Flavor thresholds and quark-mass effects in the Collins-Soper kernel.

[2405....]

P. Plößl

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work in progress in collaboration with B. Dehnadi and F.J. Tackmann

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Part I

CS-kernel: Flavor matching and mass effects.

What is the CS-kernel?

CS-kernel & rapidity anomalous dimension: rapidity evolution of TMDPDFs & soft + beam functions:

CS-kernel and rapidity anomalous dimension:

$$\frac{\mathrm{d}}{\mathrm{d}\log\zeta}\log\tilde{f}_i(x,b_T,\mu,\zeta)=\frac{1}{2}\tilde{\gamma}_\zeta(b_T,\mu)\,,\qquad \frac{\mathrm{d}}{\mathrm{d}\log\nu}\log\tilde{S}(b_T,\mu,\nu)=\tilde{\gamma}_\nu(b_T,\mu)\,.$$

Equivalence between CS-kernel and rapidity anomalous dimension:

$$\tilde{f}_i(x,b_T,\mu,\zeta)=\tilde{B}_i(x,b_T,\mu,\nu/\sqrt{\zeta})\tilde{S}(b_T,\mu,\nu)\,,\quad\longrightarrow\quad \tilde{\gamma}_\zeta(b_T,\mu)=\frac{1}{2}\tilde{\gamma}_\nu(b_T,\mu)\,.$$

RGE for the rapidity anomalous dimension:

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}\tilde{\gamma}_\nu(b_T,\mu)=-4\Gamma_{\mathrm{cusp}}(\mu)\,.$$

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How to obtain the CS-kernel?

$\Lambda_{\rm QCD} \ll 1/b_T$: perturbative region

- can be calculated perturbatively.
- available up to $\mathcal{O}(\alpha_s^4)$ for gluons and massless quarks. [Moult, Zhu, Zhu, '22; Duhr, Mistelberger, Vita, '22]
- massive quark contributions known to $\mathcal{O}(\alpha_s^2)$. [Pietrulewicz et al, '17]

$1/b_T \ll \Lambda_{\rm QCD}$: non-perturbative region

- extraction from experimental data (low q_T DY & SIDIS). [MAP collaboration, '22; Moos et al, '23]
- determination through lattice calculations. [Lattice Parton Collaboration, '23; Shu et al, '23; Avkhadiev et al, '23 & '24]

Open issues:

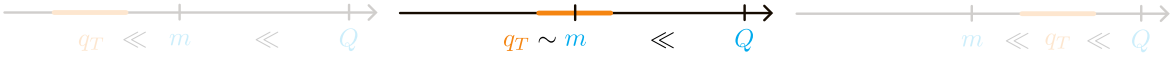
- mass effects are neglected & flavor thresholds treated improperly.
- lattice results are available for $n_f = 3$ and $n_f = 4$, but not $n_f = 5$.

CS-kernel: Flavor matching and mass effects.

Different regimes in q_T $(1/b_T)$.

[Pietrulewicz et al, '17]

Consider the impact of a massive quark (in addition to n_l massless quarks) for different q_T regimes:



regime I:

NOT massless n_l case with $\mathcal{O}(m^{-2}b_T^{-2})$ power corrections!

Would need $m \to \infty$, have $m \ll Q$, leading to:

- soft & collinear mass mode functions,
- plus their μ & ν running.

regime II:

Generalization of massless factorization with:

- mass dependent beam & soft functions or TMDPDFs.
- mass dependent CS-kernel.

Genuine mass dependence, not power corrections!

regime III:

Massless $n_l + 1$ case with $\mathcal{O}(m^2 b_T^2)$ power corrections to:

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Mass effects are absorbed in $n_l + 1$ flavor PDFs!

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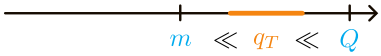

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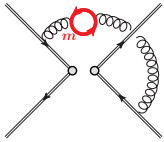
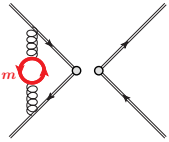
Massive contributions to the soft function and CS-kernel.

[Pietrulewicz et al, '17]

Massive quark contributions to the soft function – and the CS-kernel – can be calculated perturbatively:





These contributions are perturbative even for large b_T, the mass acts as a perturbatively large scale!

Massive contribution to the CS-kernel:

$$\tilde{\gamma}_\nu^{(h)}(b_T, m, \mu) = \left(\frac{\alpha_s}{4\pi}\right)^2 C_F\, T_F \Bigg\{ -\frac{32}{3} L_b L_m - \frac{16}{3} L_m^2 - \frac{160}{9} L_m - \frac{448}{27} + \frac{8\sqrt{\pi}}{3}\left[2G_{1,3}^{3,0}\left(\begin{smallmatrix}\frac{3}{2}\\0,0,0\end{smallmatrix}\middle| m^2 b_T^2\right) + G_{1,3}^{3,0}\left(\begin{smallmatrix}\frac{5}{2}\\0,0,1\end{smallmatrix}\middle| m^2 b_T^2\right)\right]\Bigg\} + \mathcal{O}(\alpha_s^3)$$

where:

$$L_m = 2\log\left(\frac{m}{\mu}\right), \qquad L_b = 2\log\left(\frac{b_T\,\mu}{b_0}\right).$$

Perturbative nature of massive contributions.

