Studying NLP Effects with Jets

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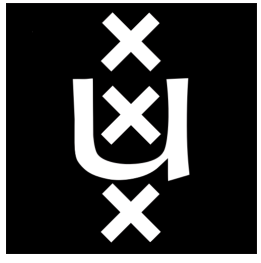
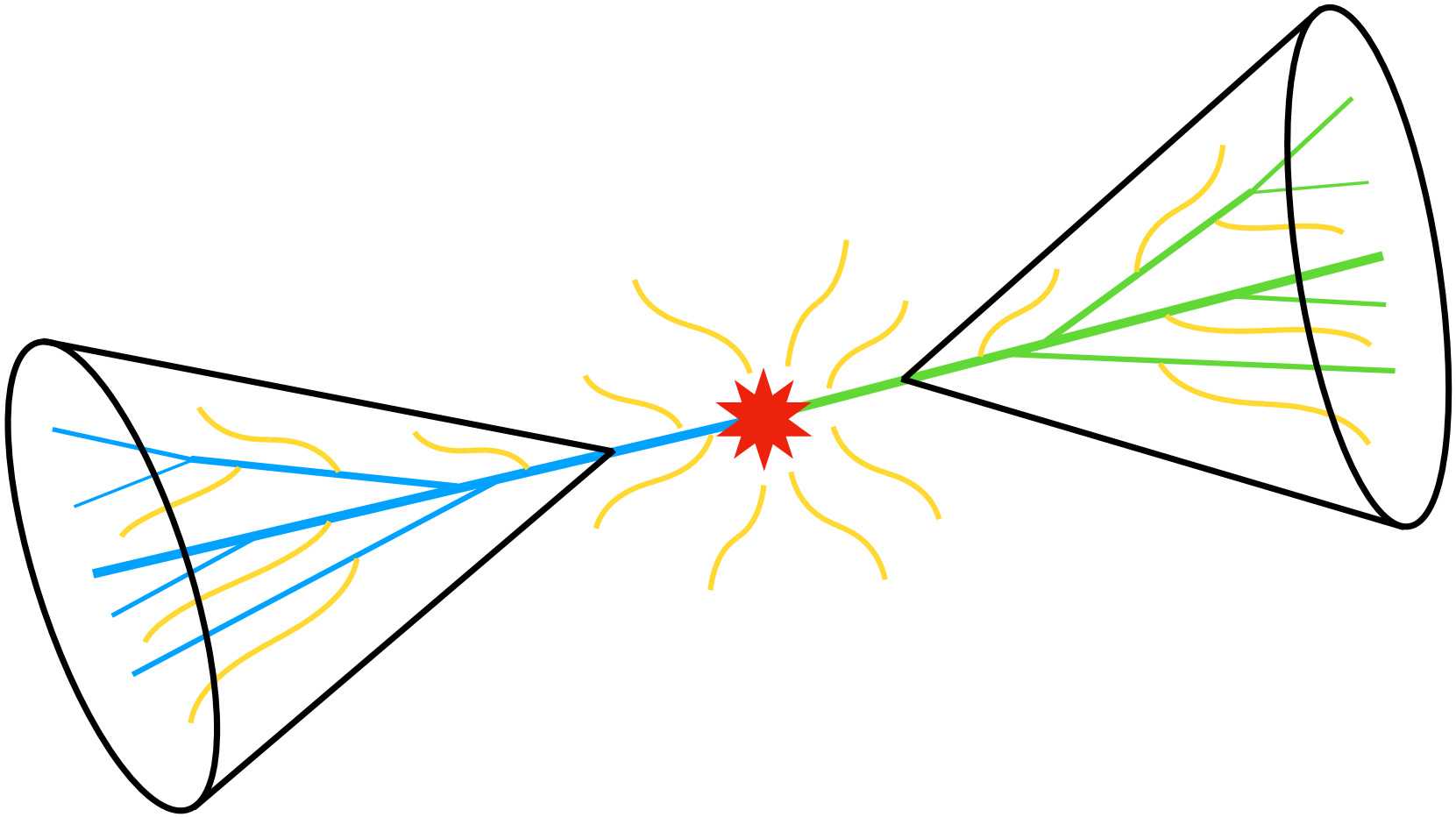


What to expect from this talk?



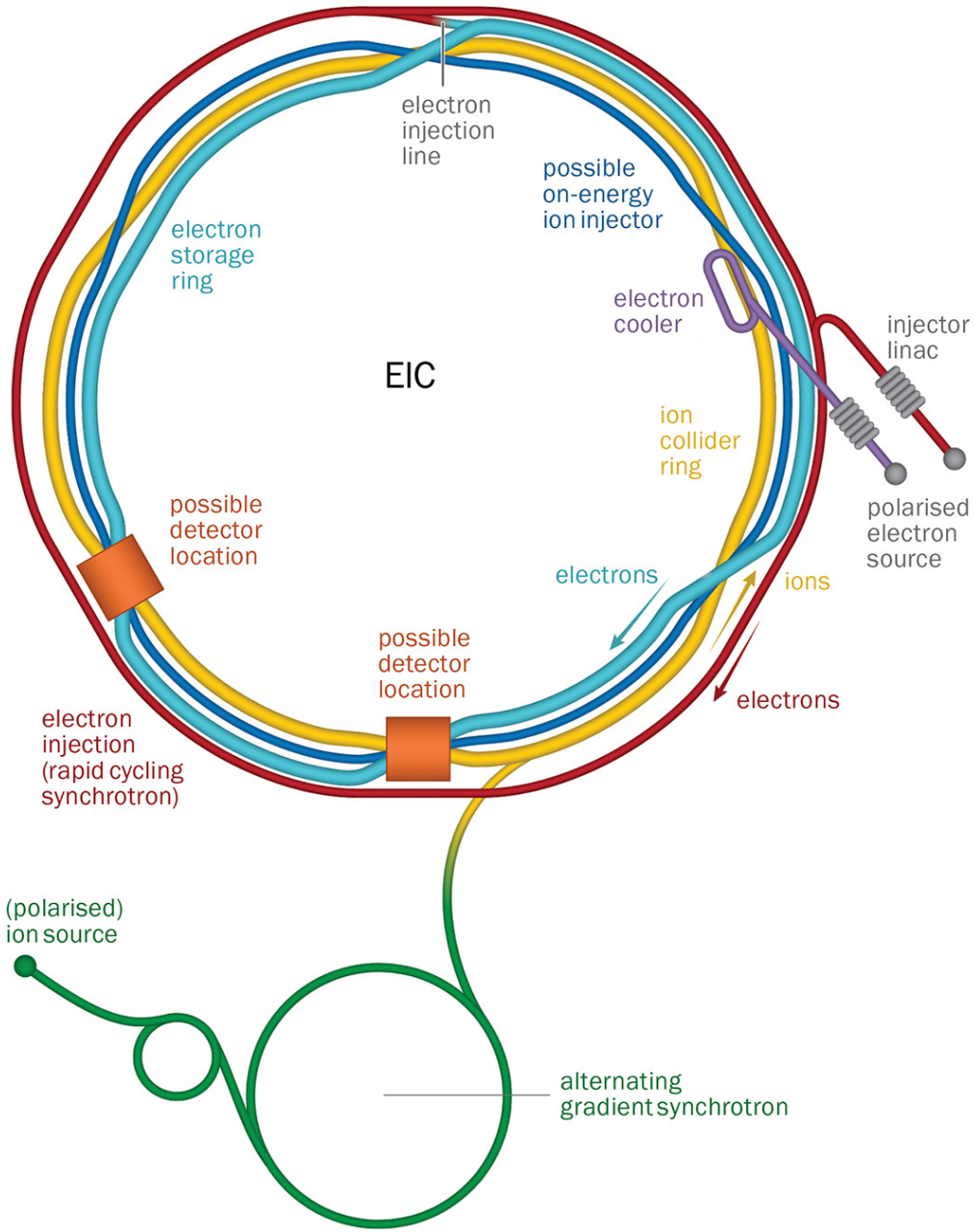
$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}\mathbf{q}} = \underbrace{H \otimes J \otimes J \otimes S}_{\text{LP}} + \sum \underbrace{\tilde{H} \otimes \tilde{J} \otimes \tilde{J} \otimes S}_{\text{NLP}} + \dots$$

Why study this?

