Perturbative Evolution of Hadronization Effects in Energy Correlators

Hao Chen
Zhejiang University

based on work with Pier Monni, Zhen Xu and HuaXing Zhu

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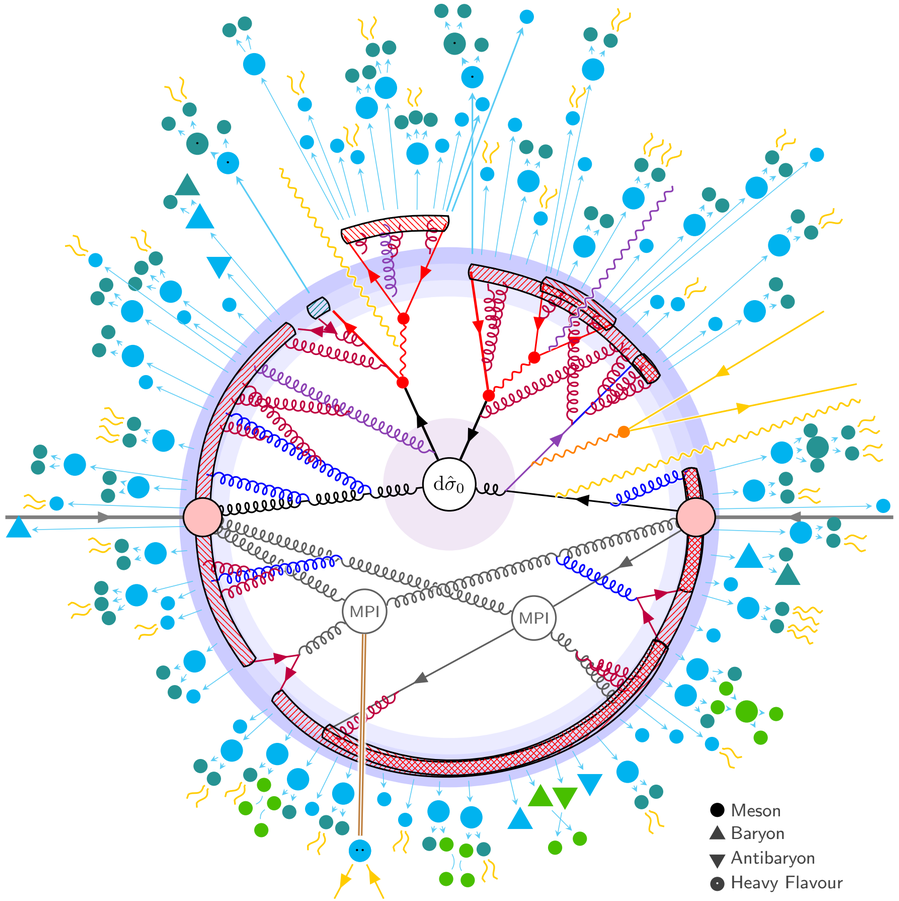
Typical QCD Dynamics at Colliders

QCD Lagrangian is simple, but real-world QCD dynamics looks complicated.



Pythia Manual

initial-state dynamics
(e.g. PDF and evolution)

⊗

perturbative scattering

⊗

final-state dynamics
(e.g. parton shower,
hadronization)

e^+e^-
annihilation

this talk

[also see Andre Hoang's talk]

Methods for Hadronization Corrections

- Operator Product Expansion/Factorization

  [Korchemsky, Sterman, 1999; Lee, Sterman, 2006; Abbate, Fickinger, Hoang, Mateu, Stewart, 2011; Hoang, Kolodrubetz, Mateu, Stewart, 2014; Mateu, Stewart, Thaler, 2012; Hoang, Mantry, Pathak, Stewart, 2019; …]

- Renormalon [see Zhiquan Sun's talk]

  [Nason, Seymour, 1995; Beneke, 1998; Schindler, Stewart, Sun, 2023;…]

- Dispersive approach

  [Dokshitzer, Marchesini, Webber, 1996; Dokshitzer, Lucenti, Marchesini, Salam, 1997; Dokshitzer, Marchesini, Webber, 1999; …]

- …

- Monte Carlo Models: e.g. Lund string model (Pythia) and Cluster model (Herwig)

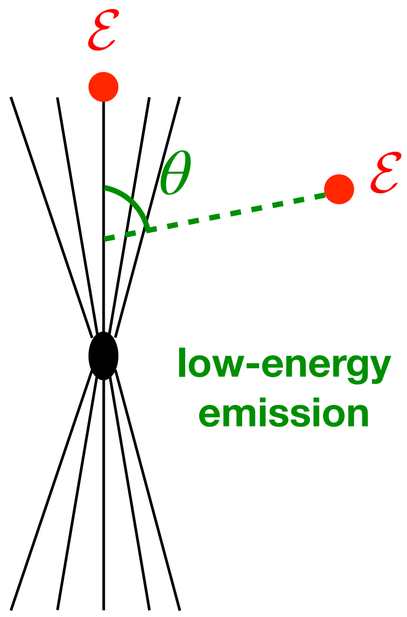
One Interesting Example

[Korchemsky, Sterman, 1999]

The IR shape functions are different for different event shapes, and depend on a factorization scale, but are independent of the c.o.m. energy. They … carry information on a class of universal matrix elements of the energy-momentum tensor in QCD, directly related to the energy-energy correlations.

Thrust:
$$\frac{d\sigma(t)}{dt} = \frac{d\sigma_{\rm PT}(t-\lambda_1/Q)}{dt} + \mathcal{O}(1/(tQ)^2)$$

$$\lambda_1 = \int d\vec{n}\; w_T(\vec{n}) \boxed{\langle 0|W^\dagger(0)\mathcal{E}(\vec{n})W(0)|0\rangle} \quad \propto \frac{1}{\sin^3\theta}$$



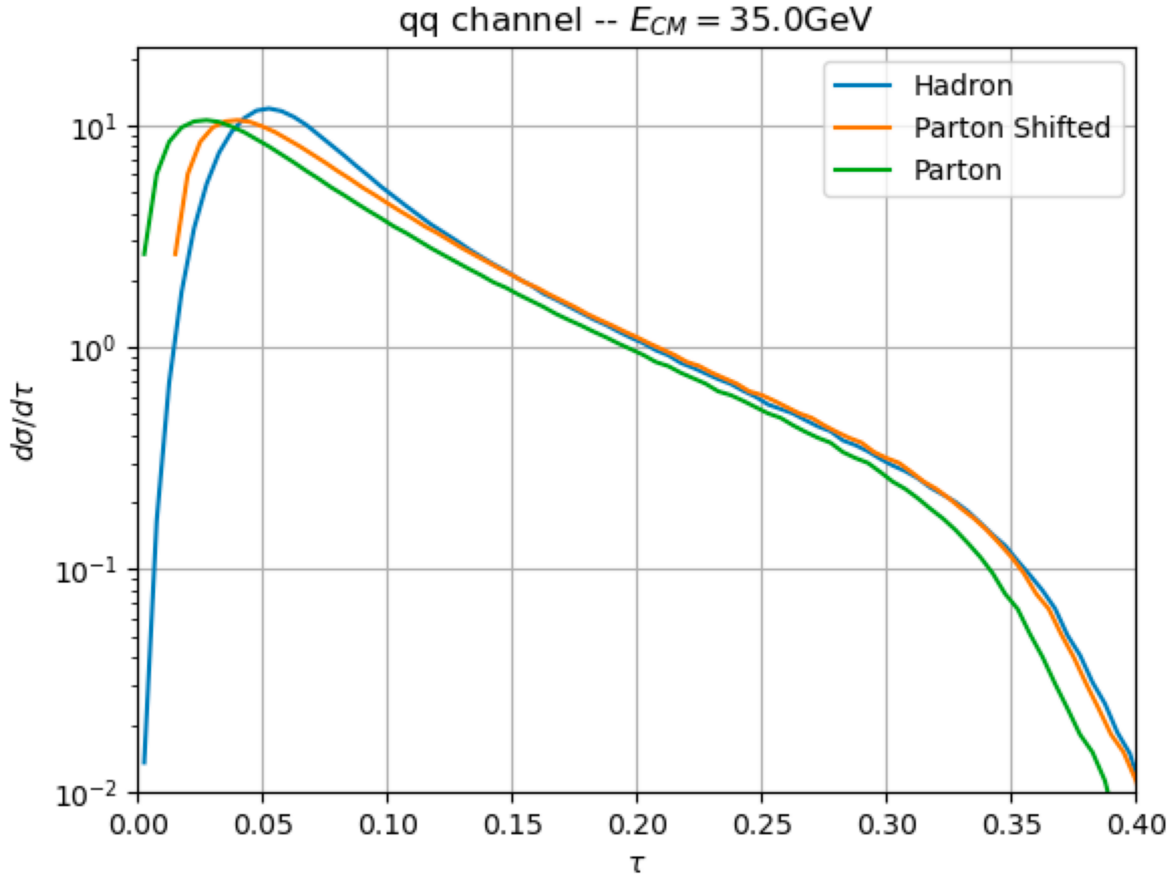
EEC power correction: matrix element of a energy detector with Wilson lines

