Factorization and transverse phase-space parton distributions

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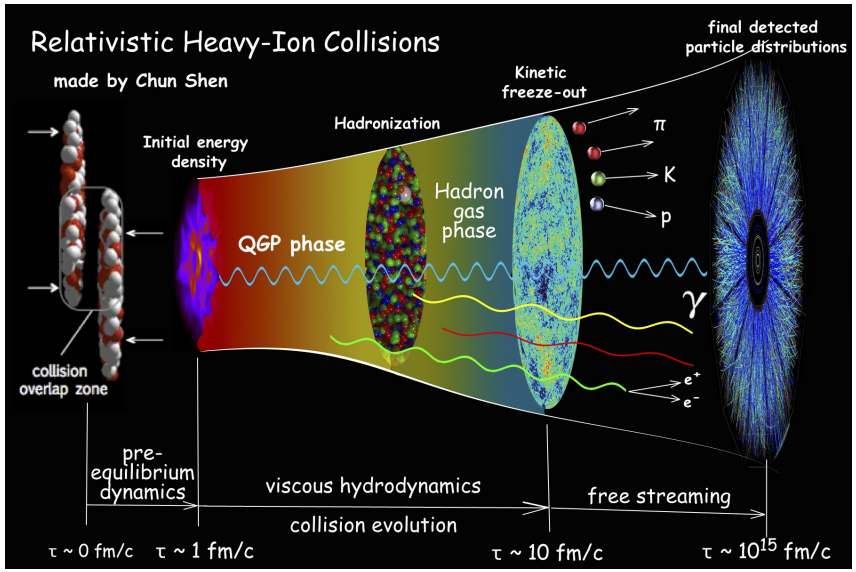
Motivation:

QCD factorization in heavy-ion collisions?

What is more in heavy-ion collisions?

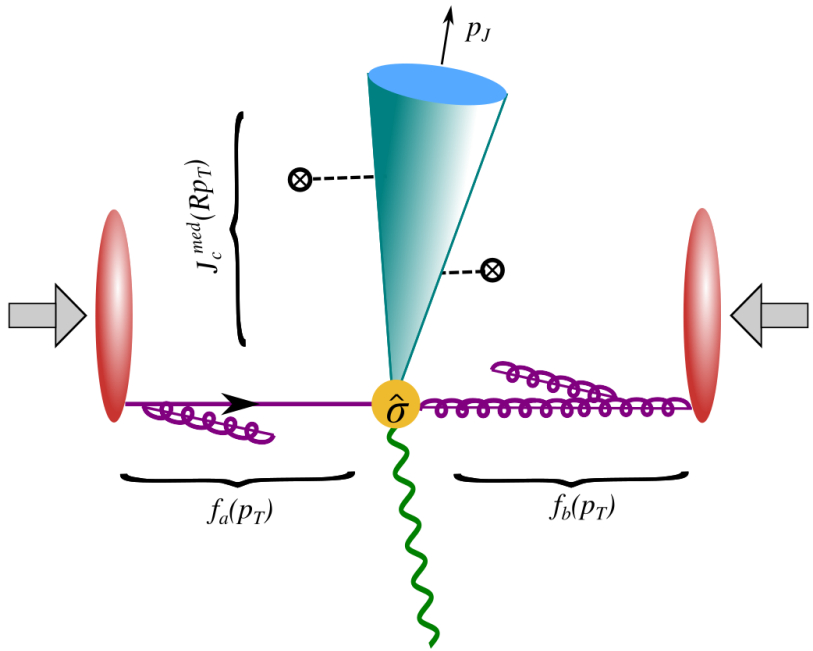


Compared to pp collisions:

1. Impact-parameter dependence at initial states
2. Interactions with QCD matter ($\sim$ 10 fm for Pb, Au, etc)

QCD factorization in heavy-ion collisions?

Factorization is assumed in the studies of hard processes:

