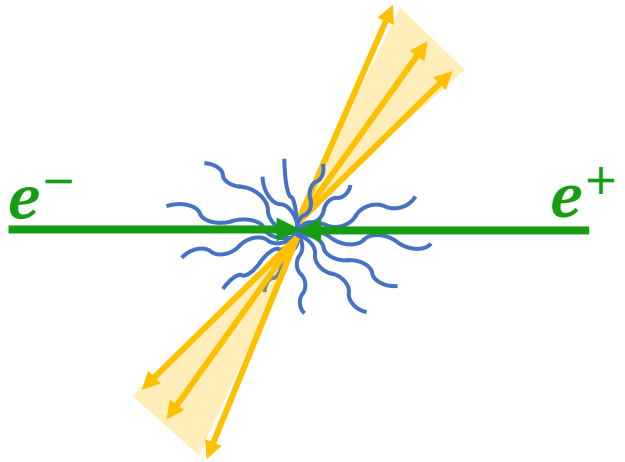
SCET and bHQET jet functions and matching coefficients in the large-β_0 limit



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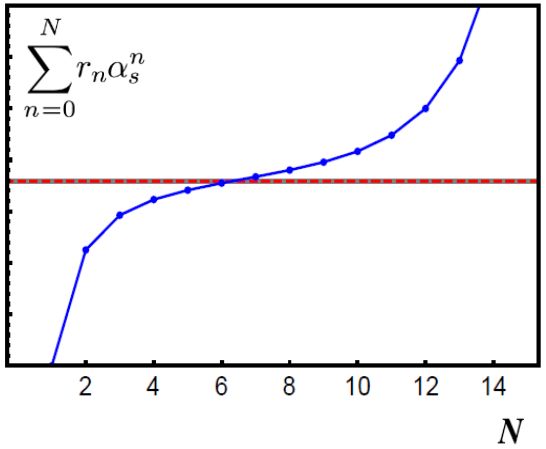
N. G. Gracia and V. Mateu. Toward massless and massive event shapes in the large-β0 limit. *Journal of High Energy Physics*, 2021(7), jul 2021.

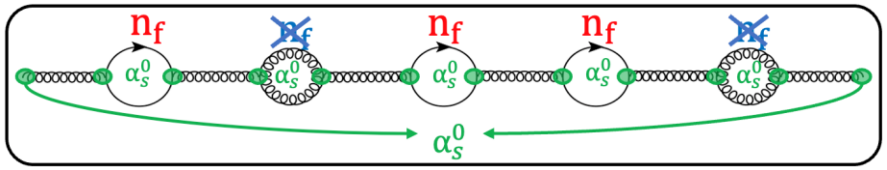
Context and motivation

- Perturbative series $\sum_{n=0}^{\infty} r_n \alpha_s^n$ in QCD (and EFTs) are in general divergent but asymptotic



- Studies of their large order behavior provide useful information
  - Estimate of the sum of the series (plateau)
  - Investigate the (size of) the non-perturbative corrections associated with the series (hadronization)
  - Proper renormalization schemes to improve convergence (leading renormalon cancellation)

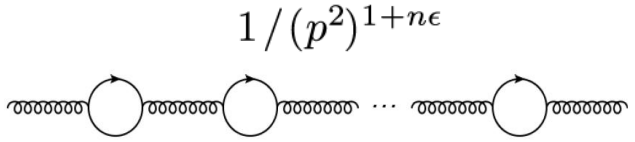
- These studies require the knowledge of all coefficients, so they are usually carried out in the large-β_0 limit, in which one can compute all $\mathcal{O}(\alpha_s^n n_f^{n-1})$ contributions to $\sum_{n=0}^{\infty} r_n \alpha_s^n$



Coefficients of highest n_f power

$$\sum_{n=0}^{\infty} r_n \alpha_s^n = 1 + \frac{1}{\beta_0} \sum_{i=1}^{\infty} c_i \left(\frac{\alpha_s \beta_0}{4\pi} \right)^i + \mathcal{O}\left(\frac{1}{\beta_0^2} \right)$$

Only contributions from



Our work

N. G. Gracia and V. Mateu. Toward massless and massive event shapes in the large-β0 limit. *Journal of High Energy Physics*, 2021(7), jul 2021.

- Factorization theorems exist for the event-shape cross-section of $e^+e^- \longrightarrow$ Hadrons in the peak region

PHYSICAL REVIEW D **77,** 074010 (2008)

Jets from massive unstable particles: Top-mass determination

Sean Fleming,[1,*] Andre H. Hoang,[2,†] Sonny Mantry,[3,‡] and Iain W. Stewart[4,§]

in SCET

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times J_n \otimes J_{\bar{n}} \otimes S$$

Hard function (universal QCD matching onto SCET)

SCET jet function (hemisphere mass and trust)

PHYSICAL REVIEW D **78,** 034027 (2008)

Factorization of e^+e^- event shape distributions with hadronic final states in soft collinear effective theory

Christian W. Bauer,[1,*] Sean Fleming,[2,+] Christopher Lee,[1,‡] and George Sterman[3,§]

and in bHQET

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}e} = H_Q \times H_m \times B_n \otimes B_{\bar{n}} \otimes S$$

Mass-scale hard function (SCET with masses onto bHQET)

bHQET jet function for hemisphere mass

- We tackled the large-β_0 computation of the jet functions for thrust and matching coefficients

  1) Derived a framework of general formulas for a systematic study

  2) Applied it to H_Q, H_m, J_n and B_n

Our framework

In preparation we derived some general relations

Our framework

For series with cusp anomalous dimension (1-loop divergences start as $1/\epsilon^2$) the following relations hold

$$\beta_0\delta A = \frac{\pi^2}{6}G_{1,0}+G_{0,0}\log\left(\frac{\mu^2}{\mathcal{Q}^2}\right)+\left[\frac{G_{0,0}}{\beta_{\mathcal{Q}}}-G_{0,1}\right]\log\left(\frac{\beta}{\beta_{\mathcal{Q}}}\right)+\int_0^\infty \mathrm{d}\tau\left(\frac{\Lambda_{\mathrm{QCD}}}{\mathcal{Q}}\right)^{2\tau}\left[\frac{G(0,\tau)-G(0,0)}{\tau^2}-\frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}u}G(0,u)\Big|_{u=0}\right]$$
$$-\int_{-\beta}^0 \mathrm{d}\tau\left\{\log\left(1+\frac{\tau}{\beta}\right)\frac{G(\tau,0)-G(0,0)}{\tau^2}-\frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}u}[G(\tau,u)-G(0,u)]\Big|_{u=0}-\log\left(\frac{\mu^2}{\mathcal{Q}^2}\right)\frac{G(\tau,0)-G(0,0)}{\tau}\right\}.$$

Renormalized series

$$\gamma_A(\beta) = \frac{2}{\beta_0}\int_{-\beta}^0 \mathrm{d}\tau\frac{G(\tau,0)-G(-\beta,0)}{\beta+\tau}+\frac{2\beta}{\beta_0}\frac{\mathrm{d}G(-\beta,s)}{\mathrm{d}s}\Big|_{s=0}$$
$$\Gamma_A(\beta) = \frac{2\beta}{\beta_0}G(-\beta,0)$$

Cusp and non-cusp anomalous dimensions

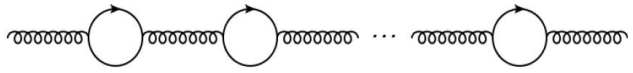
$$\mu\frac{\mathrm{d}A}{\mathrm{d}\mu}=\gamma_A+\log\left(\frac{\mu^2}{\omega^2}\right)\Gamma_A$$

Comments

1) Notation

$G^\mu(\epsilon,u)$ comes from step 1 of computation:

$$\delta A\equiv A-1$$
$$\beta_{\mathcal{Q}}\equiv\alpha_s(\mathcal{Q})\beta_0/(4\pi)$$
$$\beta_\mu\equiv\beta$$