- NLP $\rightarrow$ Theoretical precision
- NLP $\rightarrow$ Azimuthal asymmetries
- NLP $\rightarrow$ qgq correlations
- Jets $\rightarrow$ Playground for theorists
- Jets $\rightarrow$ Everything perturbative*

*Under some assumptions

Why care?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detector
location
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injection
(rapid cycling
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(polarised)
ion source
alternating
gradient synchrotron

Outline

- Part I: Main story
  - Kinematics
  - Factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}\mathbf{q}} = H \otimes J \otimes J \otimes S$$

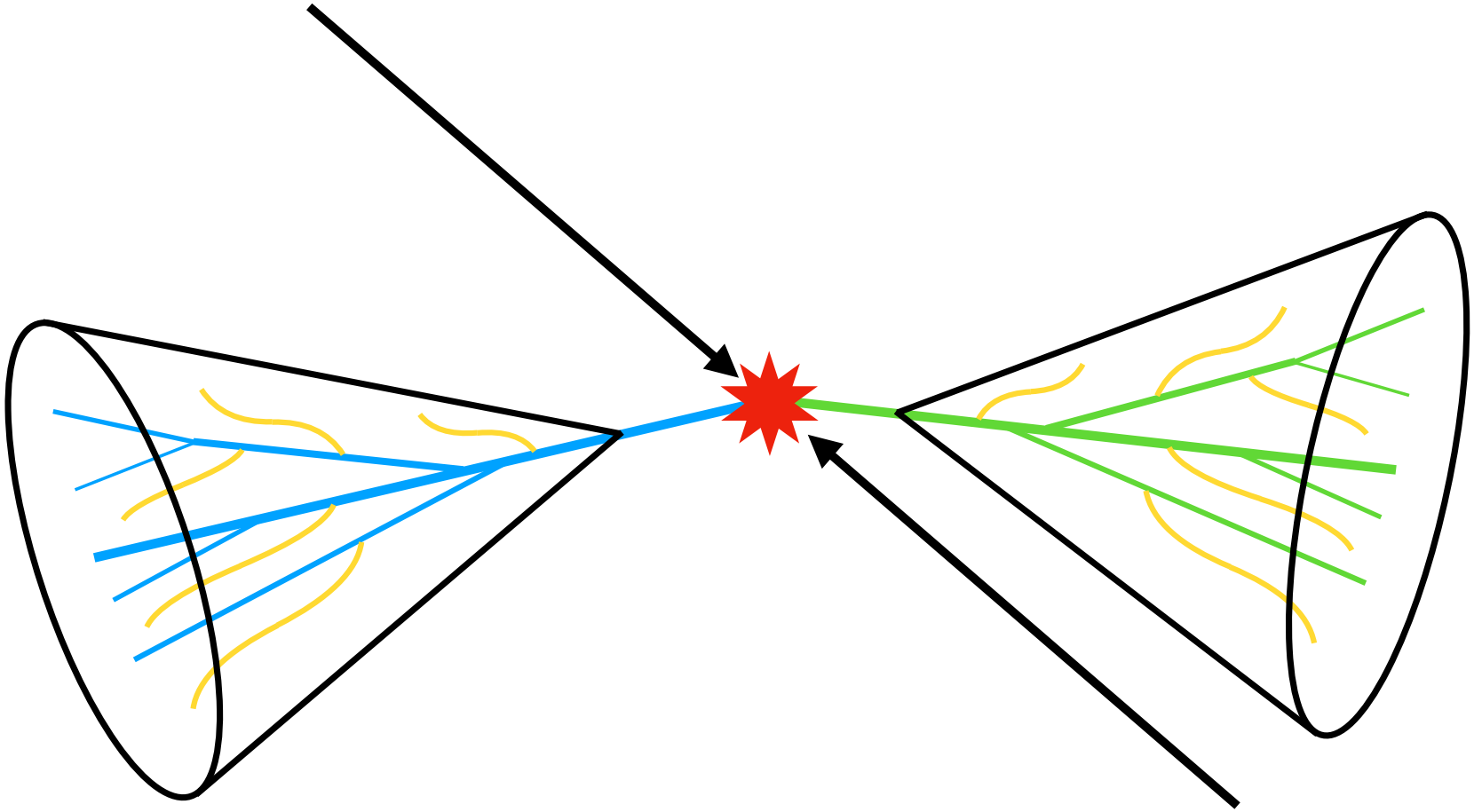
- Part II: Subtleties
  - Rapidity divergences
  - Endpoint divergences