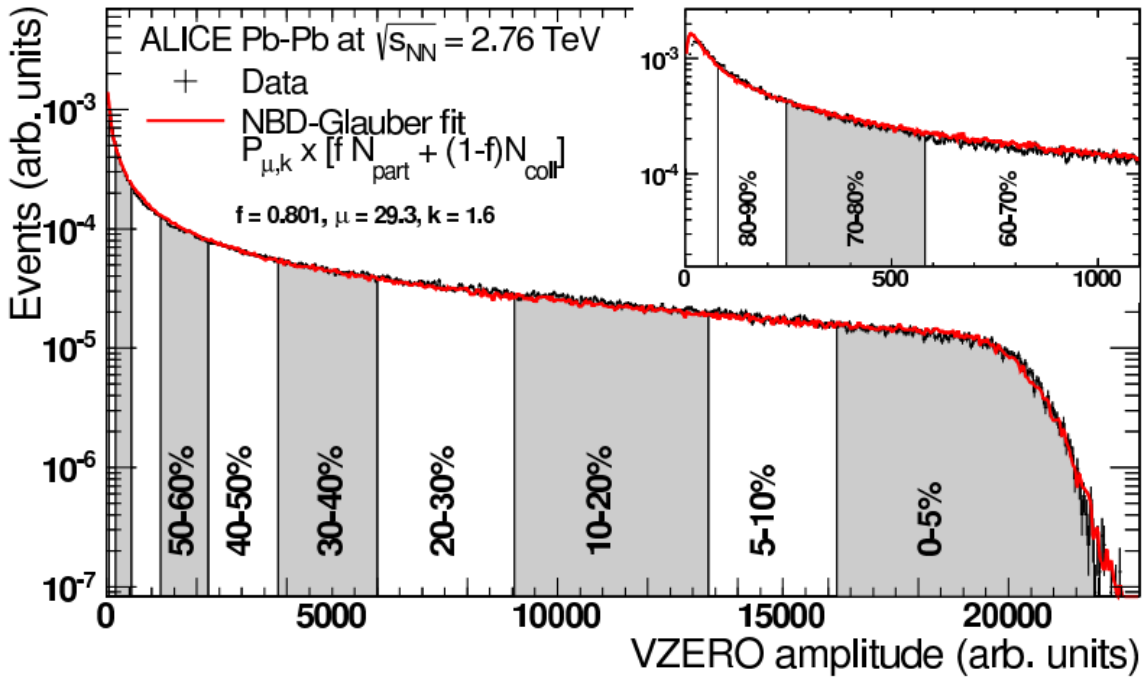


$$\frac{d\sigma}{dp_T d\eta} = \sum_{abc} f_a \otimes f_b \otimes \hat{\sigma}_{ab\to c} \otimes \underbrace{J_c^{med}}_{\text{final-state interactions}}$$

Is it valid and can it be formulated using SCET?

Impact parameter in heavy-ion collisions

Centrality c:

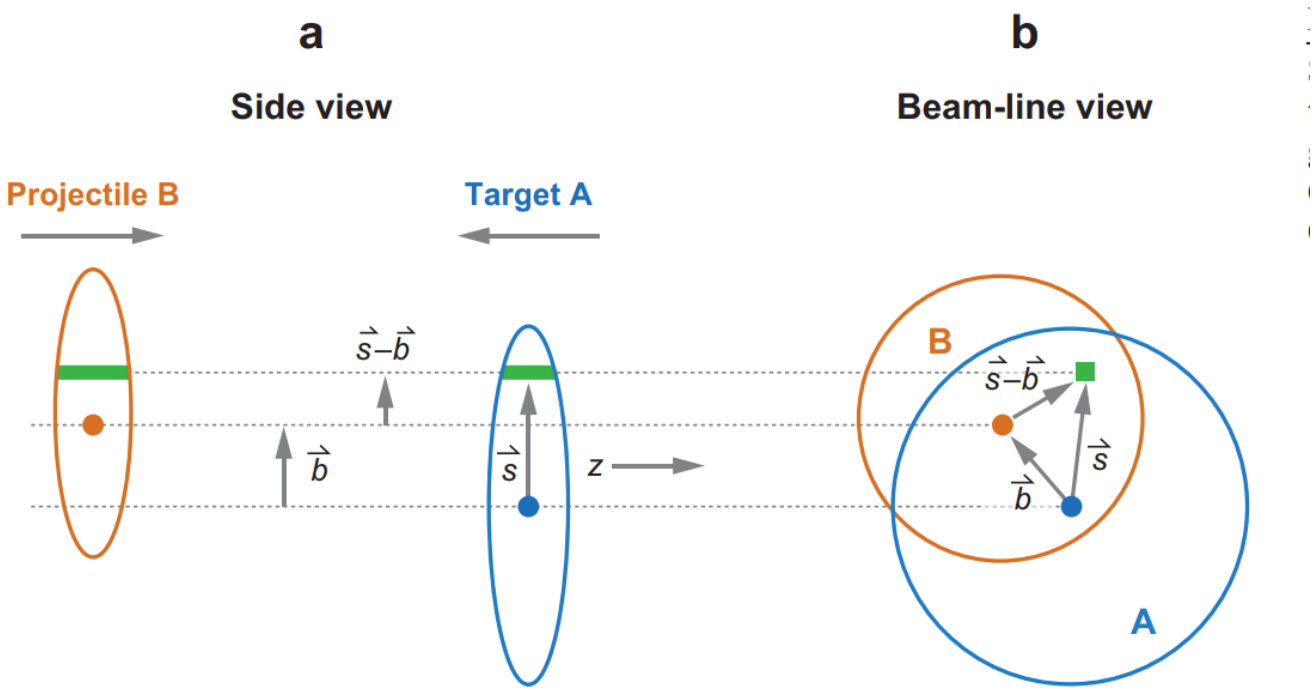


$$c \approx \underbrace{\frac{1}{\sigma}\int_{N_{ch}^{THR}}^{\infty}\frac{d\sigma}{dNch'}\,dNch' \approx \frac{1}{\sigma}\int_{0}^{E_{ZDC}^{THR}}\frac{\mathrm{d}\sigma}{\mathrm{d}E'_{ZDC}}\,\mathrm{d}E'_{ZDC}}_{\text{measurement}} \approx \underbrace{\frac{1}{\sigma}\int_{0}^{b}\frac{\mathrm{d}\sigma}{\mathrm{d}b'}\,\mathrm{d}b'}_{\text{theory}}$$

For example, ALICE, Phys. Rev. C **88**, no.4, 044909 (2013) [arXiv:1301.4361 [nucl-ex]].

Impact parameter in heavy-ion collisions

The CLASSICAL picture of collision geometry in the Glauber model:



