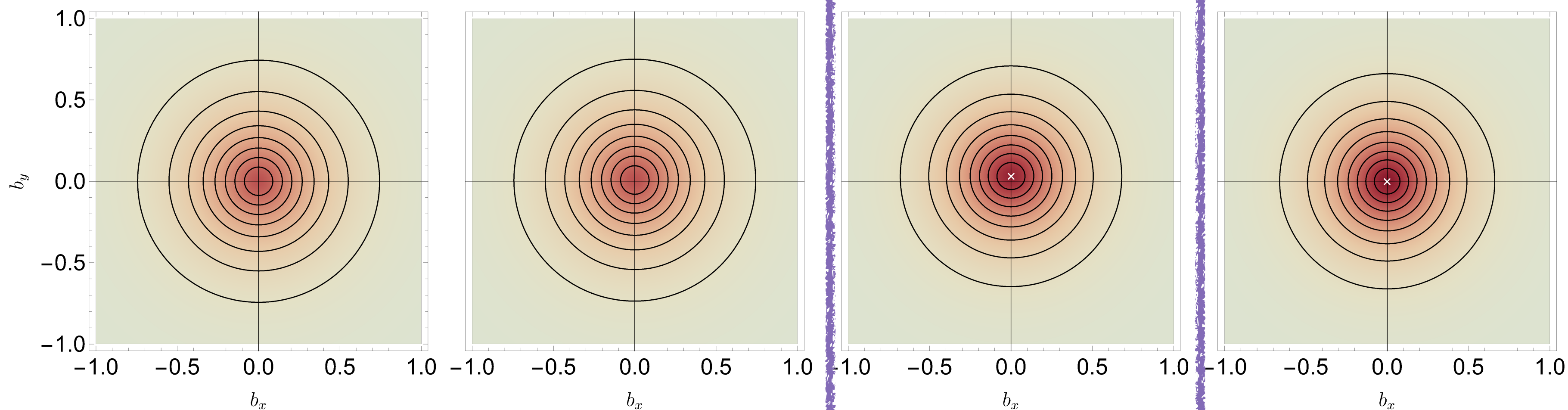
Energy, T^{00}/γ_P

$\rho^P (P_z = 0 \text{ GeV})$

$\rho^P (P_z = 0.1 \text{ GeV})$

$\rho^P (P_z = 2 \text{ GeV})$

$\rho^P (P_z = \infty \text{ GeV})$



$$\int d^2 b_{\perp} \rho = M \delta_{s's}$$

Mass

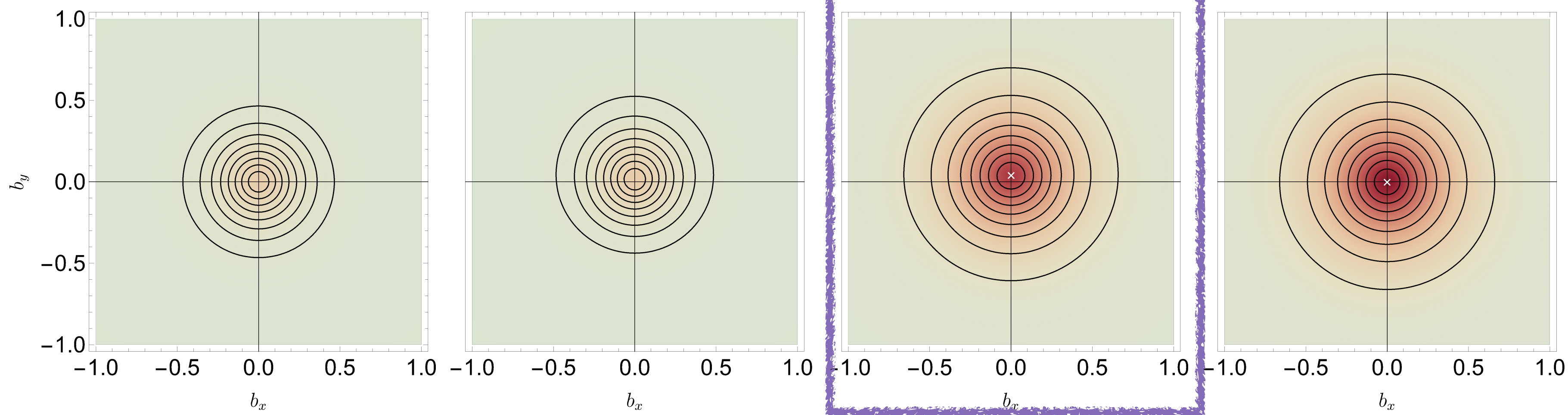
Axial momentum flux, T^{33}/γ_P

$\Pi^{zz,P} (P_z = 0 \text{ GeV})$

$\Pi^{zz,P} (P_z = 0.1 \text{ GeV})$

$\Pi^{zz,P} (P_z = 2 \text{ GeV})$

$\Pi^{zz,P} (P_z = \infty \text{ GeV})$



$$\int d^2 b_{\perp} \Pi^{zz} = M \beta_P^2 \delta_{s's}$$

Induced pressure

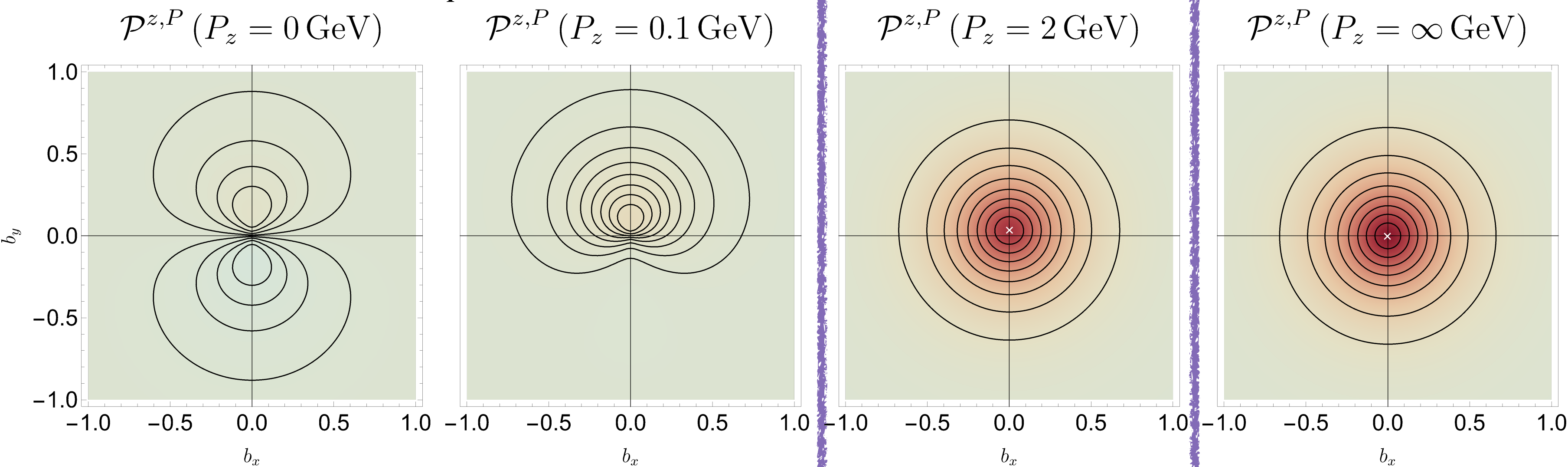
Normalized EMT distribution

$N \bullet \longrightarrow +x$

$P_z \rightarrow \infty$

Won, Lorcé, PRD 111 (2025)

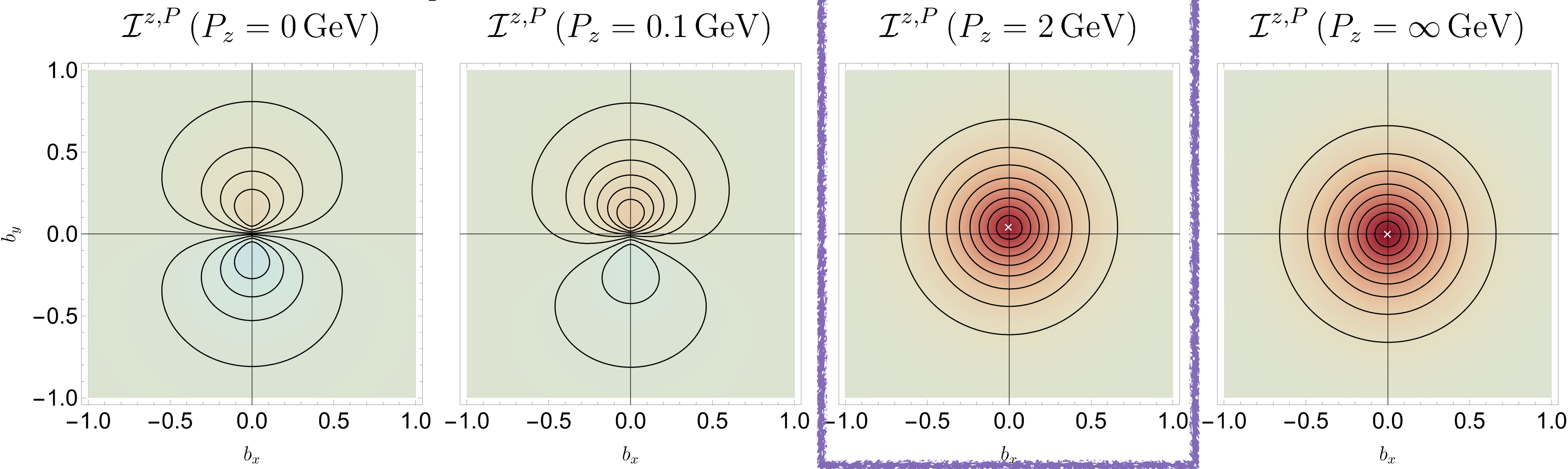
Longitudinal momentum, T^{03}/γ_P



$$\int d^2 b_{\perp} \mathcal{P}^z = M \beta_P \delta_{s's}$$

Momentum

Longitudinal energy flux, T^{30}/γ_P



$$\int d^2 b_{\perp} \mathcal{I}^z = M \beta_P \delta_{s's}$$

Energy flux

In infinite momentum frame

Won, Lorcé, PRD 111 (2025)

In the EMT distributions,

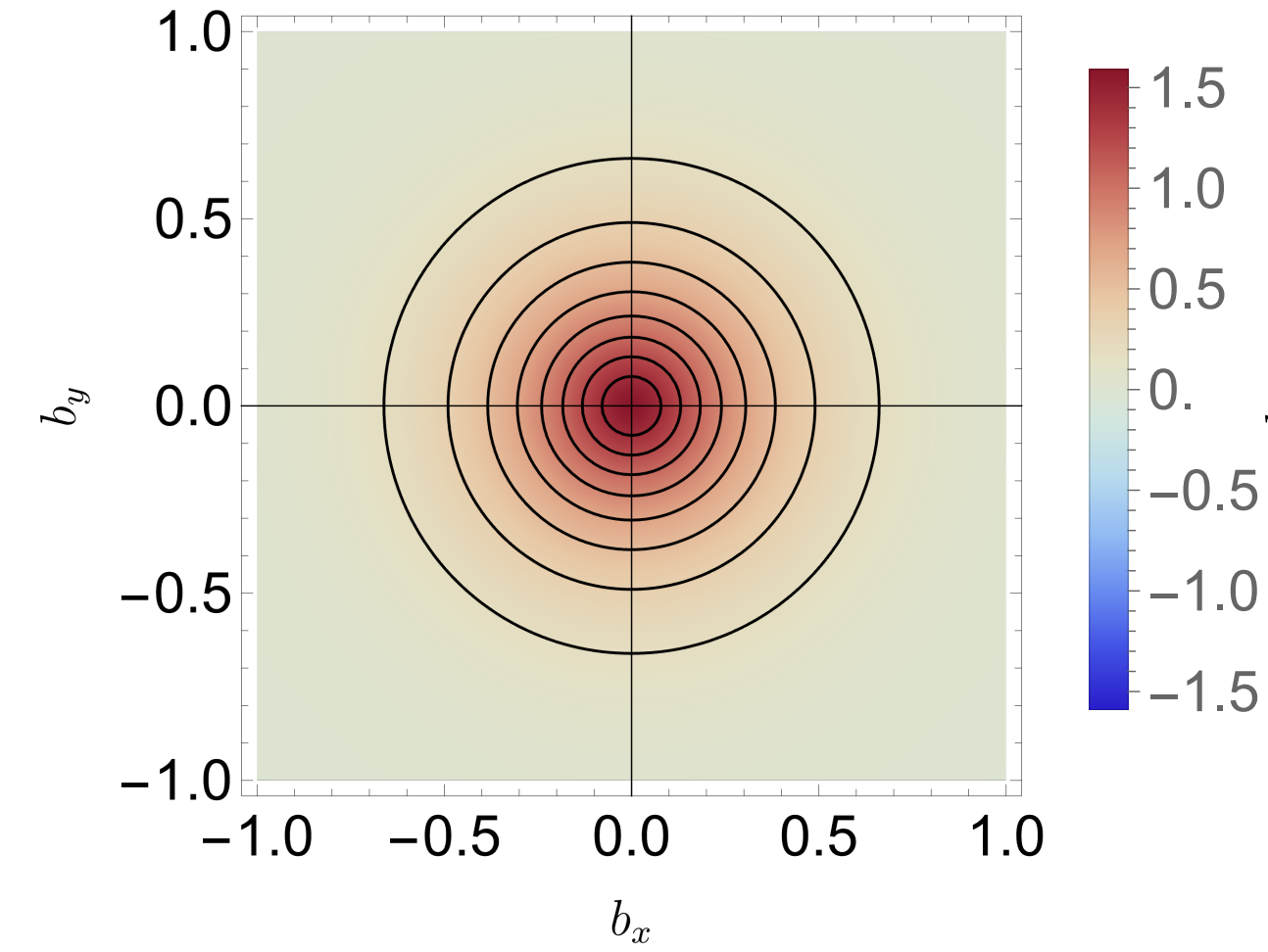
$$\begin{aligned} \rho(\mathbf{b}_\perp, \infty) &= \mathcal{P}^z(\mathbf{b}_\perp, \infty) = \mathcal{I}^z(\mathbf{b}_\perp, \infty) = \Pi^{zz}(\mathbf{b}_\perp, \infty) \\ &= \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \left[A(t) + \frac{(\boldsymbol{\sigma} \times i\mathbf{\Delta}_\perp)_z}{2M} B(t) \right] \end{aligned}$$

$B(t) = 0$ in our multi-pole ansatz since we don't know $B_G(t) = 0$ tendency.