More general cases, we need

$$\mathcal{G}(\vec{n}_1,\ldots,\vec{n}_N;\mu) = \langle 0|W^\dagger(0)\mathcal{E}(\vec{n}_1)\ldots\mathcal{E}(\vec{n}_N)W(0)|0\rangle \sim \prod_{i=1}^{N}\langle 0|W^\dagger(0)\mathcal{E}(\vec{n}_i)W(0)|0\rangle$$

[see also Miguel Benitez-Rathgeb and Guido Bell's talks for detailed discussion on thrust]

Test the Relation on Pythia



1. By comparing “Parton” and “Hadron” level results, we extract the non-parameter λ_1 from **collinear EEC**

2. Using the extracted parameter to shift the parton-level **thrust** distribution, the shifted curve agrees with hadron-level curve.

Lesson: the hadronization corrections of EECs might serve as a fundamental basis for other event shape observables.

Goal: explore hadronization effects of EECs in the collinear limit with light-ray OPE framework.

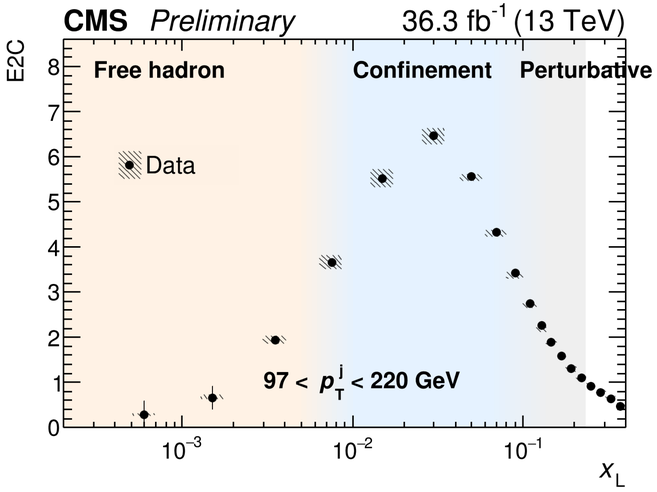
Outline

1. Introduction to Energy Correlators

    - Collinear Limit and Light-ray OPE

2. Hadronization Effect in the Collinear Limit

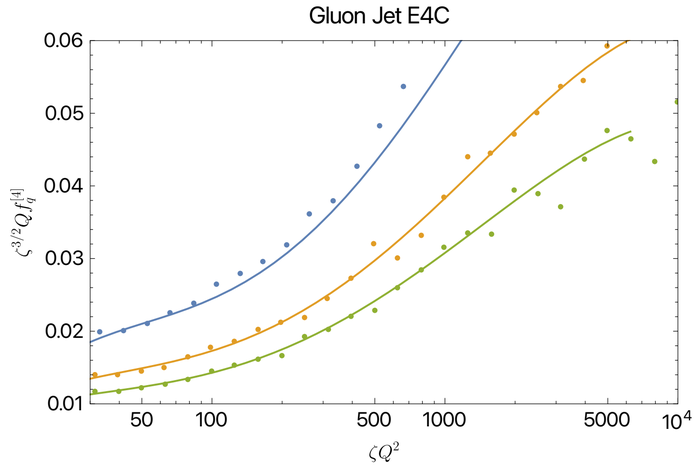
    - Parametrization and Evolution from Light-ray OPE



Highlights

- Light-ray Bjorken Scaling and Violation

- Providing a way of extracting Wilson-line expectation value of energy-flow operator

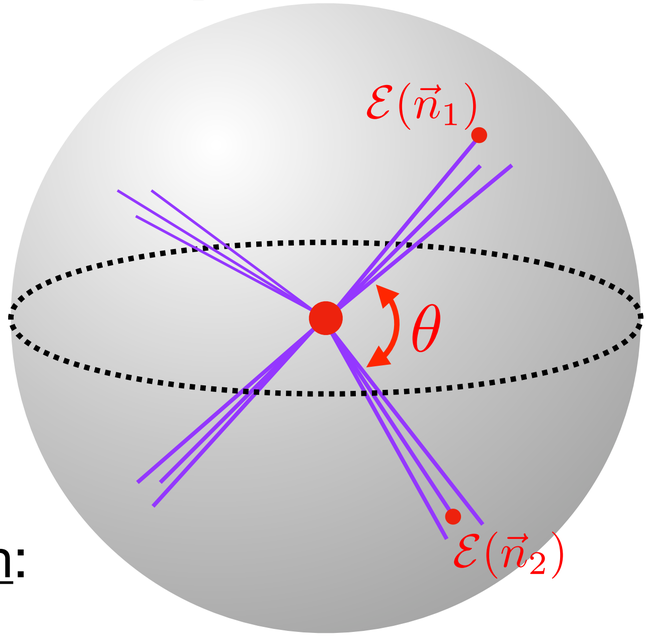


Energy-energy correlator (EEC)

Energy-energy correlation measures the correlation of two energy detectors at spatial infinity

[Basham, Brown, Ellis and Love, 1978]

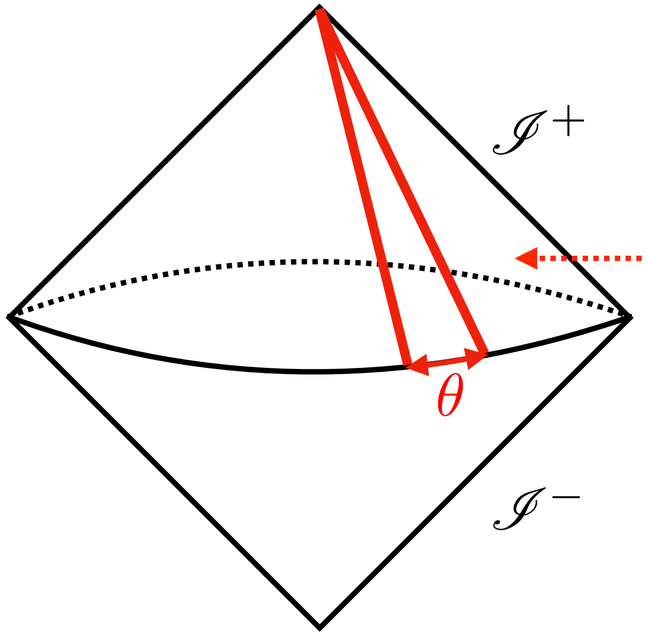
$$\frac{d\Sigma}{d\zeta} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta\left(\zeta - \frac{1-\cos\theta_{ij}}{2}\right)$$



The energy detector has a nice operator definition:

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, n^i T_{0i}(t, r\vec{n})$$

[Sveshnikov, Tkachov, 1995; Hofman, Maldacena, 2008]



It is a non-local operator on a *light-ray* — light-ray operator

EEC and its multi-point generalization are **correlation functions** of light-ray operators