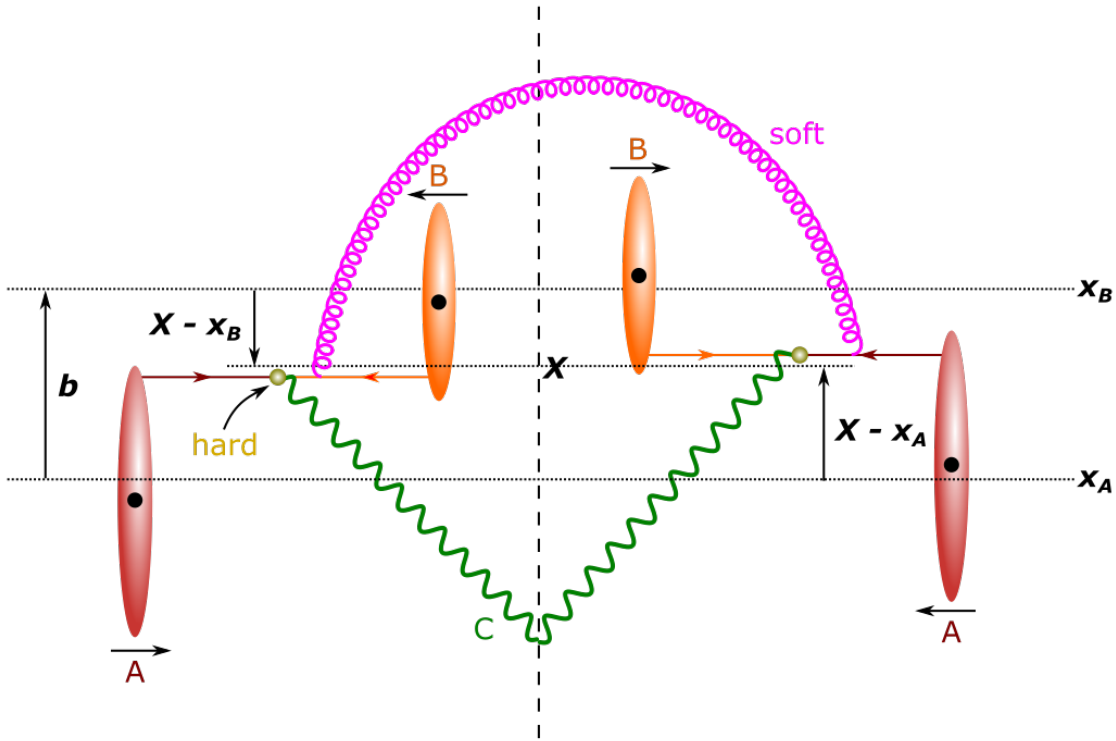
Miller, Reygers, Sanders and Steinberg, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 205 [arXiv:nucl-ex/0701025].

Topic 1: Impact-parameter dependent cross section in QFT

BW, JHEP 07 (2021) 002 [arXiv:2102.12916].

The Drell-Yan process in heavy-ion collisions

The quantum picture for A+B$\to$ C + anything else:

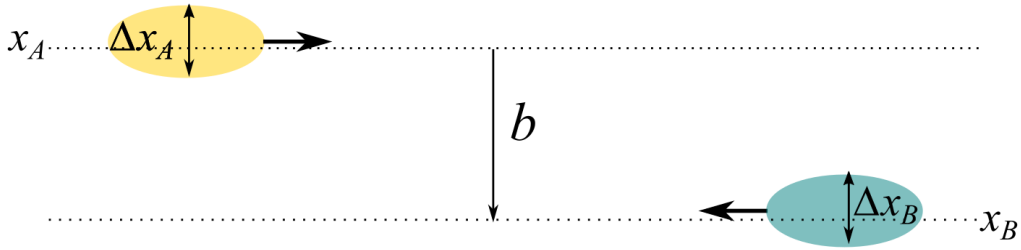


Here, C stands for some colorless final-state object.

Topic 2: Verify factorization formula in the Drell-Yan process

Impact-parameter dependent collisions

The impact-parameter dependent cross section



The probability for producing any observable O:

$$\frac{d\sigma}{d^2\mathbf{b}dO} \equiv \int \prod_f [d\Gamma_{p_f}]\, \delta(O - O(\{p_f\}) \langle \phi_1\phi_2|\hat{S}^\dagger|\{p_f\}\rangle\langle\{p_f\}|\hat{S}|\phi_1\phi_2\rangle, \qquad (1)$$

where the wave packages of the colliding particles with $\mathbf{x}_i$ the transverse location

$$|\phi_i\rangle = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{e^{-i\mathbf{p}\cdot\mathbf{x}_i}}{\sqrt{2E_p}} \underbrace{\phi_i(\vec{p})}_{\text{unknown}} |\vec{p}\rangle,$$

which are required to be sufficiently localized in space in order to define the impact parameter $\mathbf{b} \equiv \mathbf{x}_B - \mathbf{x}_A$: $\Delta x_A, \Delta x_B \ll b \equiv |\mathbf{b}|$.

BW, JHEP 07 (2021) 002 [arXiv:2102.12916].

The impact-parameter dependent cross section

$\frac{d\sigma}{d^2\mathbf{b}dO}$ defines cross section only if ϕ_i's can be integrated out!

ϕ_i's depend on how the beams are prepared.

M.E. Peskin and D.V. Schroeder, Addison-Wesley, Reading, USA (1995).

There are $3 \times 4 = 12$ momenta to integrate out:

1. Use an overall δ function of 4-momentum$\Rightarrow$ 8 momenta
2. Impose two conditions:

   i) $|P_{iz}| \gg |\mathbf{P}_i|, \Delta p_T, \Delta p_z,$ ii)$|\mathbf{b}| \gg \Delta x_T.$

3. Expand eq. (1) in the above limit:

   As a result, the 3-momenta of each ϕ_i in the amplitude and conjugate amplitude are identified.