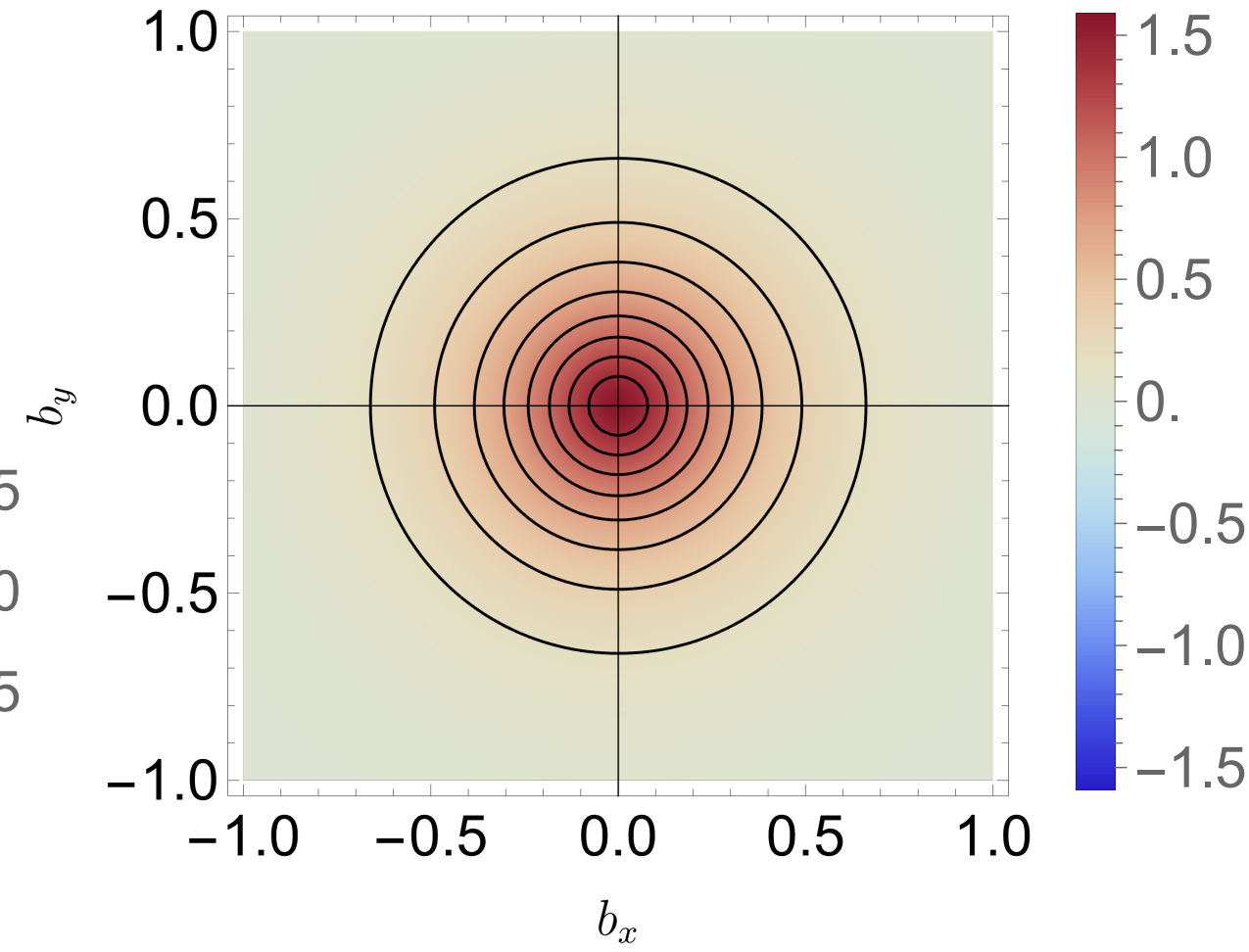
In the electric charge distribution,

$$\rho_{\text{IMF}}^{\text{ch}}(\mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \left[F_1(t) + \frac{(\boldsymbol{\sigma} \times i\mathbf{\Delta}_\perp)_z}{2M} F_2(t) \right]$$

