Transverse Momentum Dependent PDFs in Chiral Perturbation Theory

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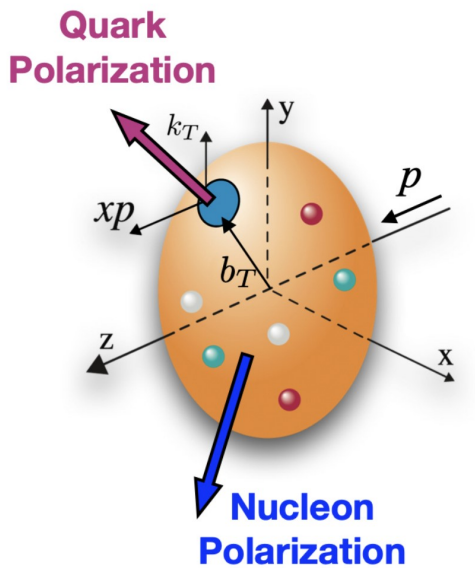
TMD PDFs

Transverse Momementum Dependent PDFs (TMD PDFs)

Generalizations of PDFs that describe the 3D structure of hadrons.

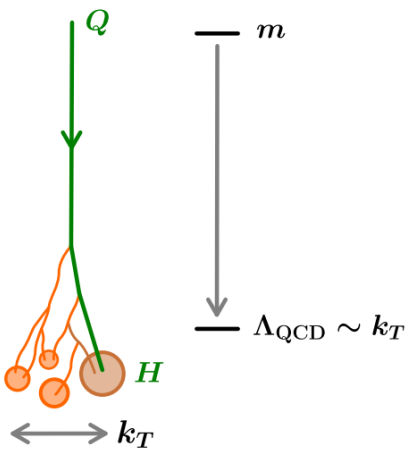
$$f_{iH}(x, b_T) = \int \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \langle H| \overline{\psi_i}(b^-, b_T) W_{\sqsubset}(b, 0) \not{n} \psi_i(0) |H\rangle$$

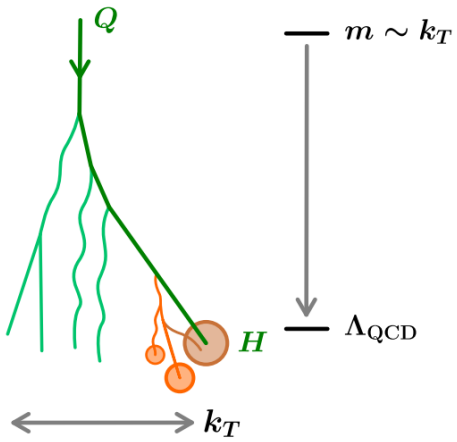


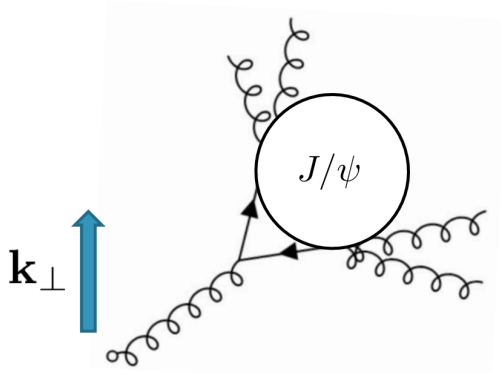
[1] TMD Handbook. 2304.03302 [hep-ph]

Can you calculate TMD PDFs in an effective theory?

- Systems with heavy quarks use heavy quark effective theory and non-relativistic QCD to study TMDs [2,3,4].







[2] R. von Kuk, J. Michel, and Z. Sun. JHEP 09 (2023) 205

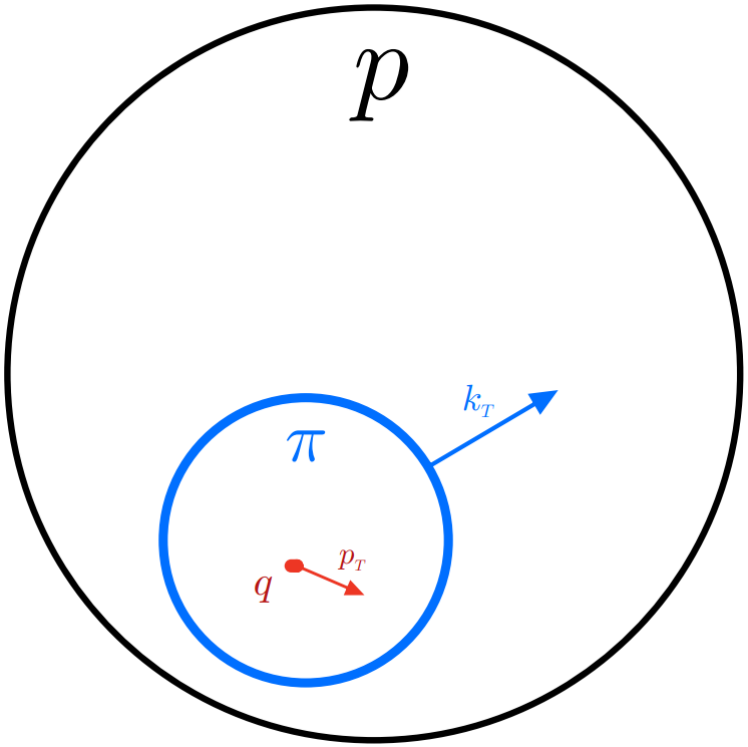
[3] R. von Kuk, J. Michel, and Z. Sun. 2404.08622 (See Monday's talk)

[4] M. Copeland, S. Fleming, R. Gupta, R. Hodges, and T. Mehen. PRD 109 (2024) 5, 5

[5] M. Echevarria, S. Romera, I. Scimemi. JHEP 12 (2023) 181 (See this morning's talk)