4. Integrate the two wave packets$\Rightarrow$ 2-momenta conjugate to **b**

The impact-parameter dependent cross section

The cross section for producing any observable O:

$$\frac{d\sigma}{d^2\mathbf{b}dO} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} \int \prod_f [d\Gamma_{p_f}]\, \delta(O - O(\{p_f\})) \tag{2}$$
$$\times \frac{1}{2s} M(p_A, p_B \to \{p_f\}) M^*(\bar{p}_A, \bar{p}_B \to \{p_f\}) (2\pi)^4 \delta^{(4)}(p_A + p_B - \sum p_f)$$

with the incoming momenta

$$p_A^\mu = \bar{n}_A \cdot P_A \frac{n_A^\mu}{2} + \frac{q_T^\mu}{2} - \frac{q_T^2}{4\bar{n}_A \cdot P_A} \frac{\bar{n}_A^\mu}{2}, \qquad p_B^\mu = \bar{n}_B \cdot P_B \frac{n_B^\mu}{2} - \frac{q_T^\mu}{2} - \frac{q_T^2}{4\bar{n}_B \cdot P_B} \frac{\bar{n}_B^\mu}{2},$$
$$\bar{p}_A^\mu = \bar{n}_A \cdot P_A \frac{n_A^\mu}{2} - \frac{q_T^\mu}{2} - \frac{q_T^2}{4\bar{n}_A \cdot P_A} \frac{\bar{n}_A^\mu}{2}, \qquad \bar{p}_B^\mu = \bar{n}_B \cdot P_B \frac{n_B^\mu}{2} + \frac{q_T^\mu}{2} - \frac{q_T^2}{4\bar{n}_B \cdot P_B} \frac{\bar{n}_B^\mu}{2}.$$

Factorization and TPS PDFs

Factorization using SCET

Consider A+B$\to$ C + anything else at impact parameter b, which generically involves:

1. **SCET scales:**

$$n_i\text{-collinear: } p_{n_i}^{\mu} \sim Q\,(\lambda^2, 1, \lambda)_{n_i \bar{n}_i}\,,$$
$$\text{soft: } \quad p_s^{\mu} \sim \begin{cases} \lambda^2 Q & \text{for SCET}_{\text{I}} \\ \lambda Q & \text{for SCET}_{\text{II}} \end{cases}\,, \tag{3}$$

I.W. Stewart, Lectures on the Soft-Collinear Effective Theory, in MIT EFT Course 8.851, 2003;

T. Becher, A. Broggio and A. Ferroglia, Lect. Notes Phys. **896**, pp.1-206 (2015) Springer, 2015, [arXiv:1410.1892 [hep-ph]].

2. **Impact parameter:** $1/q_T \sim b\ (= 0 - 10\ \text{fm})$

3. **Nuclear/medium scales**:

   nuclear size $R \approx 10$ fm (for e.g., Pb), nuclear density ρ, $\cdots$

Let us first consider scales 1 & 2 and colorless final states C.

Factorization using SCET

Simply insert the S-matrix in position space into

$$\frac{d\sigma}{d^2\mathbf{b}dO} \equiv \int \prod_f [d\Gamma_{p_f}]\, \delta(O - O(\{p_f\}) \langle \phi_1\phi_2|\hat{S}^\dagger|\{p_f\}\rangle\langle\{p_f\}|\hat{S}|\phi_1\phi_2\rangle$$

where

$$\langle p_C, \{p_X\}|\hat{S}|\phi_A\phi_B\rangle = \int d^4x e^{ip_C\cdot x}\langle\{p_X\}|i\hat{M}(x)|\phi_A\phi_B\rangle.$$

with

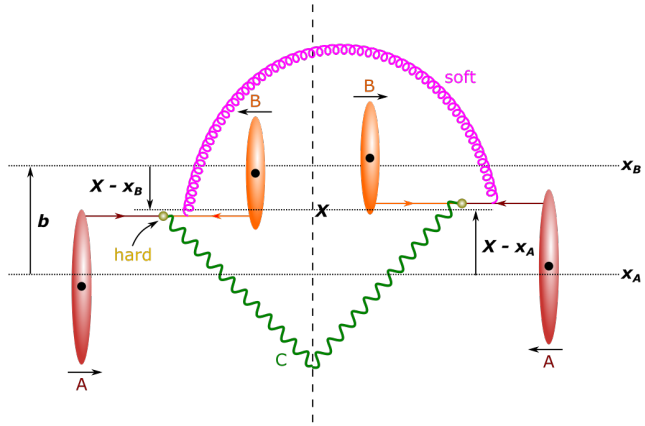
$$i\hat{M}(x) = \overbrace{\int dt_A dt_B \underbrace{C^{a_A a_B}_{\alpha_A\alpha_B}(\epsilon, t_A, t_B)}_{\text{hard}} \underbrace{[S_{n_A}\phi_{n_A}(x + t_A\bar{n}_A)]^{\alpha_A}_{a_A}[S_{n_B}\phi_{n_B}(x + t_B\bar{n}_B)]^{\alpha_B}_{a_B}}_{\text{soft-collinear factorization}}}^{\text{hard-collinear factorization}},$$

S_{n_i}: the soft Wilson line along the collinear direction n_i

$$\phi_{n_i}: \chi_{n_i}(x) \equiv W^\dagger_{n_i}(x)\frac{n_i\bar{n}_i}{4}\psi_{n_i}(x), \qquad \bar{\chi}_{n_i}(x), \qquad \mathcal{B}^\mu_{n_i T} = \frac{1}{g_s}W^\dagger_{n_i}(x)iD^\mu_{n_i T}W_{n_i}(x)$$

with $D^\mu_{n_i T} \equiv \partial^\mu_T - ig_s A^\mu_{n_i T}$ and W_{n_i} the n_i-collinear Wilson line.