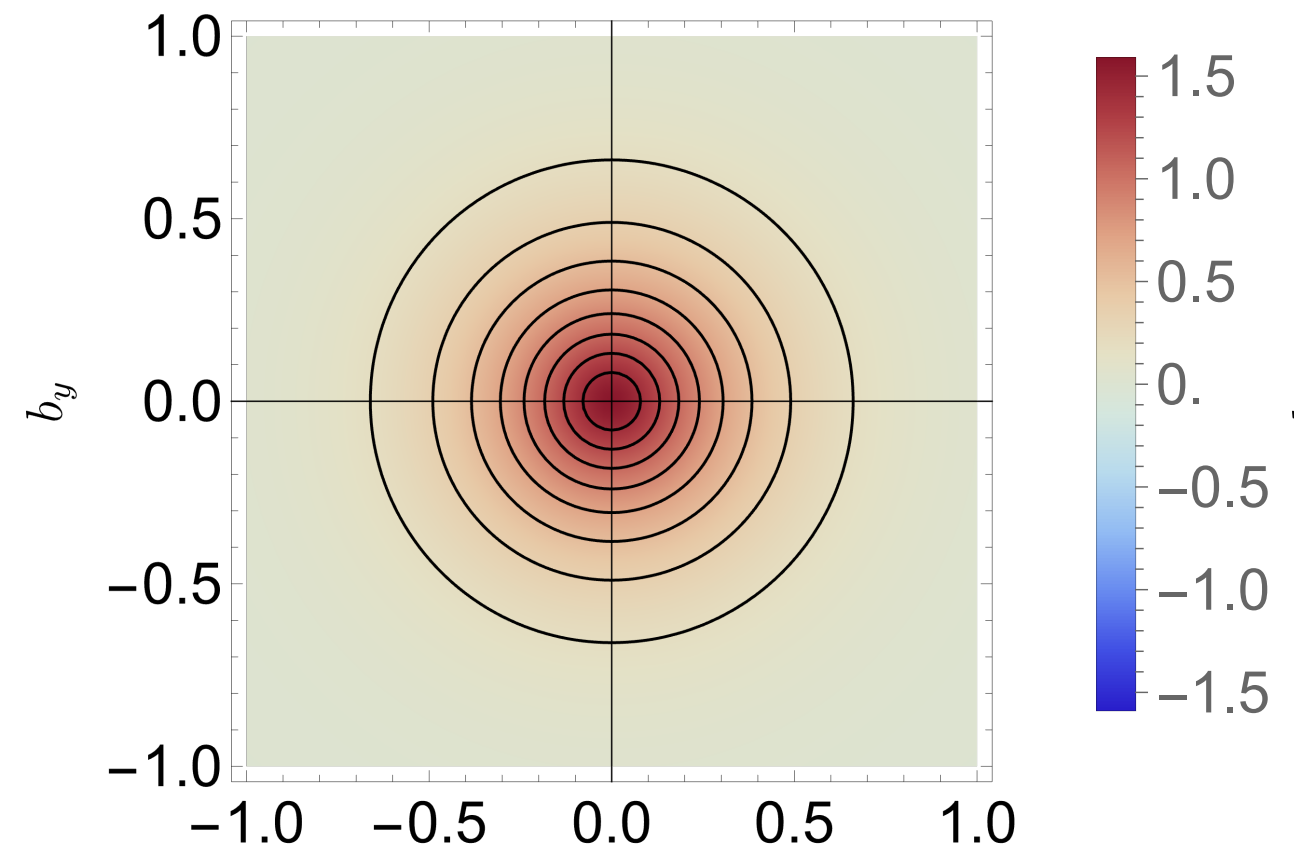
$\rho^P(P_z = \infty \text{ GeV})$



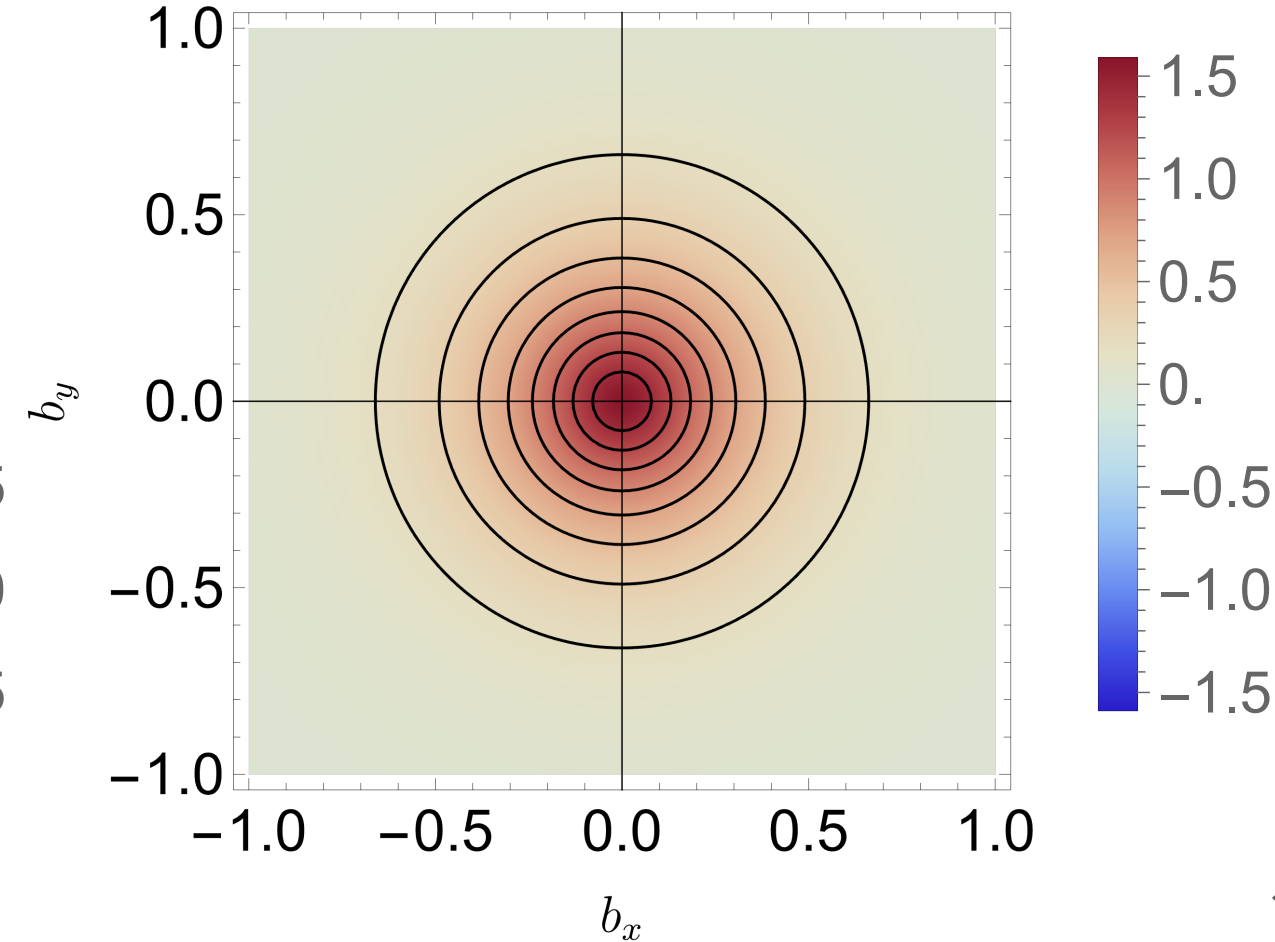
$\Pi^{zz,P}(P_z = \infty \text{ GeV})$



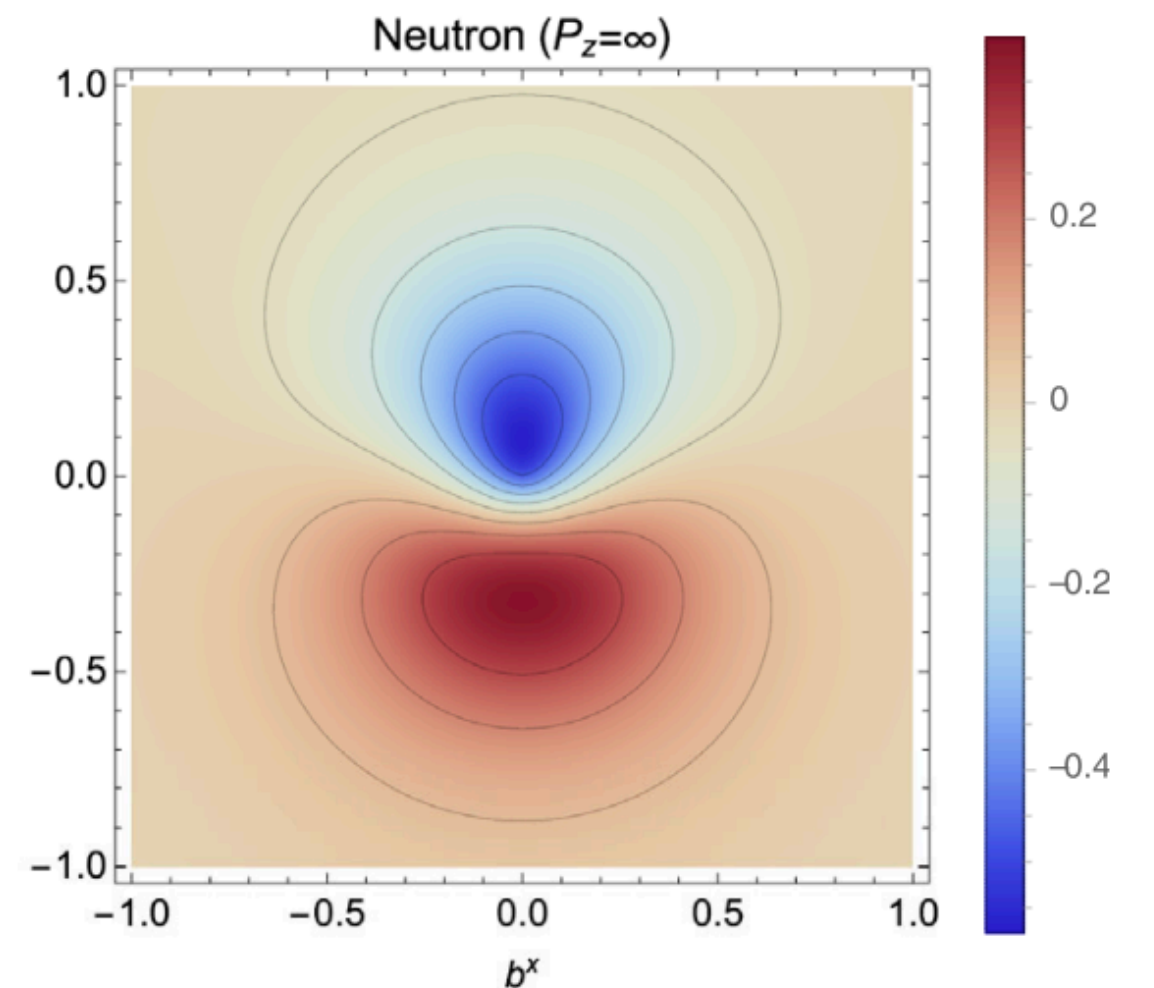
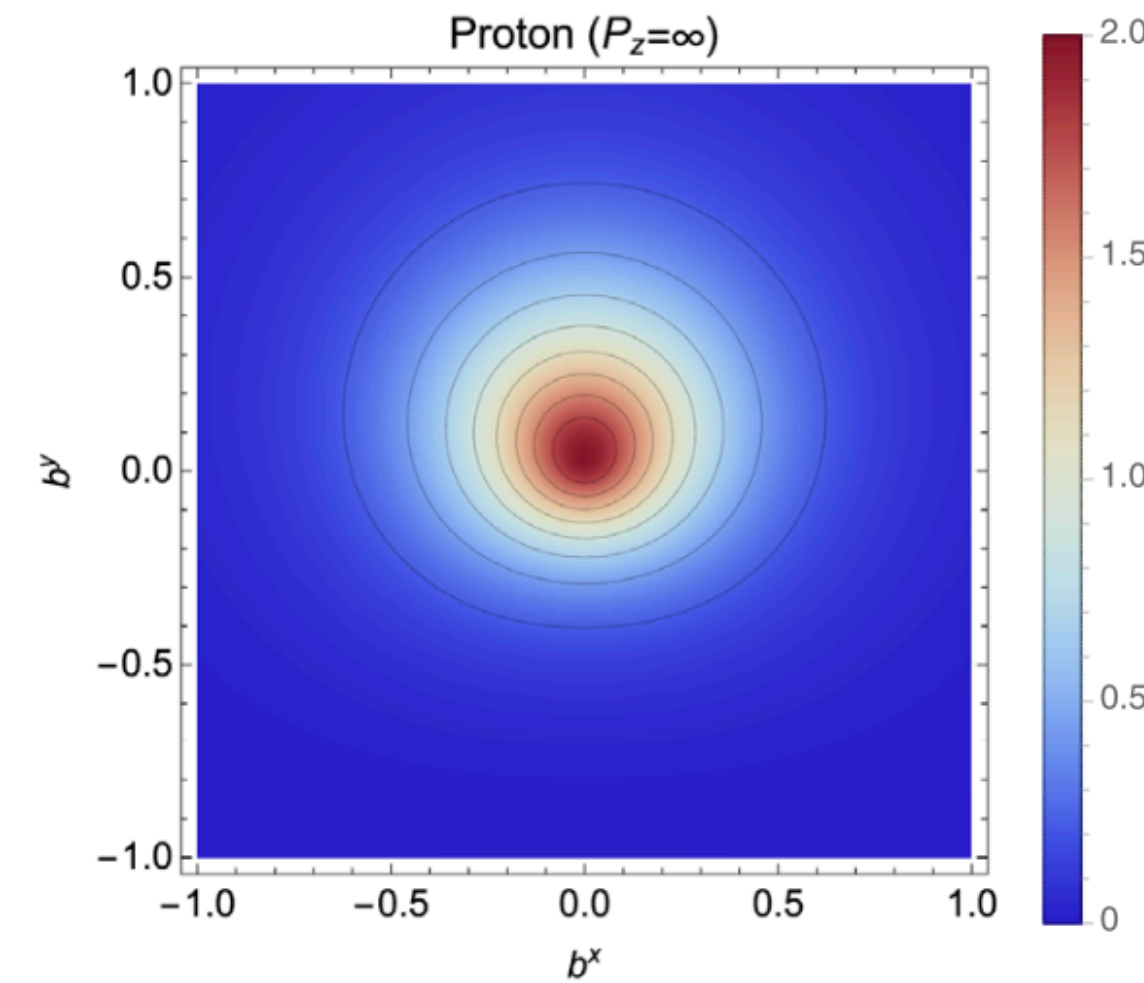
$\mathcal{P}^{z,P}(P_z = \infty \text{ GeV})$



$\mathcal{I}^{z,P}(P_z = \infty \text{ GeV})$



$N \bullet \longrightarrow +x$



Conclusion

Conclusion

- We review the **energy-momentum tensor** and related physical observables.
- Quantum phase-space formalism developed the relativistic spatial distribution in the **elastic frame**, which is a generic frame to interpolate both **rest** and **infinite momentum frames**, as well as to include the relativistic spin effect emerged as **Wigner rotation**.
- We show the relativistic spatial distributions of the **EMT** for transversely polarized nucleons in the elastic frame.
- This work can be expanded to study the **parity-odd EMT**, including spin-orbit correlation and unknown quantities.

Parity-odd EMT operator in QCD

$$\hat{T}_5^{\mu\nu} = \bar{\psi}\gamma^\mu\gamma_5\frac{i}{2}\overleftrightarrow{D}^\nu\psi - 2i\text{Tr}\tilde{F}^{\mu\lambda}F^\nu{}_\lambda + \frac{i}{2}g^{\mu\nu}\text{Tr}\tilde{F}^{\lambda\rho}F_{\lambda\rho}$$

Thank you for listening

Normalized EMT distribution

Relativistic EMT quantities

$$\int d^2 b_{\perp} T_a^{\mu\nu}(\mathbf{b}_{\perp}, P_z; s', s)$$

Lorentz boost factors

$$\gamma_P = \frac{E_P}{M}, \quad \beta_P = \frac{P_z}{E_P}, \quad E_P = \sqrt{P_z^2 + M^2},$$

$$= \begin{pmatrix} M\gamma_P & 0 & 0 & M\gamma_P\beta_P \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ M\gamma_P\beta_P & 0 & 0 & M\gamma_P\beta_P^2 \end{pmatrix} \delta_{s's} = M \frac{u^{\mu} u^{\nu}}{u^0} \delta_{s's},$$

Four-velocity of the system
 $u = \gamma_P (1, \mathbf{0}_{\perp}, \beta_P)$

Longitudinal EMT componenets

$$\hat{T}^{00} \sim P_z^2 \longrightarrow \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} \frac{\langle p', s' | \hat{T}^{00}(0) | p, s \rangle}{2P^0} \sim P_z^1$$

$P^0 \propto P_z^1$

 It goes infinity.

c.f. electric charge

$$\hat{j}^0 \sim P_z^1 \longrightarrow \rho^{\text{ch}}(\mathbf{b}_{\perp}) = \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} \frac{\langle p', s' | \hat{j}^0(0) | p, s \rangle}{2P^0} \sim P_z^0$$

Normalized EMT distributions

Using the Lorentz transformation on a generic second-rank tensor,

$$t_a^{\mu\nu} := \gamma_P T_a^{\mu\nu} = \begin{pmatrix} \gamma_P^2 \rho_a & \gamma_P \mathcal{P}_a^i & \gamma_P^2 \mathcal{P}_a^z \\ \gamma_P \mathcal{I}_a^i & \Pi_a^{ij} & \gamma_P \Pi_a^{iz} \\ \gamma_P^2 \mathcal{I}_a^z & \gamma_P \Pi_a^{zi} & \gamma_P^2 \Pi_a^{zz} \end{pmatrix}, \quad i, j = 1 \sim 2, a = q, G$$

Purely transverse: normal and shear forces

$$\Pi^{ij} = \delta_{\perp}^{ij} \sigma_a + \left(\frac{b_{\perp}^i b_{\perp}^j}{|\mathbf{b}_{\perp}|^2} - \frac{1}{2} \delta_{\perp}^{ij} \right) \Pi_a$$

Radial and tangential forces

$$\sigma_{r/t,a} = \sigma_a \pm \frac{1}{2} \Pi_a$$

With the normalizations $h = \mathcal{P}^z, \mathcal{I}^z$

$$\int d^2 b_{\perp} \sum_{a=q,G} \rho_a(\mathbf{b}_{\perp}, P_z; s', s) = M \delta_{s's}, \quad \text{Mass}$$

$$\int d^2 b_{\perp} \sum_{a=q,G} h_a(\mathbf{b}_{\perp}, P_z; s', s) = M \beta_P \delta_{s's}, \quad \text{Longitudinal momentum (energy flux)}$$

$$\int d^2 b_{\perp} \sum_{a=q,G} \Pi_a^{zz}(\mathbf{b}_{\perp}, P_z; s', s) = M \beta_P^2 \delta_{s's}. \quad \text{Axial longitudinal momentum}$$

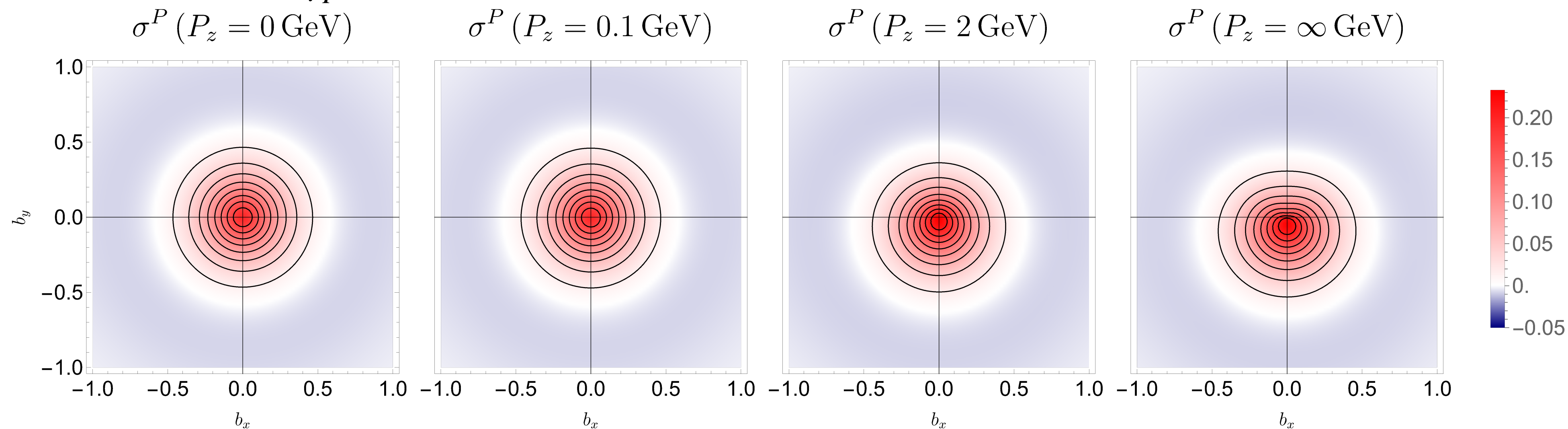
Others vanish

Normalized EMT distribution

$N \bullet \longrightarrow +x$

$P_z \rightarrow \infty$

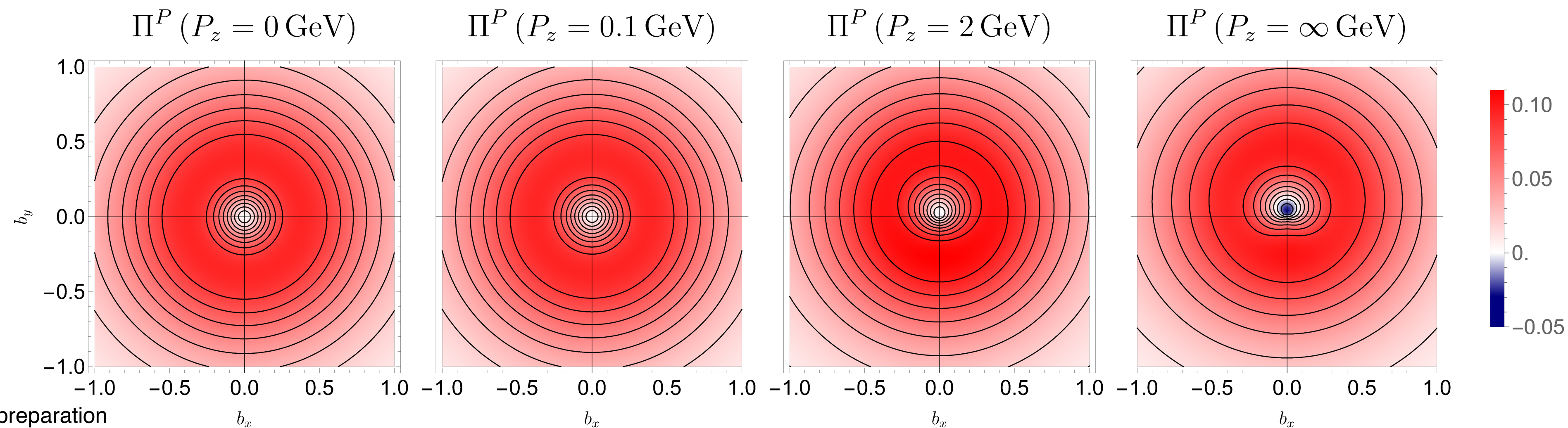
Transverse normal force, $\gamma_P T^{ii}/2$



$$\int d^2 b_{\perp} \sigma = 0$$

Global stability condition

Transverse shear force, $2(b_{\perp}^i b_{\perp}^j / b^2 - \delta_{\perp}^{ij} / 2) \gamma_P T^{ij}$



$$\int d^2 b_{\perp} b^2 \Pi = -\frac{2}{M} D(0)$$

Normalized EMT distribution

$N \bullet \longrightarrow +x$

$P_z \rightarrow \infty$

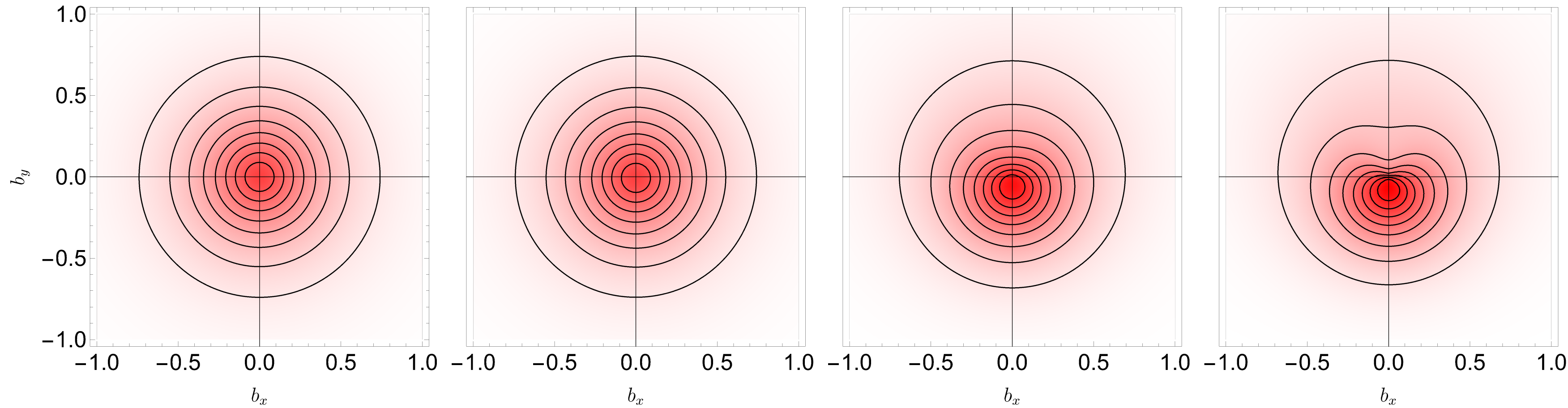
Transverse radial force, $\gamma_P T^{ij} \hat{n}_r^i = \sigma_r \hat{n}_r^j$