Insertion of  (1-loop difficulty)

QCD renormalization $\alpha_s^0=Z_\alpha\alpha_s=\frac{\alpha_s}{1+\alpha_s\beta_0/(4\pi\epsilon)}$ and $\mathcal{O}(1/\beta_0)$ expansion

$$\beta_0\delta A_0 = \sum_{n=1}^\infty\beta^n(-1)^n\epsilon^{-n}\sum_{\ell=1}^n\frac{(-1)^\ell\Gamma(n)}{\Gamma(\ell)\Gamma(n-\ell+1)}\frac{G^\mu(\epsilon,\ell\epsilon)}{\ell^2\epsilon}$$

$$G^\mu(\epsilon,u)=\left(\frac{\mu^2}{\mathcal{Q}^2}\right)^u G(\epsilon,u)$$

In preparation we derived some general relations

Our framework

For series with cusp anomalous dimension (1-loop divergences start as $1/\epsilon^2$) the following relations hold

$$\beta_0\delta A = \frac{\pi^2}{6}G_{1,0} + G_{0,0}\log\left(\frac{\mu^2}{\mathcal{Q}^2}\right) + \left[\frac{G_{0,0}}{\beta_{\mathcal{Q}}} - G_{0,1}\right]\log\left(\frac{\beta}{\beta_{\mathcal{Q}}}\right) + \int_0^\infty \mathrm{d}\tau \left(\frac{\Lambda_{\mathrm{QCD}}}{\mathcal{Q}}\right)^{2\tau}\left[\frac{G(0,\tau)-G(0,0)}{\tau^2} - \frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}u}G(0,u)\Big|_{u=0}\right]$$
$$-\int_{-\beta}^0 \mathrm{d}\tau\left\{\log\left(1+\frac{\tau}{\beta}\right)\frac{G(\tau,0)-G(0,0)}{\tau^2} - \frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}u}[G(\tau,u)-G(0,u)]\Big|_{u=0} - \log\left(\frac{\mu^2}{\mathcal{Q}^2}\right)\frac{G(\tau,0)-G(0,0)}{\tau}\right\}.$$

Renormalized series

$$\gamma_A(\beta) = \frac{2}{\beta_0}\int_{-\beta}^0 \mathrm{d}\tau \frac{G(\tau,0)-G(-\beta,0)}{\beta+\tau} + \frac{2\beta}{\beta_0}\frac{\mathrm{d}G(-\beta,s)}{\mathrm{d}s}\Big|_{s=0}$$
$$\Gamma_A(\beta) = \frac{2\beta}{\beta_0}G(-\beta,0)$$

Cusp and non-cusp anomalous dimensions

$$\mu\frac{\mathrm{d}A}{\mathrm{d}\mu} = \gamma_A + \log\left(\frac{\mu^2}{\omega^2}\right)\Gamma_A$$

Comments

2) Finite vs Divergent

- Divergences are enclosed in a single term
- Anomalous dimensions are finite (finite convergence radius as perturbative series)

In preparation we derived some general relations

Our framework

For series with cusp anomalous dimension (1-loop divergences start as $1/\epsilon^2$) the following relations hold

$$\beta_0\delta A = \frac{\pi^2}{6}G_{1,0}+G_{0,0}\log\left(\frac{\mu^2}{\mathcal{Q}^2}\right)+\left[\frac{G_{0,0}}{\beta_{\mathcal{Q}}}-G_{0,1}\right]\log\left(\frac{\beta}{\beta_{\mathcal{Q}}}\right)+\int_0^\infty \mathrm{d}\tau\left(\frac{\Lambda_{\rm QCD}}{\mathcal{Q}}\right)^{2\tau}\left[\frac{G(0,\tau)-G(0,0)}{\tau^2}-\frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}u}G(0,u)\Big|_{u=0}\right]$$
$$-\int_{-\beta}^0 \mathrm{d}\tau\left\{\log\left(1+\frac{\tau}{\beta}\right)\frac{G(\tau,0)-G(0,0)}{\tau^2}-\frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}u}[G(\tau,u)-G(0,u)]\Big|_{u=0}-\log\left(\frac{\mu^2}{\mathcal{Q}^2}\right)\frac{G(\tau,0)-G(0,0)}{\tau}\right\}.$$

Renormalized series

$$\gamma_A(\beta) = \frac{2}{\beta_0}\int_{-\beta}^0 \mathrm{d}\tau\frac{G(\tau,0)-G(-\beta,0)}{\beta+\tau}+\frac{2\beta}{\beta_0}\frac{\mathrm{d}G(-\beta,s)}{\mathrm{d}s}\Big|_{s=0}$$
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Cusp and non-cusp anomalous dimensions

$$\mu\frac{\mathrm{d}A}{\mathrm{d}\mu}=\gamma_A+\log\left(\frac{\mu^2}{\omega^2}\right)\Gamma_A$$

Comments

3) Properties of the divergent term

- Divergences are encoded by the poles $G(0,\tau)$: renormalon structure
- Easy to use **PV** prescription to define the integral and hence the sum of the series
  - The difference between jumping poles from above/below defines the ambiguity of the sum
  - It equals the sum of the residues of $G(0,\tau)$ in $\mathbb{R}^+$, which is muliplied by $(\Lambda_{\rm QCD}/\mathcal{Q})^{2\tau}$
  - No μ-dependence

Results

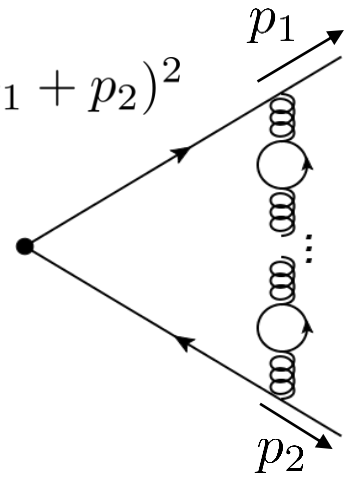
Hard function

Match QCD onto SCET (masless) $\quad H_Q(Q,\mu) = |C_H(Q^2 + i0^+, \mu)|^2$

$$G_{H_Q}(\epsilon,u) = 2C_F e^{\gamma_E u}[(u-1)\epsilon^2 + (u^2-2u+3)\epsilon - 2]\cos(\pi u) \times \frac{\Gamma(1+u)\Gamma^2(1-u)}{\Gamma(3-u-\epsilon)}\left[\frac{6\Gamma(1+\epsilon)\Gamma^2(2-\epsilon)}{\Gamma(4-2\epsilon)}\right]^{\frac{u}{\epsilon}-1}$$

- numerically interated
- poles and residues computed analitically

$Q^2 = (p_1+p_2)^2$



	Poles	Order	Crossed
$G_H(\epsilon,0)$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$G_H(0,u)$	$-n, \quad n=2,3,4\ldots$	simple	No
	$1,2$	double and simple	Yes
	$n, \quad n=3,4,5\ldots$	simple	Yes

The ambiguity can again be resumed

$$\delta_\Lambda H_Q = \frac{C_F}{\beta_0}\left\{\frac{8}{3}\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^2\left[6\log\left(\frac{\Lambda_{\rm QCD}}{Q}\right)+5\right] - \frac{2}{3}\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^4\left[12\log\left(\frac{\Lambda_{\rm QCD}}{Q}\right)+1\right]\right.$$
$$\left. + 4\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^2 - 6\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^4 + 4\left[\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^2 - 1\right]^2\log\left[1-\left(\frac{e^{\frac{5}{6}}\Lambda_{\rm QCD}}{Q}\right)^2\right]\right\}$$