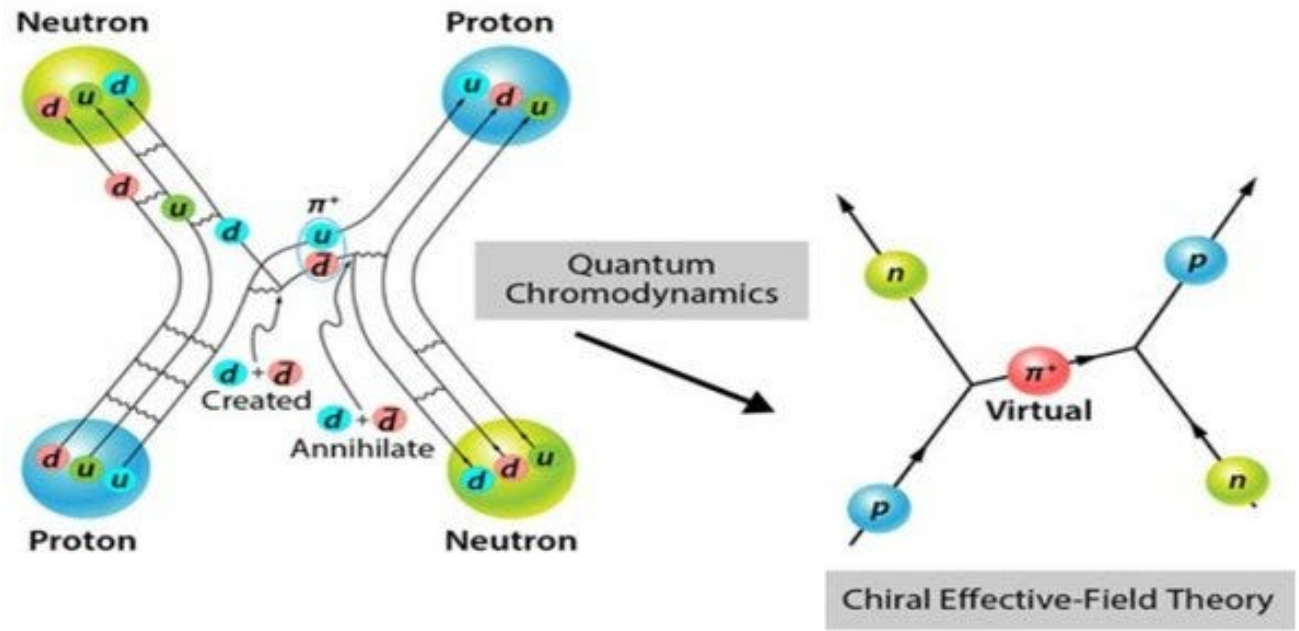
Can you calculate TMD PDFs in an effective theory?

- What about systems with only light quarks and gluons? (pions, kaons, protons, etc.)



Chiral Perturbation Theory

- Chiral Perturbation Theory (χ-PT) is an effective field theory of QCD where the interactions are written in terms of hadronic degrees of freedom.



SU(2) Chiral Fields

- The pions are in an exponential representation,

$$\Sigma = \exp\left(i\frac{\sqrt{2}M}{f_\pi}\right), \quad M = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}.$$

  Define $\xi^2 = \Sigma$. These transform like

$$\Sigma(x) \to L\Sigma(x)R^\dagger, \qquad \xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger$$

- The proton and neutron are in a doublet,

$$N = \begin{pmatrix} p \\ n \end{pmatrix}, \qquad N(x) \to U(x)N(x).$$

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Matching Collinear PDFs onto χPT (Review)

- Collinear PDFs are matched onto nonlocal chiral operators using a convolution,

$$f^a_{qX}(\zeta) = \int \frac{db^-}{2\pi} e^{-ib^-(\zeta P^+)} \langle X | \overline{\psi_i}(b^-) \tau^a W_n(b^-, 0) \not{n} \psi_i(0) | X \rangle$$

[6] J.-W. Chen and X. Ji. PRL 87 (2001) 152002

[7] S.-I. Ando, J.-W. Chen, and C.-W. Kaom, PRD 74, 094013 (2006)

[8] A. Moiseeva and A. Vladimirov, Eur. Phys. J. A 49, 23 (2013).

[9] J.-W. Chen and I. Stewart, PRL 92 (2004) 202001

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$$\to \sum_H \int_\zeta^1 \frac{d\alpha}{\alpha} q^{(0)}_H(\alpha) f^a_{HX}\left(\frac{\zeta}{\alpha}\right).$$

[6] J.-W. Chen and X. Ji. PRL 87 (2001) 152002

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$$f^a_{qX}(\zeta) \to \sum_H \int_\zeta^1 \frac{d\alpha}{\alpha} q^{(0)}_H(\alpha) f^a_{HX}\left(\frac{\zeta}{\alpha}\right).$$

- The "hadronic splitting functions", f_{HX}, are χPT distributions

$$f^a_{HX}(\beta) = \int \frac{dx^-}{2\pi} e^{-ix^-\beta P^+} \langle X| \mathcal{O}^a_H(x^-, 0) |X\rangle .$$

[6] J.-W. Chen and X. Ji. PRL 87 (2001) 152002

[7] S.-I. Ando, J.-W. Chen, and C.-W. Kaom, PRD 74, 094013 (2006)

[8] A. Moiseeva and A. Vladimirov, Eur. Phys. J. A 49, 23 (2013).