$\sigma_r^P (P_z = 0 \text{ GeV})$

$\sigma_r^P (P_z = 0.1 \text{ GeV})$

$\sigma_r^P (P_z = 2 \text{ GeV})$

$\sigma_r^P (P_z = \infty \text{ GeV})$



$$\sigma_r > 0$$

Local stability condition

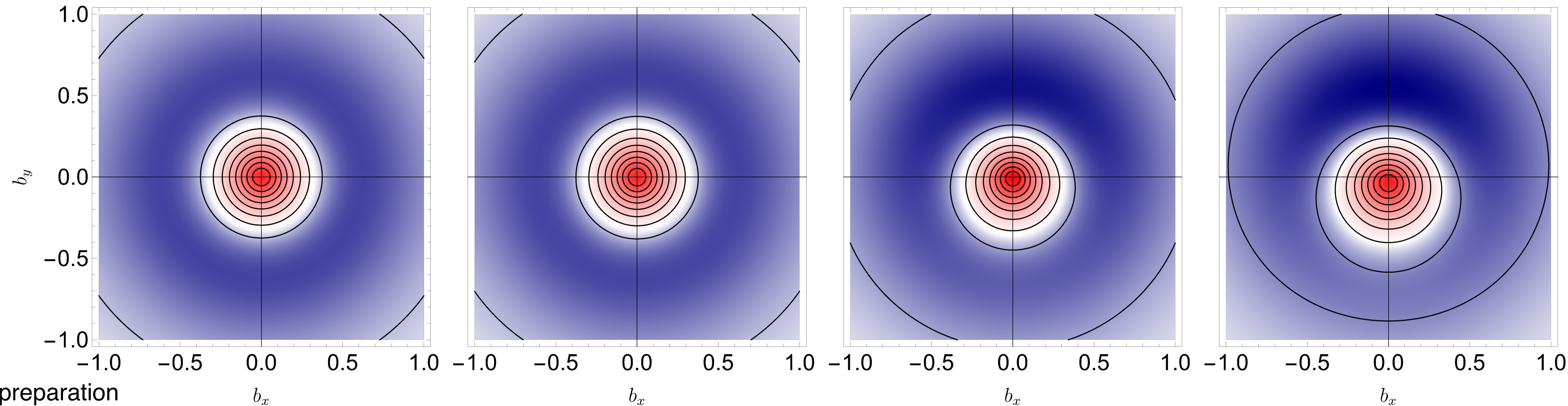
Transverse tangential force, $\gamma_P T^{ij} \hat{n}_\phi^i = \sigma_t \hat{n}_\phi^j$

$\sigma_t^P (P_z = 0 \text{ GeV})$

$\sigma_t^P (P_z = 0.1 \text{ GeV})$

$\sigma_t^P (P_z = 2 \text{ GeV})$

$\sigma_t^P (P_z = \infty \text{ GeV})$



Multipole structure in the transverse shear force distribution

In general, the transverse shear force distribution is given in the EF

$$\begin{aligned} \Pi_a(\mathbf{b}_\perp, P_z; s', s) = & \underbrace{\delta_{s's}}_{\text{Spin monopole}} \int \frac{dQ}{2\pi} Q \tau \underbrace{J_2(Qb)}_{\text{Position quadrupole}} \left(-\frac{\gamma_P}{\gamma} \cos \theta M D_a(Q^2) \right) \\ & + \underbrace{\epsilon^{ij3} \sigma_{s's}^i \frac{b_\perp^j}{b}}_{\text{Spin dipole} \quad \text{Dipolar pattern}} \int \frac{dQ}{2\pi} Q \tau^{3/2} \underbrace{[J_1(Qb) - J_3(Qb)]}_{\text{Mixing of position dipole and octupole}} \left(-\frac{\gamma_P}{\gamma} \frac{1}{2\sqrt{\tau}} \sin \theta M D_a(Q^2) \right), \end{aligned}$$

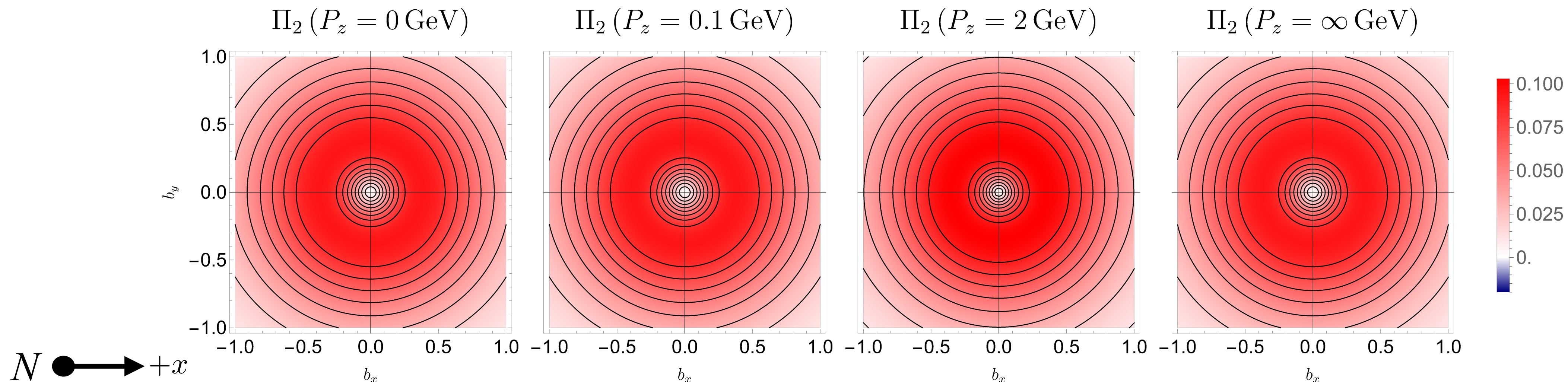
Wigner rotation angle

$$\begin{aligned} \cos \theta &= \frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}}, \\ \sin \theta &= -\frac{\sqrt{\tau} P_z}{(P^0 + M)\sqrt{1 + \tau}}, \end{aligned}$$

Quadrupole contribution

Monopolar pattern

+ oscillation deriving from J_2 in the radial direction



Multipole structure in the transverse shear force distribution

In general, the transverse shear force distribution is given in the EF

$$\begin{aligned} \Pi_a(\mathbf{b}_\perp, P_z; s', s) = & \text{Spin monopole } \boxed{\delta_{s's}} \int \frac{dQ}{2\pi} Q \tau \text{Position quadrupole } \boxed{J_2(Qb)} \left(-\frac{\gamma_P}{\gamma} \cos \theta M D_a(Q^2) \right) \\ & + \epsilon^{ij3} \text{Spin dipole } \boxed{\sigma_{s's}^i} \text{Dipolar pattern } \boxed{\frac{b_\perp^j}{b}} \int \frac{dQ}{2\pi} Q \tau^{3/2} \text{Mixing of position dipole and octupole } \boxed{[J_1(Qb) - J_3(Qb)]} \left(-\frac{\gamma_P}{\gamma} \frac{1}{2\sqrt{\tau}} \sin \theta M D_a(Q^2) \right), \end{aligned}$$

Wigner rotation angle

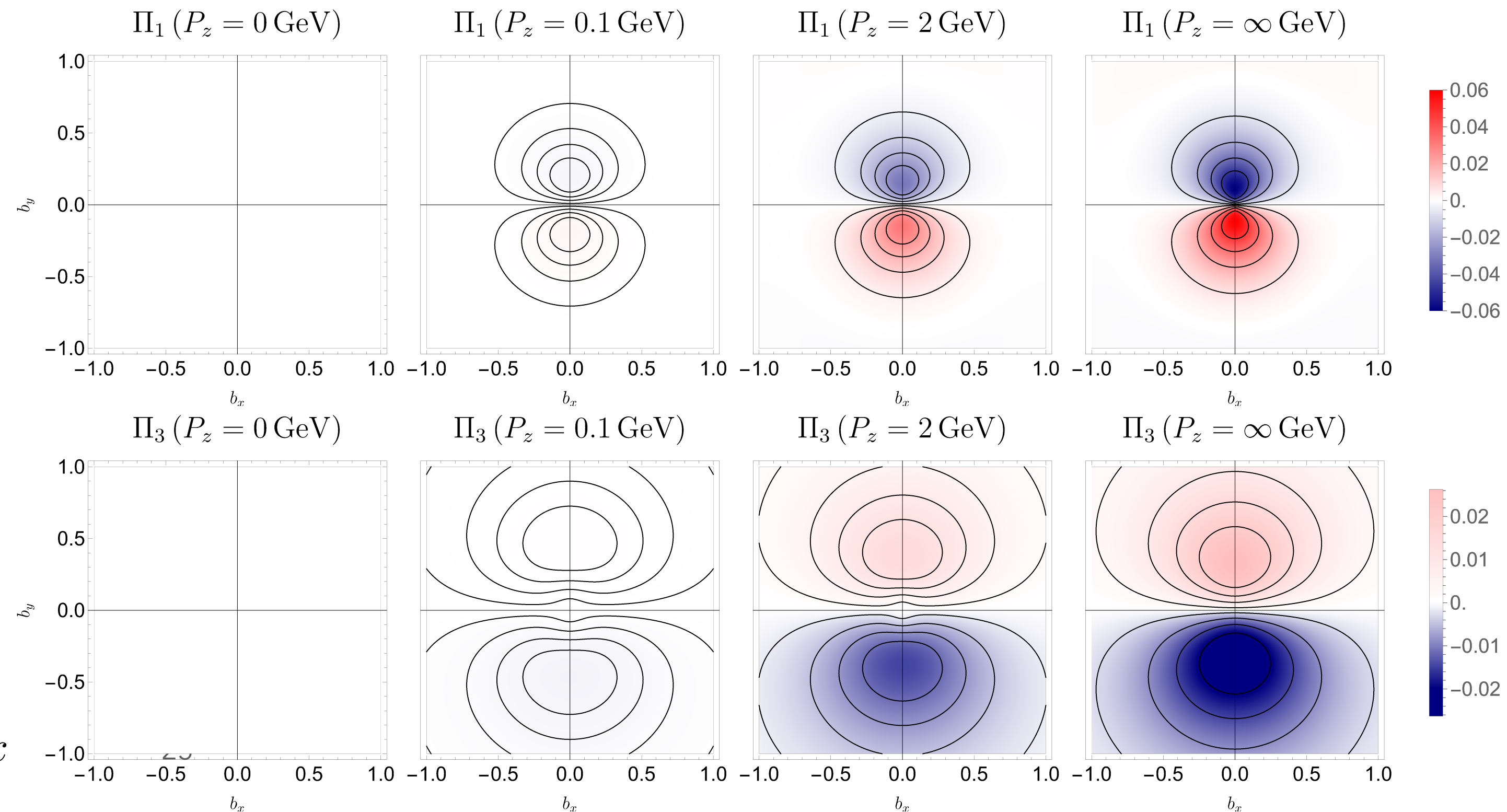
$$\begin{aligned} \cos \theta &= \frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}}, \\ \sin \theta &= -\frac{\sqrt{\tau} P_z}{(P^0 + M)\sqrt{1 + \tau}}, \end{aligned}$$