$$\frac{\ln^n \xi}{\xi - \mathrm{i}\delta^+}$$

- Conclusion & Outlook

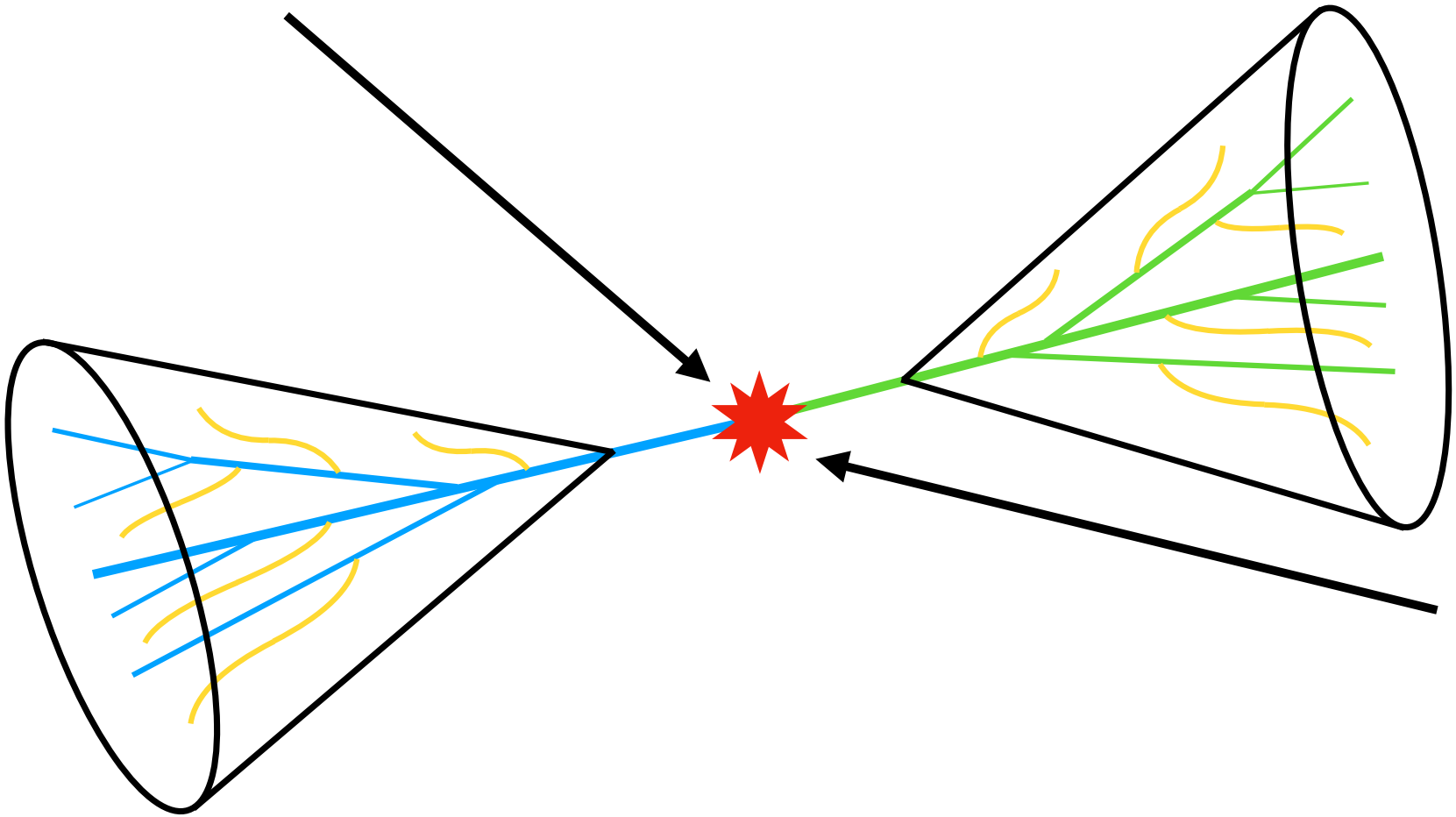
Setting the stage

Kinematics



- Recoil free and large radius
- Jets not back-to-back

Kinematics

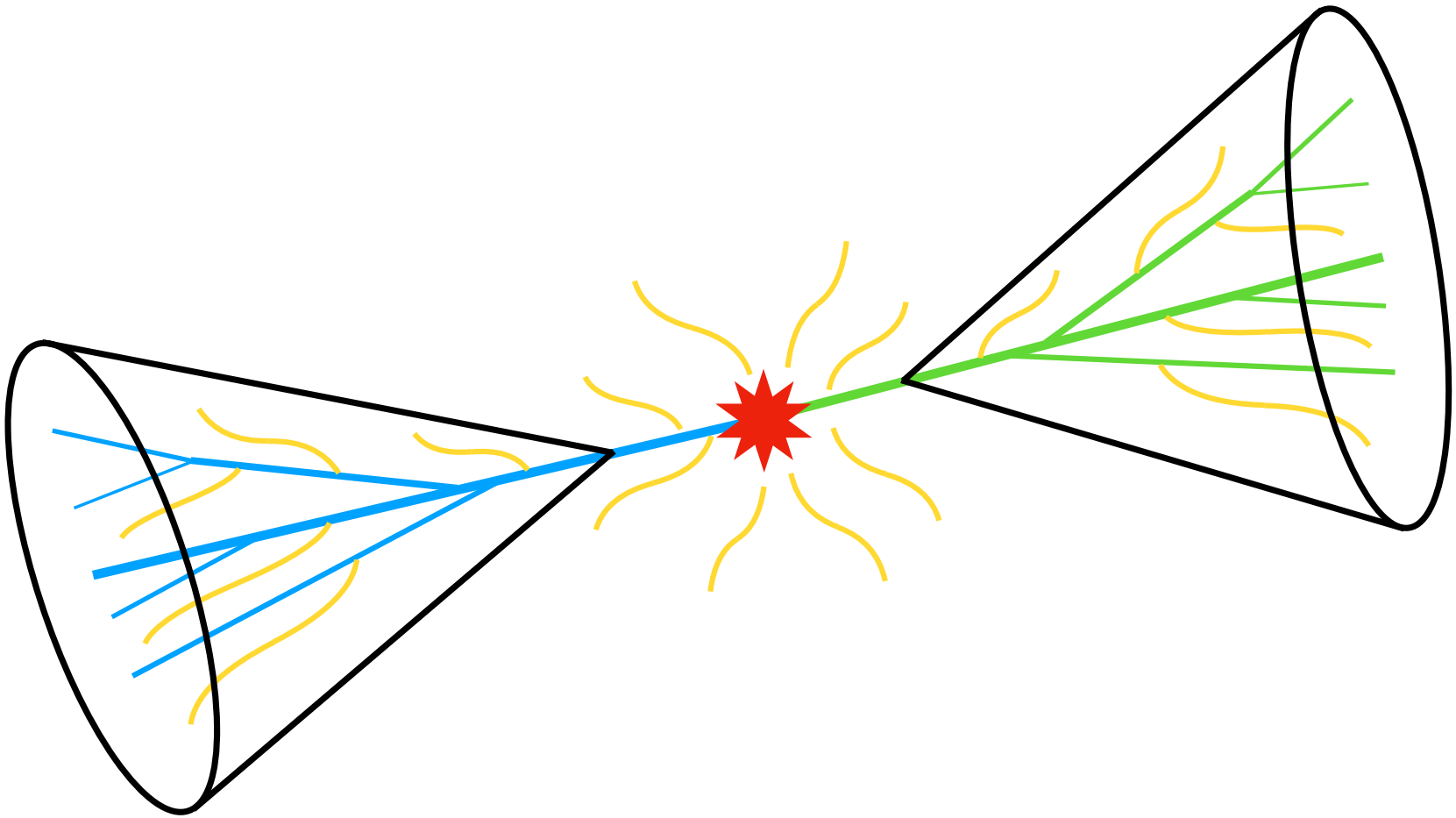


- Factorization easier in back-to-back frame
- Leptons not back-to-back $\rightarrow$ photon has $\mathbf{q}_T$

About our jets

Freedom!

Jet definition is a choice, so make a good one



About our jets

- E^n recombination scheme:

$$\mathbf{P}_{\text{Jet}}^{E^n} = \frac{k_1^0 + k_2^0}{(k_1^0)^n + (k_2^0)^n} \left[(k_1^0)^{n-1} \mathbf{k}_1 + (k_2^0)^{n-1} \mathbf{k}_2 \right] + \mathcal{O}\left(\frac{Q_T^2}{Q^2} Q_T \right)$$

- WTA recombination scheme (E^∞)

$$\mathbf{P}_{\text{Jet}}^{\text{WTA}} = \Theta(k_1^0 - k_2^0) \, \frac{k_1^0 + k_2^0}{k_1^0} \, \mathbf{k}_1 + \Theta(k_2^0 - k_1^0) \, \frac{k_1^0 + k_2^0}{k_2^0} \, \mathbf{k}_2$$

The cross section

- What we measure:
  - y: Related to jet/lepton angle
  - $\boldsymbol{q}_T$: Of intermediate photon

- How we calculate it

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\mathbf{q}} = \frac{\pi\alpha_{\mathrm{em}}^2 Q_q^2 N_c}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

What is the goal?