[9] J.-W. Chen and I. Stewart, PRL 92 (2004) 202001

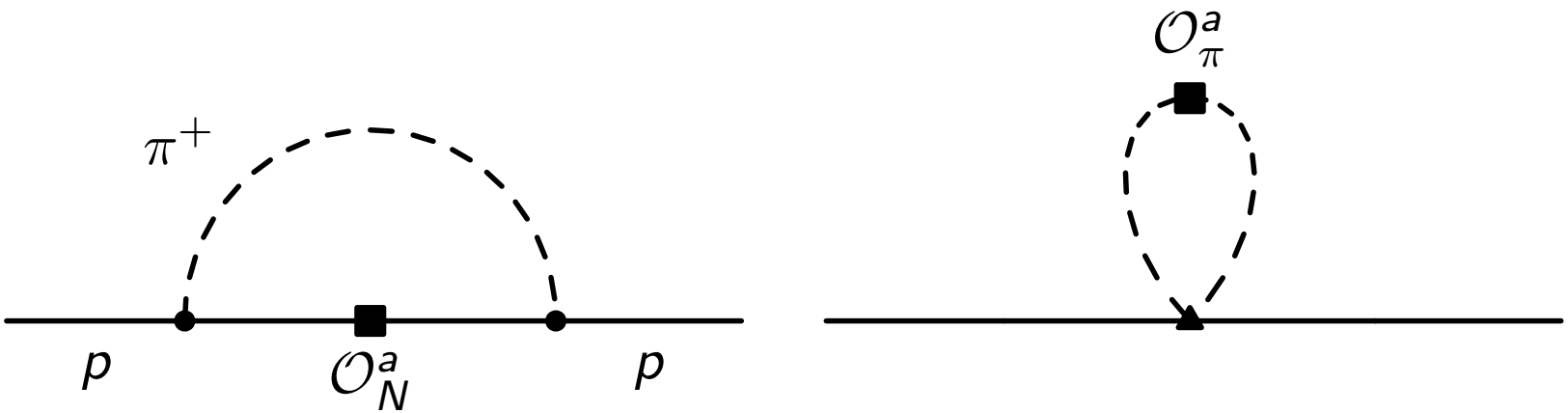
χPT operators

- In hadronic splitting functions, chiral operators are *nonlocal*,

$$\mathcal{O}_\pi^a(b^-,0) = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\Sigma^\dagger(b^-)\tau^a(in\cdot\partial)\Sigma(0) + \Sigma(b^-)\tau^a(in\cdot\partial)\Sigma^\dagger(0)\right],$$

$$\mathcal{O}_N^a(b^-,0) = \frac{1}{2}\bar{N}(b^-)\not{n}\left[\xi^\dagger(b^-)\tau^a\xi(0) + \xi(b^-)\tau^a\xi^\dagger(0)\right]N(0),$$

$$\mathcal{O}_N^{A;a}(b^-,0) = \frac{g_A}{2}\bar{N}(b^-)\not{n}\gamma_5\left[\xi^\dagger(b^-)\tau^a\xi(0) - \xi(b^-)\tau^a\xi^\dagger(0)\right]N(0).$$



Interpreting the convolution

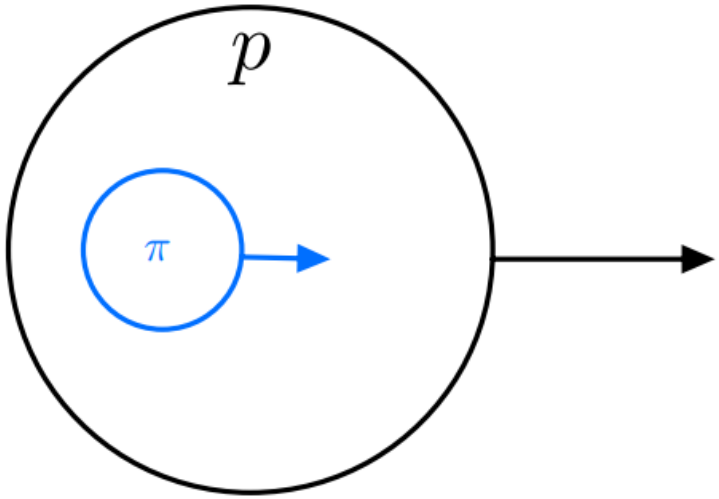
- Convolution has two pieces

$$f_{qX}(\zeta) = \sum_H \int_\zeta^1 \frac{d\alpha}{\alpha} q_H^{(0)}(\alpha) f_{HX}\left(\frac{\zeta}{\alpha}\right).$$

Interpreting the convolution

- Low-energy, χPT hadronic splitting functions ($p^2 \ll \Lambda_\chi^2 \sim 1 - 4$ GeV^2).

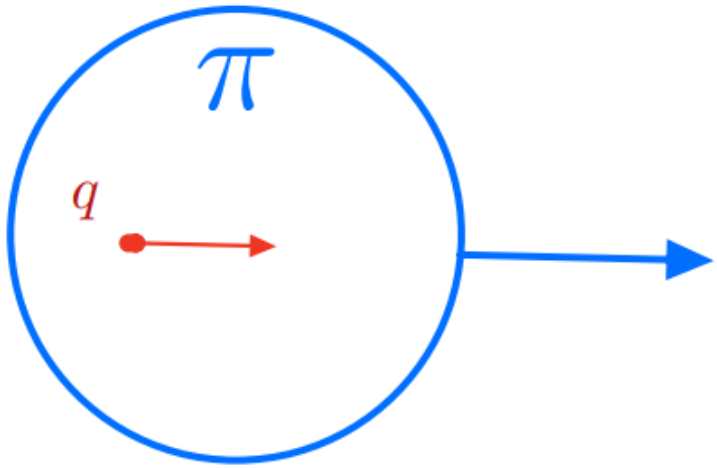
$$f_{qX}(\zeta) = \sum_H \int_\zeta^1 \frac{d\alpha}{\alpha} q_H^{(0)}(\alpha) f_{HX}\left(\frac{\zeta}{\alpha}\right).$$



Interpreting the convolution

- High-energy, matching coefficients ($p^2 \sim \Lambda_\chi^2 \sim 1 - 4$ GeV^2).
  - These are PDFs in the intermediate hadrons (in the chiral limit).