Dipole and octupole contributions

Dipolar pattern

+ oscillations deriving from $J_{1/3}$ in the radial direction

→ The oscillations are suppressed by the dipolar operator

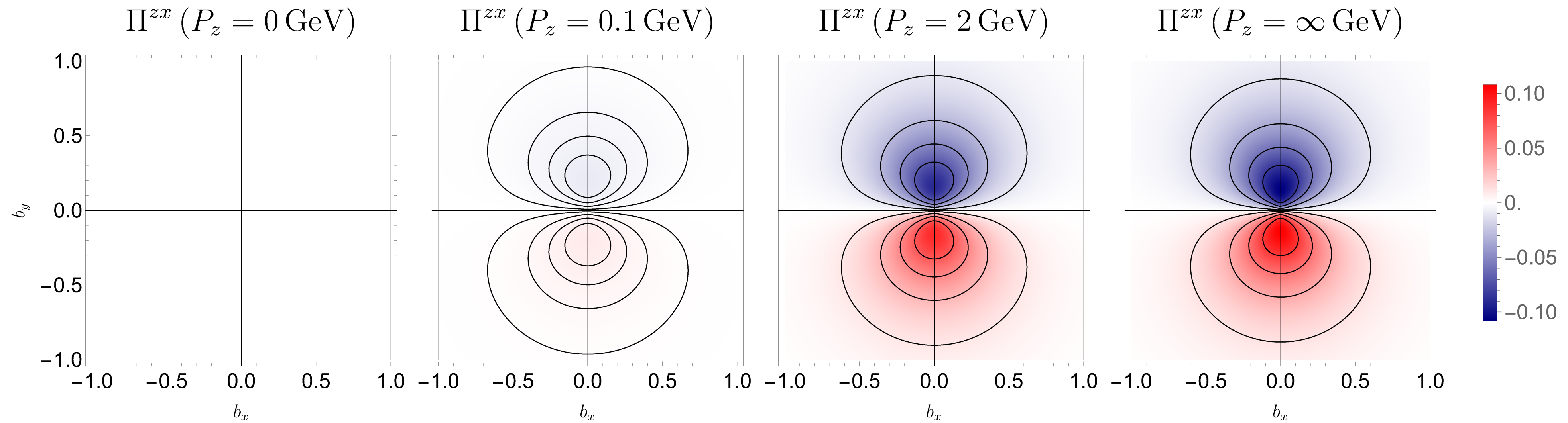


Normalized EMT distribution

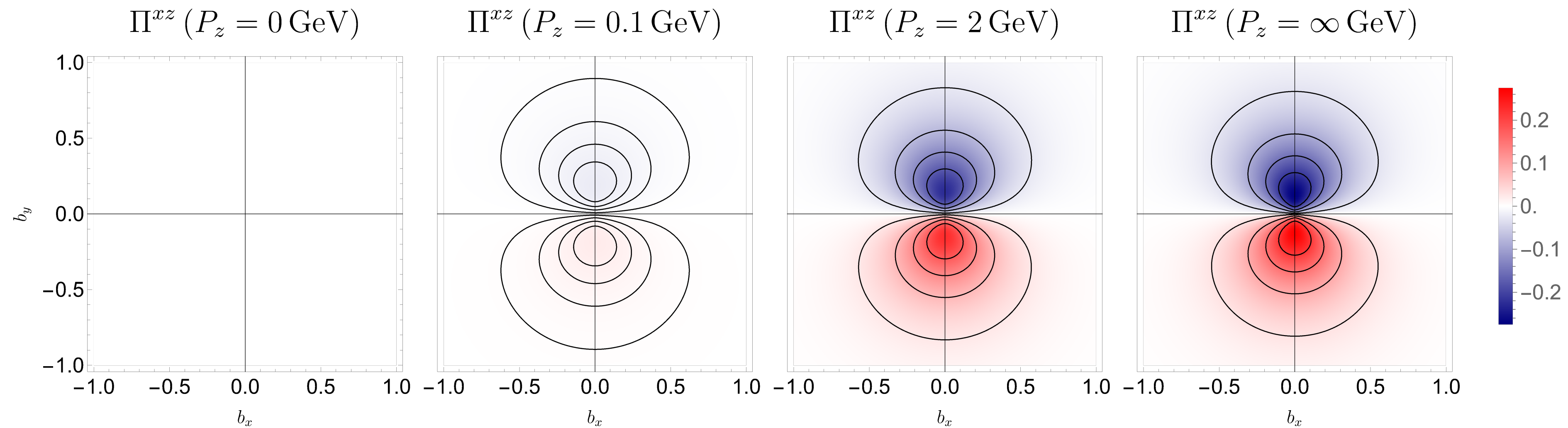
$N \bullet \longrightarrow +x$

$P_z \rightarrow \infty$

Longitudinal flux of transverse momentum, T^{3i}

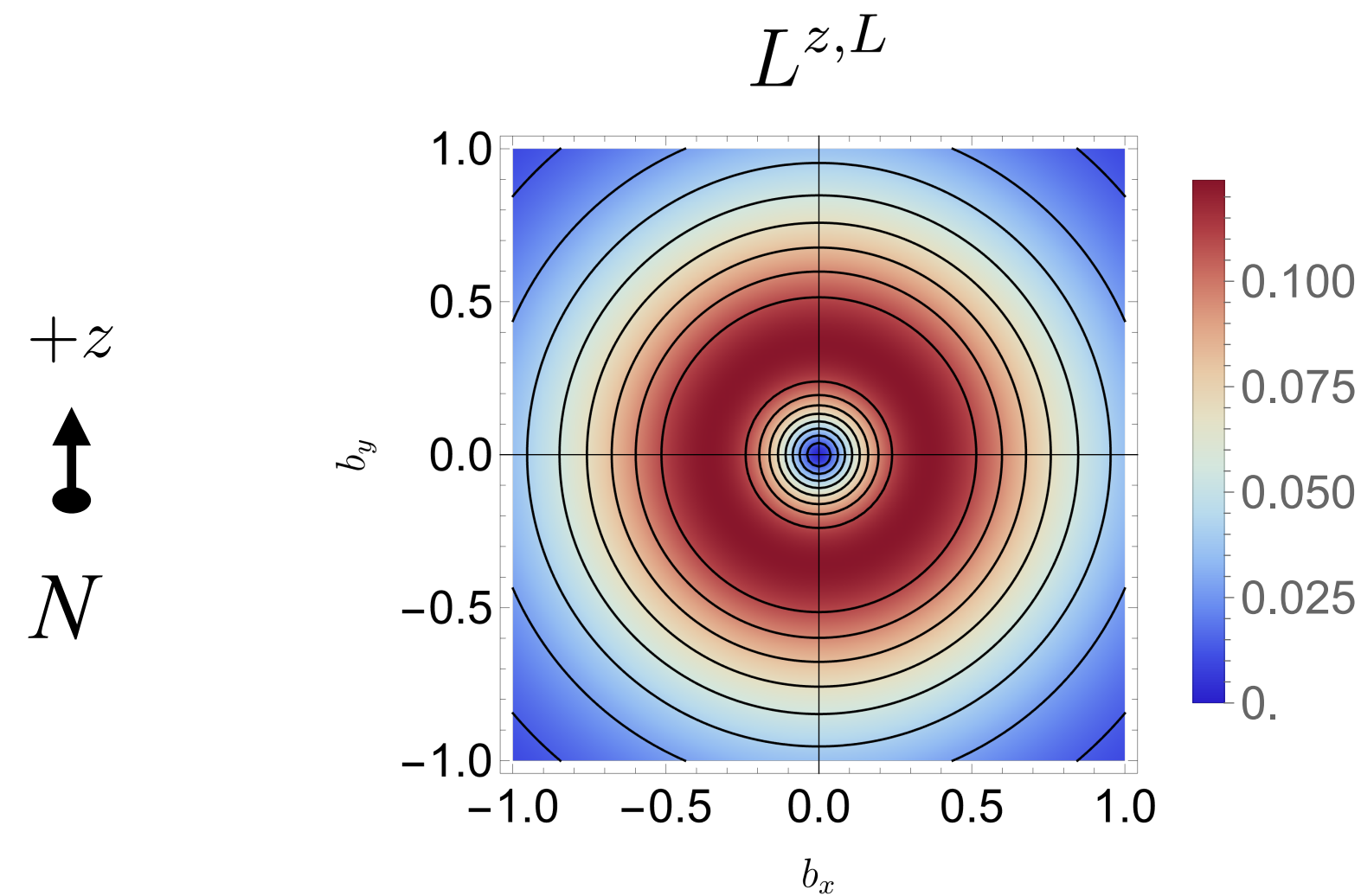


Transverse flux of longitudinal momentum, T^{i3}



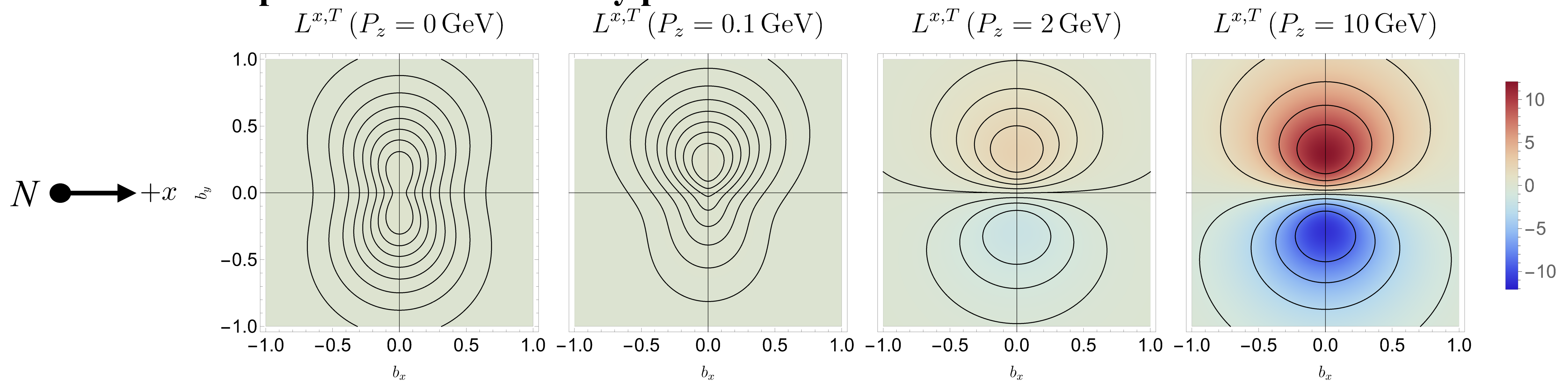
Orbital angular momentum

Longitudinal component in a longitudinal polarized nucleon



$$L^i(b_\perp; s', s) = \epsilon^{ijk} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta \cdot b_\perp} \left[-i \frac{\partial}{\partial \Delta^j} \frac{\langle p', s' | \hat{T}^{0k}(0) | p, s \rangle}{2P^0} \right]_{\Delta_z=0}$$

Transverse component in a transversely polarized nucleon



Back up

Relativistic matrix elements

Spin- j particle

$$\mathcal{M}_{s's}[\hat{O}^{\mu_1 \cdots \mu_n}] = \sum_{s'_{\text{BF}}, s_{\text{BF}}} \boxed{D_{s_{\text{BF}}s}^{(j)}(p_{\text{BF}}, \Lambda) D_{s'_{\text{BF}}s'}^{*(j')}(p'_{\text{BF}}, \Lambda)} \times \Lambda^{\mu_1}_{\alpha_1} \cdots \Lambda^{\mu_n}_{\alpha_n} \mathcal{M}_{s'_{\text{BF}}s_{\text{BF}}}[\hat{O}^{\alpha_1 \cdots \alpha_n}],$$

Non-trivial structure

Durand, DeCelles, Marr, PR 126 (1962)

Polyzou, Glöckle, Witala, FBS 54 (2013)

Irreciprocal process

$$\begin{array}{ccc} & |p, s\rangle & \xrightarrow{U(\Lambda)} |p', s'\rangle \\ U(p) \nearrow & & \nwarrow U^{-1}(\Lambda p) \\ |0, s\rangle & \neq \sum_{s'} \underbrace{D_{ss'}(p, \Lambda)}_{\text{Wigner rotation}} |0, s'\rangle \end{array} \quad p'^{\mu} = \Lambda^{\mu}_{\nu} p^{\nu}$$

$$\boxed{[K^i, K^j] = -i\epsilon^{ijk} J^k}$$

Melosh rotation

$$|p, \lambda\rangle = \sum_s \mathcal{M}_{s\lambda} |p, s\rangle$$

$$\lim_{p_z \rightarrow \infty} D(p, \Lambda) = \mathcal{M}$$

Physical spin states in the relativistic sense

$$|p, s\rangle = \sum_{s_{\text{BF}}} U(\Lambda) |p_{\text{BF}}, s_{\text{BF}}\rangle D_{s_{\text{BF}}s}(p_{\text{BF}}, \Lambda)$$

$$\rightarrow D_{s_{\text{BF}}s}(p_{\text{BF}}, \Lambda) = \langle 0, s | U^{-1}(\Lambda p_{\text{BF}}) U(\Lambda) U(p_{\text{BF}}) | 0, s_{\text{BF}} \rangle$$

Back up

Parity-odd EMT operator

$$\hat{T}_5^{\mu\nu} = \bar{\psi} \gamma^\mu \gamma_5 \frac{i}{2} \overleftrightarrow{D}^\nu \psi - 2i \text{Tr} \tilde{F}^{\mu\lambda} F^\nu{}_\lambda + \frac{i}{2} g^{\mu\nu} \text{Tr} \tilde{F}^{\lambda\rho} F_{\lambda\rho}.$$

Dual field tensor

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

Chiral decomposition

$$\psi_{R/L} = \frac{1}{2} (1 \pm \gamma_5) \psi$$

$$F_\pm^{\mu\nu} = \frac{1}{2} \left(F^{\mu\nu} \pm i \tilde{F}^{\mu\nu} \right)$$

Helicity decomposition in the field strength tensor

Matrix elements of quark P-odd EMT operator

$$\begin{aligned} \langle p', s' | \hat{T}_{q5}^{\mu\nu}(0) | p, s \rangle = & \bar{u}(p', s') \left[\frac{P^{\{\mu} \gamma^{\nu\}} \gamma_5}{2} \tilde{A}_a(Q^2) + \frac{P^{\{\mu} \Delta^{\nu\}} \gamma_5}{2} \tilde{B}_a(Q^2) \right. \\ & \left. + \frac{P^{[\mu} \gamma^{\nu]} \gamma_5}{2} \tilde{C}_a^{29}(Q^2) + \frac{P^{[\mu} \Delta^{\nu]} \gamma_5}{2} \tilde{D}_a(Q^2) + M i \sigma^{\mu\nu} \gamma_5 \tilde{F}_a(Q^2) \right] u(p, s) \end{aligned}$$

Back up

Matrix elements of the EMT

V.D. Burkert et al., RMP 95 (2023)
 Won, Lorcé, PRD 111 (2025)

$$\begin{aligned} &\langle p', s' | \hat{T}_a^{\mu\nu} (0) | p, s \rangle \\ &= \bar{u} (p', s') \left[\frac{P^\mu P^\nu}{M} A_a (Q^2) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} D_a (Q^2) + M g^{\mu\nu} \bar{C}_a (Q^2) \right. \\ &\quad \left. + \frac{i P^{\{\mu} \sigma^{\nu\} \rho} \Delta_\rho}{2M} J_a (Q^2) - \frac{i P^{[\mu} \sigma^{\nu] \rho} \Delta_\rho}{2M} S_a (Q^2) \right] u (p, s) \end{aligned}$$

Multi-pole ansatz

$$F_a(t) = \frac{F_a(0)}{(1 - t/\Lambda_{F_a}^2)^{n_F}}$$

C. Lorcé, Moutarde, Trawiński, EPJC 79 (2019)
 Won, Lorcé, PRD 111 (2025)

$\mu = 2 \text{ GeV}$					
F_a	n_F	$F_q(0)$	$\Lambda_{F_q} \text{ (GeV)}$	$F_G(0)$	$\Lambda_{F_G} \text{ (GeV)}$
A_a	2	0.55	0.91	0.45	0.91
B_a	3	−0.07	0.80	0.07	0.80
D_a	3	−1.28	0.80	−2.24	0.80
$\bar{C}_a$	2	−0.11	0.91	0.11	0.91
S_a	2	0.33	1.00	—	—

Energy form factor

$$\langle p', s' | \hat{T}_a^{00}(0) | p, s \rangle \Big|_{\text{BF}} = 2P_{\text{BF}}^0 M \textcolor{blue}{E_a(Q^2)} \delta_{s'_{\text{BF}} s_{\text{BF}}},$$