- Only one unknown ingredient

$$W^{\mu\nu}(q) = \int \frac{\mathrm{d}^4 y}{(2\pi)^4}\, e^{\mathrm{i}q\cdot y} \sum_X \langle 0|\, J^{\mu\,\dagger}(y)\, |P_1, P_2, X\rangle \langle P_1, P_2, X|\, J^{\nu}(0)\, |0\rangle$$

- Factorize the hadronic tensor at NLP in q_T/Q

$$\begin{aligned} W^{\mu\nu} = &\sum \quad (\dots)^{\mu\nu} \quad H \otimes J \otimes J \otimes S \qquad \Big\}\text{LP} \\ &+ \sum \quad (\dots)^{\mu\nu} \quad \tilde{H} \otimes \tilde{J} \otimes \tilde{J} \otimes \tilde{S} \qquad \Big\}\text{NLP} \end{aligned}$$

Background Field Method

Observation

External states are dominated by collinear and anti-collinear modes

- Momentum of external states scales as

$$P_1 \sim (\lambda^2, 1, \lambda)Q \qquad P_2 \sim (1, \lambda^2, \lambda)Q$$

Vladimirov, Moos, Scimemi (2021)

$$v = v^+ \bar{n} + v^- n + v_\perp \qquad v = (v^+, v^-, v_\perp)$$

$$n^2 = \bar{n}^2 = 0 \qquad n \cdot \bar{n} = 1$$

Idea

Split off the collinear and anti-collinear modes from the rest

- Split up the field

$$\phi = \phi + \phi + \varphi$$

- Count powers

$$\partial^\mu \phi \sim (\lambda^2, 1, \lambda)\phi$$
$$\partial^\mu \phi \sim (1, \lambda^2, \lambda)\phi$$

- Integrate out the rest

Factorization: 5-step process

1. Expand the current
2. Factorize the matrix elements
3. Define a set of jet functions
4. Derive evolution equations
5. Combine results

Step 1: Expand the current

- Integrate out the dynamical fields

$$J^\mu_{\text{eff}}[\phi,\phi] = \int \mathcal{D}\varphi\,\mathcal{D}\bar{\varphi}\,\mathcal{D}B\; e^{iS}\;(\bar{\psi}+\bar{\psi}+\bar{\varphi})\gamma^\mu(\psi+\psi+\varphi)$$

- Expand in λ

$$J_{\text{eff}} = \bar{\chi}\gamma^\mu_\perp\chi - \bar{\chi}\left(\frac{\bar{n}^\mu i\overleftarrow{\not{\partial}_\perp}}{i\overleftarrow{\partial^-}} + \frac{n^\mu i\overrightarrow{\not{\partial}_\perp}}{i\overrightarrow{\partial^+}}\right)\xi + \bar{\xi}\not{\mathcal{A}}_\perp\left(\frac{\bar{n}^\mu}{i\overleftarrow{\partial^-}} + \frac{n^\mu}{i\overrightarrow{\partial^+}}\right)\chi + \dots$$

- Higher-orders in α_s absorbed in matching coefficients

Beneke, Campanario, Mannel, Pecjak (2005)
Ebert, Gao, Stewart (2021)
Vladimirov, Moos, Scimemi (2021)

Step 2: Factorize

- Go back to the hadronic tensor

$$W^{\mu\nu}(q) = \int \frac{\mathrm{d}^4 y}{(2\pi)^4} e^{\mathrm{i}q\cdot y} \sum_X \langle 0| J^{\mu\,\dagger}(y) |P_1, P_2, X\rangle \langle P_1, P_2, X| J^{\nu}(0) |0\rangle$$

- Factorize the matrix elements

$$\langle P_1, P_2, X| J^{\mu} |0\rangle = (\gamma_T^{\mu})_{ij} \langle P_1, X| \bar{\chi}_i |0\rangle \langle P_2, X| \chi_j |0\rangle + \dots$$

- Do the bookkeeping
  - Fierz identities
  - Lorentz decomposition

Step 3: Define jet functions

Jets are much simpler than individual hadrons

Symmetries allow us to relate and discard many of the matrix elements

- Invariant under charge and parity conjugation
- Jet measurement is spin independent
- Large radius jet $\rightarrow P^{+}_{\text{jet}} = q^{+}$
- Recoil free jet $\rightarrow$ Jet axis insensitive to soft

Bertolini, Chan, Thaler (2014)

Step 3: Define jet functions

- J_{11}: Leading power χ
- J'_{11}: Kinematic correction $\partial\!\!\!/_\perp \chi$
  - Derivative of LP jet function
- J_{21}: Higher-twist contribution $\mathcal{B}\!\!\!/_\perp \chi$
  - Additional momentum fraction