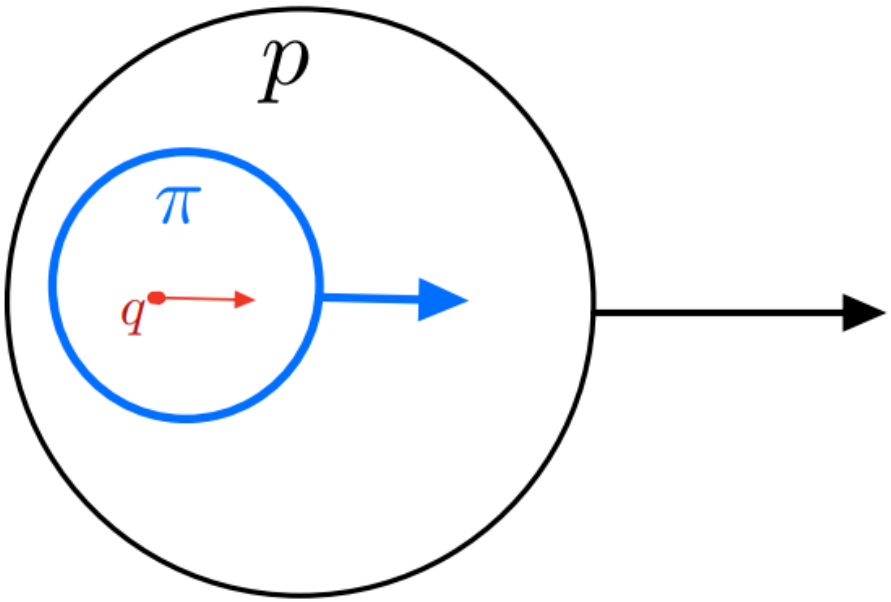
$$f_{qX}(\zeta) = \sum_H \int_\zeta^1 \frac{d\alpha}{\alpha} q_H^{(0)}(\alpha) f_{HX}\left(\frac{\zeta}{\alpha}\right).$$



Interpreting the convolution

- Putting them together paints the picture,

$$f_{qX}(\zeta) = \sum_H \int_\zeta^1 \frac{d\alpha}{\alpha} q_H^{(0)}(\alpha) f_{HX}\left(\frac{\zeta}{\alpha}\right).$$



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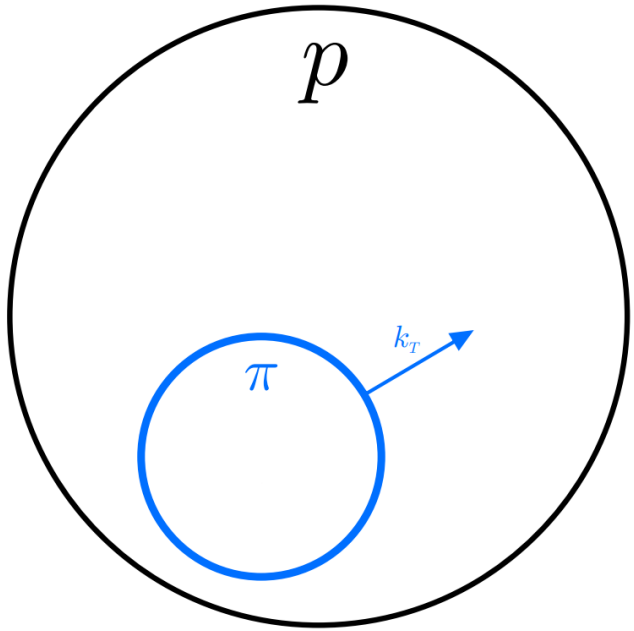
TMD Splitting Functions

- Define *TMD hadronic splitting functions*

$$\tilde{f}^a_{HX}(\beta, \mathbf{b}_T) = \int \frac{db^-}{2\pi} e^{-ib^-\beta P^+} \langle X | \mathcal{O}^a_H(b, 0) | X \rangle .$$

where the chiral operators have transverse and lightlike separation, $b = (b^-, 0, \mathbf{b}_T)$.



TMD Splitting Functions

- Chiral operators are nonlocal, with $\mathbf{b}_T$,

$$\mathcal{O}_\pi^a(b,0) = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\Sigma^\dagger(b)\tau^a(n\cdot\partial)\Sigma(0) + \Sigma(b)\tau^a(n\cdot\partial)\Sigma^\dagger(0)\right],$$

$$\mathcal{O}_N^a(b,0) = \frac{1}{2}\bar{N}(b)\not{n}\left[\xi^\dagger(b)\tau^a\xi(0) + \xi(b)\tau^a\xi^\dagger(0)\right]N(0),$$

$$\mathcal{O}_N^{A;a}(b,0) = \frac{g_A}{2}\bar{N}(b)\not{n}\gamma_5\left[\xi^\dagger(b)\tau^a\xi(0) - \xi(b)\tau^a\xi^\dagger(0)\right]N(0).$$

Side note: Chiral invariance of nonlocal operators

Remark

The nonlocal operators are still chirally invariant.

- The $\xi(x)$ fields cancel out the $U(x)$ dependence from the nucleon's transformation.

$$N(x) \to U(x)N(x), \quad \xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger$$

$$so$$

$$[\xi(x)N(x)] \to L[\xi(x)N(x)], \quad [\xi^\dagger(x)N(x)] \to R[\xi^\dagger(x)N(x)].$$

[8] A. M. Moiseeva and A. A. Vladimirov, Eur. Phys. J. A 49, 23 (2013), 1208.1714

Side Note: Building Blocks in χPT

- Use $\xi(x)$ fields to make chirally invariant objects.

Example

$$N(x) \to U(x)N(x), \quad \xi(x) \to L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger$$

so

$$[\overline{N}(x)\xi(x)][\xi^\dagger(y)N(y)] + [\overline{N}(x)\xi^\dagger(x)][\xi(y)N(y)]$$

is chirally invariant!