Total angular momentum

$$\langle p', s' | \hat{T}_a^{03}(0) | p, s \rangle \Big|_{\text{BF}} = 2P_{\text{BF}}^0 \frac{(\boldsymbol{\sigma}_{s'_{\text{BF}} s_{\text{BF}}}} \times i\boldsymbol{\Delta})^k}{2M} [\textcolor{blue}{J_a(Q^2)} - S_a(Q^2)],$$

$$\langle p', s' | \hat{T}_a^{30}(0) | p, s \rangle \Big|_{\text{BF}} = 2P_{\text{BF}}^0 \frac{(\boldsymbol{\sigma}_{s'_{\text{BF}} s_{\text{BF}}}} \times i\boldsymbol{\Delta})^k}{2M} [J_a(Q^2) + \textcolor{blue}{S_a(Q^2)}],$$

Intrinsic spin

$$\langle p', s' | \hat{T}_a^{33}(0) | p, s \rangle \Big|_{\text{BF}} = 2P_{\text{BF}}^0 M \textcolor{blue}{F_a(Q^2)} \delta_{s'_{\text{BF}} s_{\text{BF}}},$$

Force

Back up

EMT distributions in the EF

$$g_a(\mathbf{b}_\perp, P_z; s', s) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \left[\delta_{s's} \tilde{g}_a^U(Q^2, P_z) + \frac{(\boldsymbol{\sigma}_{s's} \times i\mathbf{\Delta}_\perp)_z}{2M} \tilde{g}_a^T(Q^2, P_z) \right],$$

Energy

$$\begin{aligned} \tilde{\rho}_a^U(Q^2, P_z) &= \frac{M}{\sqrt{P_z^2 + M^2}} \frac{P^0 [P^0 + M(1 + \tau)]}{(P^0 + M)(1 + \tau)} \left\{ E_a(Q^2) + \frac{2\tau P_z^2}{P^0 [P^0 + M(1 + \tau)]} J_a(Q^2) + \left(\frac{P_z}{P^0} \right)^2 F_a(Q^2) \right\} \\ \tilde{\rho}_a^T(Q^2, P_z) &= \frac{M}{\sqrt{P_z^2 + M^2}} \frac{P^0 P_z}{(P^0 + M)(1 + \tau)} \left\{ -E_a(Q^2) + \frac{2[P^0 + M(1 + \tau)]}{P^0} J_a(Q^2) - \left(\frac{P_z}{P^0} \right)^2 F_a(Q^2) \right\}, \end{aligned}$$

$$E_a(Q^2) = A_a(Q^2) - \tau B_a(Q^2) + \bar{C}_a(Q^2) + \tau D_a(Q^2),$$

$$J_a(Q^2) = \frac{1}{2} [A_a(Q^2) + B_a(Q^2)],$$

$$F_a(Q^2) = -\tau D_a(Q^2) - \bar{C}_a(Q^2),$$

$$\begin{aligned} \langle p', s' | \hat{T}_a^{00}(0) | p, s \rangle \Big|_{\text{BF}} &= 2P_{\text{BF}}^0 M \underbrace{E_a(Q^2)}_{\text{Energy form factor}} \delta_{s'_{\text{BF}} s_{\text{BF}}}, \\ \langle p', s' | \hat{T}_a^{03}(0) | p, s \rangle \Big|_{\text{BF}} &= 2P_{\text{BF}}^0 \frac{(\boldsymbol{\sigma}_{s'_{\text{BF}} s_{\text{BF}}} \times i\mathbf{\Delta})^k}{2M} \underbrace{[J_a(Q^2) - S_a(Q^2)]}_{\text{Total angular momentum}}, \\ \langle p', s' | \hat{T}_a^{30}(0) | p, s \rangle \Big|_{\text{BF}} &= 2P_{\text{BF}}^0 \frac{(\boldsymbol{\sigma}_{s'_{\text{BF}} s_{\text{BF}}} \times i\mathbf{\Delta})^k}{2M} [J_a(Q^2) + \underbrace{S_a(Q^2)}_{\text{Intrinsic spin}}], \\ \langle p', s' | \hat{T}_a^{33}(0) | p, s \rangle \Big|_{\text{BF}} &= 2P_{\text{BF}}^0 M \underbrace{F_a(Q^2)}_{\text{Force}} \delta_{s'_{\text{BF}} s_{\text{BF}}}, \end{aligned}$$

Back up

Longitudinal momentum

$$\begin{aligned}\tilde{\mathcal{P}}_a^{z,U}(Q^2, P_z) &= \frac{M}{\sqrt{P_z^2 + M^2}} \frac{P_z [P^0 + M(1 + \tau)]}{(P^0 + M)(1 + \tau)} \\ &\quad \times \left\{ E_a(Q^2) + \frac{\tau P^2}{P^0 [P^0 + M(1 + \tau)]} \left[\left(\frac{2P_z^2}{P^2} + 1 \right) J_a(Q^2) - S_a(Q^2) \right] + F_a(Q^2) \right\}, \\ \tilde{\mathcal{P}}_a^{z,T}(Q^2, P_z) &= \frac{M}{\sqrt{P_z^2 + M^2}} \frac{P_z^2}{(P^0 + M)(1 + \tau)} \\ &\quad \times \left\{ -E_a(Q^2) + \frac{P^2 [P^0 + M(1 + \tau)]}{P^0 P_z^2} \left[\left(\frac{2P_z^2}{P^2} + 1 \right) J_a(Q^2) - S_a(Q^2) \right] - F_a(Q^2) \right\},\end{aligned}$$

Axial momentum flux

$$\begin{aligned}\tilde{\Pi}_a^{zz,U}(Q^2, P_z) &= \frac{M}{\sqrt{P_z^2 + M^2}} \frac{P^0 [P^0 + M(1 + \tau)]}{(P^0 + M)(1 + \tau)} \left[\left(\frac{P_z}{P^0} \right)^2 E_a(Q^2) + \frac{2\tau P_z^2}{P^0 [P^0 + M(1 + \tau)]} J_a(Q^2) + F_a(Q^2) \right] \\ \tilde{\Pi}_a^{zz,T}(Q^2, P_z) &= \frac{M}{\sqrt{P_z^2 + M^2}} \frac{P^0 P_z}{(P^0 + M)(1 + \tau)} \left[- \left(\frac{P_z}{P^0} \right)^2 E_a(Q^2) + \frac{2 [P^0 + M(1 + \tau)]}{P^0} J_a(Q^2) - F_a(Q^2) \right]\end{aligned}$$

Back up

Normal force

$$\tilde{\sigma}_a^U (Q^2, P_z) = \frac{\gamma_P}{\gamma} \cos \theta M F_{0,a} (Q^2) , \quad F_{0,a} = -\bar{C}_a - \frac{\tau}{2} D$$

$$\tilde{\sigma}_a^T (Q^2, P_z) = \frac{\gamma_P}{\gamma} \frac{1}{\sqrt{\tau}} \sin \theta M F_{0,a} (Q^2)$$

Shear force

$$\Pi_a (\mathbf{b}_\perp, P_z; s', s) = \frac{1}{M^2 b^2} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i \mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \left[\tilde{\Pi}_a^U (Q^2, P_z) \delta_{s' s} + \frac{(\boldsymbol{\sigma}_{s' s} \times i \mathbf{\Delta}_\perp)_z}{2M} \tilde{\Pi}_a^T (Q^2, P_z) \right]$$

$$\tilde{\Pi}_a^U (Q^2, P_z) = - \left(\frac{d}{dQ^2} \right)^2 \left[Q^4 \frac{\gamma_P}{\gamma} \cos \theta M F_{2,a} (Q^2) \right] ,$$