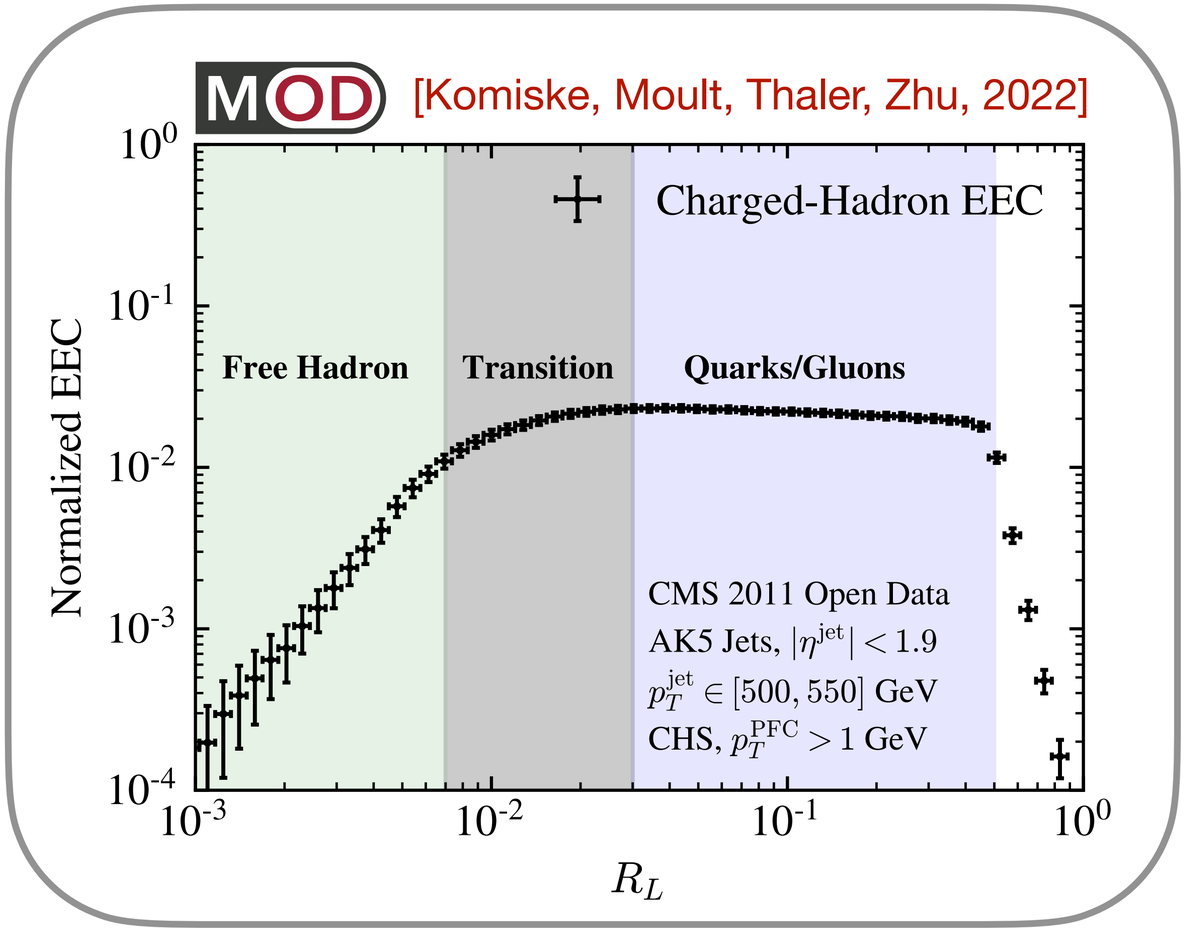
$$\langle O'(-q)|\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ldots|O(q)\rangle$$

Source with total momentum $q = (Q,0,0,0)$

Collinear Limit Result from CMS

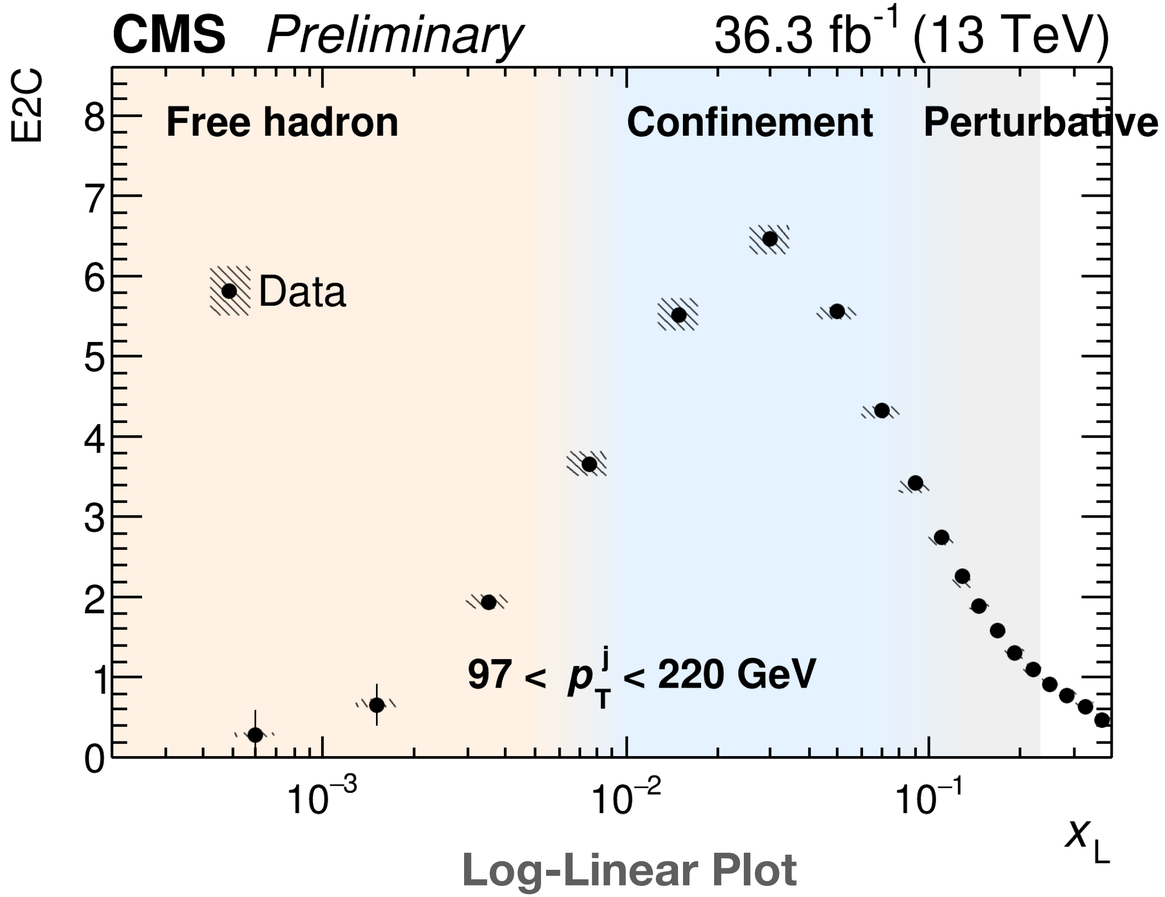
[Meng Xiao et al, 2023]

Previous exploration from CMS Open Data



Log-Log Plot

See illustration on
https://cms.cern/news/jets-elucidate-how-partons-evolve-hadrons

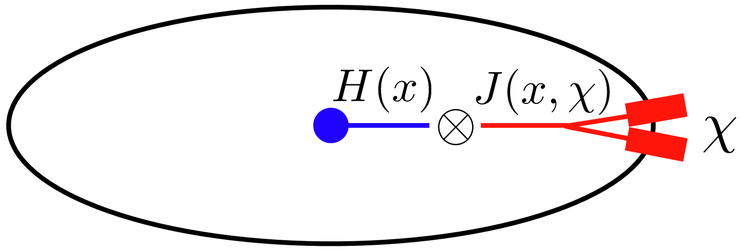


Log-Linear Plot

Enables precision measurement of α_s in jet substructure

Universality in the Collinear Limit

EEC exhibits collinear universality in pQCD, described by the factorization formula