Logaritmic enhancement from double poles

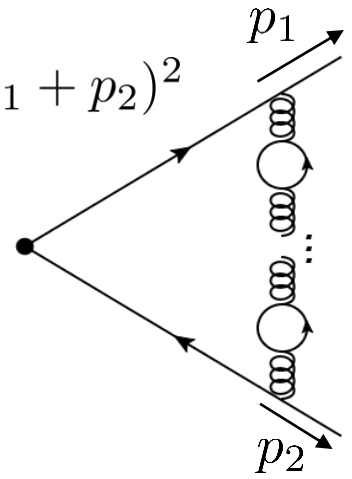
Hard function

Match QCD onto SCET (masless) $\quad H_Q(Q,\mu) = |C_H(Q^2 + i0^+, \mu)|^2$

$$G_{H_Q}(\epsilon,u) = 2C_F e^{\gamma_E u}[(u-1)\epsilon^2 + (u^2-2u+3)\epsilon - 2]\cos(\pi u) \times \frac{\Gamma(1+u)\Gamma^2(1-u)}{\Gamma(3-u-\epsilon)}\left[\frac{6\Gamma(1+\epsilon)\Gamma^2(2-\epsilon)}{\Gamma(4-2\epsilon)}\right]^{\frac{u}{\epsilon}-1}$$

- numerically interated
- poles and residues computed analitically

$Q^2 = (p_1+p_2)^2$

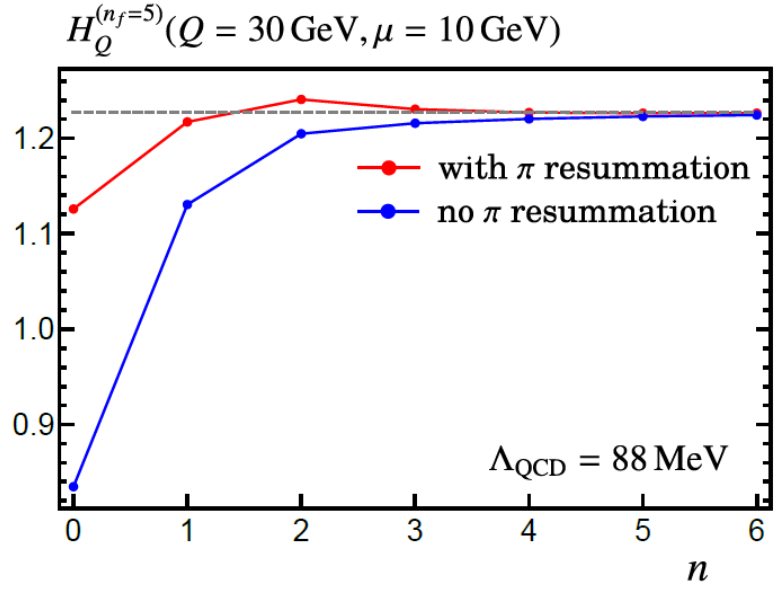


	Poles	Order	Crossed
$G_H(\epsilon,0)$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$G_H(0,u)$	$-n, \quad n=2,3,4\ldots$	simple	No
	$1,2$	double and simple	Yes
	$n, \quad n=3,4,5\ldots$	simple	Yes

π-resummation improves convergence

$\mu_0 = -iQ$

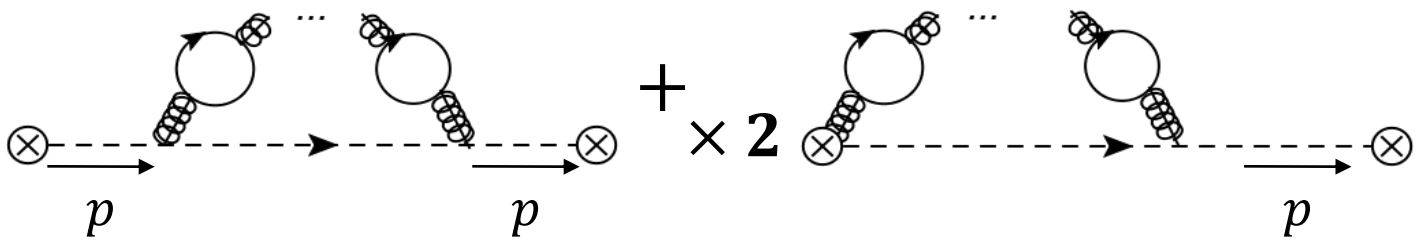
$\mu_0 = Q$



Despite logarithmic enhancement, the ambiguity is very small

Jet field in SCET

SCET jet function



	Poles	Order	Crossed
$G_{\tilde{J}}(\epsilon, 0)$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$G_{\tilde{J}}(0, u)$	$1, 2$	simple	Yes

We take the Fourier transform w.r.t. $p^2 = s$ to avoid distributions

There are only two contributions to the ambiguity

$$\delta_{\tilde{J}} = -\frac{2C_F}{\beta_0}\left[\mathrm{i}ye^{5/3}\Lambda^2_{\mathrm{QCD}} + \left(\frac{1}{2}\mathrm{i}ye^{5/3}\Lambda^2_{\mathrm{QCD}}\right)^2 \right]$$