The factorization formula



$$\frac{d\sigma}{d^2\mathbf{b}dy_C d^2\mathbf{p}_C} = \frac{1}{2(2\pi)^3}\int d^4X d^4x\; e^{-ip_C\cdot x} \times \sum_{\{p_X\}} \langle\phi_A\phi_B|\hat{M}^\dagger\left(X+\frac{x}{2}\right)|\{p_X\}\rangle\langle\{p_X\}|\hat{M}\left(X-\frac{x}{2}\right)|\phi_A\phi_B\rangle$$

$$= \frac{1}{4\pi s}\sum_{j,k}\int d^2\mathbf{X}\int d^2\mathbf{x}e^{i\mathbf{p}_C\cdot\mathbf{x}}\int_0^1 \frac{dz_A}{z_A}\frac{dz_B}{z_B}\underbrace{\mathcal{T}_{j/A}(\mathbf{X},z_A,\mathbf{x})\mathcal{T}_{k/B}(\mathbf{X}-\mathbf{b},z_B,\mathbf{x})}_{\text{Thickness beam functions}}$$

$$\times\int\prod_f[d\Gamma_{p_f}]\prod_{i=A,B}\delta(z_i\bar{n}_i\cdot P_i-\bar{n}_i\cdot p_C-\sum\bar{n}_i\cdot p_f)$$

$$\times\underbrace{H^{\bar{a}_A\bar{a}_B}_{a_Aa_B}(z_AP_A,z_BP_B\to p_C,\{p_f\})}_{\text{hard function}}\underbrace{\mathcal{S}^{\bar{a}_A\bar{a}_B}_{a_Aa_B}(\mathbf{x})}_{\text{soft function}}.$$

BW, JHEP 07 (2021) 002 [arXiv:2102.12916].

Thickness beam functions

Definition:

$$\mathcal{T}_{j/i}(\mathbf{r},z,\mathbf{x}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{r}} \int \frac{dt}{2\pi} e^{-izt\bar{n}\cdot P} \times \left\langle \bar{n}\cdot P, -\frac{\mathbf{q}}{2}\right| [\phi_n^\dagger]_a^{\alpha'} \left(\frac{t\bar{n}}{2}+\frac{x_T}{2}\right) \Gamma_{\alpha'\alpha} [\phi_n]_a^{\alpha} \left(-\frac{t\bar{n}}{2}-\frac{x_T}{2}\right) \left|\bar{n}\cdot P, \frac{\mathbf{q}}{2}\right\rangle$$

where Γ: some projector in spin/polarization.

The operators are the same as beam functions!

I. W. Stewart, F. J. Tackmann and W. J. Waalewijn, Phys. Rev. D **81**, 094035 (2010) [arXiv:0910.0467 [hep-ph]].

For example,

$$\mathcal{T}_{q/i}(\mathbf{r},z,\mathbf{x}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{r}} \int \frac{dt}{2\pi} e^{-izt\bar{n}\cdot P} \times \left\langle \bar{n}\cdot P, -\frac{\mathbf{q}}{2}\right| \bar{\chi}_n \left(\frac{t\bar{n}}{2}+\frac{x_T}{2}\right) \frac{\bar{n}}{2} \chi_n \left(-\frac{t\bar{n}}{2}-\frac{x_T}{2}\right) \left|\bar{n}\cdot P, \frac{\mathbf{q}}{2}\right\rangle \tag{4}$$

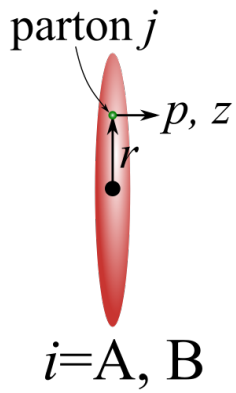
$$\mathcal{T}_{g/i}(\mathbf{r},z,\mathbf{x}) = z\bar{n}\cdot P(-g_{T\,\alpha'\alpha}) \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{r}} \int \frac{dt}{2\pi} e^{-izt\bar{n}\cdot P} \times \left\langle \bar{n}\cdot P, -\frac{\mathbf{q}}{2}\right| \mathcal{B}_{nT}^{a\alpha'} \left(\frac{t\bar{n}}{2}+\frac{x_T}{2}\right) \mathcal{B}_{nT}^{a\alpha} \left(-\frac{t\bar{n}}{2}-\frac{x_T}{2}\right) \left|\bar{n}\cdot P, \frac{\mathbf{q}}{2}\right\rangle .$$

Thickness beam functions

Physical interpretation:

Fourier transform of transverse phase space (TPS) PDF with respect to **x**

$$f_{j/i}(\mathbf{r}, z, \mathbf{p}) = \quad = \int d^2\mathbf{x} e^{i\mathbf{p}\cdot\mathbf{x}} \mathcal{T}_{j/i}(\mathbf{r}, z, \mathbf{x}).$$



TMD PDF at r! (up to gauge link prescription etc)

In the limit x $\to$ 0: $\mathcal{T}_{j/i}(\mathbf{r}, z, \mathbf{0})$ admits the interpretation as the PDF at **r**

The Glauber model

Nuclei are modelled as uncorrelated nucleons:

the nucleons are distributed according to the charge distribution

$$\rho_A(r)/\rho_0 = \frac{1}{1+e^{\frac{r-R_{ws}}{a}}} \qquad \text{with } R_{ws} = 6.62 \text{ fm and } a = 0.55 \text{ fm for } {}^{208}\text{Pb}.$$

where ρ_0 is the density at the center.

C. W. De Jager, H. De Vries and C. De Vries, Atom. Data Nucl. Data Tabl. **14** (1974), 479-508.

The thickness function

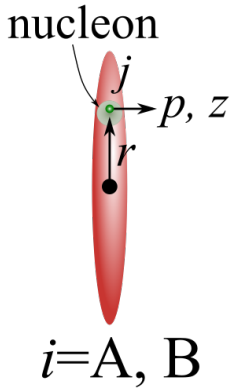
$$T_i(\mathbf{r}_i) \equiv \int dz \rho_i(\mathbf{r}_i, z). \tag{5}$$

Miller, Reygers, Sanders and Steinberg, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 205 [arXiv:nucl-ex/0701025].