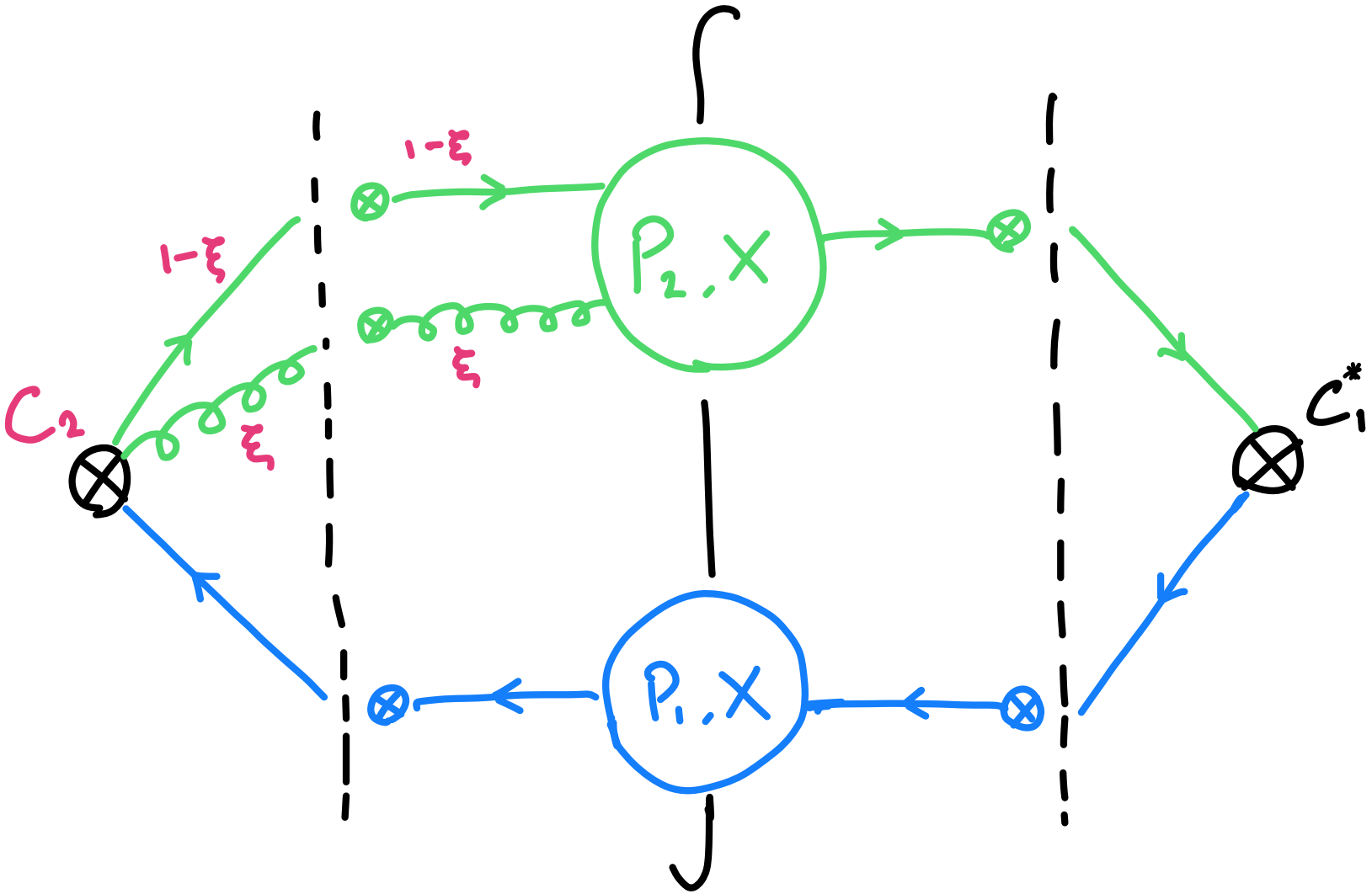
Step 3: Define jet functions

$$J_{11}(\mathbf{b}^2) = \frac{1}{2N_c} \int \frac{\mathrm{d}y^-}{2\pi}\, e^{\mathrm{i}y^- q^+}\, \mathrm{tr}\Big[\not{n}\, \langle 0|\, \chi(y^-, \mathbf{b})\, |P_2, X\rangle\, \langle P_2, X|\, \bar{\chi}(0)\, |0\rangle\Big]$$

$$J'_{11}(\mathbf{b}^2) = b^\mu \frac{\partial}{\partial b^\mu} J_{11}(\mathbf{b}^2)$$

$$J_{21}(\xi, \mathbf{b}^2) = \frac{1}{\xi - \mathrm{i}\delta_q^+} \frac{b^\mu \epsilon_\perp^{\mu\alpha}}{2N_c} \int \frac{\mathrm{d}y_1^-}{2\pi} \frac{\mathrm{d}y_2^-}{2\pi}\, e^{\mathrm{i}(\bar{\xi}y_1^- + \xi y_2^-)q^+}$$
$$\times\, \mathrm{tr}\Big[\sigma^{\alpha+}\gamma^5\, \langle 0|\, \not{\mathcal{B}}_\perp(y_2^-, \mathbf{b})\xi(y_1^-, \mathbf{b})\, |P_2, X\rangle\, \langle P_2, X|\, \bar{\xi}(0)\, |0\rangle\Big]$$

Step 3: Define jet functions



Step 4: Write down evolution equations

Why this step?

Large logs can be resummed by evolving the jet functions to the hard scale

- Evolution of J_{11} has long been known
- Evolution of J'_{11} is related to J_{11}
- Evolution of J_{21} follows from previous work

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} J_{21}(\xi, \mathbf{b}^2) = \int \mathrm{d}\xi' \gamma(\xi, \xi')\, J_{21}(\xi', \mathbf{b}^2)$$

Braun, Manashov, Pirnay (2009)
Freedman, Goerke (2014)
Goerke, Inglis-Whalen (2017)
Beneke, Garny, Szafron, Wang (2017)
Vladimirov, Rodini (2022)

Step 5: Combine results and simplify

$$
\begin{aligned}
W^{\mu\nu} = & -N_c\, g_T^{\mu\nu}\, H_1(Q^2) \int_0^\infty \mathrm{d}\mathbf{b}^2\, \frac{J_0(|\mathbf{b}||\mathbf{q}|)}{2\pi}\, J_{11}(\mathbf{b}^2)\, J_{11}(\mathbf{b}^2) \\
& + N_c \left[\frac{n^\mu q_T^\nu}{q^+} + \frac{n^\nu q_T^\mu}{q^+} \right] H_1(Q^2) \int_0^\infty \mathrm{d}\mathbf{b}^2\, \frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\, J'_{11}(\mathbf{b}^2)\, J_{11}(\mathbf{b}^2) \\
& + N_c \left[\frac{\bar{n}^\mu q_T^\nu}{q^-} - \frac{n^\mu q_T^\nu}{q^+} \right] \int_0^1 \mathrm{d}\xi\, H_2(\xi, Q^2) \int_0^\infty \mathrm{d}\mathbf{b}^2\, \frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|} \\
& \times \Big\{ J_{21}(\xi, \mathbf{b}^2)\, J_{11}(\mathbf{b}^2) - J_{11}(\mathbf{b}^2)\, J_{21}(\xi, \mathbf{b}^2) \Big\}
\end{aligned}
$$

Result

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\mathbf{q}} = {} & \frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(2Q^2\left(1-2y+2y^2\right) - 4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right) \\
& \quad \times \int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_0(|\mathbf{b}||\mathbf{q}|)}{2\pi}\,H_1\,J_{11}\,J_{11} \\
& + \frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right) \\
& \quad \times \int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}2H_1\left(J'_{11}\,J_{11} - J_{11}\,J'_{11}\right) \\
& + \frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right) \\
& \quad \times \int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\left[\int_0^1 \mathrm{d}\xi\,H_2\left(J_{21}\,J_{11} - J_{11}\,J_{21}\right) + \mathsf{c.c.}\right]
\end{aligned}
$$

LP contribution

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\mathbf{q}} = \frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(2Q^2\left(1-2y+2y^2\right) - 4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_0(|\mathbf{b}||\mathbf{q}|)}{2\pi}\,H_1\,J_{11}\,J_{11}$$
$$+\frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\,2H_1\left(J'_{11}\,J_{11} - J_{11}\,J'_{11}\right)$$
$$+\frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\left[\int_0^1 \mathrm{d}\xi\,H_2\left(J_{21}\,J_{11} - J_{11}\,J_{21}\right) + \text{c.c.}\right]$$

NLP correction: kinematical factors

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\mathbf{q}} = \frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\Big(2Q^2\left(1-2y+2y^2\right) - 4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\Big)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_0(|\mathbf{b}||\mathbf{q}|)}{2\pi}\,H_1\,J_{11}\,J_{11}$$
$$+\frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\Big(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\Big)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\,2H_1\left(J'_{11}\,J_{11} - J_{11}\,J'_{11}\right)$$
$$+\frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\Big(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\Big)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\left[\int_0^1 \mathrm{d}\xi\,H_2\left(J_{21}\,J_{11} - J_{11}\,J_{21}\right) + \text{c.c.}\right]$$

NLP correction: kinematic

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\mathbf{q}} = \frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(2Q^2\left(1-2y+2y^2\right) - 4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_0(|\mathbf{b}||\mathbf{q}|)}{2\pi}\,H_1\,J_{11}\,J_{11}$$
$$+\frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}2H_1\left(J'_{11}\,J_{11} - J_{11}\,J'_{11}\right)$$
$$+\frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\left[\int_0^1 \mathrm{d}\xi\,H_2\left(J_{21}\,J_{11} - J_{11}\,J_{21}\right) + \text{c.c.}\right]$$

NLP correction: higher-twist

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\mathbf{q}} = \frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(2Q^2\left(1-2y+2y^2\right) - 4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right)$$
$$\times\int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_0(|\mathbf{b}||\mathbf{q}|)}{2\pi}\,H_1\,J_{11}\,J_{11}$$
$$+\frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right)$$
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\begin{aligned}
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& \quad \times \int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_0(|\mathbf{b}||\mathbf{q}|)}{2\pi}\,H_1\,J_{11}\,J_{11} \\
& + \frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right) \\
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& + \frac{\pi\alpha_{\mathsf{em}}^2 Q_q^2 N_c}{Q^4}\left(4Q_T Q\,\cos(\phi_1)\,(1-2y)\sqrt{y(1-y)}\right) \\
& \quad \times \int_0^\infty \mathrm{d}\mathbf{b}^2\,\frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\left[\int_0^1 \mathrm{d}\xi\,H_2\left(J_{21}\,J_{11} - J_{11}\,J_{21}\right) + \text{c.c.}\right]
\end{aligned}
$$

Always vanishes?

