FeynCalc

An Introduction to Mathematica Tools for HEP

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What is FeynCalc (FC) ?

FeynCalc is a Mathematica package for algebraic calculations in Quantum Field Theory and semi-automatic evaluation of Feynman Diagrams.

-One-loop integration

-Dirac algebra, contractions, color algebra, etc

Preface

We count with three main tools to understand FeynArts, FeynCalc, etc:

1. Command InputForm[X] in Mathematica. Shows how Mathematica understands the expression X.
2. Command ?X . Shows the definition of X.
3. User Guide provided by the creators. FC Guide: https://feyncalc.github.io/reference

Using InputForm

Typical amplitude for a Feynman diagram

```
hhtohhtreeamp2 = PickLevel[Particles][hhtohhtreeamp] // ToFA1Conventions
```

$$\text{FAFeynAmpList}\left(\text{Process} \to \begin{pmatrix} S(1) & p1 & FCGV(MH) & \{\} \\ S(1) & p2 & FCGV(MH) & \{\} \end{pmatrix} \to \begin{pmatrix} S(1) & k1 & FCGV(MH) & \{\} \\ S(1) & k2 & FCGV(MH) & \{\} \end{pmatrix}\right), \text{Model} \to \{\text{SM}\}, \text{GenericModel} \to \{\text{Lorentz}\}, \text{AmplitudeLevel} \to \{\text{Particles}\}, \text{ExcludeParticles} \to \{\}, \text{ExcludeFieldPoints} \to \{\}, \text{LastSelections} \to \{\}\Bigg)$$

$$\left(\text{FAFeynAmp}\left(\text{GraphName}(, \text{T1}, \text{G1C1P1}, \text{N1}), -\frac{3\,FCGV(EL)^2\,FCGV(MH)^2}{4\,FCGV(MW)^2\,FCGV(SW)^2}\right), \text{FAFeynAmp}\left(\text{GraphName}(, \text{T2}, \text{G1C2P1}, \text{N2}), -\frac{9\,FCGV(EL)^2\,FCGV(MH)^4\,\frac{1}{(k1+k2)^2-FCGV(MH)^2}}{4\,FCGV(MW)^2\,FCGV(SW)^2}\right),\right.$$

$$\left.\text{FAFeynAmp}\left(\text{GraphName}(, \text{T3}, \text{G1C3P1}, \text{N3}), -\frac{9\,FCGV(EL)^2\,FCGV(MH)^4\,\frac{1}{(k2-p2)^2-FCGV(MH)^2}}{4\,FCGV(MW)^2\,FCGV(SW)^2}\right), \text{FAFeynAmp}\left(\text{GraphName}(, \text{T4}, \text{G1C4P1}, \text{N4}), -\frac{9\,FCGV(EL)^2\,FCGV(MH)^4\,\frac{1}{(k1-p2)^2-FCGV(MH)^2}}{4\,FCGV(MW)^2\,FCGV(SW)^2}\right)\right)$$

```
InputForm[hhtohhtreeamp2]
```

forma de entrada

```
InputForm=
FAFeynAmpList[Process -> {{S[1], p1, FCGV["MH"], {}}, {S[1], p2, FCGV["MH"], {}}} -> {{S[1], k1, FCGV["MH"], {}}, {S[1], k2, FCGV["MH"], {}}}, Model -> {"SM"}, GenericModel -> {"Lorentz"},
  AmplitudeLevel -> {Particles}, ExcludeParticles -> {}, ExcludeFieldPoints -> {}, LastSelections -> {}][FAFeynAmp[GraphName["", "T1", "G1C1P1", "N1"],
  (-3*FCGV["EL"]^2*FCGV["MH"]^2)/(4*FCGV["MW"]^2*FCGV["SW"]^2)], FAFeynAmp[GraphName["", "T2", "G1C2P1", "N2"], (-9*FAPropagatorDenominator[k1 + k2, FCGV["MH"]]*FCGV["EL"]^2*FCGV["MH"]^4)/
  (4*FCGV["MW"]^2*FCGV["SW"]^2)], FAFeynAmp[GraphName["", "T3", "G1C3P1", "N3"], (-9*FAPropagatorDenominator[k2 - p2, FCGV["MH"]]*FCGV["EL"]^2*FCGV["MH"]^4)/(4*FCGV["MW"]^2*FCGV["SW"]^2)],
 FAFeynAmp[GraphName["", "T4", "G1C4P1", "N4"], (-9*FAPropagatorDenominator[k1 - p2, FCGV["MH"]]*FCGV["EL"]^2*FCGV["MH"]^4)/(4*FCGV["MW"]^2*FCGV["SW"]^2)]]
```