Thickness beam functions in the Glauber model

In terms of the thickness functions

$$\mathcal{T}_{j/i}(\mathbf{r}_i, z, \mathbf{x}) \rightarrow$$



$$= T_i(\mathbf{r}_i)\left[\frac{Z_i}{Z_i+N_i}B_{j/p}(z,\mathbf{x}) + \frac{N_i}{Z_i+N_i}B_{j/n}(z,\mathbf{x})\right]$$

$=$ the thickness functions $\times$ the beam functions

where $B_{j/p}$ and $B_{j/n}$ are the beam functions for protons and neutrons, respectively, and the nucleus is assumed to contain Z_i protons and N_i neutrons.

The factorization formula gives

$$R_{AA} \equiv \frac{\frac{d\sigma_{AB}}{d^2\mathbf{b}dy_C d^2\mathbf{p}_C}}{T_{AB}(\mathbf{b})\frac{d\sigma_{nn}}{dy_C d^2\mathbf{p}_C}} = 1,$$

with $T_{AB}(\mathbf{b}) \equiv \int d^2\mathbf{X}\, T_A(\mathbf{X})\, T_B(\mathbf{X}-\mathbf{b})$.

Soft and hard functions

The soft function:

$$\mathcal{S}^{\bar{a}_A \bar{a}_B}_{a_A a_B}(\mathbf{x}) = \langle 0|\, \bar{T}[S^{\dagger a'_B \bar{a}_B}_{n_B}(\mathbf{x}) S^{\dagger a'_A \bar{a}_A}_{n_A}(\mathbf{x})]\, T[S^{a_A a'_A}_{n_A}(0) S^{a_B a'_B}_{n_B}(0)]|0\rangle$$

The hard function:

$$H^{\bar{a}_A \bar{a}_B}_{a_A a_B} \equiv \frac{P^{\bar{\alpha}_A \alpha_A}}{d_{c_A}} \frac{P^{\bar{\alpha}_B \alpha_B}}{d_{c_B}} \tilde{\mathcal{C}}^{*\bar{a}_A \bar{a}_B}_{\bar{\alpha}_A \bar{\alpha}_B} \tilde{\mathcal{C}}^{a_A a_B}_{\alpha_A \alpha_B}$$

with $\tilde{\mathcal{C}}$ given by

$$\tilde{\mathcal{C}}(\epsilon, z_A \bar{n}_A \cdot P_A, z_B \bar{n}_B \cdot P_B) = \int dt_A dt_B e^{i(t_A z_A \bar{n}_A \cdot P_A + t_B z_B \bar{n}_B \cdot P_B)} \mathcal{C}(\epsilon, t_A, t_B).$$

The same as for pp collisions!

Verification of factorizaiton

The Drell-Yan process at parton level

The impact-parameter dependent Drell-Yan cross section

$$q\bar{q} \to \gamma^* + \text{anything elseat impact parameter } b$$

Verification up to one loop

the definition in eq. (2) gives

$$\begin{aligned}\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} =& \frac{\pi}{s}\int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} \int \prod_f [d\Gamma_{p_f}]\, \delta(p_C^2 - Q^2) \\ & \times M(p_A, p_B \to p_C, \{p_f\}) M^*(\bar{p}_A, \bar{p}_B \to p_C, \{p_f\}) \end{aligned} \tag{6}$$

and the factorization formula reduces to

$$\begin{aligned}\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} =& \frac{\pi}{s}\sum_{j,k}\int \frac{dz_A}{z_A}\frac{dz_B}{z_B} \int d^2\mathbf{X} \mathcal{T}_{j/q}(\mathbf{X}, z_A)\mathcal{T}_{k/\bar{q}}(\mathbf{X}-\mathbf{b}, z_B) \int \prod_f [d\Gamma_{p_f}] \\ & \times H\left(z_A P_A, z_B P_B \to p_C, \{p_f\}\right) \delta(\bar{n}_A\cdot p_C \bar{n}_B \cdot p_C - Q^2) \end{aligned} \tag{7}$$

to be compared up to leading order in $\lambda = 1/(bQ) \ll 1$.

At the Born level

Expanding the cross section at leading order in λ yields

$$\frac{d\sigma^{(0)}}{d^2\mathbf{b}} = \pi^2 e_q^2 \delta(s - Q^2)\delta^{(2)}(\mathbf{b}).$$

The hard scattering is initiated by the quark and antiquark only when they pass very close to one another at a distance $\sim 1/Q$, which defines "point-like" in our calculation.

For the factorization formula:

Inserting into the factorization formula the thickness beam function

$$\mathcal{T}^{(0)}_{q/q}(\mathbf{r}, z) = \mathcal{T}^{(0)}_{\bar{q}/\bar{q}}(\mathbf{r}, z) = \delta(1 - z)\delta^{(2)}(\mathbf{r}),$$

and the hard function

$$H^{(0)}(z_A P_A, z_B P_B \to p_C) = \overline{|M^{(0)}|}(z_A P_A, z_B P_B \to p_C) = e_q^2(1 - \epsilon) z_A z_B s,$$

confirms the validity of factorization at the Born level.