Jet function Hard function

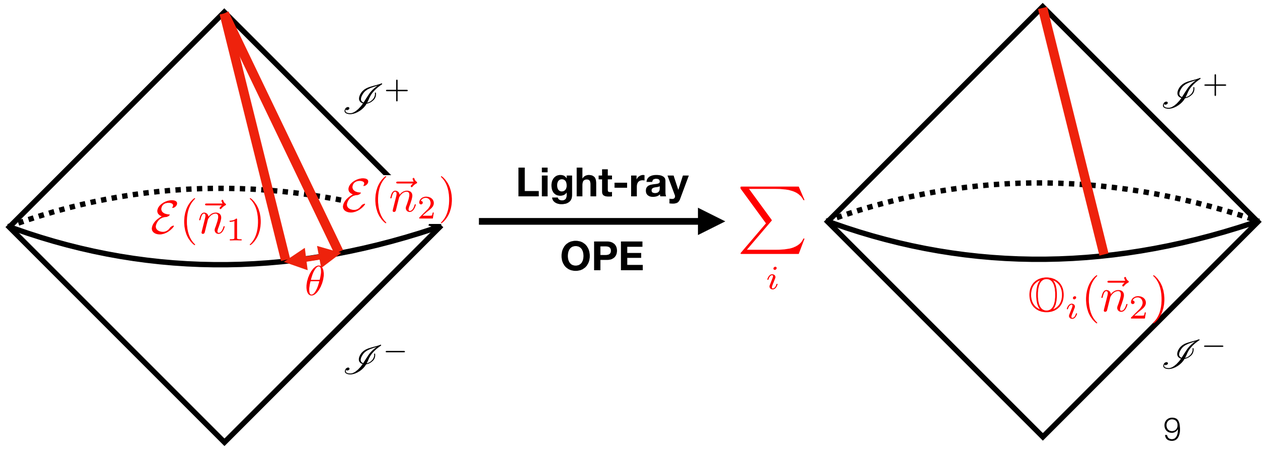
$$\Sigma(\zeta, \log\frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx\, x^2 \vec{J}(\log\frac{\zeta x^2 Q^2}{\mu^2}, \mu)\cdot \vec{H}(x, \frac{Q^2}{\mu^2}, \mu)$$

convolution in momentum fraction x

[Dixon, Moult, Zhu, 2019]

As a correlation function of light-ray operators, collinear limit has the interpretation of light-ray OPE.

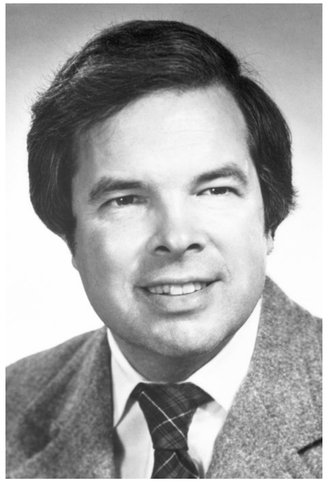
[Hofman, Maldacena, 2008; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]



[HC, 2023]

- Lorentz Symmetry
- RG invariance
- Dimensional Analysis

Various Operator Product Expansion



Short distance scaling behavior is determined by local OPE.

Local Operator OPE

$\mathcal{O}_2(x_2)$

$\mathcal{O}_1(x_1)$

$$\sim \quad \sum \mathcal{O}(x_1)$$

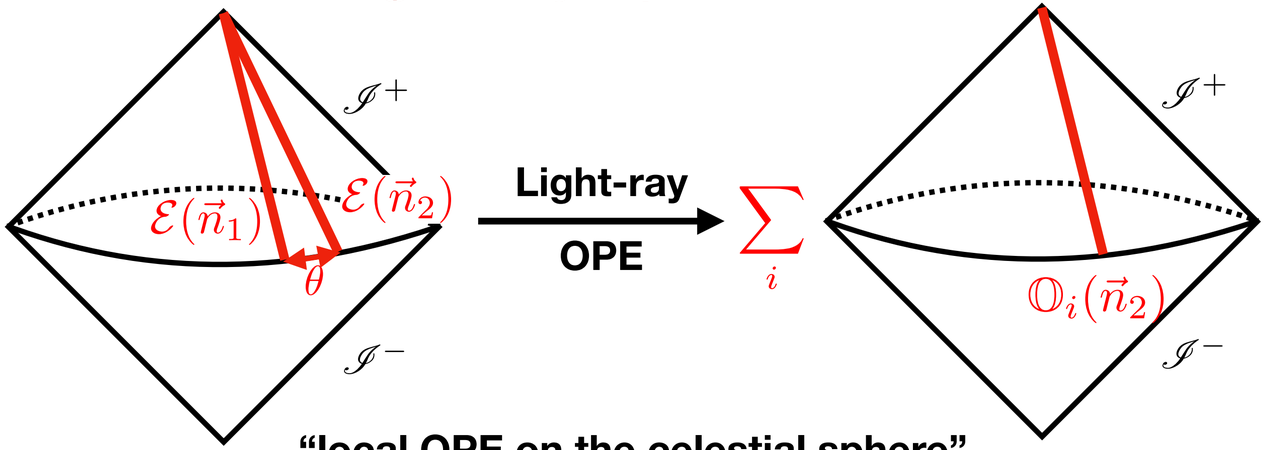
Euclidean distance OPE

is an **operator dimension** expansion

Lightcone OPE

is controlled by **twist expansion**

Small angle behavior is controlled by the OPE of light-ray operators.



"local OPE on the celestial sphere"

Light-ray OPE is a fixed-spin twist expansion in CFT

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \sim \sum_i c_i \, \theta^{\tau_i - 4} \mathbb{O}_i^{[J=3]}(\vec{n}_2)$$

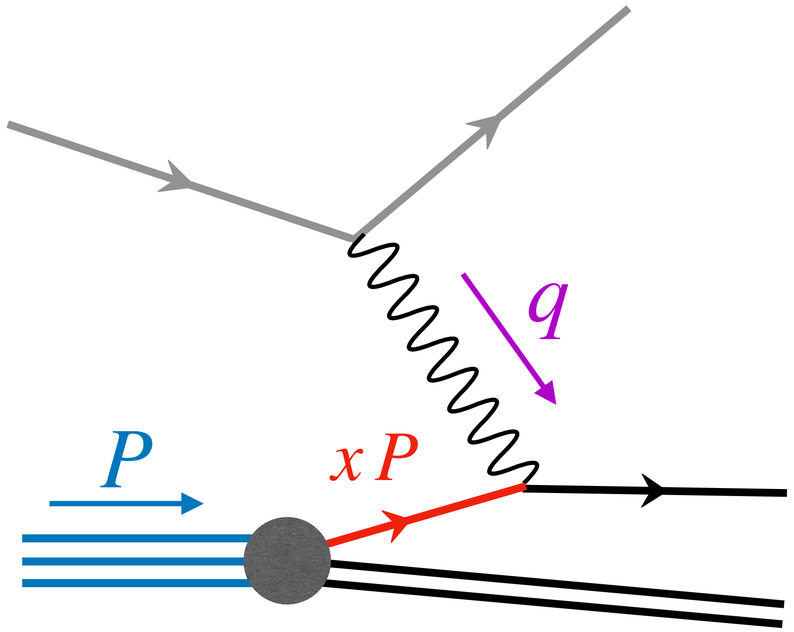
spin-3 light-ray operators

[Hofman, Maldacena, 2008]

Review of Deep Inelastic Scattering

In DIS, we have the notion of Parton Distribution Functions $f_a(x;\mu)$

momentum fraction — PDFs are scale dependent



$$x = \frac{-q^2}{2P\cdot q}$$

- PDFs are non-perturbative functions
  [from experiment data or lattice calculation]
- Their evolution is perturbatively calculable, called DGLAP equation