[Pietrulewicz et al, '17]

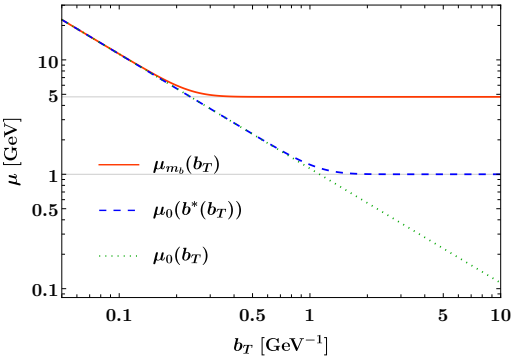
Canonical scale for $\tilde{\gamma}_\nu^{(h)}(b_T, m, \mu)$:

$$\mu_m(b_T \ll 1/m) \sim \frac{b_0}{b_T}\,, \qquad \mu_m(b_T \gg 1/m) \sim m\,.$$

→ makes it possible to use the perturbative result for all b_T while resumming all logarithms!

Massless canonical scale:

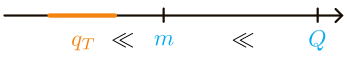
$$\mu_0(b_T) \sim \frac{b_0}{b_T}\,.$$



→ for large b_T the canonical scale becomes non-perturbative!

→ use of $\mu_0(b^*(b_T))$ induces large unresummed logs for large b_T!

Large and small b_T limits of the massive soft function and CS-kernel.

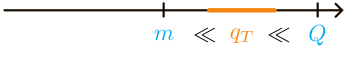


$1/b_T \ll m$:

$$\tilde{S}^{(n_l+1)}(b_T, m, \mu, \nu) = \tilde{S}^{(n_l)}(b_T, \mu, \nu)\, H_s(m, \mu) + \mathcal{O}\left(\frac{1}{m^2 b_T^2}\right)$$

$$\tilde{\gamma}_\nu^{(n_l+1)}(b_T, m, \mu) = \tilde{\gamma}_\nu^{(n_l)}(b_T, \mu) + \gamma_{\nu, H_s}(m, \mu) + \mathcal{O}\left(\frac{1}{m^2 b_T^2}\right)$$

→ mass does not decouple, mass dependence remains in H_s and γ_{ν,H_s}!



$m \ll 1/b_T$:

$$\tilde{S}_i^{(n_l+1)}(b_T, m, \mu, \nu) = \tilde{S}_i^{(n_l+1)}(b_T, \mu, \nu) + \mathcal{O}(m^2 b_T^2)$$

$$\tilde{\gamma}_i^{(n_l+1)}(b_T, m, \mu) = \tilde{\gamma}_i^{(n_l+1)}(b_T, \mu) + \mathcal{O}(m^2 b_T^2)$$

→ massless limit is recovered!

Flavor matching for the CS-kernel.

Using the massive quark contributions one can construct a flavor matching relation for all values of b_T:

Flavor matching:

$$\tilde{\gamma}_\nu^{(n_l+1)}(b_T, m, \mu) = \tilde{\gamma}_\nu^{(n_l)}(b_T, \mu_0) - 4\eta_\Gamma^{(n_l)}(\mu_0, \mu) + \Delta\tilde{\gamma}_\nu(b_T, m, \mu_m) + \Delta\eta_\Gamma(\mu_m, \mu)$$

where:

$$\Delta\tilde{\gamma}_\nu(b_T, m, \mu) = \tilde{\gamma}_{\nu,\mathrm{FO}}^{(n_l)}(b_T, \mu)\big|_{\alpha_s^{(n_l+1)}} - \tilde{\gamma}_{\nu,\mathrm{FO}}^{(n_l)}(b_T, \mu)\big|_{\alpha_s^{(n_l)}} + \tilde{\gamma}_\nu^{(h)}(b_T, m, \mu)\big|_{\alpha_s^{(n_l+1)}},$$

and:

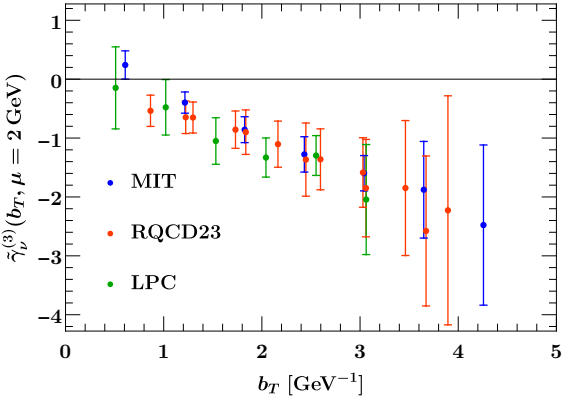
$$\Delta\eta_\Gamma^{(n_l+1)}(\mu_0, \mu) = -4\eta_\Gamma^{(n_l+1)}(\mu_0, \mu) + 4\eta_\Gamma^{(n_l)}(\mu_0, \mu).$$

Part II

CS-kernel: Lattice determination.