- Sub-leading jet functions only come in this combination:

$$2H_1 \left(J'_{11}\, J_{11} - J_{11}\, J'_{11}\right)$$
$$\int_0^1 \mathrm{d}\xi\, H_2 \left(J_{21}\, J_{11} - J_{11}\, J_{21}\right)$$

- Vanishes if Jet 1 and Jet 2 have the same jet definition

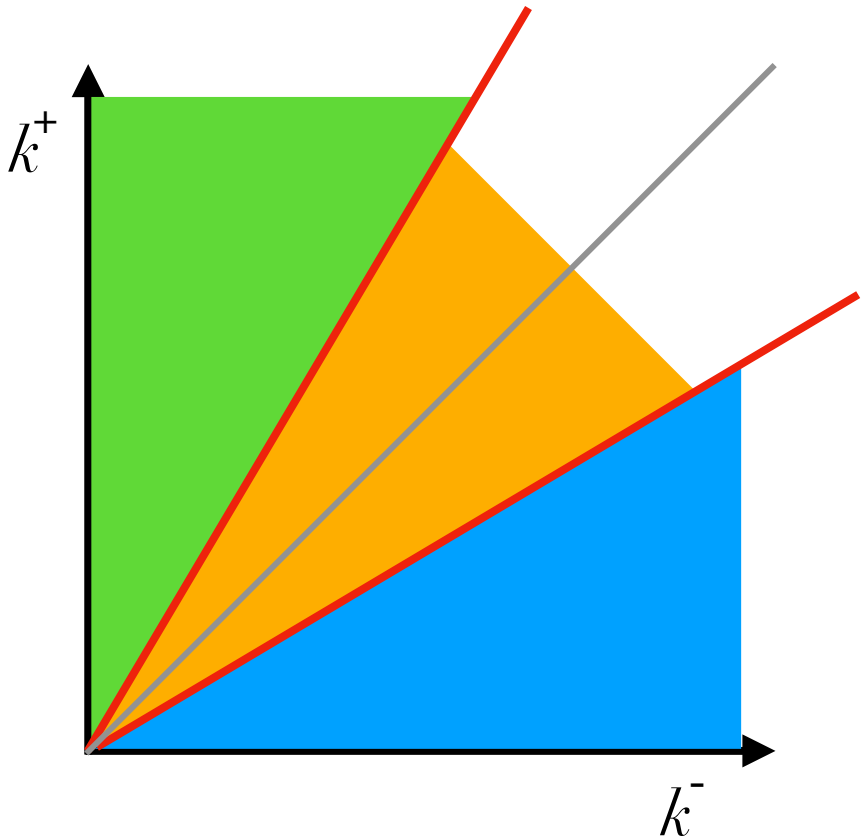
Summary of Part I

- Factorization for $e^+e^- \to 2$ jets with $\boldsymbol{q}_T$ measurement
  - Recoil free jets with large radius make life easier
  - Background field method for expanding the current

- Factorization at NLP accomplished
  - Only 2 independent jet functions
  - All ingredients for NLL resummation are available
  - Corrections vanish if $\text{alg}_1 = \text{alg}_2$

Rapidity divergences

Where do they come from?



- Collinear, anti-collinear, soft separated by rapidity cuts
- No explicit cut $\rightarrow$ rapidity divergences

What happens to them?

- Soft factor can be split up into two pieces

$$S(b, 2\delta^+\delta^-) = \sqrt{S\left(b, (\delta_q^+)^2\zeta\right)}\sqrt{S\left(b, (\delta_q^-)^2\zeta\right)}$$

- Absorb into jet functions

$$\frac{J(b, \delta_q^+)\, J(b, \delta_q^-)}{S(b, 2\delta^+\delta^-)} = J^{\text{sub}}(b, \zeta) J^{\text{sub}}(b, \zeta)$$

- Finite, but depends on a rapidity scale

Special rapidity divergences

- Not all rapidity divergences cancel after subtraction

$$J_{21} = 4a_s C_F \left\{ -\ln\left(\frac{\delta^+}{q^+}\right) \quad + \left[\frac{1}{\xi}\right]_+ + \text{ regular as } \xi \to 0 \right\}$$

$$J'_{11} = 2a_s C_F \left\{ 4\ln\left(\frac{\delta^+}{q^+}\right) \quad + 3 \right\}$$

- Subtleties in overlapping regions

- Cancels between collinear and anti-collinear

$$J_{21} J_{11} - J_{11} J_{21} = \text{finite}$$

$$J'_{11} J_{11} - J_{11} J'_{11} = \text{finite}$$

Endpoint divergences

Remember this?

NLP correction: higher-twist

$$W^{\mu\nu} = -N_c\, g_T^{\mu\nu}\, H_1(Q^2) \int_0^\infty \mathrm{d}\mathbf{b}^2\, \frac{J_0(|\mathbf{b}||\mathbf{q}|)}{2\pi}\, J_{11}(\mathbf{b}^2)\, J_{11}(\mathbf{b}^2)$$

$$+N_c \left[\frac{n^\mu q_T^\nu}{q^+} + \frac{n^\nu q_T^\mu}{q^+}\right] H_1(Q^2) \int_0^\infty \mathrm{d}\mathbf{b}^2\, \frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}\, J_{11}'(\mathbf{b}^2)\, J_{11}(\mathbf{b}^2)$$

$$+N_c \left[\frac{\bar{n}^\mu q_T^\nu}{q^-} - \frac{n^\mu q_T^\nu}{q^+}\right] \int_0^1 \mathrm{d}\xi\, H_2(\xi, Q^2) \int_0^\infty \mathrm{d}\mathbf{b}^2\, \frac{J_1(|\mathbf{b}||\mathbf{q}|)}{2\pi|\mathbf{b}||\mathbf{q}|}$$

$$\times \Big\{ J_{21}(\xi, \mathbf{b}^2)\, J_{11}(\mathbf{b}^2) - J_{11}(\mathbf{b}^2)\, J_{21}(\xi, \mathbf{b}^2) \Big\}$$

$$+ \ldots$$

What are they?

- Endpoint divergences arise in

$$\int_0^1 \mathrm{d}\xi \, H_2(\xi, Q^2) \, J_{21}(\xi, \mathbf{b}^2)$$

- This is ill defined

$$\int_0^1 \mathrm{d}\xi \, \ln \xi \left[\frac{1}{\xi}\right]_+$$

Endpoint divergences

Both the hard function and the jet functions are singular as $\xi \to 0$

$$H_2(\xi, Q^2) = 1 - a_s C_A \ln \xi + \dots \qquad J_{21}(\xi, \mathbf{b}^2) = 4 a_s C_F \left[\frac{1}{\xi}\right]_+ + \dots$$

What to do with them?

Pray they cancel out

Separate the singular behaviour of $J_{21}(\xi, \mathbf{b}^2)$ as $\xi \to 0$

Where to start?