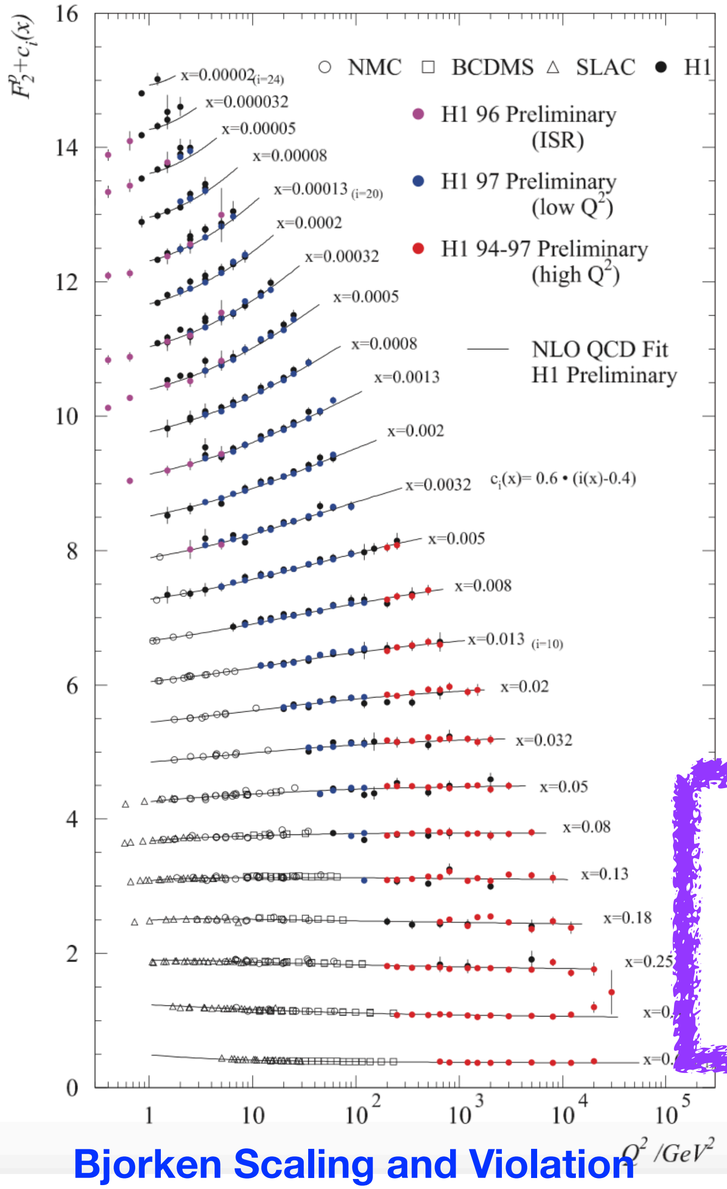
From OPE perspective,

They are the evolution of local twist-2 operators

$$\langle P|J_\mu(q)J_\mu(-q)|P\rangle \sim \frac{q_{\mu_1}\cdots q_{\mu_J}}{(-q^2)^{(\Delta+J-2)/2}}\langle P|O^{\mu_1\cdots\mu_J}|P\rangle$$

Moments of PDF

Review of Deep Inelastic Scattering



Bjorken Scaling and Violation

Parton Distribution Functions $f_a(x;\mu)$

momentum fraction

PDFs are scale dependent

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  [from experiment data or lattice calculation]

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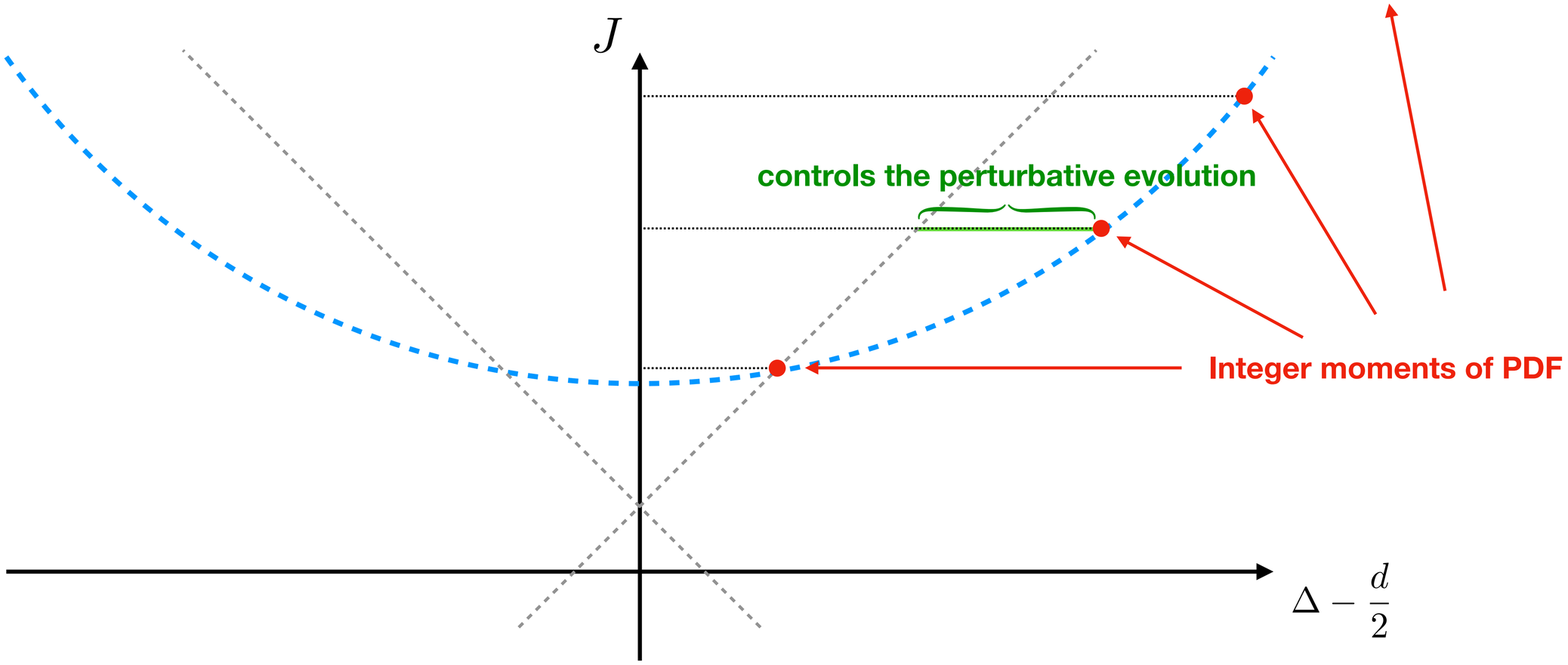
Can we find counterpart for light-ray OPE?

Non-perturbative parameters + perturbative evolution

$(-q^2)^{\ell}$

Moments of PDF

Schematic Chew-Frautschi Illustration for DIS



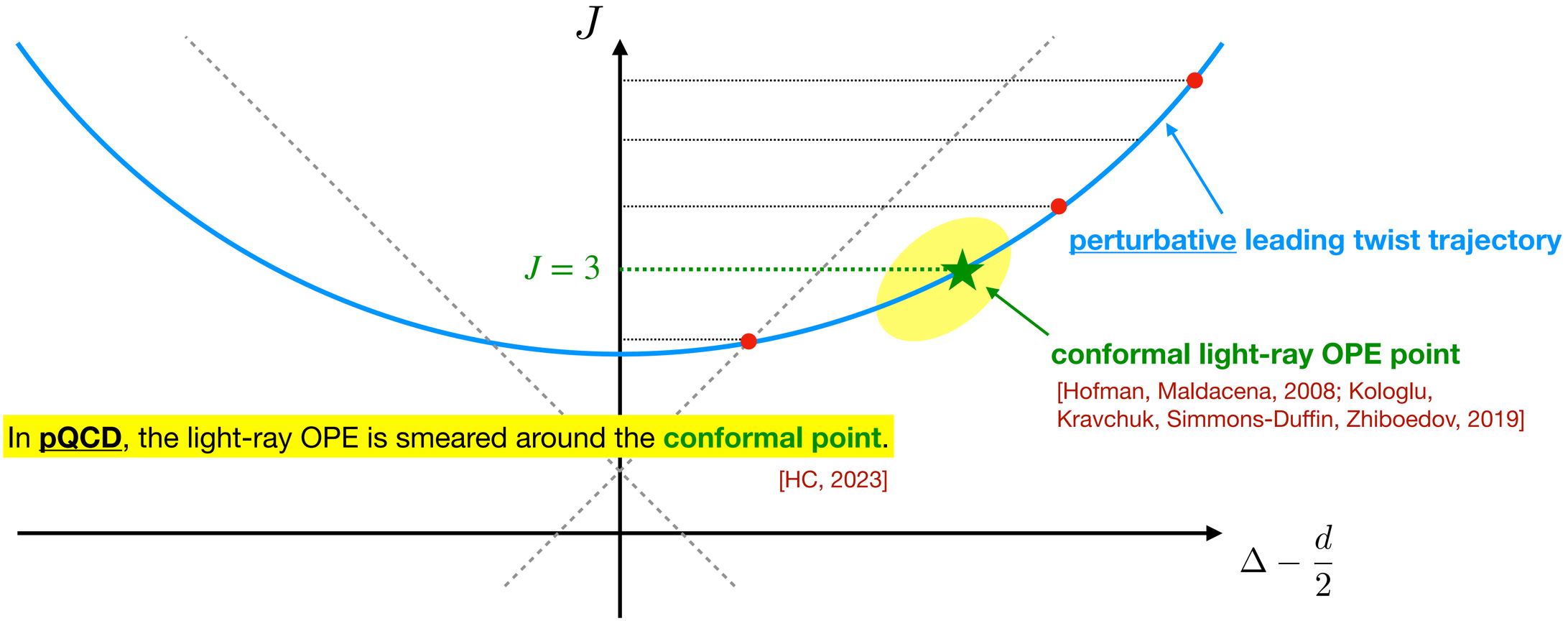
All red points on the **leading trajectory** contribute to high-energy limit of DIS.