Lattice computations of the CS-kernel.

Recent years have seen a surge of ab initio lattice QCD calculations of the CS-kernel:



Lattice determinations of the CS-kernel:

- kernel obtained using quasi-TMD approach.
- $2+1$ active flavors for RQCD23 and LPC.
  [Shu et al, 2023; Lattice Parton Collaboration, 2023]
- $2+1+1$ active flavors for MIT.
  [Avkhadiev et al, 2023]
- MIT data can be translated to $2+1$ active flavors using $\Delta\tilde{\gamma}_\nu$.

→ Make an ansatz for the $n_f = 3$ CS-kernel and fit it to the lattice data!

→ $n_f = 4$ and $n_f = 5$ kernels can be obtained using flavor matching!

Ansatz for the CS-kernel.

Ansatz for $\tilde{\gamma}_\nu^{(3)}$:

$$\tilde{\gamma}_\nu^{(3)}(b_T,\mu) = \tilde{\gamma}_{\nu,\mathrm{FO}}^{(3)}(b^*(b_T),\mu_0(b_T)) - 4\eta_\Gamma^{(3)}(\mu_0(b_T),\mu) + \tilde{\gamma}_{\nu,\mathrm{NP}}(b_T)\,,$$

where

$$\mu_0(b_T) = \frac{b_0}{b^*(b_T)}\,,$$

and

$$b^*(b_T \ll b_{\max}) = b_T\left[1+\mathcal{O}(b_T^6)\right]\,, \qquad b^*(b_T \gg b_{\max}) = b_{\max} + \mathcal{O}(b_T^{-2})\,.$$

Requirements for $\tilde{\gamma}_{\nu,\mathrm{NP}}$:

$$\tilde{\gamma}_{\nu,\mathrm{NP}}(b_T) = \mathcal{O}(b_T^2)\,, \quad \text{for small } b_T\,.$$
$$\tilde{\gamma}_{\nu,\mathrm{NP}}(b_T) = -\mathrm{const}\,, \quad \text{for large } b_T\,. \quad \text{[Collins and Rogers, '14]}$$

Functional form of $\tilde{\gamma}_{\nu,\mathrm{NP}}$.

Ansatz:

$$\tilde{\gamma}_{\nu,\mathrm{NP}}(b_T) = -\lambda_\infty \, f\left(\lambda_2 b_T^2/\lambda_\infty + \lambda_4 b_T^4/\lambda_\infty\right) ,$$

with some function:

$$f(x \ll 1) = x + \mathcal{O}(x^2) , \qquad f(x \gg 1) = 1 + \mathcal{O}(x^{-2}) ,$$

which – modified to get rid of their second and fourth moments – can also be used as b^*:

$$b^*(b_T) = b_{\max} \, f_6(b_T/b_{\max}) .$$

Square root function:

$$f(x) = x/\sqrt{1+x^2} , \qquad f_6(x) = f(x + x^3/2 + 3x^5/8) .$$

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Hyperbolic tangent function:

$$f(x) = \tanh(x) , \qquad f_6(x) = f(x + x^3/3 + x^5/5) .$$

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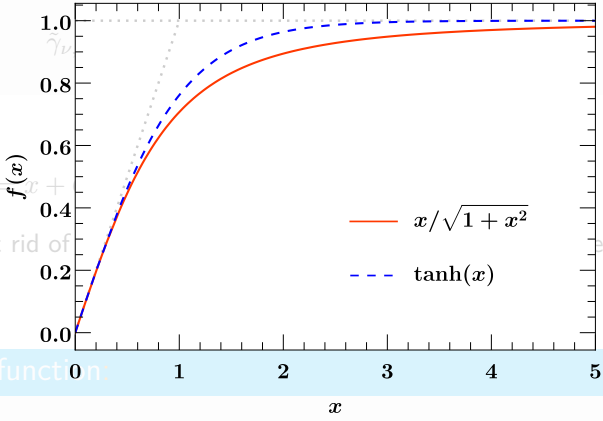

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Fit results.

The two different functional forms for $\tilde{\gamma}_{\nu,\mathrm{NP}}$ yield very similar fit results:

$$\lambda_\infty = 1.89 \pm 0.722\,,$$
$$\lambda_2 = 0.094 \pm 0.036\,,$$
$$\lambda_4 = 0.0070 \pm 0.0079\,.$$

$$\lambda_\infty = 1.67 \pm 0.49\,,$$
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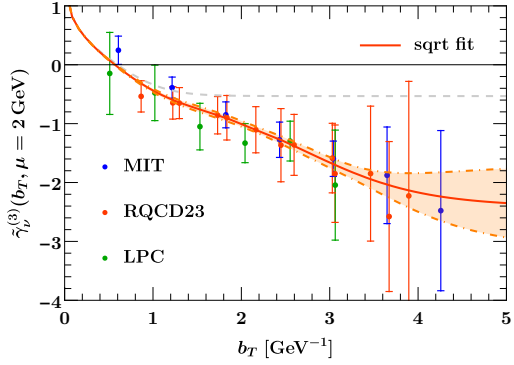