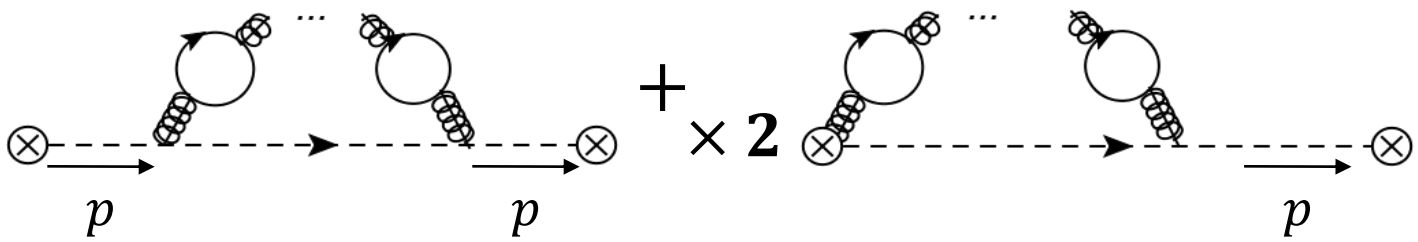
For real y

$\mathrm{Re}[\tilde{J}]$ is free from $u = 1$ renormalon

$\mathrm{Im}[\tilde{J}]$ is free from $u = 2$ renormalon

Jet field in SCET

SCET jet function



	Poles	Order	Crossed
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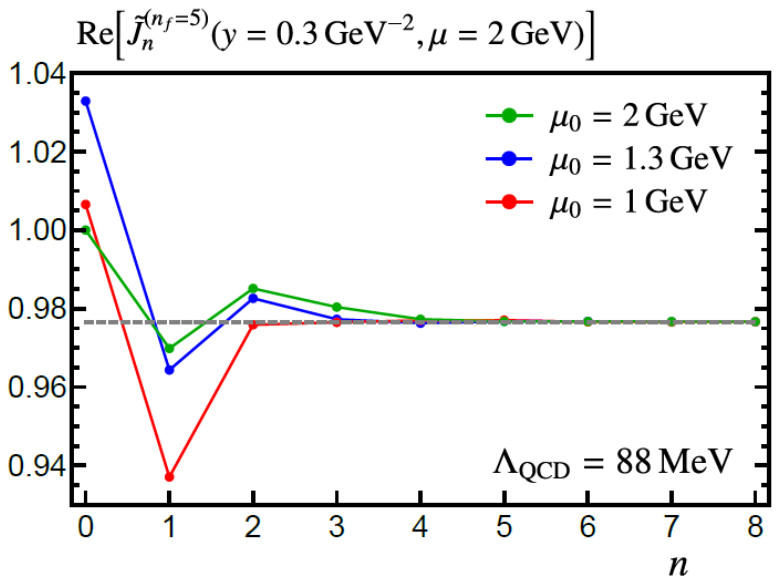
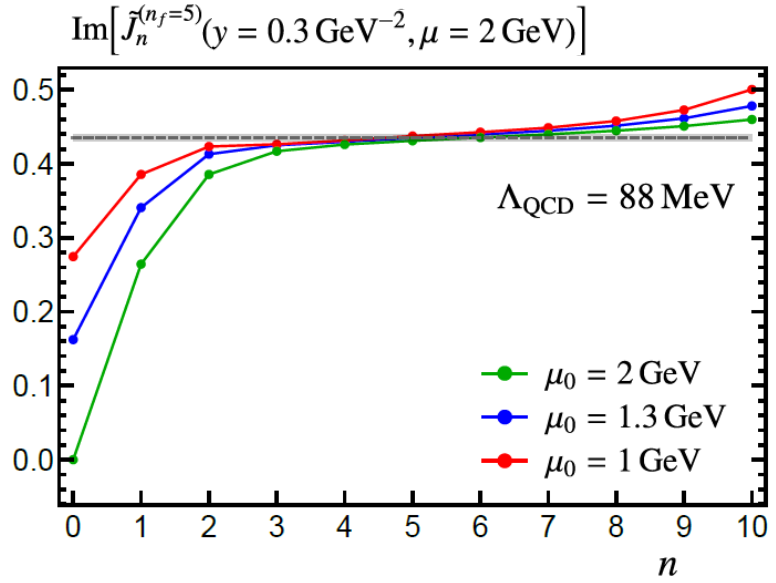
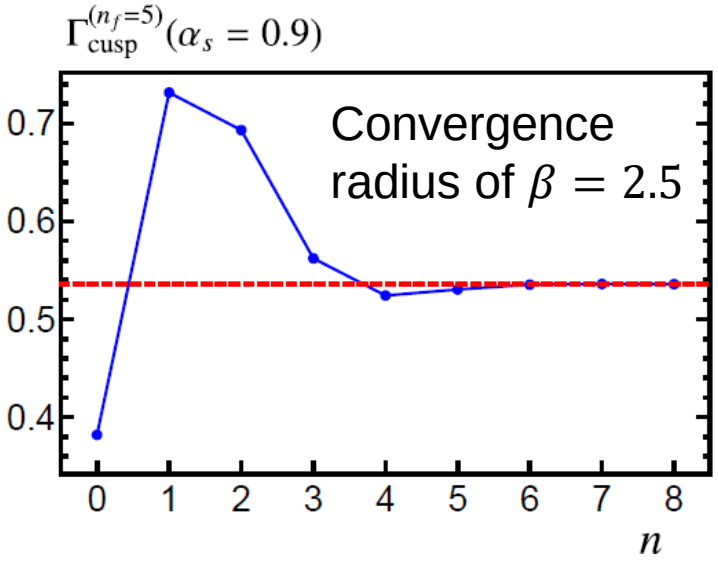
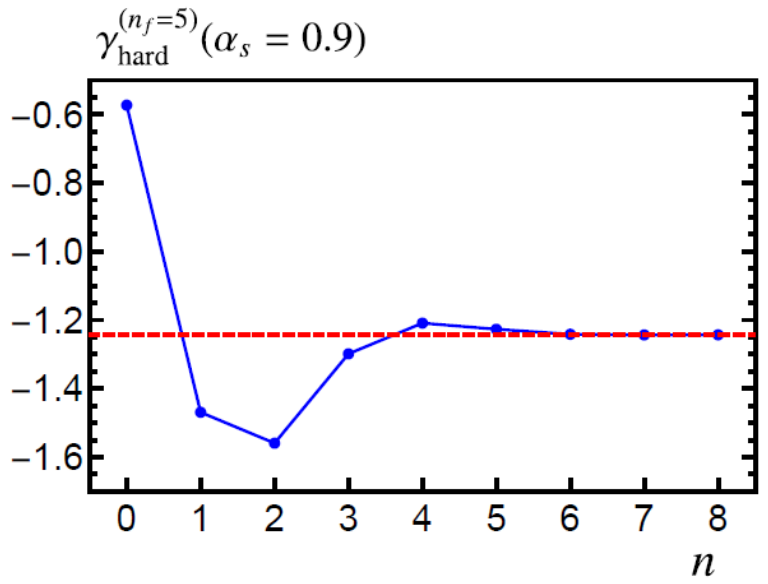
$G_{\tilde{J}}(0, u)$	$1, 2$	simple	Yes

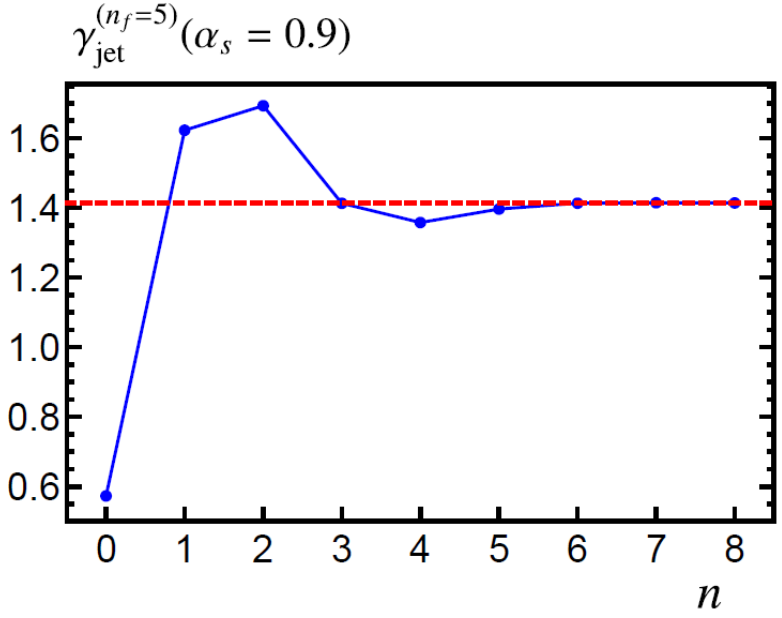


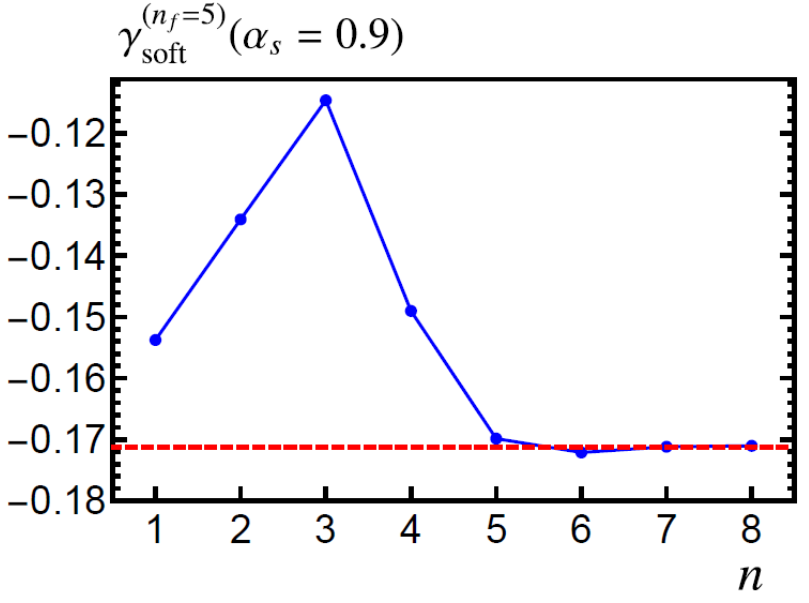


SCET anomalous dimensions









- We recover the universal cusp anomalous dimension (cross-chek)
- Compute $\gamma_{\rm hard}(\alpha_s)$, $\gamma_{\rm jet}(\alpha_s)$
- And by consistency predict $\gamma_{\rm soft}(\alpha_s)$