Side Note: Building Blocks in χPT

- Like using Wilson lines for QCD nonlocal operators.

Example

$$\psi(x) \to U(x)\psi(x), \qquad W_n(x,y) \to U(x)W_n(x,y)U^\dagger(y)$$

so

$$[\overline{\psi}(x)W_n(x,-\infty)][W_n^\dagger(y,-\infty)\psi(y)]$$

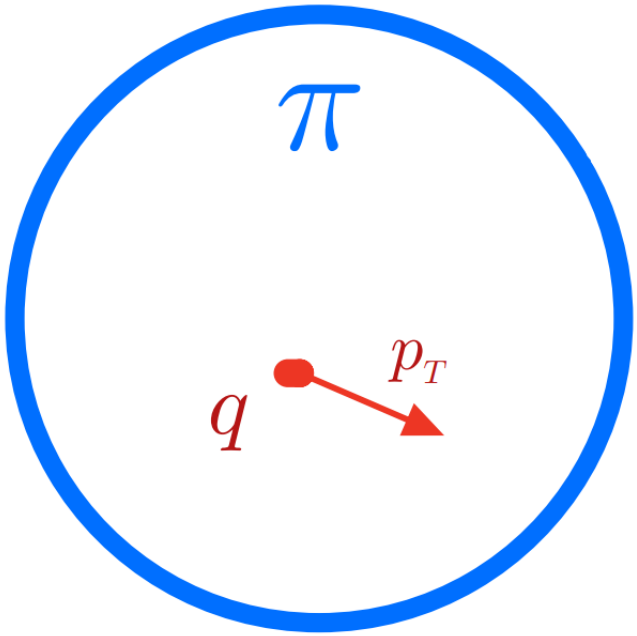
is gauge invariant.

Matching coefficients

- TMD matching coefficients: TMD distributions of intermediate hadrons (in the chiral limit)

$$\tilde{q}_H^{(0)}(\alpha, \mathbf{b}_T) = \int \frac{db^-}{2\pi} e^{-ib^-(\alpha P^+)} \langle H | \overline{\psi}(b^-, b_T) \tau^a W_{\sqsubset}(b, 0) \not{n} \psi(0) | H \rangle .$$

- Still describe high energy physics!



TMD convolution

- The collinear convolution is

$$f_{qX}(\zeta) = \sum_H \int_\zeta^1 \frac{d\alpha}{\alpha} q_H^{(0)}(\alpha) f_{HX}\left(\frac{\zeta}{\alpha}\right).$$

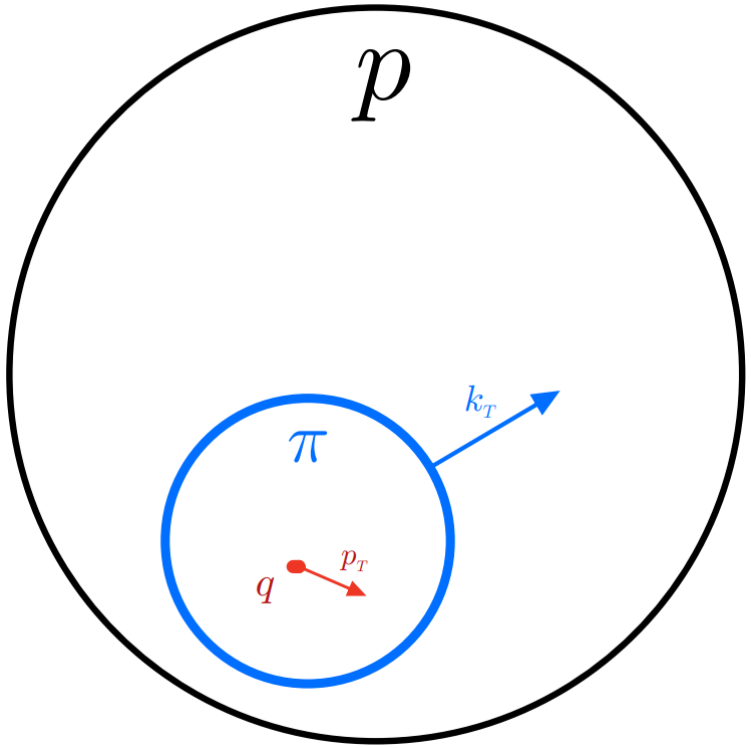
- The TMD analog is,

$$\tilde{f}_{qX}(\zeta, \mathbf{b}_T) = \sum_H \int_\zeta^1 \frac{d\alpha}{\alpha} \tilde{q}_H^{(0)}(\alpha, \mathbf{b}_T) \tilde{f}_{HX}\left(\frac{\zeta}{\alpha}, \mathbf{b}_T\right).$$

TMD convolution

- Importantly, this conserves momentum,

$$f_{qX}(\xi, \mathbf{q}_T) = \sum_H \int d^2\mathbf{p}_T d^2\mathbf{k}_T \int_\zeta^1 \frac{d\alpha}{\alpha} q_H^{(0)}(\alpha, \mathbf{p}_T) f_{HX}(\frac{\zeta}{\alpha}, \mathbf{k}_T) \times \delta^{(2)}(\mathbf{p}_T + \mathbf{k}_T - \mathbf{q}_T).$$



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Calculating splitting functions

- Want to calculate the TMD hadronic splitting functions in χPT.

$$\tilde{f}^a_{HX}(\beta, \mathbf{b}_T) = \int \frac{db^-}{2\pi} e^{-ib^-\beta P^+} \langle X | \mathcal{O}^a_H(b, 0) | X \rangle .$$

- Similar to calculation of TMD PDF, but now have hadronic operators.