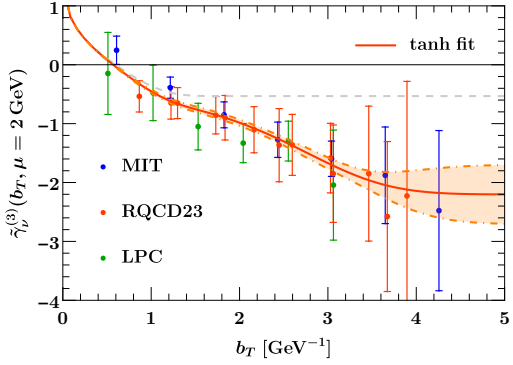

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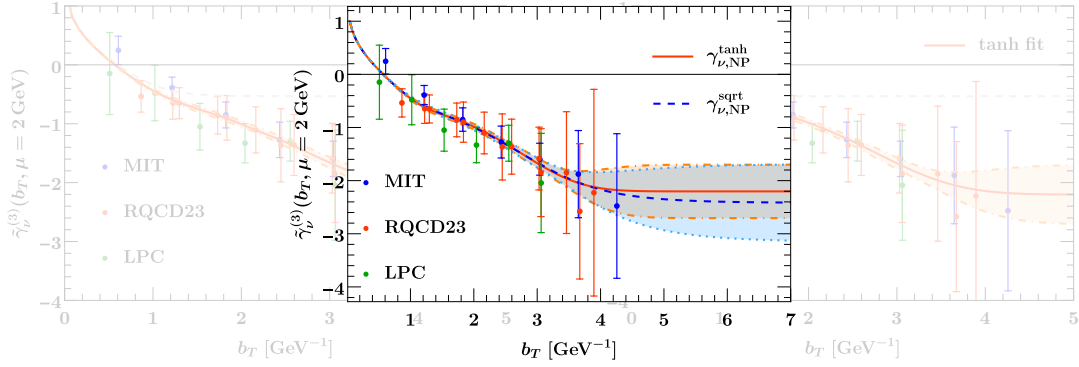
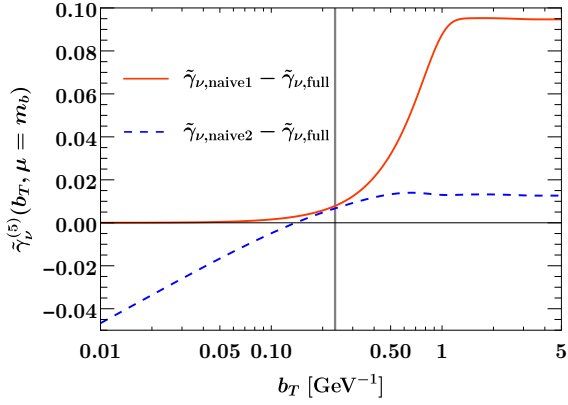


Fits start to differ noticeably – but still within errors – for large b_T where no lattice data is available!

CS-kernel with mass effects – done correctly.

Let's consider the impact of flavor matching on the CS-kernel:



Naïve approximations:

In our model for the full CS-kernel make the following changes:

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2. replace $\eta^{(3)}_\Gamma(\mu_0, \mu)$ by $\eta^{(3)}_\Gamma(\mu_0, m_c) + \eta^{(4)}_\Gamma(m_c, m_b) + \eta^{(5)}_\Gamma(m_b, \mu)$.

- 1 correct for small b_T.
- 2 works approximately for large b_T.

→ both approximations off by $\mathcal{O}(10\%)$ for $b_T \sim 1/m_b$!

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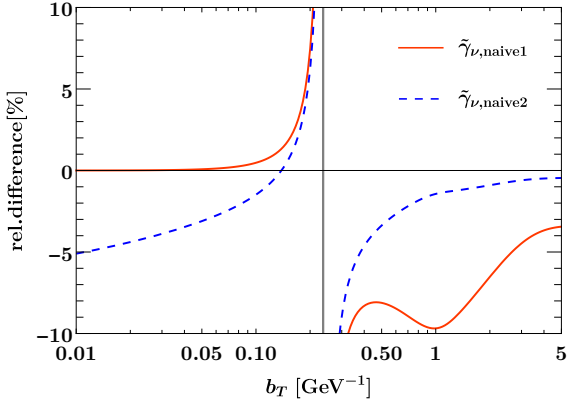

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Part III

Impact of the massive CS-kernel on the Z q_T–spectrum.

Factorization in different q_T regimes.