$$\Gamma_{\rm cusp}(\beta) = \frac{2C_F}{3\pi}\frac{\sin(\pi\beta)\Gamma(4+2\beta)}{\beta_0\Gamma(2+\beta)^2}$$

agrees with

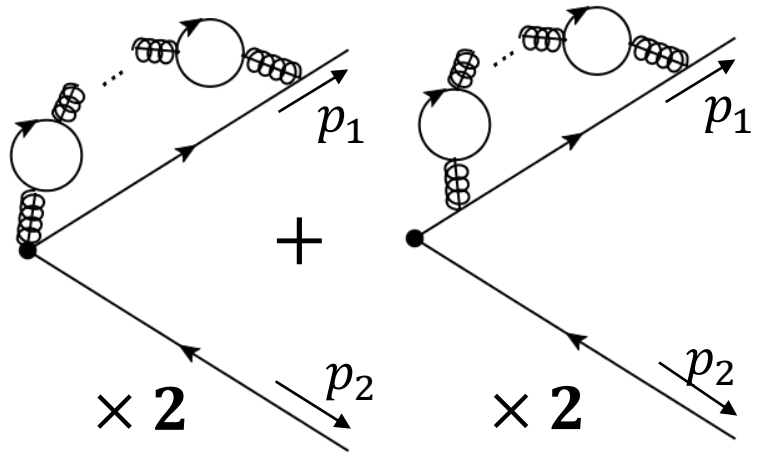
Ignazio Scimemi and Alexey Vladimirov. Power corrections and renormalons in Transverse Momentum Distributions. *JHEP*, 03:2, 2017.

Mass-scale hard function

Match massive SCET onto bHQET

$$H_m(m,\mu) = |C_H(m,\mu)|^2$$

	Poles	Order	Crossed
$G_{H_m}(\epsilon,0)$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$G_{H_m}(0,u)$	$(2n+1)/2, \quad n=0,1,2\ldots$	simple	Yes
	$-n, \quad n=2,3,4\ldots$	simple	No
	$1,2$	simple	Yes



The leading renormalon for H_m lays then at u =1/2 and its ambiguity is

$$\delta_{H_m} = -\frac{6e^{\frac{5}{6}}C_F}{\beta_0}\frac{\Lambda_{\rm QCD}}{m}$$

This is three times higher tan the pole's mass ambiguity: $\delta_{m_p} = -\frac{2C_F}{\beta_0}e^{5/6}\Lambda_{\rm QCD}$

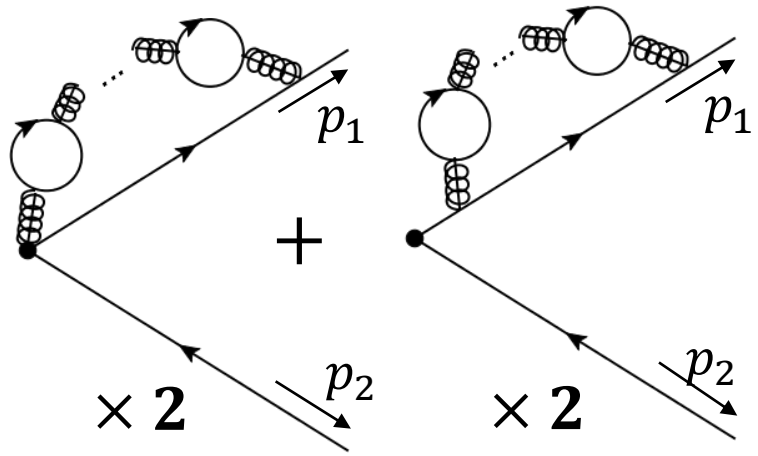
Therefore the combination H_m/m_p^3 is free from the leading ambiguity

Mass-scale hard function

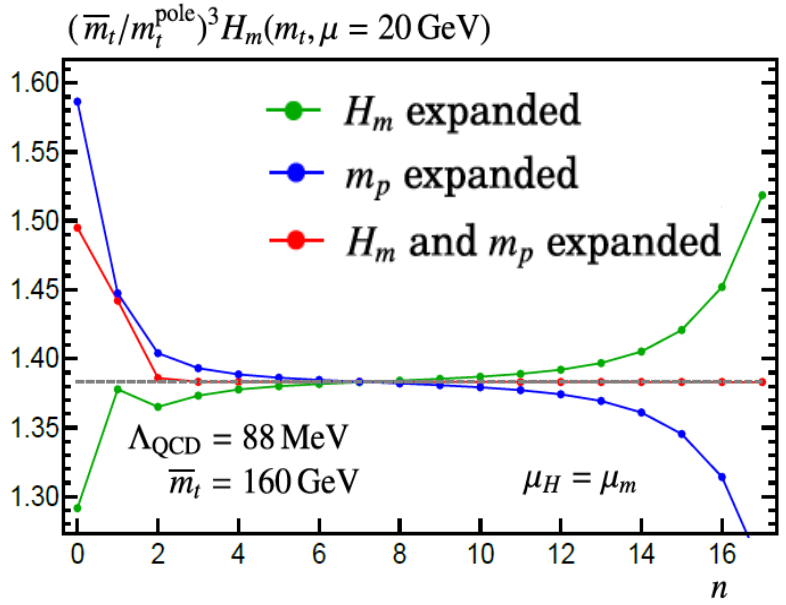
Match massive SCET onto bHQET

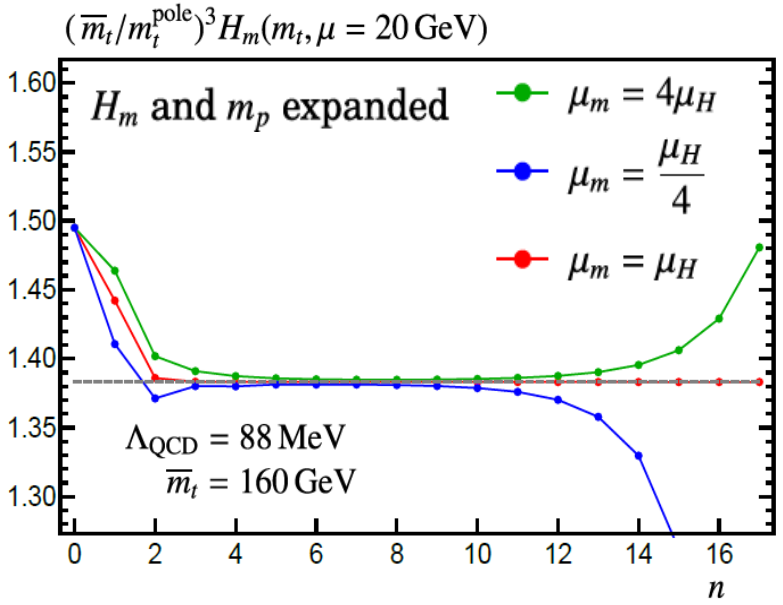
$$H_m(m,\mu) = |C_H(m,\mu)|^2$$

	Poles	Order	Crossed
$G_{H_m}(\epsilon, 0)$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$G_{H_m}(0, u)$	$(2n+1)/2, \quad n=0,1,2\ldots$	simple	Yes
	$-n, \quad n=2,3,4\ldots$	simple	No
	$1, 2$	simple	Yes