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$$\mathcal{O}^a_\pi(b,0) = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\Sigma^\dagger(b)\tau^a(n\cdot\partial)\Sigma(0) + \Sigma(b)\tau^a(n\cdot\partial)\Sigma^\dagger(0)\right],$$

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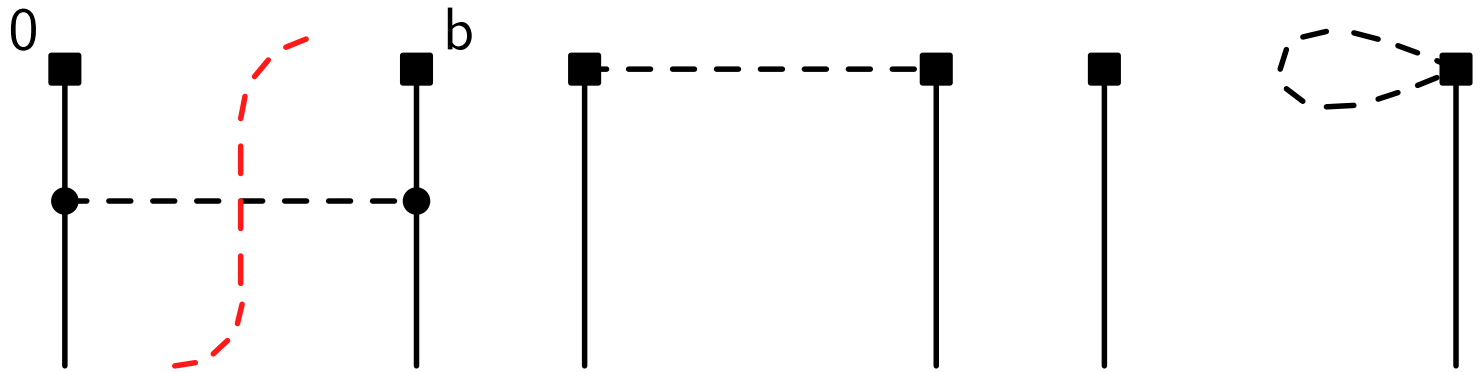
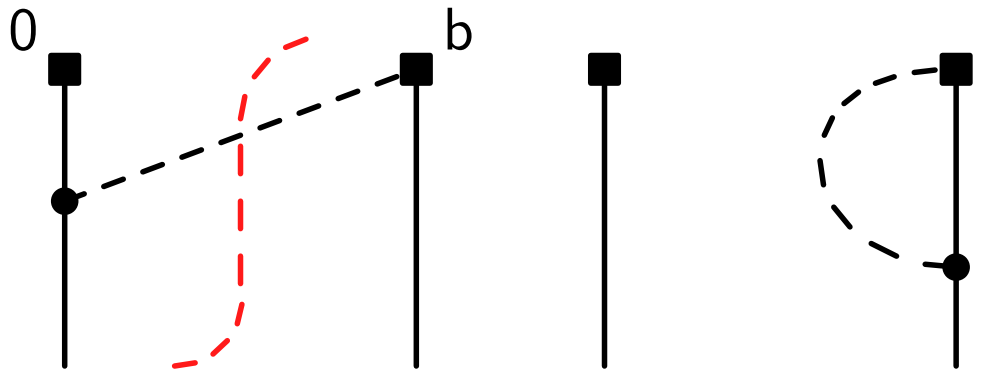
All Diagrams

$\mathcal{O}_N^a$:



$\mathcal{O}_N^{A,a}$:



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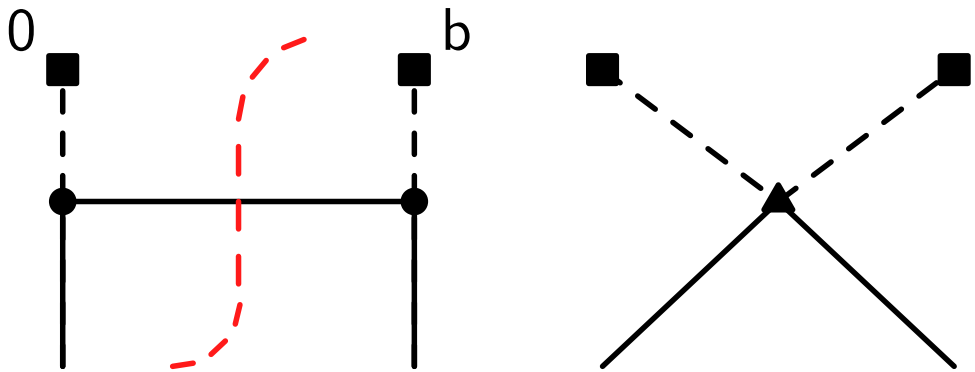
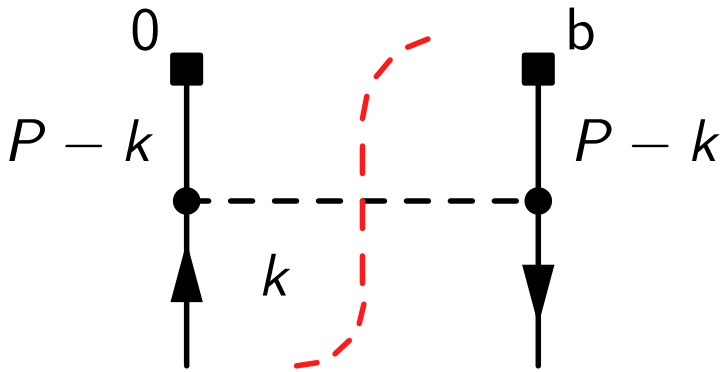


Example: Nucleon Splitting Function

- Using χPT Feynman rules, the first diagram gives

$$f^a_{NN}(\beta, \mathbf{q}_T) = \frac{g_A^2 I^a_{NN}}{4f_\pi^2} \int \frac{d^d k}{(2\pi)^d} \int \frac{db^- d^2\mathbf{b}_T}{(2\pi)^3} e^{-ib^-[\beta P^+ - (P^+ - k^+)]} e^{-i\mathbf{b}_T \cdot (\mathbf{k}_T - \mathbf{q}_T)}$$
$$\times \bar{u}(P) \not{k} \gamma_5 \frac{i(\not{P} - \not{k} + M)}{(P-k)^2 - M^2 + i\epsilon} \frac{i}{k^2 - m_\pi^2 + i\epsilon} \not{n} \frac{i(\not{P} - \not{k} + M)}{(P-k)^2 - M^2 + i\epsilon} \gamma_5 \not{k} u(P).$$