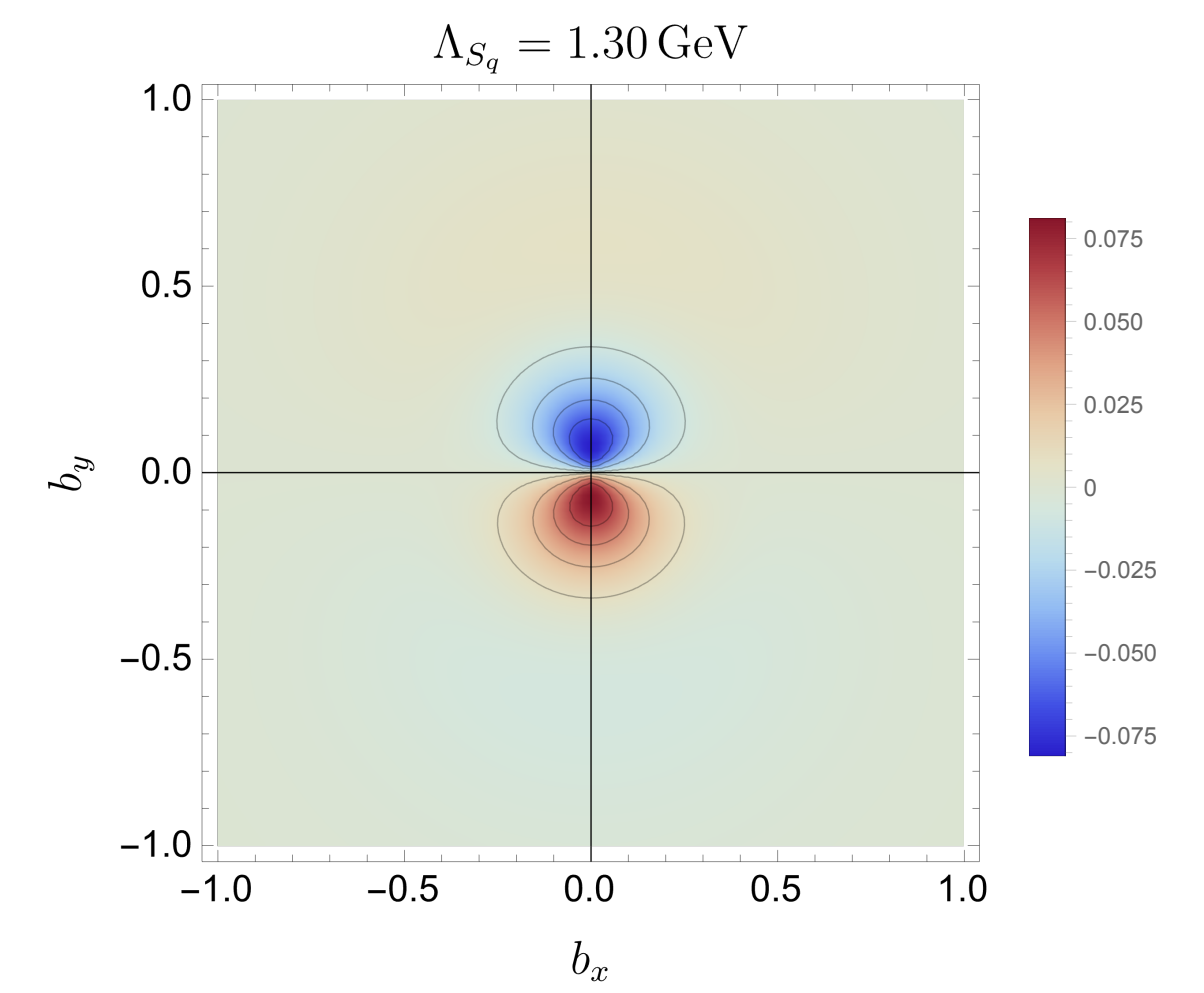
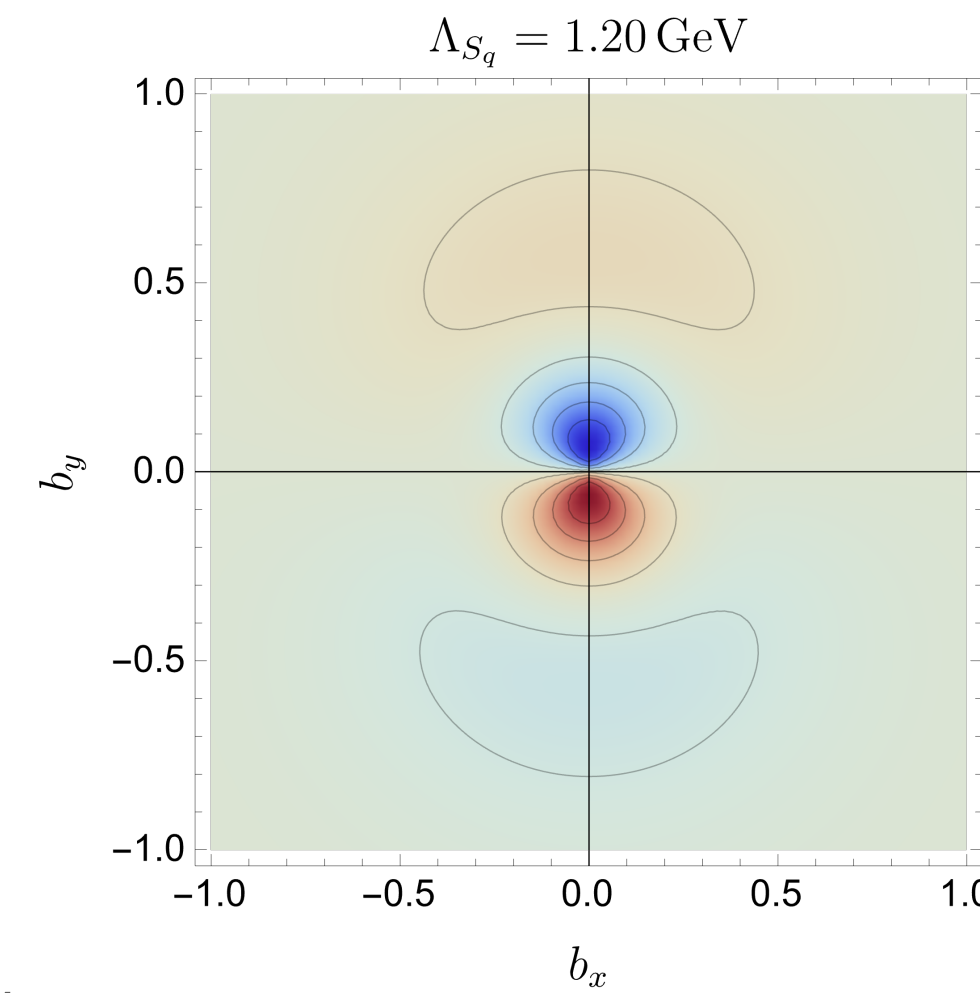
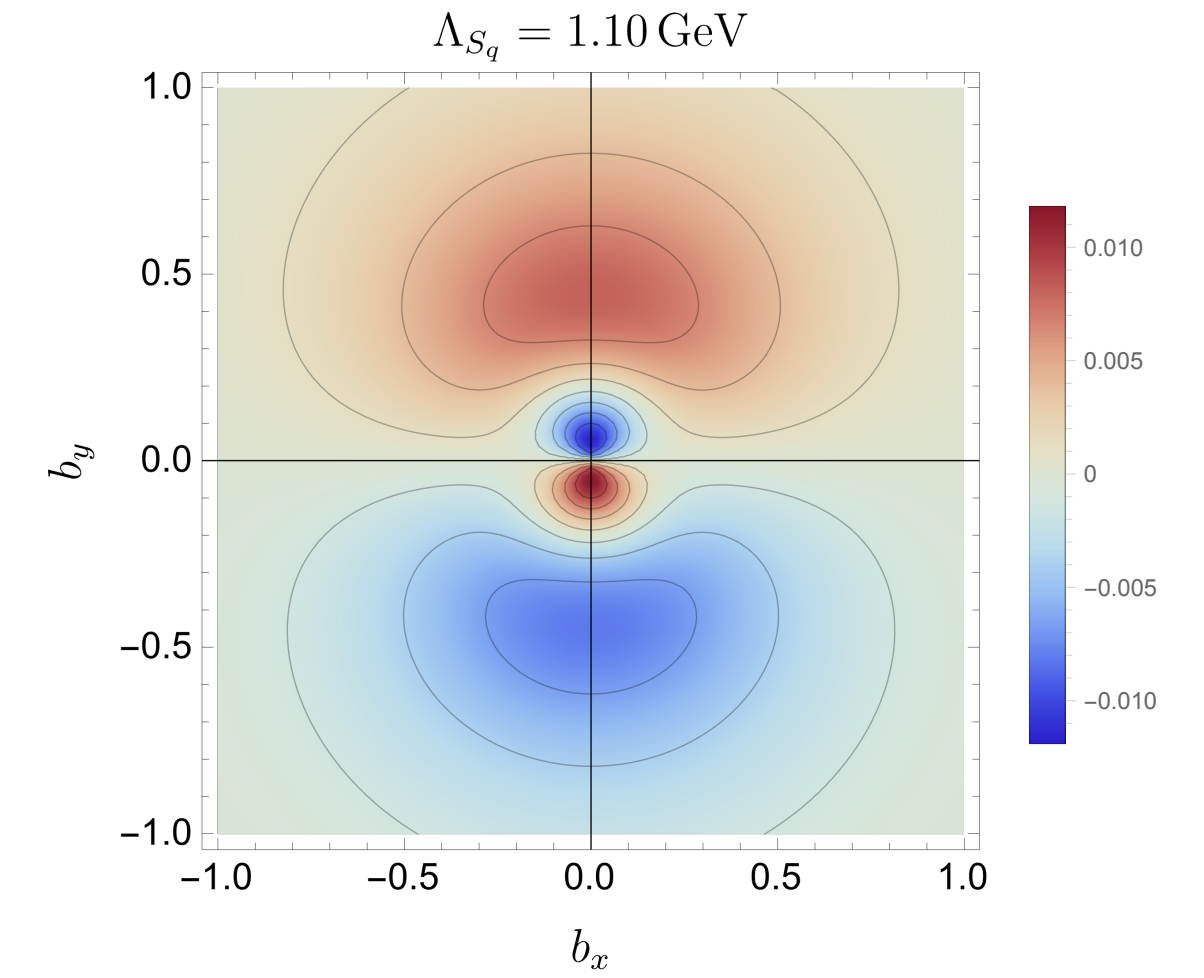
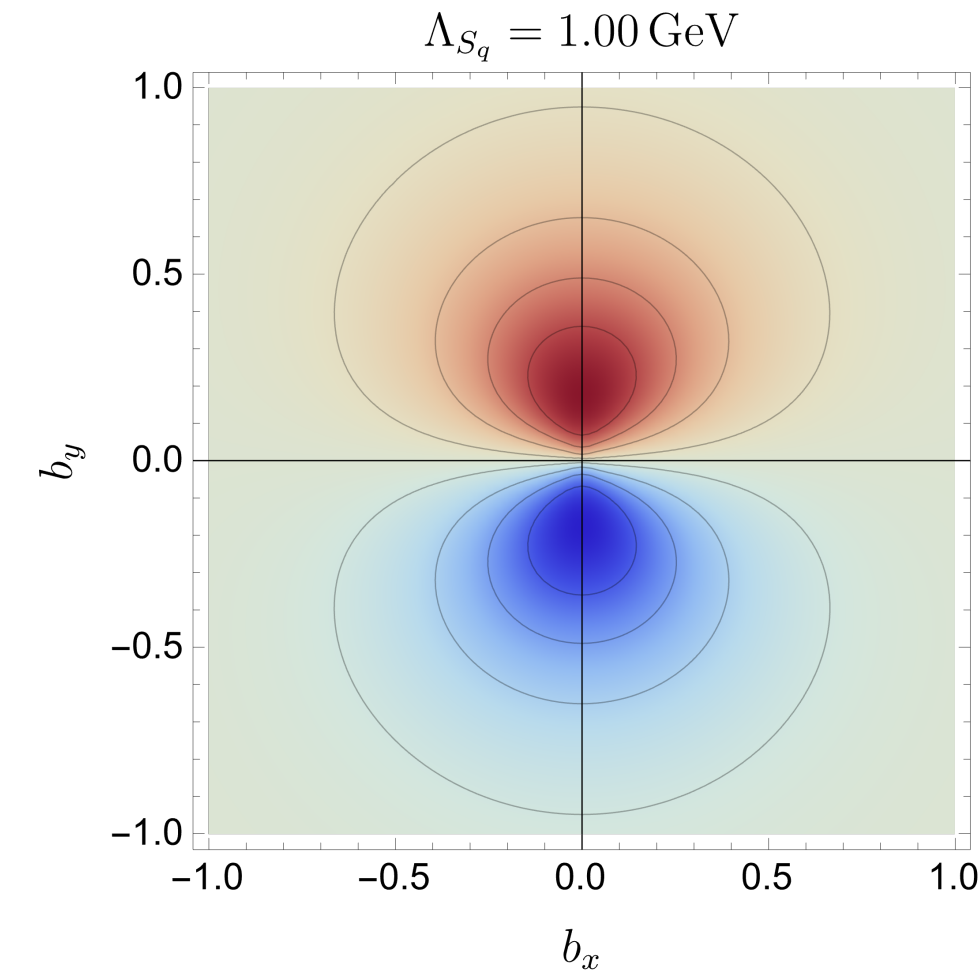
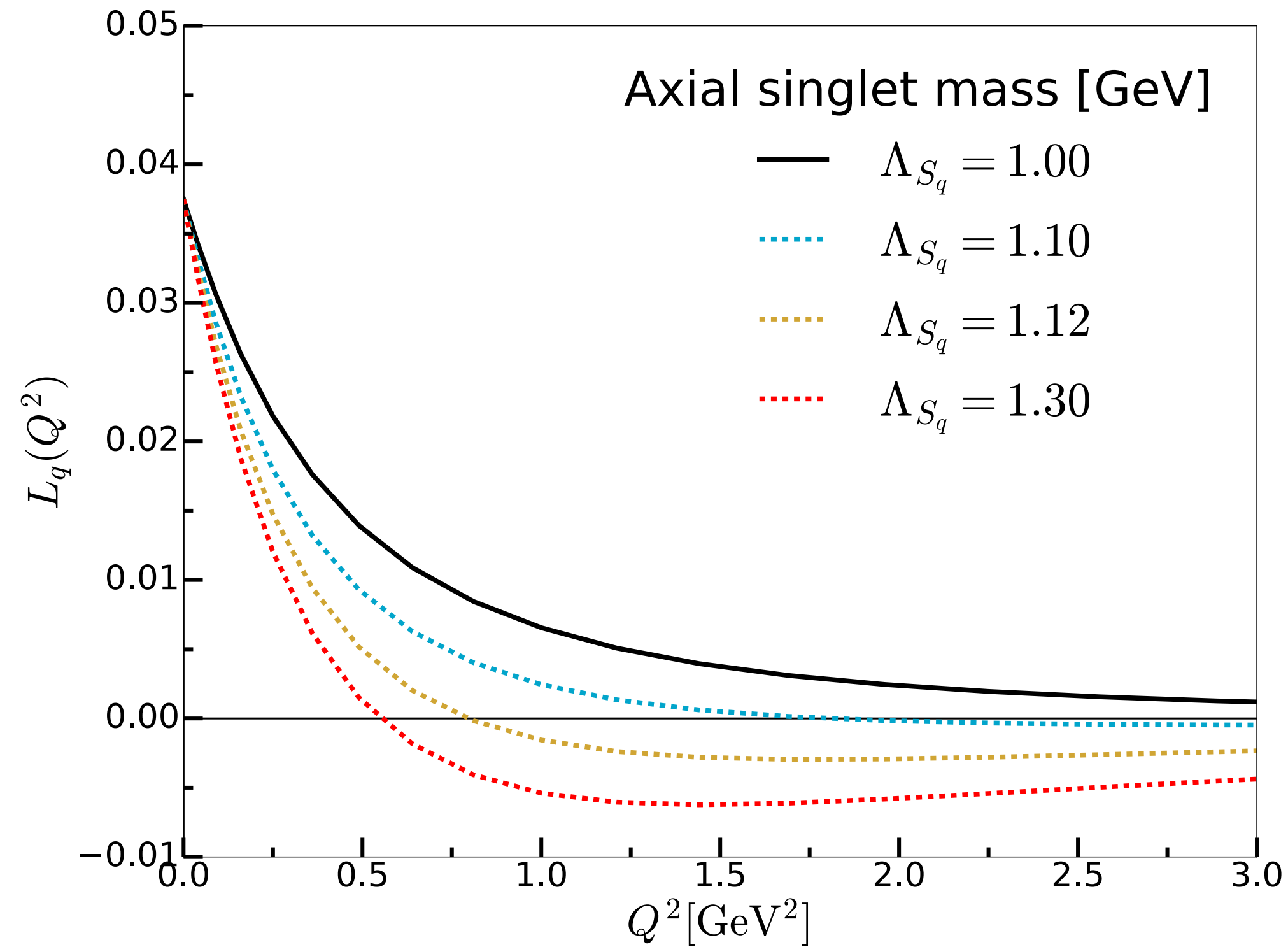
$$\tilde{\Pi}_a^T (Q^2, P_z) = - \left(\frac{d}{dQ^2} + \frac{1}{Q^2} \right) \frac{d}{dQ^2} \left[Q^4 \frac{\gamma_P}{\gamma} \frac{\sin \theta}{\sqrt{\tau}} M F_{2,a} (Q^2) \right]$$

Back up

Longitudinal OAM distribution

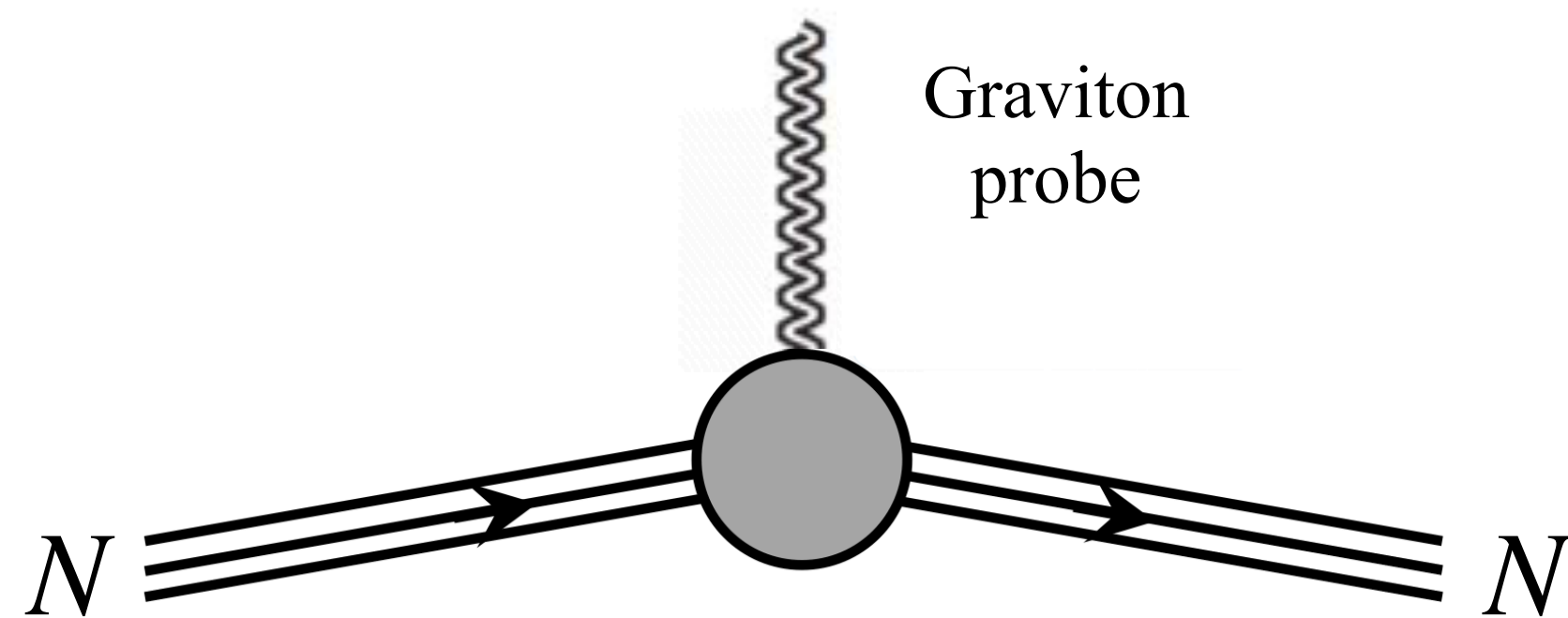
For transversely polarized nucleon along the x-axis

$$\mathcal{P}_a^{P,z}(\mathbf{b}_\perp, 0) = M \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \frac{i\Delta_y}{2M} L_a(Q^2),$$