This allows us to write substitutions /. FCGV[“MH”] -> MH

How does it work?

Takes the output of FeynArts written in FeynCalc language created first by CreateFeynAmp and then translate to FeynCalc

```
omegaomegatoomegaomegaamp = CreateFeynAmp[omegaomegatoomegaomega, AmplitudeLevel → Particles, GaugeRules → {}]  /. M$FACouplingsHEFT
```

```
omegaomegatoomegaomegaampnew = PickLevel[Particles][omegaomegatoomegaomegaamp] // ToFA1Conventions
```

$$\text{FAFeynAmpList}\left(\text{Process} \to \begin{pmatrix} S(3) & p1 & Mz & \{\} \\ S(3) & p2 & Mz & \{\} \end{pmatrix} \to \begin{pmatrix} S(3) & k1 & Mz & \{\} \\ S(3) & k2 & Mz & \{\} \end{pmatrix}, \text{Model} \to \{\text{HEFT3_FA}\}, \text{GenericModel} \to \{\text{HEFT3_FA}\},\right.$$

$$\text{AmplitudeLevel} \to \{\text{Particles}\}, \text{ExcludeParticles} \to \{\}, \text{ExcludeFieldPoints} \to \{\}, \text{LastSelections} \to \{\}\Big)\Big($$

$$\text{FAFeynAmp}\Big(\text{GraphName}(, \text{T1}, \text{G1C1P1}, \text{N1}),$$

$$-i\left(\frac{8\,i\,(a4+a5)\,(-k1).(-k2)\,p1.p2}{v^4} + \frac{8\,i\,(a4+a5)\,(-k1).p2\,(-k2).p1}{v^4} + \frac{8\,i\,(a4+a5)\,(-k1).p1\,(-k2).p2}{v^4} - \frac{2\,i\,(-k1).(-k2)}{v^2} - \frac{2\,i\,(-k1).p1}{v^2} - \frac{2\,i\,(-k1).p2}{v^2} - \frac{2\,i\,(-k2).p1}{v^2} - \frac{2\,i\,(-k2).p2}{v^2} - \frac{2\,i\,p1.p2}{v^2}\right)\Big),$$

$$\text{FAFeynAmp}\left(\text{GraphName}(, \text{T2}, \text{G1C2P1}, \text{N2}), -\frac{4\,a^2\,(-k1).(-k2)\,p1.p2\,\frac{1}{(k1+k2)^2-\text{FCGV(MH)}^2\,(\text{xiSubscript[S(4)]})}}{v^2}\right), \text{FAFeynAmp}\Big(\text{GraphName}(, \text{T3}, \text{G1C3P1}, \text{N3}),$$

$$-\frac{4\,a^2\,(-k1).p1\,(-k2).p2\,\frac{1}{(k2-p2)^2-\text{FCGV(MH)}^2\,(\text{xiSubscript[S(4)]})}}{v^2}\Big), \text{FAFeynAmp}\left(\text{GraphName}(, \text{T4}, \text{G1C4P1}, \text{N4}), -\frac{4\,a^2\,(-k1).p2\,(-k2).p1\,\frac{1}{(k1-p2)^2-\text{FCGV(MH)}^2\,(\text{xiSubscript[S(4)]})}}{v^2}\right)\Big)$$