Parton Distribution Function ⟷ **matrix element of the whole trajectory**

Similar Illustration for Collinear EEC

Light-ray operators are expected to be the **analytic continuation** of local operators.

[Kravchuk, Simmons-Duffin, 2018]



In **pQCD**, the light-ray OPE is smeared around the **conformal point**.

[HC, 2023]

Considering other effects (e.g. hadronization), we may probe points along the trajectory.

(restricted in the perturbative region)

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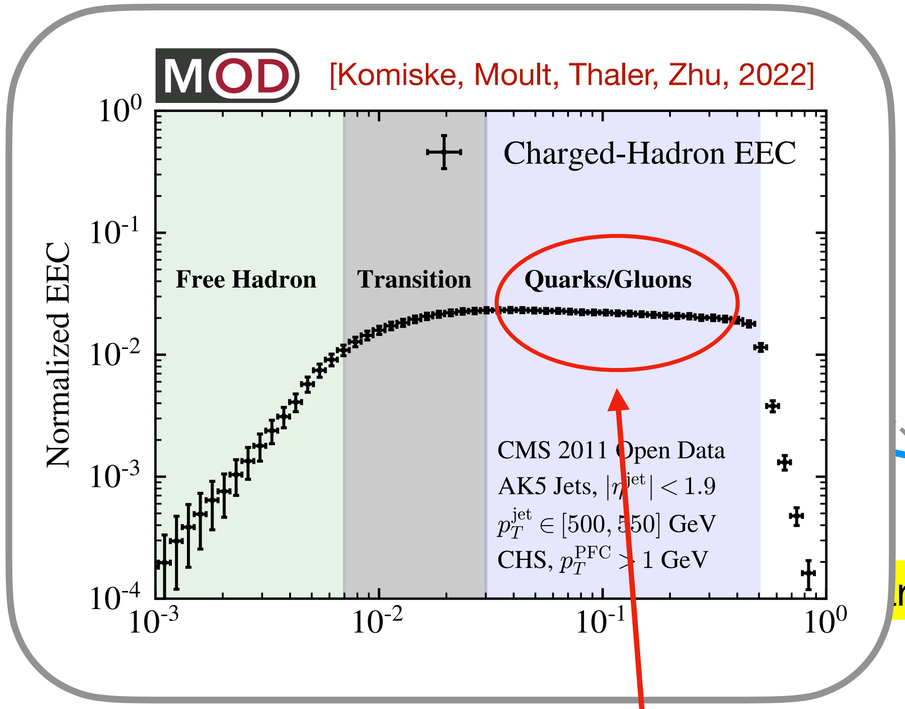


Considering other effects (e.g. hadronization), we may probe points along the trajectory.

(restricted in the perturbative region)

Phenomenological Light-ray OPE

In the perturbative region, we expect

$$\text{EEC} = \text{perturbative QCD prediction} + \text{power corrections in } \frac{\Lambda_{QCD}}{Q}$$

Previous studies on hadronization effects of EEC show similar feature in the collinear limit

consider the region

$$\Lambda_{QCD} \ll \theta Q \ll Q$$

$$\sim \frac{1}{\theta^3 Q}$$

Fragmentation model: Basham, Brown, Ellis, Love, 1979;
Soft gluon approximation: Korchemsky and Sterman, 1999;
Renormalon analysis: Schindler, Stewart, Sun, 2023

Following the **dispersive approach**, we also obtained the same behavior in the collinear limit.

Understanding the θ scaling from the light-ray OPE (controlled by twist expansion)

Perturbative QCD **Hadronization effects**

$$\mathcal{E}(n)\mathcal{E}(n') \sim \frac{C}{\theta^2}\mathbb{O}^{[J=3]}_{\tau=2}(n) + \text{dim=1}\ \Lambda_{\text{QCD}}\frac{B}{\theta^3}\mathbb{O}^{[J=2]}_{\tau=2}(n) + \cdots$$

fixed by boost symmetry

fixed by dimensional analysis

Multi-point cases and Relation to Factorization

Generalizing to multiple projected cases

Perturbative

Hadronization effects

$$\int_{\max\{\theta_{ij}\}=\theta_L} \mathcal{E}(n_1)\cdots\mathcal{E}(n_k) \sim \boxed{\frac{C}{\theta_L^2}\mathbb{O}_{\tau=2}^{[J=k+1]}(n)} + \boxed{\Lambda_{\rm QCD}\frac{B}{\theta_L^3}\mathbb{O}_{\tau=2}^{[J=k]}(n)+\cdots}$$

- Both OPE coefficients and light-ray operators should contain scale dependence.
- Light-ray OPE can be viewed as integrating out IR physics. It is also a factorization of long-distance and short-distance physics.
- We can map them to the factorization formula

$$F^{[k]}(\zeta_L, Q) = \frac{1}{\zeta_L}\int_0^1 dx\, x^k \vec{j}^{[k]}(\log\frac{\zeta_L x^2 Q^2}{\mu^2}, \mu)\cdot\vec{H}(x, \frac{Q^2}{\mu^2}, \mu)$$

← Generalization of perturbative factorization in [Dixon, Moult, Zhu, 2019]

$$f^{[k]}(\zeta_L, Q) = \frac{1}{\zeta_L^{3/2}}\int_0^1 dx\, x^{k-1} \vec{\mathcal{J}}^{[k]}(\frac{\zeta_L x^2 Q^2}{\mu^2}, \mu)\cdot\vec{H}(x, \frac{Q^2}{\mu^2}, \mu)$$

← **New proposal for the leading non-perturbative correction.**

LL approximation

We expect the perturbative evolution of light-ray operators relates the EEC curves at different energy scales.

$$f_a^{[k]}(\zeta, Q) = \frac{1}{\zeta^{3/2} Q} \sum_{I=1,2} B_I^{[k]}(\zeta Q^2) \left[\frac{\alpha_s(Q^2/4)}{\alpha_s(\zeta Q^2)}\right]^{\frac{\tilde{\gamma}_I^{(0)}(k)}{\beta_0}} V_{I;a}(k)$$

Evolution of light-ray operators

- Wilson coefficients are promoted to be scale dependent
- Quark-gluon mixing are diagonalized at leading-logarithmic accuracy