Therefore the combination H_m/m_p^3 is free from the leading ambiguity



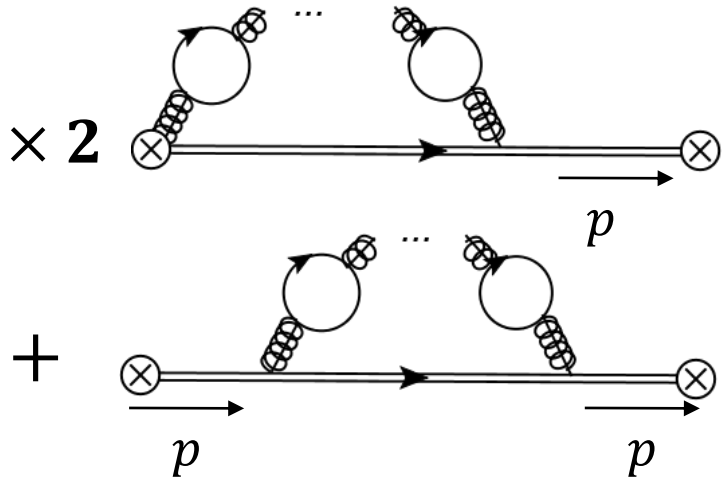


Both series need to be expanded (left) in terms of the same $\alpha_s(\mu)$ (right)

Jet field in bHQET

bHQET jet function

	Poles	Order	Crossed
$G_{\tilde{B}}(\epsilon, 0)$	$(2n+1)/2, \quad n=2,3,4\ldots$	simple	No
$G_{\tilde{B}}(0, u)$	$n, \quad n=2,3,4\ldots$	simple	Yes
	$1/2$	simple	Yes

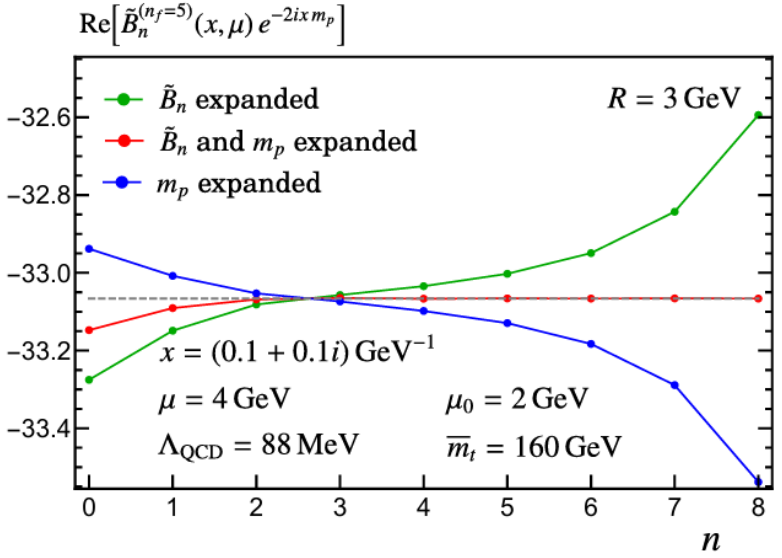


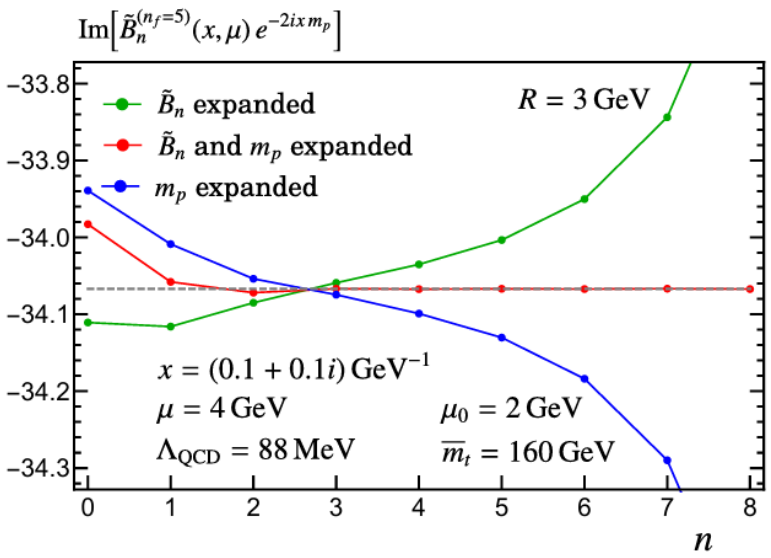
This time the leading ambiguity is twice that of the pole

$$\delta_{\tilde{B}} = -\frac{4C_F e^{5/6}}{\beta_0} i x \Lambda_{\rm QCD} \qquad \delta_{m_p} = -\frac{2C_F}{\beta_0} e^{5/6} \Lambda_{\rm QCD}$$

Fourier transform w.r.t. $\hat{s} = (s - m^2)/m$ to avoid distributions

Therefore the combination $\tilde{B}(x)e^{-2ixm_p}$ is free from the leading renormalon





Conclussions and future work

- We have developed a formalism to analyze the asymptotic properties for series without and with cusp anomalous dimension (EFTs)
    - All the perturbative coefficients agree with the up-to-date literature
    - The resummed results have led to strong observations:
        - matrix elements are all divergent, while anomalous dimensions are convergent
        - renormalon structure of the factorization theorems
        - the role of μ in the asymptotic behavior and renormalon cancelations

Extensions and further work

- mass corrections with one heavy bubble can be related to our EGP **[Bris&Mateu]**
- investigate the soft function and develop a resummed factorization theorem
- investigate the implementation of pi resummation and the found leading renormalon cancelations

Back up slides

The situation in QCD

$$\bar{R}(\alpha) \asymp R(\alpha) = 1 + \sum_{n=1}^{\infty} r_n \alpha^n$$

- The function $\bar{R}(\alpha)$ is unknown
- The power series is divergent: $r_n \sim n! a^n n^b$

To estimate $\bar{R}(\alpha)$ with the power series:

Borel summation

$$1 + \sum_{n=1}^{\infty} r_n \alpha^n = 1 + \sum_{n=1}^{\infty} \frac{r_n}{(n-1)!} \alpha^n \int_0^{\infty} \mathrm{d}t\, e^{-t} t^{n-1}$$

$$\Gamma(n) = (n-1)! = \int_0^{\infty} \mathrm{d}t\, t^{n-1} e^{-t}$$

$$R_B(\alpha) \equiv 1 + \alpha \int_0^{\infty} \mathrm{d}t\, e^{-t} B[R(\alpha)](\alpha t)$$

$$B[R(\alpha)](u) \equiv \sum_{n=0}^{\infty} \frac{r_{n+1}}{n!} u^n$$

Improves the convergence of the original series

The situation in QCD

Borel summation

$$R_B(\alpha) \equiv 1 + \alpha \int_0^\infty \mathrm{d}t\, e^{-t} B[R(\alpha)](\alpha t) \qquad B[R(\alpha)](u) \equiv \sum_{n=0}^{\infty} \frac{r_{n+1}}{n!} u^n$$

If $R(\alpha)$ has a ***non-zero radius*** of convergence in α, $B[R](u)$ has an infinite radius of convergence in u, and $R_B(\alpha)$ returns the analytical continuation of $R(\alpha)$

Example: geometric series

$$R(\alpha) = \sum_{n=0}^{\infty} (-2)^n \alpha^n = 1/(1+2\alpha) \text{ for } |\alpha| < 1/2$$

$$R_B(\alpha) = 1 - 2\alpha \int_0^\infty \mathrm{d}t\, e^{-(2\alpha+1)t} = \frac{1}{1+2\alpha}, \text{ for } \mathrm{Re}(\alpha) > -1/2$$