[Pietrulewicz et al, '17]

Consider for now n_l light quarks and a single massive quark:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\mathrm{d}Y\,\mathrm{d}b_T} = H_{ij}(Q,\mu) \qquad \tilde{S}(b_T,m,\mu,\nu) \quad \tilde{B}_i(x_a,b_T,m,\mu,\tfrac{\nu}{\omega_a}) \quad \tilde{B}_j(x_b,b_T,m,\mu,\tfrac{\nu}{\omega_b})$$

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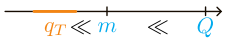
I 

$\tilde{S}^{(n_l)}(b_T,\mu,\nu) H_s(m,\mu,\nu) + \mathcal{O}\left(\frac{1}{m^2 b_T^2}\right)$ $\quad$ $\tilde{B}_j^{(n_l)}(x_b,b_T,\mu,\frac{\nu}{\omega_b}) H_c(m,\mu,\frac{\nu}{\omega_b}) + \mathcal{O}\left(\frac{1}{m^2 b_T^2}\right)$

II 

$\tilde{S}(b_T,m,\mu,\nu)$ $\quad$ $\tilde{B}_i(x_a,b_T,m,\mu,\frac{\nu}{\omega_a})$ $\quad$ $\tilde{B}_j(x_b,b_T,m,\mu,\frac{\nu}{\omega_b})$

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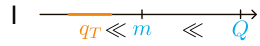
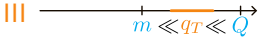


$\tilde{S}^{(n_l)}(b_T,\mu,\nu) H_s(m,\mu,\nu) + \mathcal{O}\left(\frac{1}{m^2 b_T^2}\right)$ $\quad$ $\tilde{B}_j^{(n_l)}(x_b,b_T,\mu,\frac{\nu}{\omega_b}) H_c(m,\mu,\frac{\nu}{\omega_b}) + \mathcal{O}\left(\frac{1}{m^2 b_T^2}\right)$

$\tilde{S}(b_T,m,\mu,\nu)$ $\quad$ $\tilde{B}_i(x_a,b_T,m,\mu,\frac{\nu}{\omega_a})$ $\quad$ $\tilde{B}_j(x_b,b_T,m,\mu,\frac{\nu}{\omega_b})$



$\tilde{S}^{(n_l+1)}(b_T,\mu,\nu) + \mathcal{O}(m^2 b_T^2)$ $\quad$ $\tilde{B}_j^{(n_l+1)}(x_b,b_T,\mu,\frac{\nu}{\omega_b}) + \mathcal{O}(m^2 b_T^2)$

Factorization in different q_T regimes.

[Pietrulewicz et al, '17]

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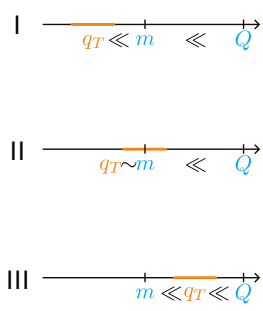

$$\tilde{S}^{(n_l)}(b_T,\mu,\nu) H_s(m,\mu,\nu) + \mathcal{O}\left(\tfrac{1}{m^2 b_T^2}\right) \qquad \tilde{B}_j^{(n_l)}(x_b, b_T, \mu, \tfrac{\nu}{\omega_b}) H_c(m,\mu,\tfrac{\nu}{\omega_b}) + \mathcal{O}\left(\tfrac{1}{m^2 b_T^2}\right)$$

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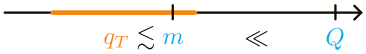
$$\tilde{S}^{(n_l+1)}(b_T,\mu,\nu) + \mathcal{O}(m^2 b_T^2) \qquad \tilde{B}_j^{(n_l+1)}(x_b, b_T, \mu, \tfrac{\nu}{\omega_b}) + \mathcal{O}(m^2 b_T^2)$$

For $q_T \sim 1/b_T \gg m$ and $q_T \sim 1/b_T \ll m$ get large unresummed logs of q_T/m or $m\, b_T$!

→ Refactorize to allow for resummation of these logs!

Combining factorization theorems.

First match (*"resummation + power corrections"*) regimes I + II (n_l like) and II + III ($n_l + 1$ like):



regime IV:

- Combine I with power corrections from II.
- virtuality evolution of beam and soft functions with n_l massless flavors.
- rapidity evolution of beam and soft functions with:

$$\tilde{\gamma}_\nu^{(n_l+1)}(b_T, m, \mu) - \gamma_{\nu,H_s}(m, \mu)$$

regime V:

- Combine III with power corrections from II.
- virtuality evolution of beam and soft functions with $n_l + 1$ massless flavors.
- rapidity evolution of beam and soft functions with:

$$\tilde{\gamma}_\nu^{(n_l+1)}(b_T, m, \mu)$$

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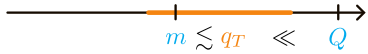

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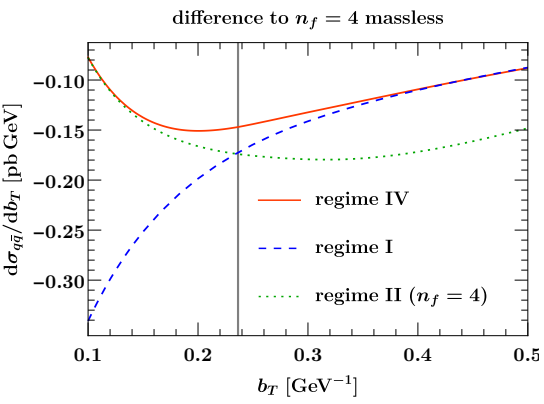
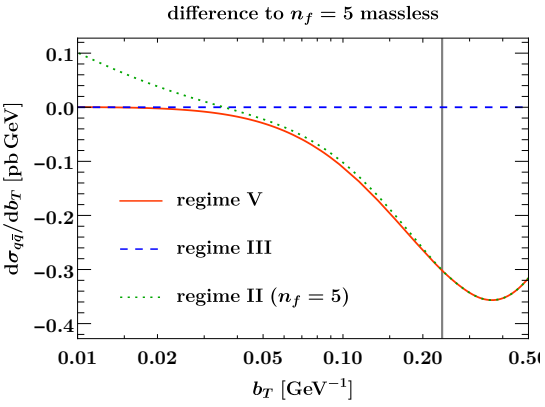
Consider now $n_l = 4$ and massive b quark (@ NNLL to avoid mass effects in FO boundary conditions):





→ The matched IV and V interpolate smoothly between I and II and II and III, respectively!