Example 1: Goldstone scattering

```
omegaomegatoomegaomegaampnew[[2, 2]]
```

$$-\frac{4\,a^2\,(-\text{k1}).(-\text{k2})\,\text{p1.p2}\,\frac{1}{(\text{k1}+\text{k2})^2-\text{FCGV(MH)}^2\,\text{xi}_{S(4)}}}{v^2}$$

```
omegaomegatoomegaomegaampnew[[2, 2]]  // InputForm
```

forma de entrada

```
InputForm=
(-4*a^2*FAPropagatorDenominator[k1 + k2, Sqrt[FAGaugeXi[S[4]]]*FCGV["MH"]]*FAScalarProduct[-k1, -k2]*
  FAScalarProduct[p1, p2])/v^2
```

```
omegaomegatoomegaamps =

Table[omegaomegatoomegaomegaampnew[[i, 2]], {i, 1, 4}] /. {FCGV[x_] → ToExpression[x]} /. {p2 → k4, p1 → k3} /. FAPropagatorDenominator[x_, y_] → 1/(x^2 - y^2) /.

  {(k1 + k2)^2 → S, (k2 - k4)^2 → T, (k1 - k4)^2 → U} //
```

tabla

convierte en expresión

$$\left\{-i\left(\frac{8\,i\,(\text{a4}+\text{a5})\,(-\text{k1}).(-\text{k2})\,\text{k3.k4}}{v^4}+\frac{8\,i\,(\text{a4}+\text{a5})\,(-\text{k1}).\text{k4}\,(-\text{k2}).\text{k3}}{v^4}+\frac{8\,i\,(\text{a4}+\text{a5})\,(-\text{k1}).\text{k3}\,(-\text{k2}).\text{k4}}{v^4}-\frac{2\,i\,(-\text{k1}).(-\text{k2})}{v^2}-\frac{2\,i\,(-\text{k1}).\text{k3}}{v^2}-\frac{2\,i\,(-\text{k1}).\text{k4}}{v^2}-\frac{2\,i\,(-\text{k2}).\text{k3}}{v^2}-\frac{2\,i\,(-\text{k2}).\text{k4}}{v^2}-\frac{2\,i\,\text{k3.k4}}{v^2}\right),\right.$$

$$\left.-\frac{4\,a^2\,(-\text{k1}).(-\text{k2})\,\text{k3.k4}}{v^2\left(S-\text{MH}^2\,(\text{xiSubscript}[S(4)])\right)},-\frac{4\,a^2\,(-\text{k1}).\text{k3}\,(-\text{k2}).\text{k4}}{v^2\left(T-\text{MH}^2\,(\text{xiSubscript}[S(4)])\right)},-\frac{4\,a^2\,(-\text{k1}).\text{k4}\,(-\text{k2}).\text{k3}}{v^2\left(U-\text{MH}^2\,(\text{xiSubscript}[S(4)])\right)}\right\}$$

```
omegaomegatoomegaamps[[2]] // InputForm
```
forma de entrada

InputForm=

```
(-4*a^2*FAScalarProduct[-k1, -k2]*FAScalarProduct[k3, k4])/(v^2*(S - "MH"^2*FAGaugeXi[S[4]]))
```