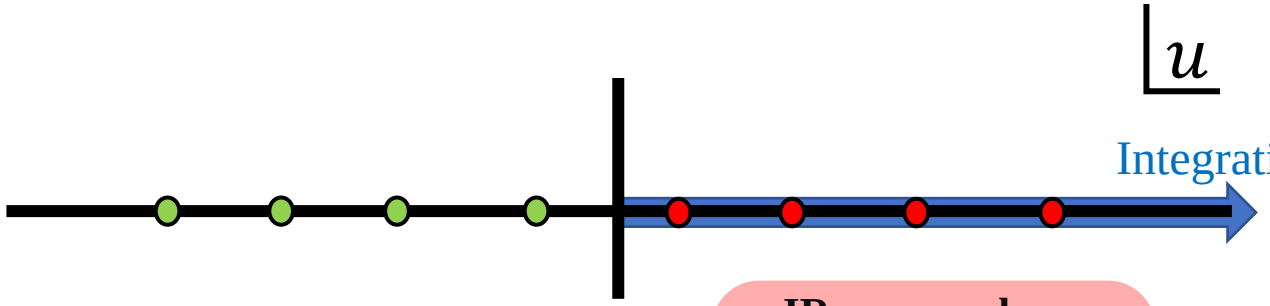
The situation in QCD

Borel summation

$$R_B(\alpha) \equiv 1 + \alpha \int_0^\infty \mathrm{d}t\, e^{-t} B[R(\alpha)](\alpha t) \qquad B[R(\alpha)](u) \equiv \sum_{n=0}^{\infty} \frac{r_{n+1}}{n!} u^n$$

If $R(\alpha)$ has a ***zero radius*** of convergence in α, the existence of $R_B(\alpha)$ is not guaranteed

The case in QCD ($r_n \sim n! a^n n^b$) is that $B[R](u)$ has poles in u



UV renormalons

u = -1/2, -1, -3/2,-2…

Alternating sign series (summable)

IR renormalons

u =1/2, 1, 3/2, 2…

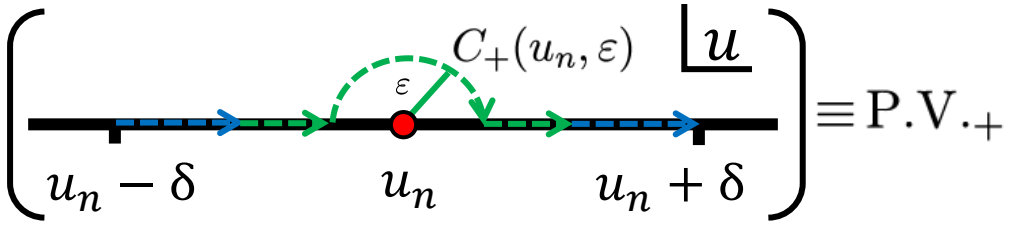
Non-alternating sign series (non-summable)

Need a prescription

The situation in QCD

Principal value prescription

Dodge poles by infinitesimally deforming the integration path



$$\equiv \text{P.V.}_+$$

$$\text{P.V.}_\pm\left\{\int_{x_n-\delta}^{x_n+\delta} \mathrm{d}x \frac{1}{(x-x_n)^k}\right\} = \begin{cases} \mp i\pi, & k=1 \\ \dfrac{-2}{(k-1)\delta^{k-1}}, & k \text{ even} \\ 0, & k \text{ odd}, \quad k>1 \end{cases}$$

Simple poles add an imaginary part

Exact value

$$\text{P.V.}\{R_B\} \equiv \frac{1}{2}[\text{P.V.}_+\{R_B\} + \text{P.V.}_-\{R_B\}]$$

Estimated resummed value of the series

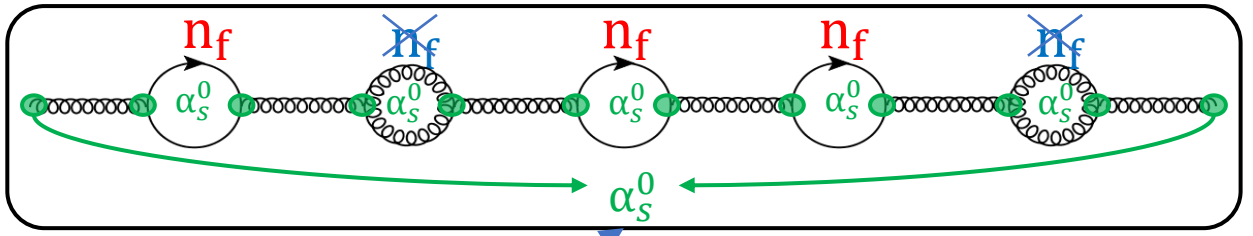
Ambiguity

$$\delta_{R_B} \equiv \frac{1}{2\pi i}[\text{P.V.}_+\{R_B\} - \text{P.V.}_-\{R_B\}]$$

Estimated resummation error
Given by sum of residues of IR renormalons.

The large-β_0 limit

We start with a series in the bare coupling



$$A_0 = 1 + \sum_{l=1}^{\infty} \left(\frac{g_0^2}{(4\pi)^2} \right)^l \sum_{n=0}^{l-1} a_{l,n} n_f^n$$

Large-β_0 expansion

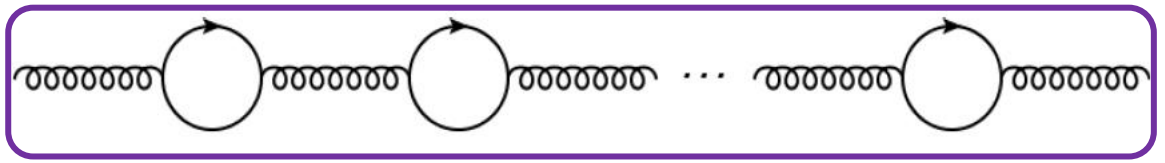
- Keep track of powers of n_f by $\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f$
- Introduce the renormalized coupling constant $\alpha_s^0 = Z_\alpha \alpha_s = \frac{\alpha_s}{1 + \alpha_s \beta_0 / (4\pi\epsilon)}$
- Expand assuming $\mathcal{O}(\alpha_s \beta_0) \sim 1$

Coefficients of highest n_f power

Only contributions from

Same functional form as gluon propagator with shifted exponent (computations are 1-loop dificulty)

$$A_0 = 1 + \frac{1}{\beta_0} \sum_{i=1}^{\infty} c_{i,l} \left(\frac{\alpha_s \beta_0}{4\pi} \right)^i + O\left(\frac{1}{\beta_0^2} \right)$$



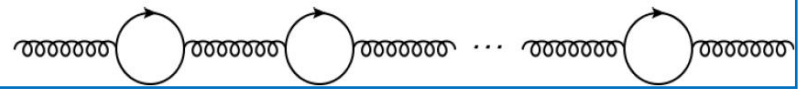
The large-β_0 limit

by only algebraic manipulations

(defining $\delta A_0 \equiv A_0 - 1$ and $\beta \equiv \alpha_s \beta_0/(4\pi)$)

$$\beta_0 \delta A_0 = \sum_{i=1}^{\infty} \beta^i (-1)^i \epsilon^{-i} \sum_{l=1}^{i} \frac{(-1)^l (l)_{i-l}}{(i-l)!} \frac{F^\mu(\epsilon, l\epsilon)}{l}$$

Insertion of efective gluon propagator with l loops



where $F^\mu(\epsilon, u) = \left(\frac{\mu^2}{\omega^2}\right)^u F(\epsilon, u)$ and ω is an external energy-scale (e.g. mass or CM energy).

From here we distinguish when $u \to 0$

- Finite series (no renormalization) $\longrightarrow F(\epsilon, u)$ starts at $O(u)$.
- Non-cusp series (1-loop divergences start as $1/\epsilon$) $\longrightarrow F(\epsilon, u)$ starts at $O(1)$
- Cusp series (1-loop divergences start as $1/\epsilon^2$) $\longrightarrow F(\epsilon, u)$ starts at $O(1/u)$

in all cases $F(\epsilon, u)$ starts at $O(1)$ when $\epsilon \to 0$.