- J_{21} describes a jet initiated by a collinear qg pair
- ξ is the momentum fraction of the gluon
- $\xi \to 0$ corresponds to gluon becoming soft

It works!

- Singular behaviour can be captured by a soft function

$$\lim_{\xi\to 0} J_{21}(\xi, \mathbf{b}^2) = S_{21}(\xi, \mathbf{b}^2)\, J_{11}(\mathbf{b}^2) + \text{regular}$$

$$\lim_{\xi\to 0} J_{21}(\xi, \mathbf{b}^2) = S_{21}(\xi, \mathbf{b}^2)\, J_{11}(\mathbf{b}^2) + \text{regular}$$

- Same soft factor for collinear and anti-collinear

- Singular behaviour cancels in the cross section

$$\lim_{\xi\to 0} J_{21}(\xi, \mathbf{b}^2)\, J_{11}(\mathbf{b}^2) - J_{11}(\mathbf{b}^2)\, J_{21}(\xi, \mathbf{b}^2) = \text{regular}$$

Ebert, Gao, Stewart (2021)
Talk by Anjie Gao at SCET 2023
Talk by Johannes Michel at ESI in 2023

But...

- Soft factors differ in rapidity regulator

$$\int \mathrm{d}\xi \, H_2(\xi) \, J_{11} \Big[S_{21}(\xi, \delta^+) - S_{21}(\xi, \delta^-) \Big] J_{11}$$

- Terms like these remain:

$$\int_0^1 \mathrm{d}\xi \, \ln \xi \left(\frac{1}{\xi - \mathrm{i}\delta^+/q^+} - \frac{1}{\xi - \mathrm{i}\delta^-/q^-} \right)$$

- And result in

$$\ln^2\left(\frac{\delta^+}{q^+}\right) - \ln^2\left(\frac{\delta^-}{q^-}\right)$$

Cancellation of endpoint divergences

All orders in a_s cancellation requires $\delta^+/q^+ = \delta^-/q^-$

Approach of [arXiv:2204.03856]

- Special rapidity (endpoint) divergences arise in

$$\int \mathrm{d}x_2\, H(x_1, x_2)\, \Phi_{21}(x_1, x_2, x_3; b) \Phi_{11}(x; b)$$

- Define physical distribution

$$\mathbf{\Phi}_{21}(x_1, x_2, x_3; b) = \Phi_{21}(x_1, x_2, x_3; b) - \left[\mathcal{R}_{21} \otimes \Phi_{11}\right](x_1, x_2, x_3; b)$$

- Construct $\mathcal{R}$ order-by-order such that

$$H \otimes \mathbf{\Phi}_{21} = \text{finite}$$

- Cancel subtraction terms

Rodini, Vladimirov (2022)
Rodini, Vladimirov (2023)

What do others say?

Background Field

- Soft function arises from overlap subtraction
- No new soft functions appear at NLP
- Define physical distribution using a perturbative kernel $\mathcal{R}$
- Finite cross section constructed order-by-order in α_s

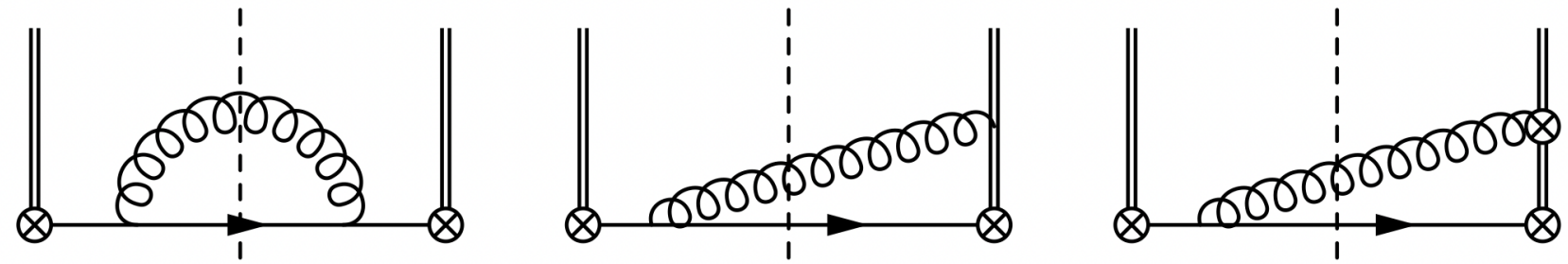
SCET

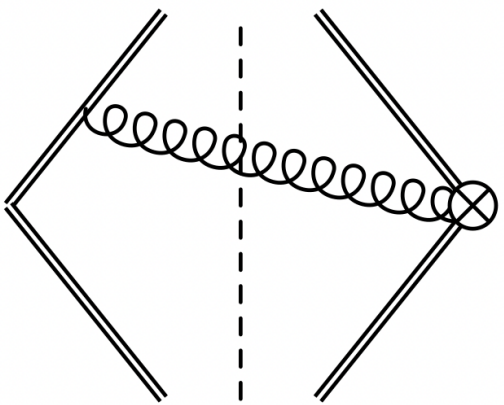
- Soft function arises from soft Wilson lines in J_{eff}
- Sub-leading soft functions appear, but cancel
- Define physical distribution using the NLP soft functions
- Convolution with H finite to all-orders in α_s

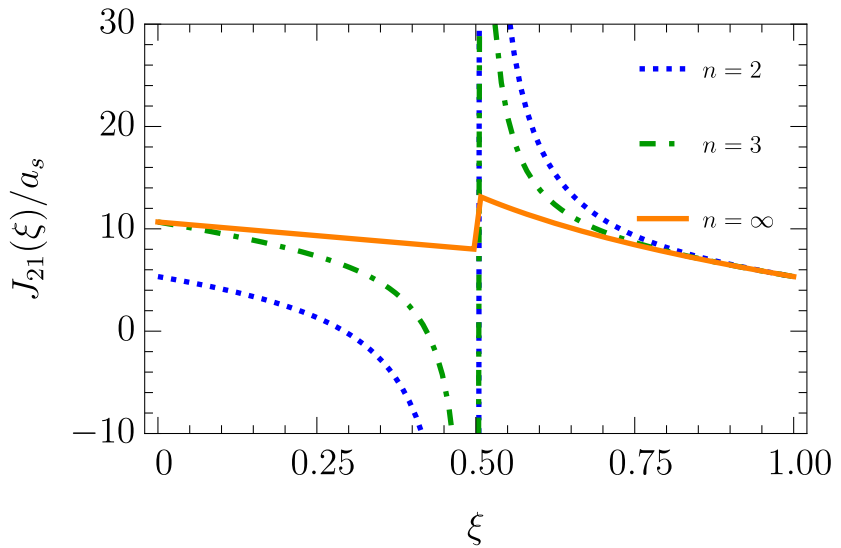
Comparison

Agreement on bare matrix elements
Different treatment of divergences

Ready for results!







Now what?

Ready for resummation!

All ingredients for LL resummation are available
NLL requires more careful treatment of divergences

Whishlist:

- Consistency at higher orders in the coupling
- Understand role of sub-leading soft operators
- Understand the structure of the cancellation

Conclusions

Conclusion

Factorization at NLP achieved!

With well defined (finite and regular) jet functions

- What to take from this talk:
  - Behaviour of $\xi \to 0$ can be separated and cancels
  - Sub-leading soft operators may be necessary
- What next?
  - Does this all work at a_s^2?
  - Resummation
  - Can jets be used to probe sub-leading TMDs?