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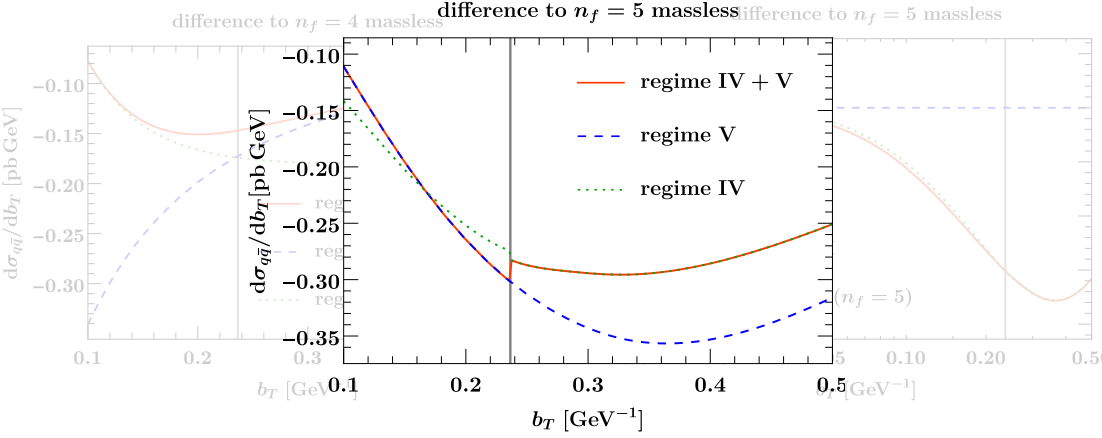
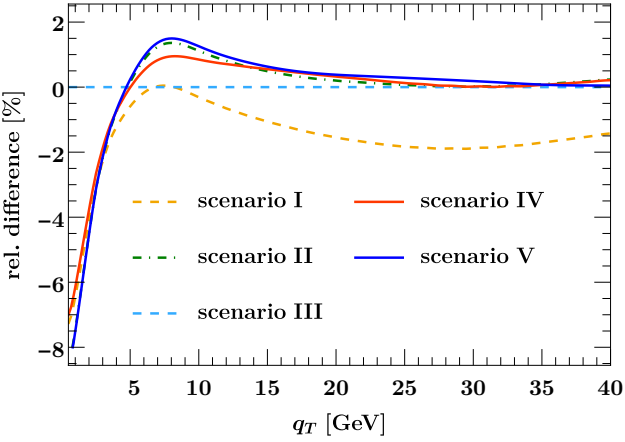


→ The matched IV and V interpolate smoothly between I and II and II and III, respectively!

→ Have a discontinuity between IV and V (n_l vs. $n_l + 1$)!

Comparing different regimes in q_T space.

Use the $n_f = 5$ massless q_T spectrum as a reference:



Small and large q_T limits:

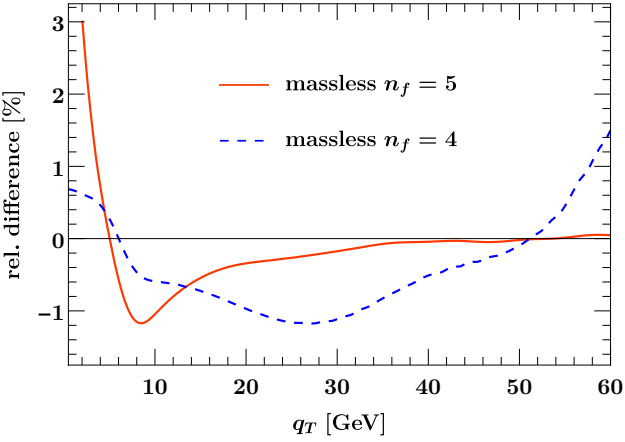
As expected:

- IV interpolates between I and II.
- V interpolates between II and III.
- III and thus also V agree with $n_f = 5$ massless for $q_T \gg m_b$.

→ Smoothly transition from IV to V around $q_T \sim m_b$!

Final result for massive b.

Compare the combined IV + V q_T space result to the naive $n_f = 4$ and $n_f = 5$ massless results:



Results:

- mass effects from rapidity resummation are of $\mathcal{O}(\%)$.
- massless $n_f = 5$ is a good approximation only for $q_T \gg m_b$.
- massless $n_f = 4$ is a not recovered for $q_T \ll m_b$ (rapidity resummation of H_s and H_c).

→ Effect persists at higher orders and should be included if we aim for percent level precision!