Series without cusp anomalous dimension

For non-cusp series we have regular $F^{\mu}(\epsilon, u)$:

$$F^{\mu}(\epsilon,u) \equiv \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \epsilon^i u^j F^{\mu}_{i,j}, \qquad F^{\mu}_{i,0} \equiv F_{i,0}$$

Sketch of the comptuation:

$$\beta_0 \delta A_0 = \sum_{i=1}^{\infty} \beta^i \left[\Gamma(i) F^{\mu}_{0,i} - \frac{(-1)^i F_{i,0}}{i} \right] - \sum_{j=1}^{\infty} \frac{1}{\epsilon^j} \sum_{i=0}^{\infty} (-\beta)^{i+j} \frac{F_{i,0}}{i+j}$$

Finite term

UV divergences

Series without cusp anomalous dimension

For non-cusp series we have regular $F^\mu(\epsilon, u)$:

$$F^\mu(\epsilon,u)\equiv\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\epsilon^i u^j F^\mu_{i,j},\qquad F^\mu_{i,0}\equiv F_{i,0}$$

Sketch of the comptuation:

Integral tricks

Regularity condition (backwards)

$$\sum_{i=1}^{\infty}\beta^i\Gamma(i)F^\mu_{0,i}=\sum_{i=1}^{\infty}F^\mu_{0,i}\int_0^\infty \mathrm{d}\tau\tau^{i-1}\mathrm{e}^{-\tau/\beta}=\int_0^\infty \mathrm{d}\tau\,\mathrm{e}^{-\tau/\beta}\frac{F^\mu(0,\tau)-F(0,0)}{\tau},$$

Series without cusp anomalous dimension

UV substractions

Borel-like, ambiguous integral

Finite, unambiguous integral

$$\beta_0 \delta A = F(0,0)\log\left(\frac{\beta_\omega}{\beta}\right) + \int_0^\infty \mathrm{d}\tau \left(\frac{\Lambda_{\mathrm{QCD}}}{\omega}\right)^{2\tau} \frac{F(0,\tau)-F(0,0)}{\tau} + \int_{-\beta}^0 \mathrm{d}\tau \frac{F(\tau,0)-F(0,0)}{\tau}$$

Renormalized series

No μ-dependence

$$\mathrm{e}^{-\tau/\beta}\left(\frac{\mu}{\omega}\right)^{2\tau} = \left(\frac{\Lambda_{\mathrm{QCD}}}{\omega}\right)^{2\tau}$$

$$\beta_0 \delta Z_A = -\int_{-\beta}^0 \mathrm{d}\tau \frac{F(\tau,0)}{\epsilon-\tau}$$

Renormalization factor

$$\gamma_A(\beta) = \frac{2\beta}{\beta_0}\sum_{i=0}^{\infty}\beta^i(-1)^i F_{i,0} = \frac{2\beta}{\beta_0}F(-\beta,0)$$

Anomalous dimension

Convergent series
(non-integral form)

Residues of $F(0,\tau)$ multiplied by $(\Lambda_{\mathrm{QCD}}/\omega)^{2\tau}$

Ambiguity linked to non-perturbative power corrections.

We present an extension of the previous formalism to series with cusp-anomalous dimension

Series with cusp anomalous dimension

N. G. Gracia and V. Mateu. Toward massless and massive event shapes in the large-β0 limit. *Journal of High Energy Physics*, 2021(7), jul 2021.

$F^\mu(\epsilon,u)$ starts at $\mathcal{O}(1/u)$ so we define the regular function $G^\mu(\epsilon,u)\equiv uF^\mu(\epsilon,u)$

$$G^\mu(\epsilon,u)\equiv\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}G^\mu_{i,j}\epsilon^i u^j$$

$$\beta_0 A_0=\sum_{i=1}^{\infty}\beta^i(-1)^i\epsilon^{-i}\sum_{l=1}^{i}\frac{(-1)^l(l)_{i-l}}{(i-l)!}\frac{F^\mu(\epsilon,l\epsilon)}{l}$$

$$\beta_0\delta A_0=\sum_{i=1}^{\infty}\beta^i(-1)^i\epsilon^{-i}\sum_{l=1}^{i}\frac{(-1)^l(l)_{i-l}}{(i-l)!}\frac{G^\mu(\epsilon,l\epsilon)}{l^2\epsilon}$$

The presence of an extra l^2 complicates manipulations…

Perturbative coefficients

This term provides the anomalous dimension

$$\beta_0\delta A_0=\sum_{i=1}^{\infty}(-\beta)^i\left[G^\mu_{0,i+1}(-1)^i\Gamma(i)-\frac{H_iG_{i+1,0}}{i}-\frac{G^\mu_{i,1}}{i}\right]-\frac{1}{\epsilon}\sum_{i=1}^{\infty}\frac{(-\beta)^i}{i}(H_iG_{i,0}+G^\mu_{i-1,1})$$

$$-\sum_{j=2}^{\infty}\frac{1}{\epsilon^j}\sum_{i=0}^{\infty}\left[(-\beta)^{i+j-1}\frac{H_{i+j-1}G_{i,0}}{i+j-1}+(-\beta)^{i+j}\frac{G^\mu_{i,1}}{i+j}\right]$$

New nuisance! Harmonic numbers

We present an extension of the previous formalism to series with cusp-anomalous dimension

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$$\beta_0\delta A_0=\sum_{i=1}^{\infty}\beta^i(-1)^i\epsilon^{-i}\sum_{l=1}^{i}\frac{(-1)^l(l)_{i-l}}{(i-l)!}\frac{G^\mu(\epsilon,l\epsilon)}{l^2\epsilon}$$

The presence of an extra l^2 complicates manipulations…

The relevant (and harder to find) integral representations involving H_i are:

$$(-\beta)^i H_i = \int_{-\beta}^{0}\mathrm{d}\tau\frac{\tau^i-(-\beta)^i}{\beta+\tau}$$

$$\frac{(-\beta)^i}{i}H_i = \int_{-\beta}^{0}\mathrm{d}\tau\tau^{i-1}\log\left(1+\frac{\tau}{\beta}\right)$$

Series with cusp anomalous dimension

Series = Ambiguous + Finite convergence radius

$$\beta_0\delta A = \frac{\pi^2}{6}G_{1,0} + G_{0,0}\log\left(\frac{\mu^2}{\omega^2}\right) + \left[\frac{G_{0,0}}{\beta_\omega} - G_{0,1}\right]\log\left(\frac{\beta}{\beta_\omega}\right)$$

Renormalized cusp series

$$+\int_0^\infty \mathrm{d}\tau \left(\frac{\Lambda_{\mathrm{QCD}}}{\omega}\right)^{2\tau}\left[\frac{G(0,\tau)-G(0,0)}{\tau^2} - \frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}u}G(0,u)\Big|_{u=0}\right]$$

$$-\int_{-\beta}^0 \mathrm{d}\tau\left\{\log\left(1+\frac{\tau}{\beta}\right)\frac{G(\tau,0)-G(0,0)}{\tau^2} - \frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d}u}[G(\tau,u)-G(0,u)]\Big|_{u=0}\right\}$$

$$+\log\left(\frac{\mu^2}{\omega^2}\right)\int_{-\beta}^0 \mathrm{d}\tau\frac{G(\tau,0)-G(0,0)}{\tau}.$$

UV substractions: some terms need two of them

$$\mu\frac{\mathrm{d}A}{\mathrm{d}\mu} = \gamma_A + \log\left(\frac{\mu^2}{\omega^2}\right)\Gamma_A$$

$$\gamma_A(\beta) = \frac{2}{\beta_0}\int_{-\beta}^0 \mathrm{d}\tau\frac{G(\tau,0)-G(-\beta,0)}{\beta+\tau} + \frac{2\beta}{\beta_0}\frac{\mathrm{d}G(-\beta,s)}{\mathrm{d}s}\Big|_{s=0}$$

$$\Gamma_A(\beta) = \frac{2\beta}{\beta_0}G(-\beta,0)$$

Cusp and non-cusp anomalous dimensions

Non-cusp needs UV substraction