omegaomegatoomegaampsbis =

Simplify[
simplifica

```
omegaomegatoomegaamps /. {a4 → 0, a5 → 0} //. {FAScalarProduct[-x_, y_] → -FAScalarProduct[x, y], FAScalarProduct[x_, -y_] → -FAScalarProduct[x, y]} /.
  {FAScalarProduct[k3, k4] → (S - FAScalarProduct[k3, k3] - FAScalarProduct[k4, k4])/2,
   FAScalarProduct[k1, k2] → (S - FAScalarProduct[k1, k1] - FAScalarProduct[k2, k2])/2,
   FAScalarProduct[k3, k4] → (S - FAScalarProduct[k3, k3] - FAScalarProduct[k4, k4])/2,
   FAScalarProduct[k1, k3] → (-T + FAScalarProduct[k1, k1] + FAScalarProduct[k3, k3])/2,
   FAScalarProduct[k2, k4] → (-T + FAScalarProduct[k2, k2] + FAScalarProduct[k4, k4])/2,
   FAScalarProduct[k1, k4] → (-U + FAScalarProduct[k1, k1] + FAScalarProduct[k4, k4])/2,
   FAScalarProduct[k2, k3] → (-U + FAScalarProduct[k2, k2] + FAScalarProduct[k3, k3])/2} /.
  {FAScalarProduct[k1, k1] → 0, FAScalarProduct[k2, k2] → 0, FAScalarProduct[k3, k3] → 0, FAScalarProduct[k4, k4] → 0}]
```

=

$$\left\{-\frac{2\,(S+T+U)}{v^2},\ -\frac{a^2\,S^2}{v^2\left(S-\mathrm{MH}^2\,(\mathrm{xiSubscript}[S(4)])\right)},\ -\frac{a^2\,T^2}{v^2\left(T-\mathrm{MH}^2\,(\mathrm{xiSubscript}[S(4)])\right)},\ -\frac{a^2\,U^2}{v^2\left(U-\mathrm{MH}^2\,(\mathrm{xiSubscript}[S(4)])\right)}\right\}$$

Then the final amplitude is the known expression for HEFT

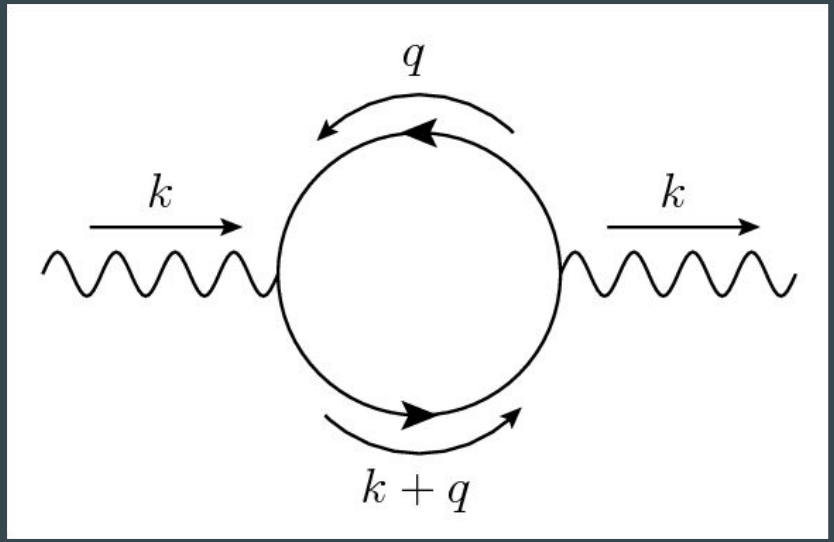
```
omegaomegatoomegaamptotal = Total[omegaomegatoomegaampsbis /. FAGaugeXi[S[4]] → 1, 1]
                            total
```

=

$$-\frac{a^2\,S^2}{v^2\left(S-\mathrm{MH}^2\right)}-\frac{a^2\,T^2}{v^2\left(T-\mathrm{MH}^2\right)}-\frac{a^2\,U^2}{v^2\left(U-\mathrm{MH}^2\right)}-\frac{2\,(S+T+U)}{v^2}$$

Example 2: Higgs self energy at one-loop

We have an amplitudes that depends on the momentum inside the loop q (q1 in FeynCalc by default)