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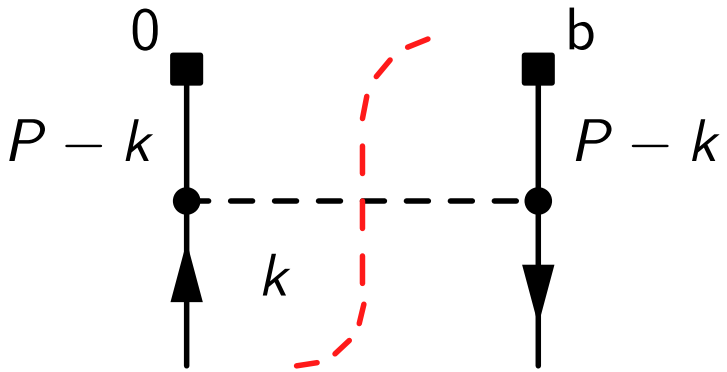
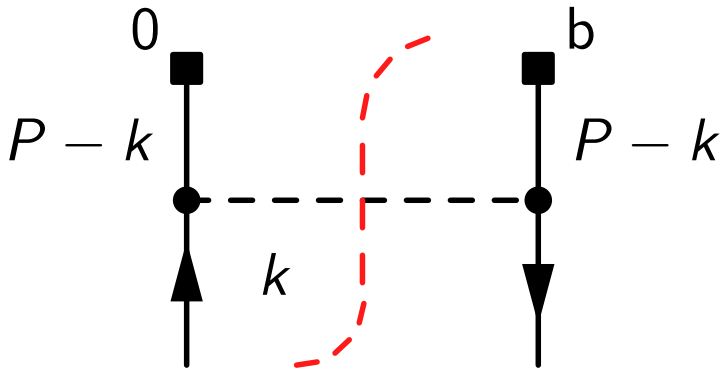
$$\times \frac{i(\not{P} - \not{k} + M)}{(P-k)^2 - M^2 + i\epsilon} \gamma_5 \not{k} u(P) \delta^{(2-2\epsilon)}(\mathbf{k}_T - \mathbf{q}_T) \delta(k^+ - (1-\beta)P^+).$$



Nucleon Splitting Function Answer

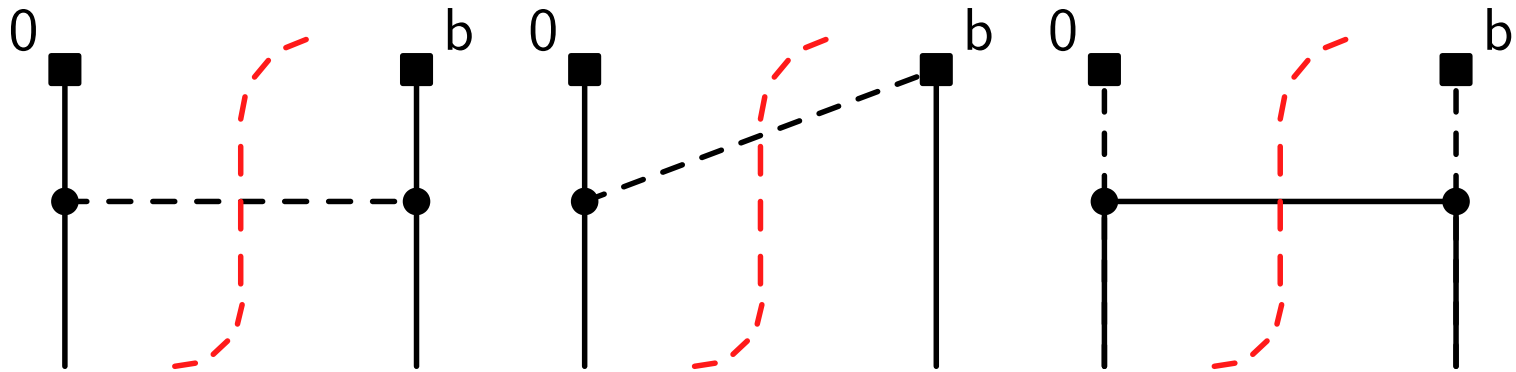
- Average over spins and evaluate the k^- integral:

$$f^a_{NN}(\beta, \mathbf{q}_T) = \frac{g_A^2 I^a_{NN}}{16\pi^3 M f_\pi^2}\left(\frac{M^2 m_\pi^2 \beta(\beta-1)}{[\mathbf{q}_T^2 + M^2(1-\beta)^2 + m_\pi^2\beta]^2} + \frac{1}{4}\left[\frac{1}{\epsilon} - \log\left(\frac{\mathbf{q}_T^2 + m_\pi^2}{\mu^2}\right)\right]\delta(1-\beta)\right)$$