At one loop

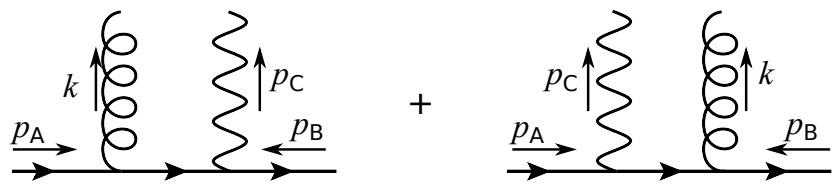
Verify using the method of regions:

From eq. (2), the contribution from real diagrams

$$\frac{d\sigma_r^{(1)}}{d^2\mathbf{b}} = \frac{\pi}{s}\int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} \int d\Gamma_k \overline{|M_r^{(1)}|^2}(p_A, p_B \to p_C, k \leftarrow \bar{p}_A, \bar{p}_B) \times \delta((p_A + p_B - k)^2 - Q^2), \tag{8}$$

with

$$iM_r^{(1)}(p_A, p_B \to p_C, k) \equiv$$



. (9)

And the contribution from virtual diagrams is denoted by

$$\frac{d\sigma_v^{(1)}}{d^2\mathbf{b}} = \frac{\pi}{s}\int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} \overline{|M_v^{(1)}|^2}(p_A, p_B \to p_C \leftarrow \bar{p}_A, \bar{p}_B) \times \delta((p_A + p_B)^2 - Q^2) \tag{10}$$

with $\overline{|M_v^{(1)}|^2}$ the spin-averaged amplitude squared which contains virtual-gluon contributions.

At one loop

1. The hard region: $k^\mu \sim Q$

$$\frac{d\sigma_h^{(1)}}{d^2\mathbf{b}} = \delta^{(2)}(\mathbf{b})\sigma^{(1)}(P_A, P_B \to p_C)$$

with $\sigma^{(1)}$ the conventional one-loop cross section for $q\bar{q} \to \gamma^*$.

2. Two collinear regions: $k^\mu \sim Q(\lambda^2, 1, \lambda)_{n_i \bar{n}_i}$

Let us take for example the n_A-collinear region in which

$$p_A^\mu \sim \bar{p}_A^\mu \sim k^\mu \sim (\lambda^2, 1, \lambda)_{n_A \bar{n}_A}, \qquad p_B^\mu \sim \bar{p}_B^\mu \sim (1, \lambda^2, \lambda)_{n_A \bar{n}_A}.$$

Upon expansion in λ, one has

$$\overline{|M_{r,n_A}|^2}(p_A, p_B \to p_C, k \leftarrow \bar{p}_A, \bar{p}_B) = e_q^2(1-\epsilon)s\frac{g_s^2 C_F}{2}\frac{N_{qq}(z)\left[4|\mathbf{k}|^2 - (1-z)^2|\mathbf{q}|^2\right]}{\left|\mathbf{k} + (1-z)\frac{\mathbf{q}}{2}\right|^2 \left|\mathbf{k} - (1-z)\frac{\mathbf{q}}{2}\right|^2}$$

with $N_{qq}(z) = \left(1+z^2\right) - \epsilon(1-z)^2$. Note that virtual diagrams become scaleless, and vanish in dimensional regularization.

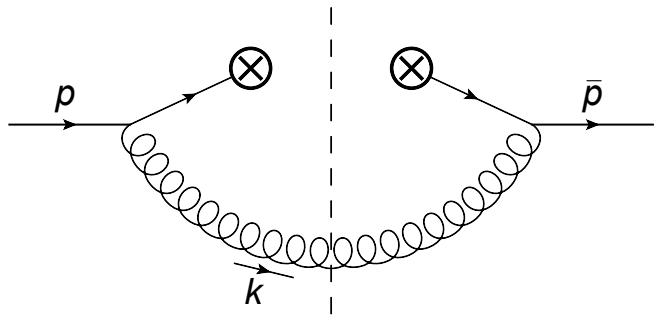
Spatial quark distribution in a quark at one loop

Perturbative series of the spatial quark distribution:

$$\mathcal{T}_{q/q}(\mathbf{r},z) = \delta(1-z)\delta^{(2)}(\mathbf{r}) + \mathcal{T}^{(1)}_{q/q}(\mathbf{r},z) + O(\alpha_s^2).$$

where

$$\mathcal{T}^{(1)}_{q/q}(\mathbf{r},z) =$$



$$= \frac{\alpha_s C_F}{2\pi^2 r^2}\left[-\left(\frac{1}{\epsilon}+L_T\right)\delta(1-z)+\frac{1+z^2}{(1-z)_+}\right].$$

with $L_T \equiv 2\log\left(\frac{r\mu}{2e^{-\gamma_E}}\right)$.

At finite b

$$\frac{d\sigma^{(1)}_{qq}}{d^2\mathbf{b}} \propto \alpha_s/(sb^2) + O(\alpha_s^2).$$

At one loop

By comparing the above results, one confirms:

$$\frac{d\sigma^{(1)}_{r,n_A}}{d^2\mathbf{b}} = \frac{\pi}{s}\int\frac{d^2\mathbf{q}}{(2\pi)^2}e^{i\mathbf{q}\cdot\mathbf{b}}\int d\Gamma_k\overline{|M_{r,n_A}|^2}(p_A,p_B\to p_C,k\leftarrow\bar{p}_A,\bar{p}_B)\delta(z_As-Q^2)$$
$$= \frac{\pi}{s}\int\frac{dz_A}{z_A}\mathcal{T}^{(1)}_{q/q}(\mathbf{b},z_A)H^{(0)}(z_AP_A,P_B\to p_C)\delta(z_As-Q^2)$$