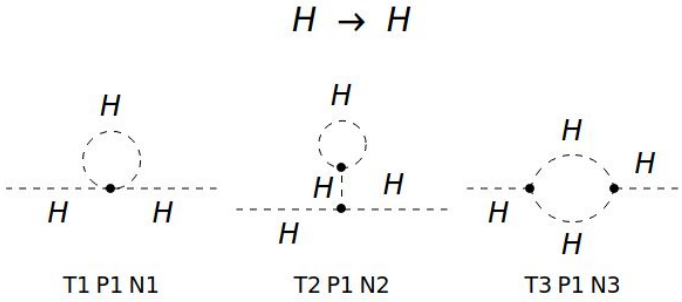

```
Paint[hhselfenergy2, PaintLevel → {Particles}, ImageSize → 500]
```

tamaño de imagen

> Top. 1 ac/bc/cc.m, 0 diagrams

> Top. 2 ac/bc/cddd.m, 0 diagrams

> Top. 3 ac/bd/cdcd.m, 0 diagrams



Example 2: Higgs self energy at one-loop

The amplitude for looks like this

```
hhselfenergy2ampnew = PickLevel[Particles][hhselfenergy2amp] // ToFA1Conventions
```

FAFeynAmpList(Process → ($S(1)$ p1 FCGV(MH) {}) → ($S(1)$ k1 FCGV(MH) {}), Model → {SM}, GenericModel → {Lorentz}, AmplitudeLevel → {Particles},
ExcludeParticles → {F, U, V, $-F(1)$, $F(1)$, $-F(2)$, $F(2)$, $-F(3)$, $F(3)$, $-F(4)$, $F(4)$, $-F(1, \{1\})$, $F(1, \{1\})$, $-F(1, \{2\})$, $F(1, \{2\})$, $-F(1, \{3\})$, $F(1, \{3\})$, $-F(2, \{1\})$, $F(2, \{1\})$, $-F(2, \{2\})$, $F(2, \{2\})$, $-F(2, \{3\})$, $F(2, \{3\})$, $-F(3, \{1, _\})$, $F(3, \{1, _\})$, $-F(3, \{2, _\})$, $F(3, \{2, _\})$, $-F(3, \{3, _\})$, $F(3, \{3, _\})$, $-F(4, \{1, _\})$, $F(4, \{1, _\})$, $-F(4, \{2, _\})$, $F(4, \{2, _\})$, $-F(4, \{3, _\})$, $F(4, \{3, _\})$, $S(2)$, $-S(3)$, $S(3)$, $-U(1)$, $U(1)$, $-U(2)$, $U(2)$, $-U(3)$, $U(3)$, $-U(4)$, $U(4)$, $V(1)$, $V(2)$, $-V(3)$, $V(3)$}, ExcludeFieldPoints → {}, LastSelections → {$S(1)$})

$$\left(\text{FAFeynAmp}\left(\text{GraphName}(, \text{T1}, \text{G1C1P1}, \text{N1}), \text{q1}, -\frac{3\, i\, \text{FCGV(EL)}^2\, \text{FCGV(MH)}^2\, \text{FAFeynAmpDenominator}\left(\frac{1}{\text{q1}^2-\text{FCGV(MH)}^2}\right)}{128\, \pi^4\, \text{FCGV(MW)}^2\, \text{FCGV(SW)}^2}\right),\right.$$

$$\text{FAFeynAmp}\left(\text{GraphName}(, \text{T2}, \text{G1C2P1}, \text{N2}), \text{q1}, -\frac{9\, i\, \text{FCGV(EL)}^2\, \text{FCGV(MH)}^4\, \text{FAFeynAmpDenominator}\left(\frac{1}{\text{q1}^2-\text{FCGV(MH)}^2}\right)\frac{1}{(\text{p1}-\text{k1})^2-\text{FCGV(MH)}^2}}{128\, \pi^4\, \text{FCGV(MW)}^2\, \text{FCGV(SW)}^2}\right),$$

$$\left.\text{FAFeynAmp}\left(\text{GraphName}(, \text{T3}, \text{G1C3P1}, \text{N3}), \text{q1}, -\frac{9\, i\, \text{FCGV(EL)}^2\, \text{FCGV(MH)}^4\, \text{FAFeynAmpDenominator}\left(\frac{1}{\text{q1}^2-\text{FCGV(MH)}^2}, \frac{1}{(\text{q1}-\text{k1})^2-\text{FCGV(MH)}^2}\right)}{128\, \pi^4\, \text{FCGV(MW)}^2\, \text{FCGV(SW)}^2}\right)\right)$$