Including a massive c.

So far considered c as massless, how to include it with finite mass?

Considerations to include a massive c:

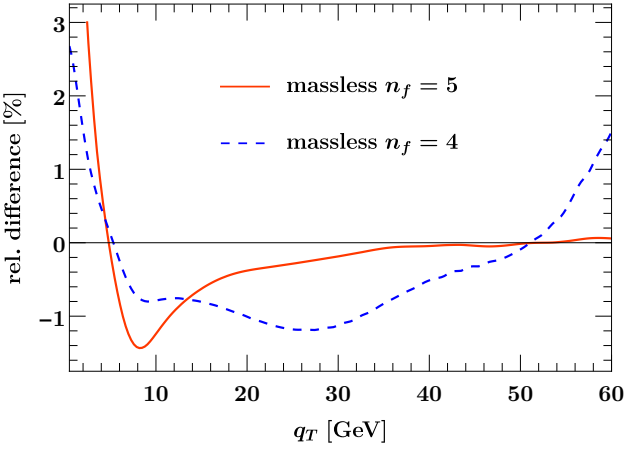
- for massless c used $n_f = 4$ massless beam and soft functions and rapidity anomalous dimensions as baseline.
- can ignore $q_T \ll m_c$ (not relevant for pheno), consider only $q_T \sim m_c$ and $q_T \gg m_c$.
- combine $q_T \sim m_c$ and $q_T \gg m_c$ analogously to V and use this as baseline for the b mass effects.

At NNLL including the c mass effects is achieved by simply replacing:

$$\tilde{\gamma}_\nu^{(4+1)}(b_T, m_b, \mu) \longrightarrow \tilde{\gamma}_\nu^{(3+1+1)}(b_T, m_c, m_b, \mu)$$

Final result for massive c and b.

Compare finally the massive c and b q_T space result to the naive $n_f = 4$ and $n_f = 5$ massless results:



Results:

- mass effects more pronounced, especially for small q_T.
- massless $n_f = 5$ remains a good approximation only for $q_T \gg m_b$.
- deviations from massless $n_f = 4$ are much stronger.

Part IV

Summary and outlook.

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- a flavor matching relation for the CS-kernel can be derived from perturbative mass corrections.
- the large b_T CS-kernel can be constrained using lattice data for $n_f = 3$.
- mass effects and flavor thresholds for the CS-kernel cannot be neglected.
- there are three different regimes of $q_T \sim 1/b_T$ w.r.t. m: $q_T \ll m$ (I), $q_T \sim m$ (II), $m \ll q_T$ (III).
- regimes can be combined in a two-step procedure:
  - first IV = I + II & V = II + III (in b_T-space).
  - then IV + V (in q_T space).
- mass effects from rapidity evolution alone are $\mathcal{O}(\%)$ (working at NNLL avoids mass effects in the beam and soft functions).

Outlook:

- study mass corrections to beam and soft functions (come in at NNLL').
- investigate differences in mass effects for the W and Z q_T spectra.

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Thank you for your attention!

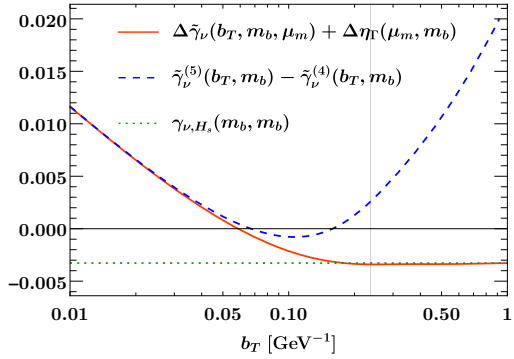
Part V

Backup.

Limits of the flavor matching kernel.

Check that the flavor matching prescription for the CS-kernel has the correct small and large b_T limits!

For definiteness consider the flavor matched 4+1 flavor CS-kernel at $\mu = m_b$:



$\tilde{\gamma}_\nu^{(n_l+1)}(b_T, m, \mu)$:

- for $m \ll 1/b_T$ the $n_l + 1$ flavor massless CS-kernel is recovered.
- for $1/b_T \ll m$ the massless n_l flavor CS-kernel plus γ_{ν,H_s} is approached.

→ The flavor matched CS-kernel has the correct limiting behavior!

Expanded vs. unexpanded $\Delta\tilde{\gamma}_\nu$.

The flavor matching expression contains fixed-order ingredients with $\alpha_s^{(n_l)}$ and $\alpha_s^{(n_l+1)}$:

$$\tilde{\gamma}^{(n_l)}_{\nu,\mathrm{FO}}(b_T,\mu)\big|_{\alpha_s^{(n_l+1)}} - \tilde{\gamma}^{(n_l)}_{\nu,\mathrm{FO}}(b_T,\mu)\big|_{\alpha_s^{(n_l)}}$$