- Inserted into $a = q, g$ sources
- Normalized by Q^k

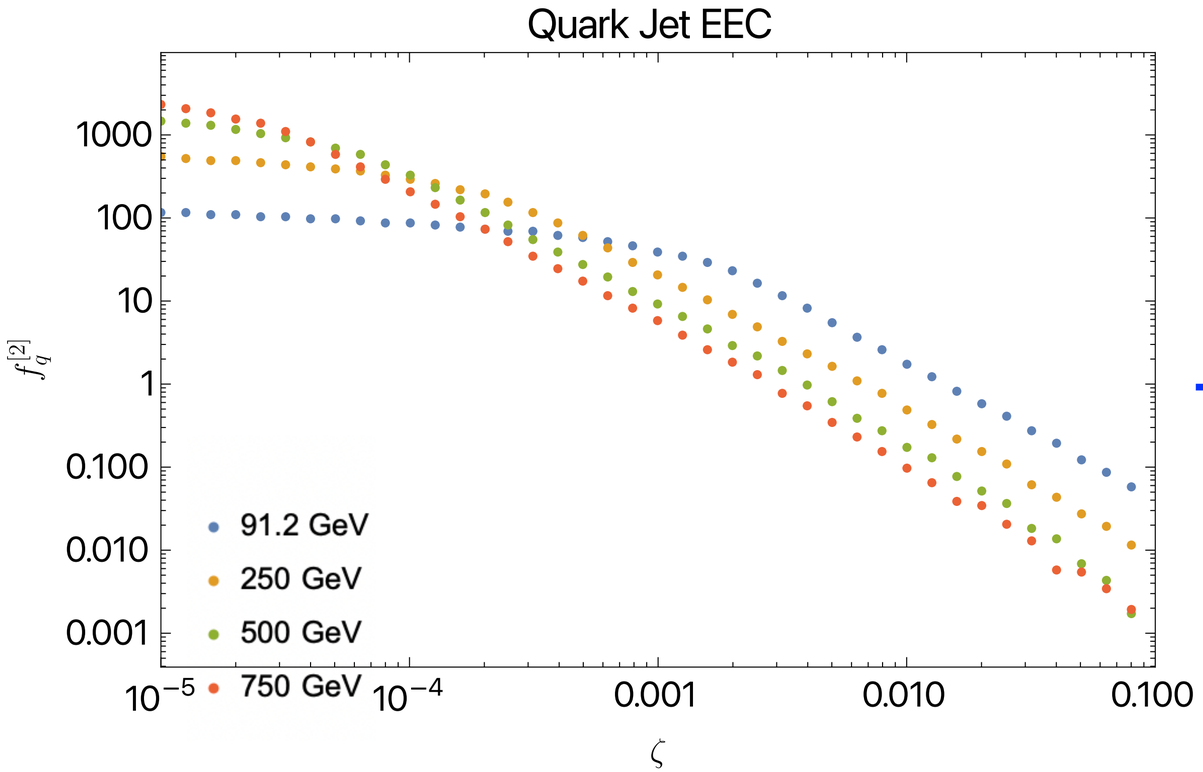
$$\int_{\max\{\theta_{ij}\}=\theta_L} \mathcal{E}(n_1) \cdots \mathcal{E}(n_k) \sim \frac{C}{\theta_L^2} \mathbb{O}_{\tau=2}^{[J=k+1]}(n) + \Lambda_{\mathrm{QCD}} \frac{B}{\theta_L^3} \mathbb{O}_{\tau=2}^{[J=k]}(n) + \cdots$$

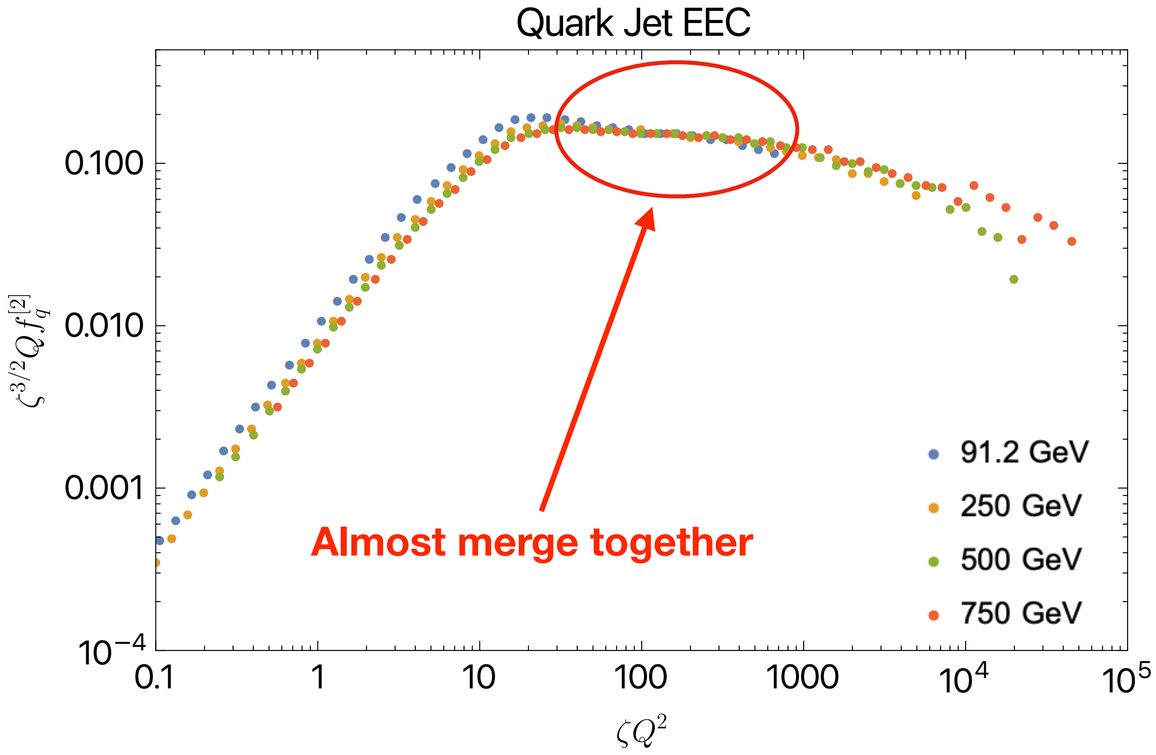
Hadronization effects

Simulation Data

In Monte Carlo event generators,

we extract power corrections from (hadron-level simulation - parton-level simulation).





change axes and strip off the expected classical behavior

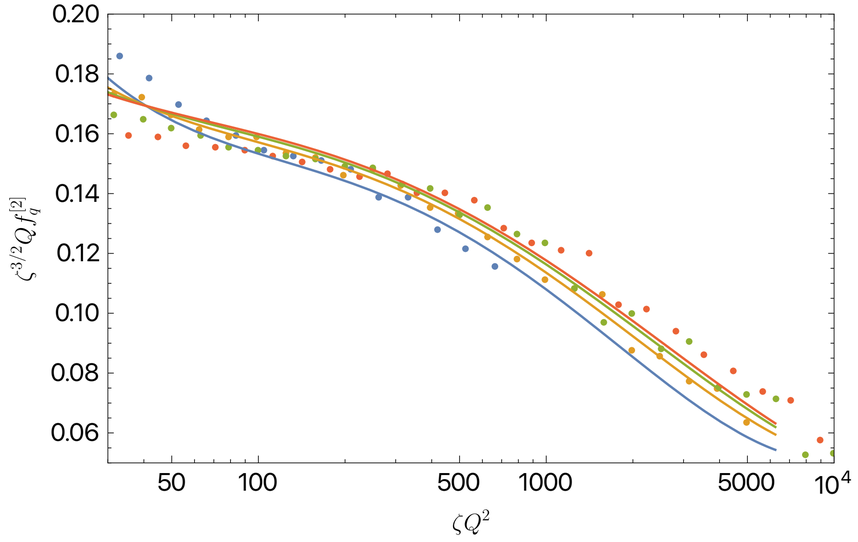
Next: input data at one scale to predict curves at other scales in the perturbative regions

Comparison

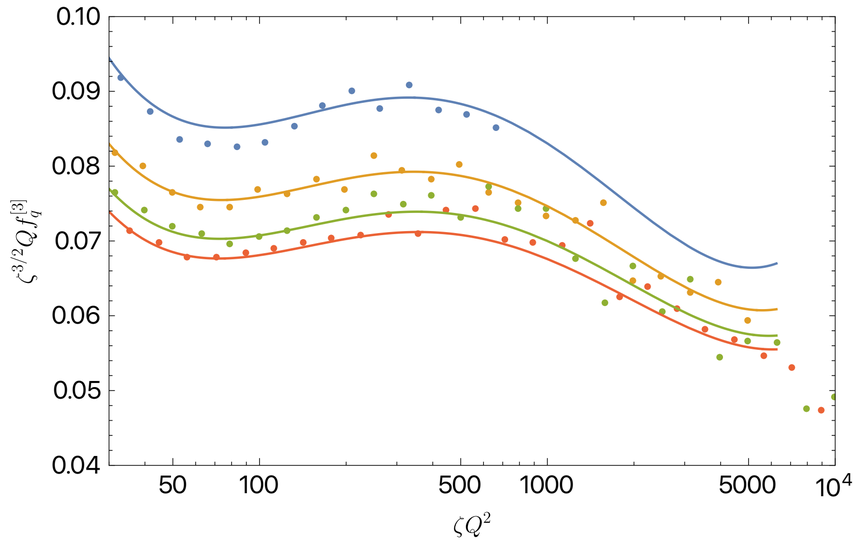
91.2 GeV
250 GeV
500 GeV
750 GeV

We fit curves from 250 GeV simulation data and predict curves at other center of mass energies.