3. The soft region: $k^\mu\sim\lambda Q$:

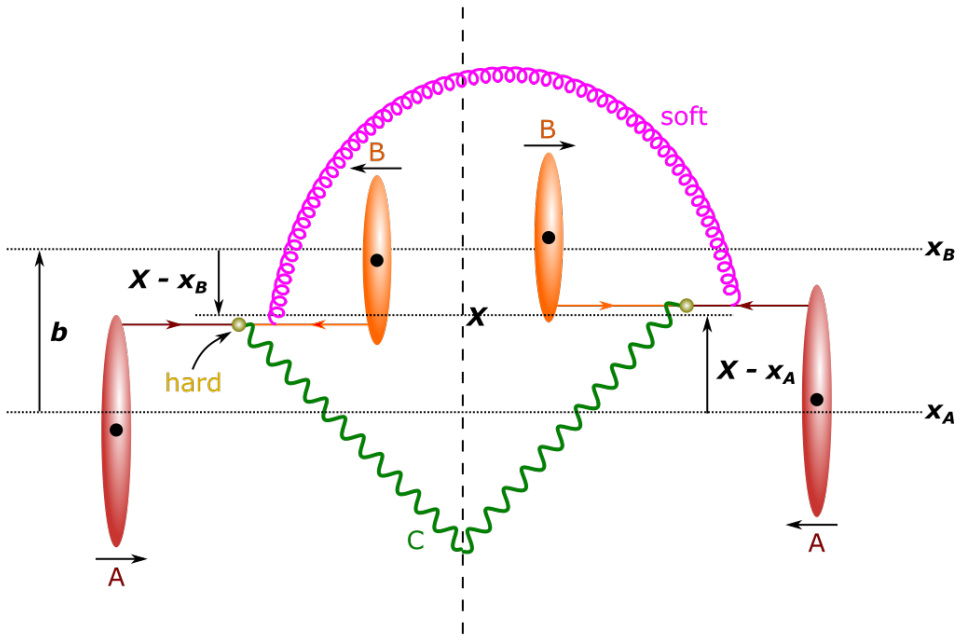
The contribution from the soft region vanishes.

In summary: at leading order in $1/(bQ)$, one has

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = \frac{\pi}{s}\int\frac{d^2\mathbf{q}}{(2\pi)^2}e^{i\mathbf{q}\cdot\mathbf{b}}\int\prod_f[d\Gamma_{p_f}]\,\delta(p_C^2-Q^2)$$
$$\times M(p_A,p_B\to p_C,\{p_f\})M^*(\bar{p}_A,\bar{p}_B\to p_C,\{p_f\})$$
$$= \frac{\pi}{s}\sum_{j,k}\int\frac{dz_A}{z_A}\frac{dz_B}{z_B}\int d^2\mathbf{X}\mathcal{T}_{j/q}(\mathbf{X},z_A)\mathcal{T}_{k/\bar{q}}(\mathbf{X}-\mathbf{b},z_B)\int\prod_f[d\Gamma_{p_f}]$$
$$\times H\left(z_AP_A,z_BP_B\to p_C,\{p_f\}\right)\delta(\bar{n}_A\cdot p_C\bar{n}_B\cdot p_C-Q^2)$$

Summary and Perspective

A unified framework for A+B$\to$ C + anything else:



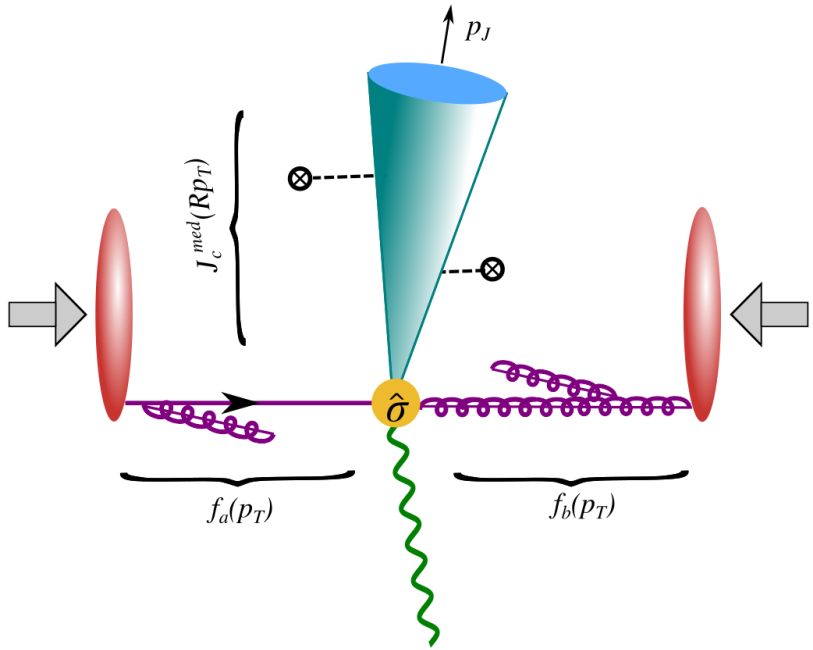
Issues solved:

1. Impact-parameter dependent cross section in QFT
2. The factorization formula is derived using SCET for $C = \gamma^*, W^{\pm}, Z^0, H^0$

One open question: the evolution equation of $\mathcal{T}_{j/i}$?

Summary and Perspective

Hard processes to be addressed:



$$\frac{d\sigma}{d^2\mathbf{b}d\eta_J dq_x dy_V d^2\mathbf{p}_V} = \sum_{ijk} \int \frac{db_x}{2\pi} e^{ib_x q_x} \int d^2\mathbf{X} \int_0^1 \frac{dz_i}{z_i} \frac{dz_j}{z_j} \mathcal{T}_{i/A}(\mathbf{X}, z_i, b_x)$$

$$\mathcal{T}_{j/B}(\mathbf{X}-\mathbf{b}, z_j, b_x) \times \mathcal{H}_{ij\to Vk}(p_{T,V}, y_V - \eta_J) \ll J_k(b_x, \mathbf{X}) S_{ijk}(b_x, \eta_J, \mathbf{X}) \gg$$

where $\ll \cdots \gg$ reminds us that the two functions are modified in the final states by QCD medium.

Y. T. Chien, R. Rahn, D. Y. Shao, W. J. Waalewijn and BW, work in progress.