TMD diagrams

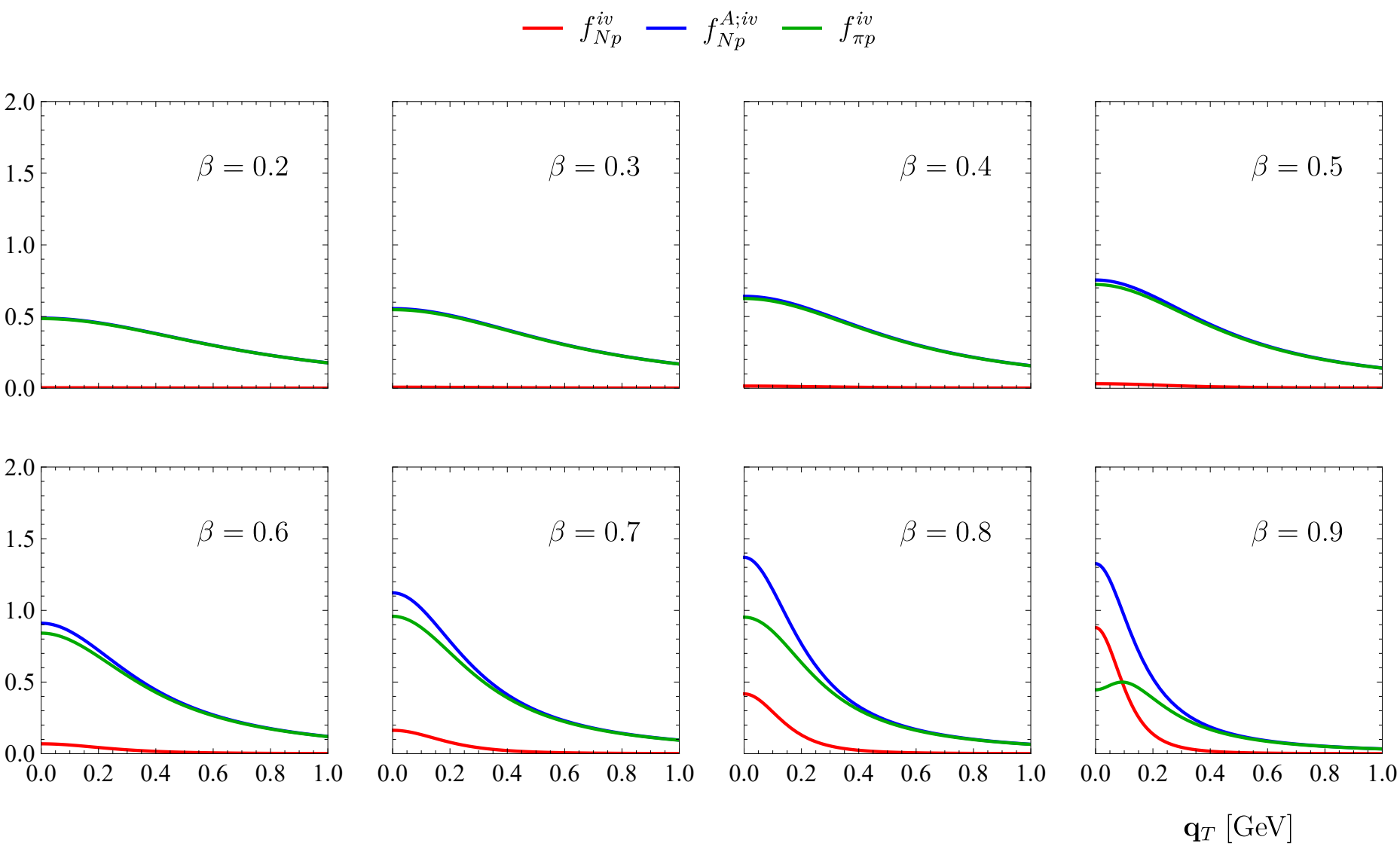
- Only the diagrams with "cuts" have nontrivial transverse momentum dependence.



Plotting

- Domain of plots:
  - Stay in small-ish $\mathbf{q}_T$ region : $0 < \mathbf{q}_T < 1$ GeV.
  - Stay away from divergent region, $\delta(1-\beta)$, so plot $0.2 < \beta < 0.9$.
- Plot isovector splitting functions for $u - d$ distribution, i.e., take $\tau^a = \tau^3 = \sigma_z$.
  - The isospin factors are $I_{Np}^{(iv)} = -1$, $I_{Np}^{A;(iv)} = -1$, and $I_{\pi p}^{(iv)} = 4$.

TMD hadronic Splitting Functions



Summary

- Reviewed how χPT can be applied to PDFs.
- Extended this framework to match TMD PDFs onto χPT.
- Calculated the TMD hadronic splitting functions at next-to-leading order in the chiral expansion.
- Studied the TMD hadronic splitting functions numerically.

Outlook

- Many future directions.
  - Include Δ resonance.
  - Extend to SU(3).
  - Study polarized TMDs.
  - Study TMDs of other hadrons (like pions).
  - Include higher order corrections.
  - Use SCET for subleading operators.
- Compare against data.
  - Test small $\mathbf{q}_T$ vs. large $\mathbf{q}_T$ approximations.

Back Up Slides

Weird k^- Integral

- The $\delta(1-\beta)$ comes from a divergence as $k^+ \to 0$ in the following integral:

$$\int dk^- \frac{1}{k^2 - m_\pi^2 + i\epsilon} = \int dk^- \frac{1}{2k^+k^- - \mathbf{k}_T^2 - m_\pi^2 + i\epsilon}$$

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$$= \begin{cases} \infty & \text{if } k^+ = 0 \\ 0 & \text{otherwise} \end{cases}$$

- Like a delta function!

Weird k^- Integral

- Like a rapidity divergence, but can be regulated with dim. reg.:

$$\int d^2k \frac{1}{k^2 - m_\pi^2 + i\epsilon} \to \int d^{d-2}k \frac{1}{k^2 - m_\pi^2 + i\epsilon}$$
$$= \frac{i\pi}{2}\left[\frac{-1}{\epsilon} + \log\left(\frac{\mathbf{k}_T^2 + m_\pi^2}{\mu^2}\right)\right]$$

- Answer is proportional to $\delta(k^+)$, so k^- integral is

$$\int dk^- \frac{1}{k^2 - m_\pi^2 + i\epsilon} = \frac{i\pi}{2}\left[\frac{-1}{\epsilon} + \log\left(\frac{\mathbf{k}_T^2 + m_\pi^2}{\mu^2}\right)\right]\delta(k^+)$$