Back up

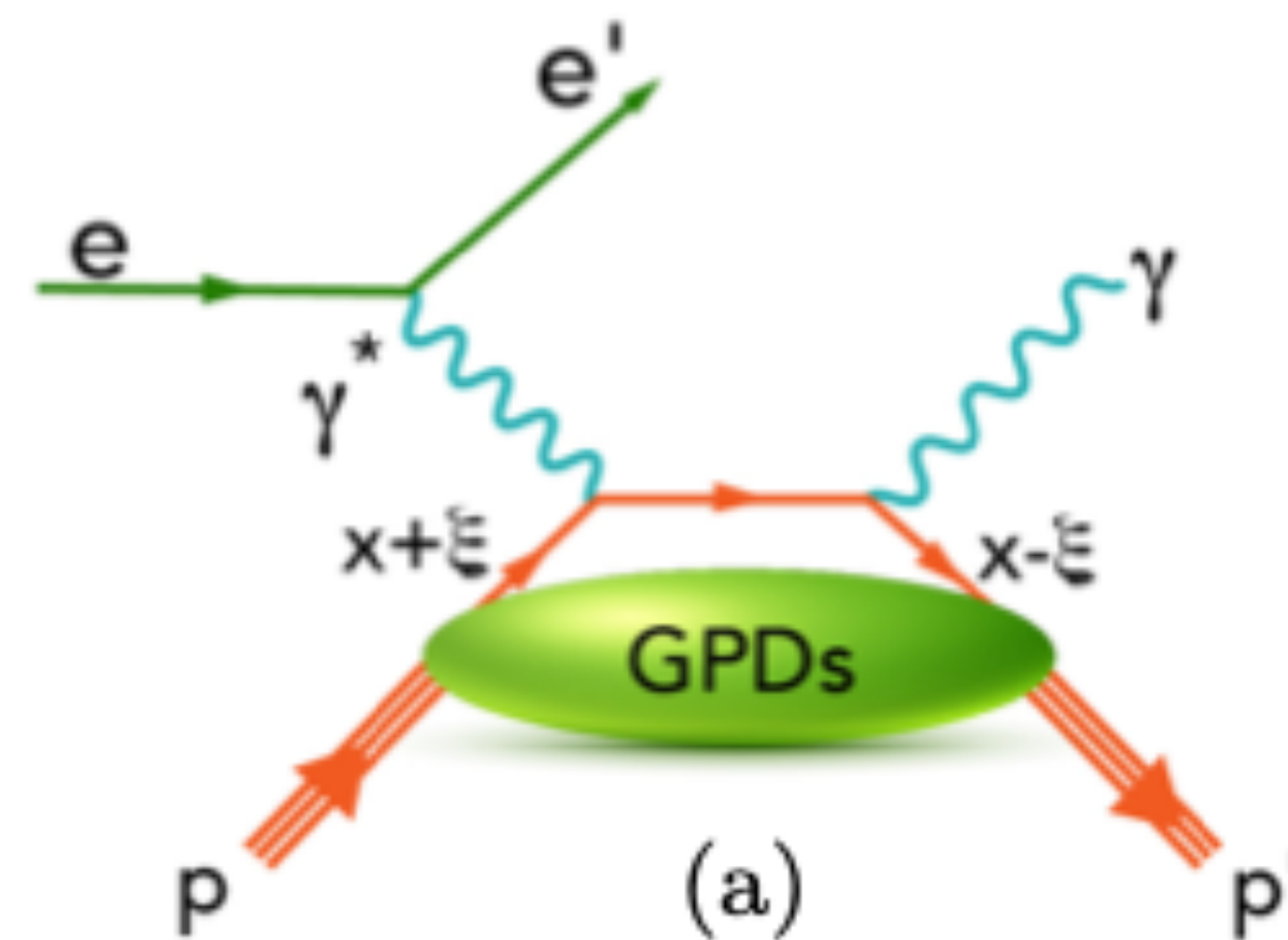
Direct measure



Too weak gravitation



Indirect measure



Deeply virtual Compton scattering

Back up

Matrix elements of non-local quark operator; Generalized Parton Distributions (GPDs)

$$\begin{aligned} \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{ix(P \cdot z)} \langle p', s' | \bar{\psi}_q \left(-\frac{\lambda n}{2} \right) \gamma^\mu n_\mu \psi_q \left(\frac{\lambda n}{2} \right) | p, s \rangle \Big|_{z=\lambda n} \\ = \frac{1}{2(P \cdot n)} \bar{u}(p', s') \left[H^q(x, \xi, t) \gamma^\mu n_\mu + E^q(x, \xi, t) \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} \right] u(p, s) \end{aligned}$$

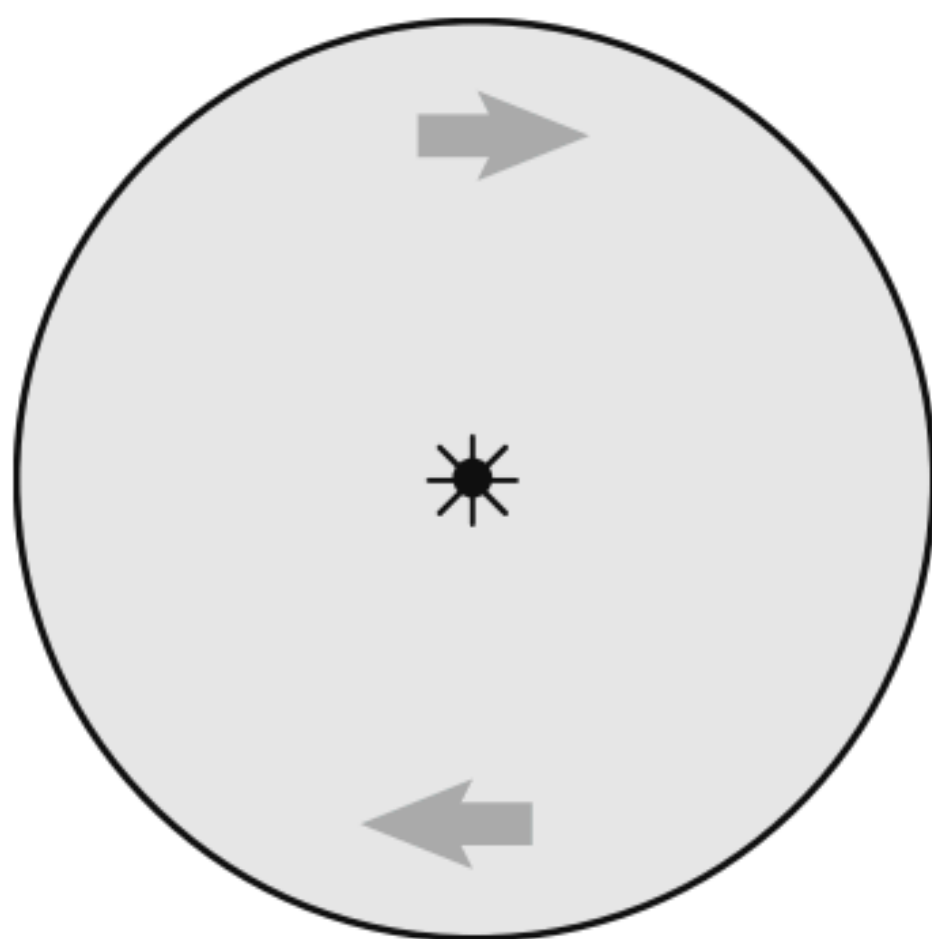
In the DVCS, the actual observables are Compton form factors (CFFs) at leading order α_s

$$\text{Re}\mathcal{H}(\xi, t) + i\text{Im}\mathcal{H}(\xi, t) = \sum_q e_q^2 \int_{-1}^1 dx \left[\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right] H_q(x, \xi, t)$$

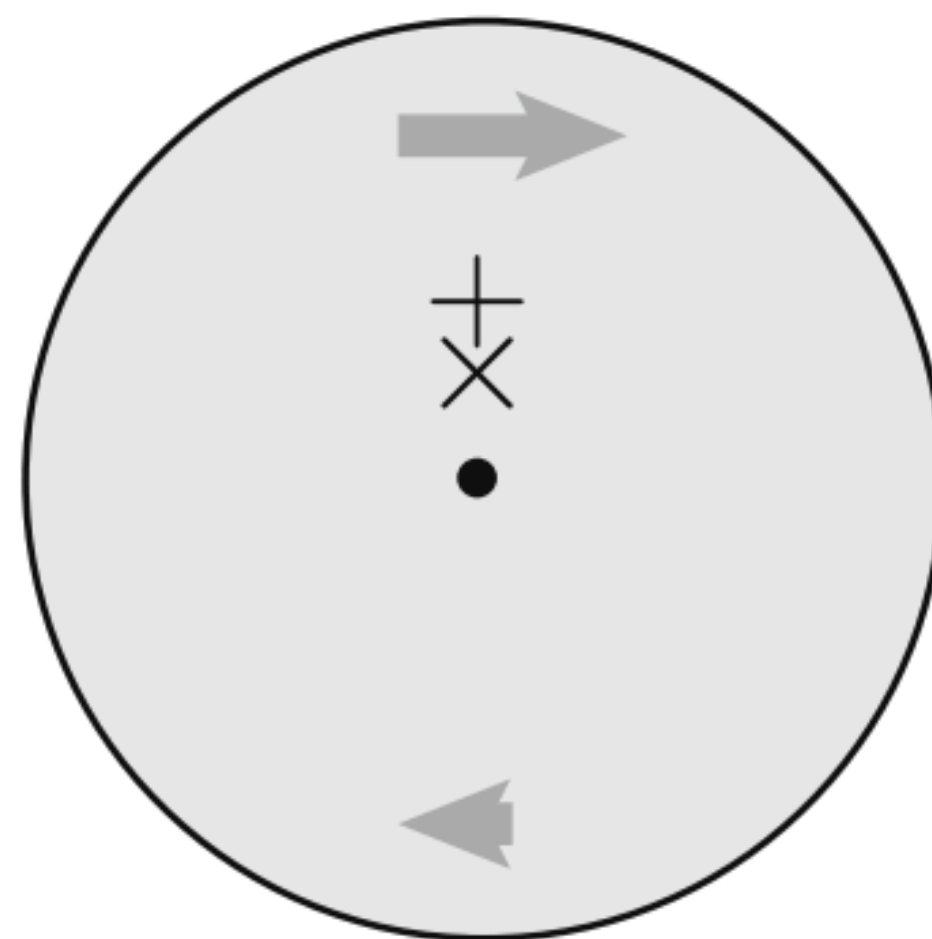
Back up

C. Lorcé, EPJC 81 (2023)

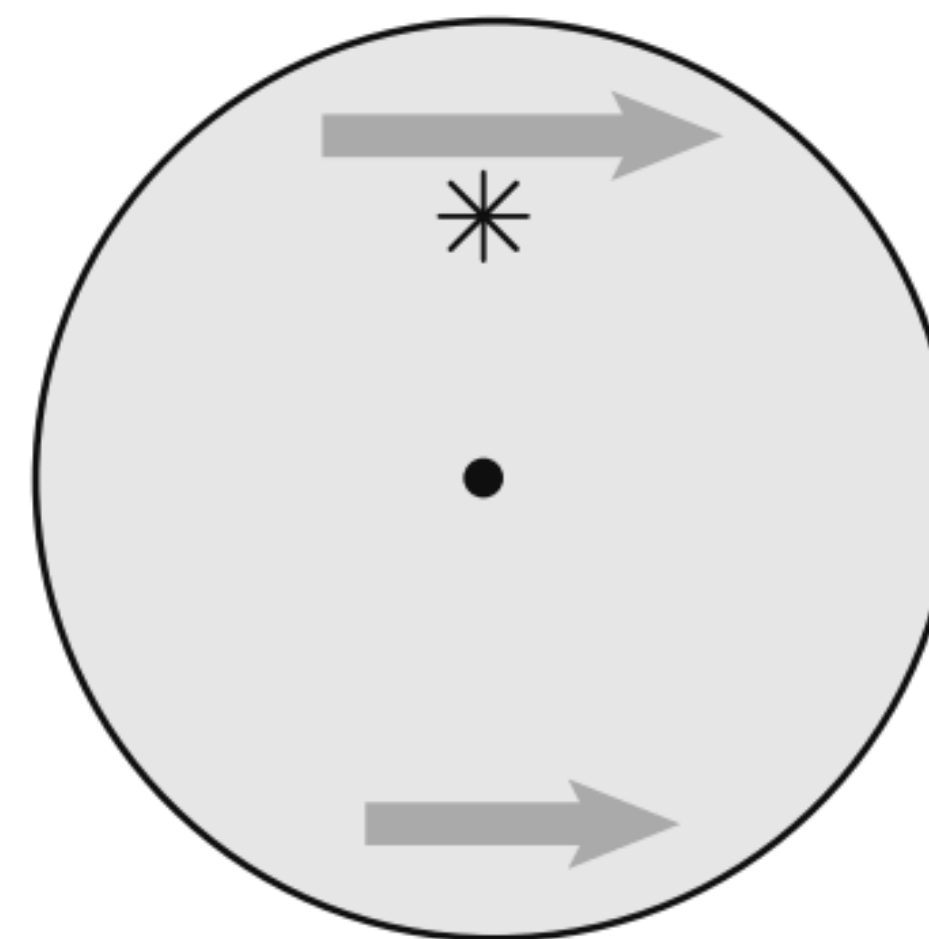
Relativistic pivots $[J^i, K^j] = i\epsilon^{ijk}K^k$



Rest frame



Moving frame



Infinite-momentum frame

Center of
 + energy
 × spin
 • mass

Back up

C. Lorcé’s DIS 2024 talk

Center	Position operator	Canonical relation $[R^i, P^j] = i\delta^{ij}$	Vector under rotations $[J^i, R^j] = i\epsilon^{ijk} R^k$	Compatibility of components $[R^i, R^j] = 0$	Four-vector transformation $R'^\mu = \Lambda^\mu_\nu R^\nu$	
$\mathbf{P}^+$	$R^i_\perp = \frac{1}{P^+} \int dx^- d^2x_\perp x^i_\perp T^{++} = -\frac{B^i_\perp}{P^+}$ $x^+ = 0$					2D
Energy	$R^i_E = \frac{1}{P^0} \int d^3x x^i T^{00} = -\frac{K^i}{P^0}$ $x^0 = 0$					
Mass	$R^\mu_M = \Lambda^\mu_\nu R^\nu_E _{\text{rest}}$					3D
Canonical	$R^i_c = \frac{P^0 R^i_E + M R^i_M}{P^0 + M}$					

Relativistic center of	$\langle \mathbf{R}_X \rangle$	$\langle \mathbf{S}_X \rangle = \langle \mathbf{J} \rangle - \langle \mathbf{R}_X \times \mathbf{P} \rangle$
Energy ($X = E$)	$\mathcal{R} + \frac{\mathbf{p} \times \mathbf{s}}{2p^0(p^0+m)}$	$\frac{m}{2p^0} \left(\mathbf{s} + \frac{\mathbf{p}(\mathbf{p} \cdot \mathbf{s})}{m(p^0+m)} \right)$
Mass ($X = M$)	$\mathcal{R} - \frac{\mathbf{p} \times \mathbf{s}}{2m(p^0+m)}$	$\frac{p^0}{2m} \left(\mathbf{s} - \frac{\mathbf{p}(\mathbf{p} \cdot \mathbf{s})}{p^0(p^0+m)} \right)$
Spin ($X = c$)	$\mathcal{R}$	$\frac{1}{2} \mathbf{s}$