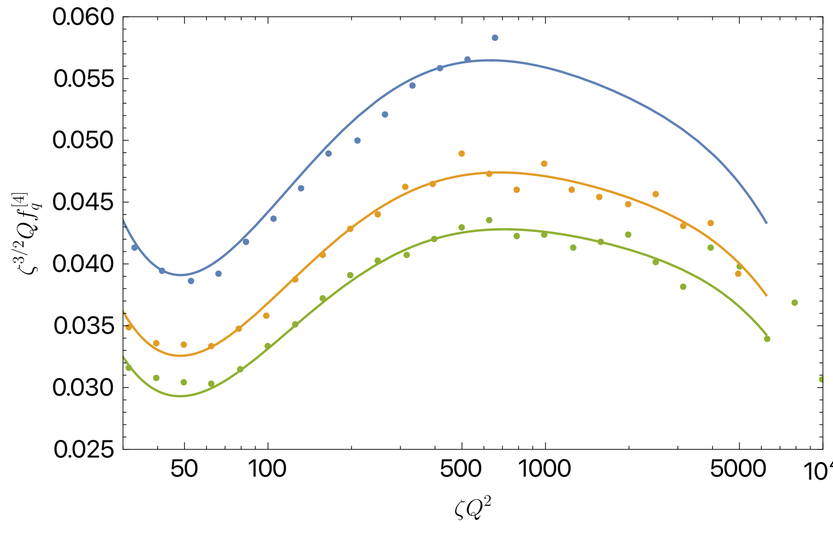
Quark Jet EEC



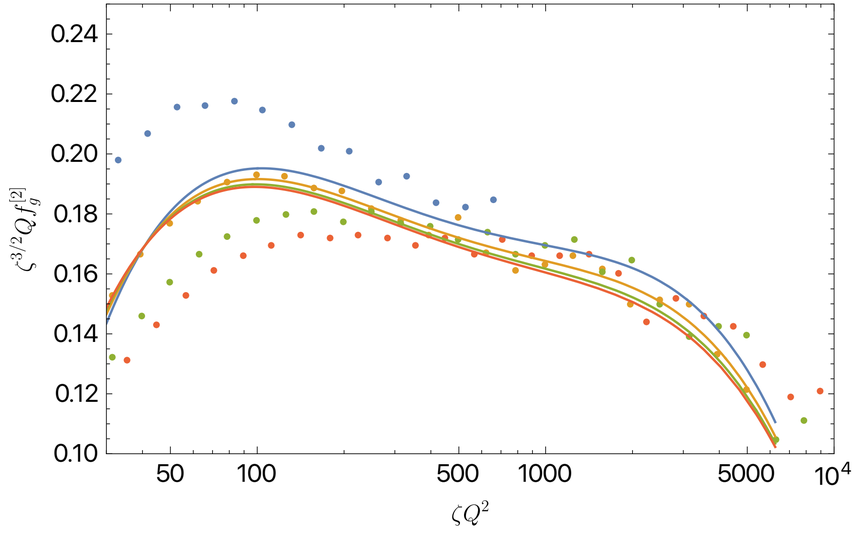
Quark Jet E3C



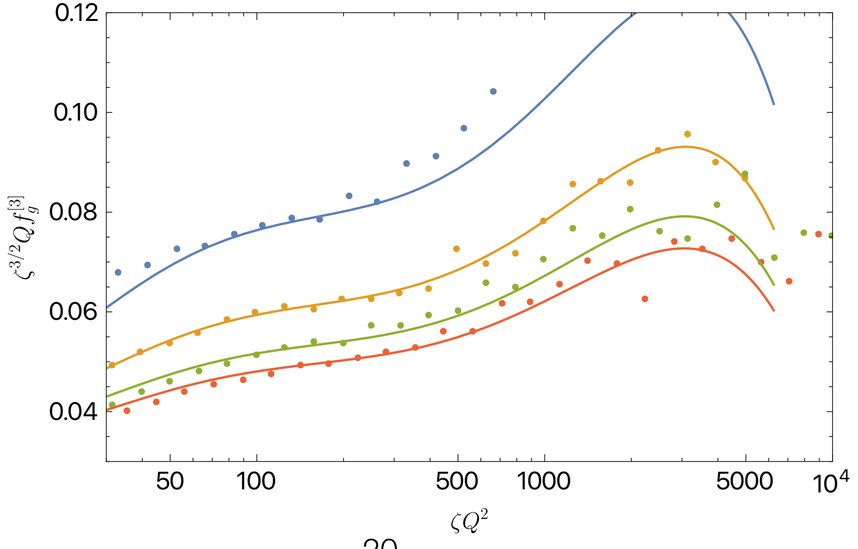
Quark Jet E4C



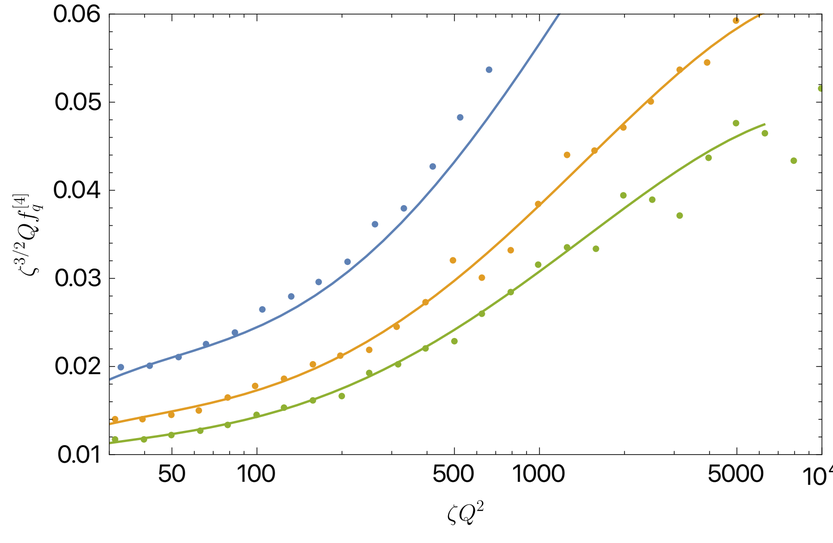
Gluon Jet EEC



Gluon Jet E3C



Gluon Jet E4C



Summary

- We have used light-ray OPE framework to study hadronization effects and introduced non-perturbative OPE functions.
- In the perturbative scaling region, we expect the hadronization corrections are described by lower-spin light-ray operators, whose anomalous dimension control the scale evolution.
- Compared with MC data, we validate our idea on the perturbative evolution for the leading hadronization effects in the collinear limit.

more precise extraction of the strong coupling from LHC data in the future

Outlook:

1. Interpretation of non-perturbative functions; Any relation to known parameters?
2. Combining with other QCD physics insights;
3. The validity of leading correction assumption $\Lambda^{1}_{\mathrm{QCD}}$ or $\Lambda^{1+\delta}_{\mathrm{QCD}}$?

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Thank you for your attention!

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