In this unexpanded form the $m \ll 1/b_T$ limit is recovered, unlike the $1/b_T \ll m$ limit, where:

$$\begin{aligned}\Delta\tilde{\gamma}_\nu(b_T,m,\mu_m) &= \tilde{\gamma}_{\nu,H_s}(m,m) \\ &+ \tilde{\gamma}^{(n_l)}_{\nu,\mathrm{FO}}(b_T,m)\big|_{\alpha_s^{(n_l+1)}} - \tilde{\gamma}^{(n_l)}_{\nu,\mathrm{FO}}(b_T,m)\big|_{\alpha_s^{(n_l)}} + \left(\frac{\alpha_s^{(n_l+1)}}{4\pi}\right)^2 \frac{4}{3} L_m \tilde{\gamma}^{(1)}_{\nu,\mathrm{FO}}(b_T,m)\end{aligned}$$

Re-expanding $\alpha_s^{(n_l)}$ in terms of $\alpha_s^{(n_l+1)}$ yields:

$$\Delta\tilde{\gamma}_\nu(b_T,m,\mu_m) = \tilde{\gamma}_{\nu,H_s}(m,m)$$

in agreement with with expected behavior!

Expanded vs. unexpanded $\Delta\tilde{\gamma}_\nu$.

In order to get both limits right use different expressions for $\Delta\tilde{\gamma}_\nu$ in the $m \ll 1/b_T$ and $1/b_T \ll m$ limits:

$$\Delta\tilde{\gamma}_{\nu,\text{unexp}}(b_T, m, \mu) = \tilde{\gamma}_{\nu,\text{FO}}^{(n_l)}(b_T, \mu)\Big|_{\alpha_s^{(n_l+1)}} - \tilde{\gamma}_{\nu,\text{FO}}^{(n_l)}(b_T, \mu)\Big|_{\alpha_s^{(n_l)}} + \tilde{\gamma}_\nu^{(h)}(b_T, m, \mu)\,,$$

$$\Delta\tilde{\gamma}_{\nu,\text{exp}}(b_T, m, \mu) = \tilde{\gamma}_\nu^{(h)}(b_T, m, \mu) + \left(\frac{\alpha_s^{(n_l+1)}}{4\pi}\right)^2 \frac{4}{3} L_m \tilde{\gamma}_{\nu,\text{FO}}^{(1)}(b_T, \mu)\,.$$

Define a "patched" version of $\Delta\tilde{\gamma}_\nu^{(n_l+1)}$ which combines the two limiting expressions as:

$$\Delta\tilde{\gamma}_{\nu,\text{patched}}^{(n_l+1)}(b_T, m, \mu) = \Delta\tilde{\gamma}_{\nu,\text{unexp}}^{(n_l+1)}(b_T, m, \mu) + f(b_T, m)\left[\Delta\tilde{\gamma}_{\nu,\text{exp}}^{(n_l+1)}(b_T, m, \mu) - \Delta\tilde{\gamma}_{\nu,\text{unexp}}^{(n_l+1)}(b_T, m, \mu)\right]\,,$$

with some interpolating function:

$$f(b_T, m) = 0 \qquad \text{for} \quad m\, b_T \ll 1\,,$$
$$f(b_T, m) = 1 \qquad \text{for} \quad m\, b_T \gg 1\,.$$

Why NNLL?

Focusing on the Z q_T spectrum at NNLL isolates the effect of the mass-dependent rapidity evolution!

At NNLL' (NNLL resummation + NLO fixed order) mass effects start to matter also for:

- beam functions:
  - primary mass effects for heavy quark beam functions ($\mathcal{O}(\alpha_s) \times \mathcal{O}(\alpha_s)$).
  - secondary mass effects for light quark beam functions($\mathcal{O}(\alpha_s^2)$).
- the soft function (secondary mass effects $\mathcal{O}(\alpha_s^2)$).

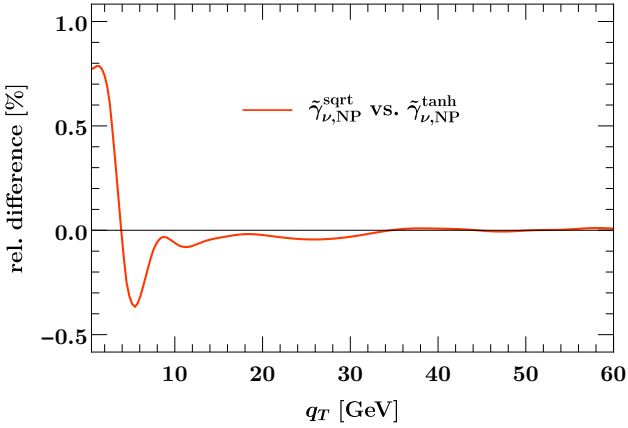
Note that this is a crucial difference between Z and W production:

For W production primary mass effects for the beam functions come in already at NNLL.

→ This is relevant for the W mass measurement!

Impact of different $\tilde{\gamma}_{\nu,\mathrm{NP}}$ fits .

Consider now how the different fits for $\tilde{\gamma}_{\nu,\mathrm{NP}}$ effect the q_T spectrum:



Observations:

- for large q_T ($\equiv$ small b_T) both fits are in agreement with each other.
- for small q_T ($\equiv$ large b_T) the q_T spectrum differs by up to almost one percent.

→ lattice data not sufficient to constrain the very large b_T behavior.

→ observed differences are small compared to the estimated mass effect.

Acknowledgement.

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