Example 2: Higgs self energy at one-loop

Let us inspect the amplitude

```
Dimensions[hhselfenergy2ampnew]
dimensiones

{3}
```

```
hhselfenergy2ampnew[[3, 3]] // InputForm
                               forma de entrada

utForm=
(((-9*I)/128)*FAFeynAmpDenominator[FAPropagatorDenominator[q1, FCGV["MH"]], FAPropagatorDenominator[-k1 + q1, FCGV["MH"]]]*FCGV["EL"]^2*FCGV["MH"]^4)/(Pi^4*FC
```

Create a table containing the amplitude of each diagram

```
hhselfenergy2amps = Table[hhselfenergy2ampnew[[i, 3]], {i, 1, 3}] /. {FCGV[x_] → ToExpression[x]} /. {p1 → k1}
                    tabla                                                  convierte en expresión
```

$$\left\{-\frac{3\,i\,\mathrm{EL}^2\,\mathrm{MH}^2\,\mathrm{FAFeynAmpDenominator}\left(\frac{1}{\mathrm{q1}^2-\mathrm{MH}^2}\right)}{128\,\pi^4\,\mathrm{MW}^2\,\mathrm{SW}^2},\ -\frac{9\,i\,\mathrm{EL}^2\,\mathrm{MH}^4\left(-\frac{1}{\mathrm{MH}^2}\right)\mathrm{FAFeynAmpDenominator}\left(\frac{1}{\mathrm{q1}^2-\mathrm{MH}^2}\right)}{128\,\pi^4\,\mathrm{MW}^2\,\mathrm{SW}^2},\ -\frac{9\,i\,\mathrm{EL}^2\,\mathrm{MH}^4\,\mathrm{FAFeynAmpDenominator}\left(\frac{1}{\mathrm{q1}^2-\mathrm{MH}^2},\frac{1}{(\mathrm{q1}-\mathrm{k1})^2-\mathrm{MH}^2}\right)}{128\,\pi^4\,\mathrm{MW}^2\,\mathrm{SW}^2}\right\}$$

Example 2: Higgs self energy at one-loop

To integrate the momenta in the loop we use the function OneLoop

```
OneLoop[q1, hhselfenergy2amps]
```

In order for OneLoop to work the Propagators and Momenta need to be defined correctly (right dimension, syntax and might change among versions of FeynCalc)

```
hhselfenergy2amps // InputForm
                     forma de entrada

InputForm=
{(((-3*I)/128)*"EL"^2*"MH"^2*FAFeynAmpDenominator[FAPropagatorDenominator[q1, "MH"]])/("MW"^2*"SW"^2*Pi^4),
 (((-9*I)/128)*"EL"^2*"MH"^4*FAFeynAmpDenominator[FAPropagatorDenominator[q1, "MH"]]*FAPropagatorDenominator[0, "MH"])/
  ("MW"^2*"SW"^2*Pi^4), (((-9*I)/128)*"EL"^2*"MH"^4*FAFeynAmpDenominator[FAPropagatorDenominator[q1, "MH"],
    FAPropagatorDenominator[-k1 + q1, "MH"]])/("MW"^2*"SW"^2*Pi^4)}
```

Example 2: Higgs self energy at one-loop

Write the Propagators and Momenta in the correct way

```
hhselfenergy2ampsmod =
 hhselfenergy2amps  /. FAFeynAmpDenominator[FAPropagatorDenominator[x_, y_], FAPropagatorDenominator[z_, w_]] →
    FAFeynAmpDenominator[FAPropagatorDenominator[x, y]] * FAFeynAmpDenominator[FAPropagatorDenominator[z, w]] /.
  FAFeynAmpDenominator[FAPropagatorDenominator[x_, y_]] → FeynAmpDenominator[PropagatorDenominator[Momentum[x, D], y]]
                                                                                                          deriva
```

$$= \left\{-\frac{3\,i\,EL^2\,MH^2}{128\,\pi^4\,MW^2\,SW^2\,(q1^2-MH^2)},\ -\frac{9\,i\,EL^2\,MH^4\left(-\frac{1}{MH^2}\right)}{128\,\pi^4\,MW^2\,SW^2\,(q1^2-MH^2)},\ -\frac{9\,i\,EL^2\,MH^4}{128\,\pi^4\,MW^2\,SW^2\,(q1^2-MH^2)\,((q1-k1)^2-MH^2)}\right\}$$

Now we can apply OneLoop to obtain the result in terms of the PaVe functions

```
hhselfenergy2ampsPaVe = Table[OneLoop[q1, hhselfenergy2ampsmod[[i]]], {i, 1, 3}]
                                                                                tabla
```

$$\left\{\frac{3\,EL^2\,MH^2\,A_0(MH^2)}{128\,\pi^2\,MW^2\,SW^2},\ \frac{9\,EL^2\,MH^4\,(-MH^2)\,\frac{1}{MH^4}\,A_0(MH^2)}{128\,\pi^2\,MW^2\,SW^2},\ \frac{9\,EL^2\,MH^4\,B_0(\overline{k1}^2,\,MH^2,\,MH^2)}{128\,\pi^2\,MW^2\,SW^2}\right\}$$

Example 2: Higgs self energy at one-loop

I can set the momentum on-shell

```
hhselfenergy2ampsPaVe2 = hhselfenergy2ampsPaVe /. Pair[Momentum[k1], Momentum[k1]] → MH^2
```

$$\left\{\frac{3\,EL^2\,MH^2\,A_0(MH^2)}{128\,\pi^2\,MW^2\,SW^2},\ \frac{9\,EL^2\,MH^4\,(-MH^2)\,\frac{1}{MH^4}\,A_0(MH^2)}{128\,\pi^2\,MW^2\,SW^2},\ \frac{9\,EL^2\,MH^4\,B_0(MH^2,\,MH^2,\,MH^2)}{128\,\pi^2\,MW^2\,SW^2}\right\}$$

```
hhselfenergy2ampsPaVetotal = Total[hhselfenergy2ampsPaVe2, 1]
```

total

$$\frac{9\,EL^2\,MH^4\,B_0(MH^2,\,MH^2,\,MH^2)}{128\,\pi^2\,MW^2\,SW^2}+\frac{3\,EL^2\,MH^2\,A_0(MH^2)}{128\,\pi^2\,MW^2\,SW^2}+\frac{9\,EL^2\,(-MH^2)\,\frac{1}{MH^4}\,MH^4\,A_0(MH^2)}{128\,\pi^2\,MW^2\,SW^2}$$

The final amplitude is the sum of the three terms


```
hhselfenergy2ampsPaVetotal = Total[hhselfenergy2ampsPaVe2, 1]
```

total

$$\frac{9\,EL^2\,MH^4\,B_0(MH^2,\,MH^2,\,MH^2)}{128\,\pi^2\,MW^2\,SW^2}+\frac{3\,EL^2\,MH^2\,A_0(MH^2)}{128\,\pi^2\,MW^2\,SW^2}+\frac{9\,EL^2\,(-MH^2)\,\frac{1}{MH^4}\,MH^4\,A_0(MH^2)}{128\,\pi^2\,MW^2\,SW^2}$$

Hands on!

Exercise 1:

- Use the amplitude from the previous session to calculate the process hh-> hh

Exercise 2:

- Use the amplitude from the previous session to calculate the one-loop self-energy of the Higgs boson with only Higgs bosons in the loop

Hands on!

Exercise 3 (extra):

- Use the amplitude from the previous session to calculate the process G0G0-> G0G0