Thank you for your attention!

Factorization at NLP achieved!

With well defined (finite and regular) jet functions

- What to take from this talk:
  - Behaviour of $\xi \to 0$ can be separated and cancels
  - Sub-leading soft operators may be necessary
- What next?
  - Does this all work at a_s^2?
  - Resummation
  - an jets be used to probe sub-leading TMDs?

Backup slides

Backup I - Background fields for QCD

- Background fields for quark and gluon

$$\psi = \psi + \psi + \varphi \qquad\qquad A^\mu = A^\mu + A^\mu + B^\mu$$

- Introduce good and bad components

$$\psi = \frac{\not{n}\bar{\not{n}}}{2}\psi + \frac{\bar{\not{n}}\not{n}}{2}\psi \equiv \xi + \eta$$

- Power counting for the fields

$$\xi \sim \lambda \qquad \eta \sim \lambda^2 \qquad A^\mu \sim p^\mu$$

Backup II - Jet functions

$$J_{11}(\mathbf{b}^2) = \frac{1}{2N_c} \int \frac{\mathrm{d}y^-}{2\pi} e^{\mathrm{i}y^- q^+} \operatorname{tr}\left[\not{n} \langle 0| \xi(0^+, y^-, \mathbf{b}) |P_2, X\rangle \langle P_2, X| \bar{\xi}(0) |0\rangle\right]$$

$$J'_{11}(\mathbf{b}^2) = b^\mu \frac{\partial}{\partial b^\mu} J_{11}(\mathbf{b}^2)$$

$$J_{21}(\xi, \mathbf{b}^2) = \frac{1}{\xi - \mathrm{i}\delta_q^+} \frac{b^\mu \epsilon_{T,\mu\alpha}}{2N_c} \int \frac{\mathrm{d}y_1^-}{2\pi} \frac{\mathrm{d}y_2^-}{2\pi} e^{\mathrm{i}(\bar{\xi}y_1^- + \xi y_2^-)q^+}$$
$$\times \operatorname{tr}\left[\sigma^{\alpha+}\gamma^5\gamma_{T,\rho} \langle 0| F^{\rho+}(0^+, y_2^-, \mathbf{b})\xi(0^+, y_1^-, \mathbf{b}) |P_2, X\rangle\right.$$
$$\left.\times \langle P_2, X| \bar{\xi}(0) |0\rangle\right]$$

Backup III - Soft factor

- Limit of $\xi \to 0$ is captured by

$$S_{21}(\xi, \mathbf{b}^2) = \frac{1}{\xi - \mathrm{i}s\delta_q^+} \frac{b_\rho}{N_c} \int \frac{\mathrm{d}y_2^-}{2\pi} e^{i\xi y_2^- q^+} \times \mathrm{tr}\Big[\langle 0| S_n^\dagger S_{\bar{n}}\, gF^{\rho+}(y_2^- n + b) |X\rangle \langle X| S_{\bar{n}}^\dagger S_n(0) |0\rangle\Big]$$

- Define a jet function that is regular as $\xi \to 0$

$$\mathbf{J}_{21}(\xi, \mathbf{b}^2) = J_{21}(\xi, \mathbf{b}^2) - S_{21}(\xi, \mathbf{b}^2)\, J_{11}(\mathbf{b^2})$$

Backup IV - Relation to Alexey

- Our twist-3 jet function is defined as

$$J_{21}(\xi,\mathbf{b}^2) = \frac{1}{\xi - \mathrm{i}\delta_q^+}\,\frac{b^\mu \epsilon_{T,\mu\alpha}}{2N_c}\int \frac{\mathrm{d}y_1^-}{2\pi}\frac{\mathrm{d}y_2^-}{2\pi}\, e^{\mathrm{i}(\bar{\xi}y_1^- + \xi y_2^-)q^+}$$
$$\times \mathrm{tr}\Big[\sigma^{\alpha+}\gamma^5\gamma_{T,\rho}\,\langle 0|\, F^{\rho+}(0^+,y_2^-,\mathbf{b})\xi(0^+,y_1^-,\mathbf{b})\,|P_2,X\rangle\,\langle P_2,X|\,\bar{\xi}(0)\,|0\rangle\Big]$$

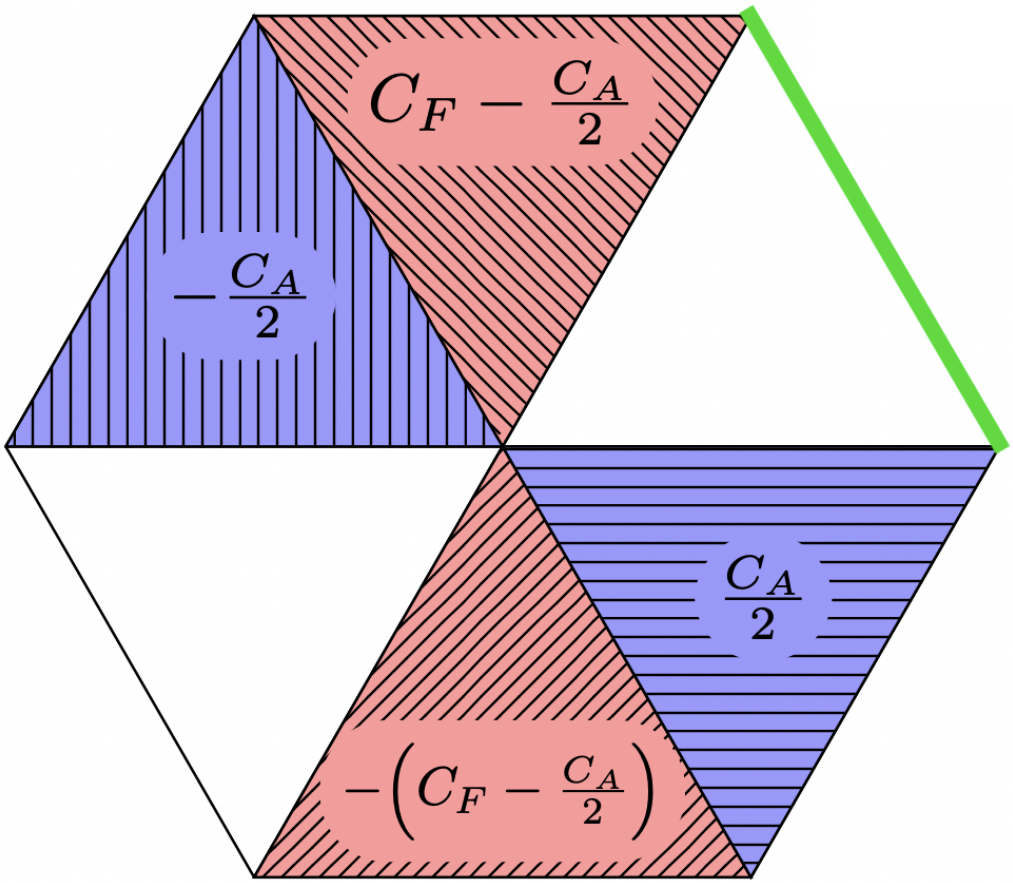
- Fragmentation functions from [2306.09495] are given by

$$\Delta_{21}^{[\Gamma]}(z_1,z_2,z_3;b) = \frac{1}{2N_C|z_1 z_2 z_3|}\int \frac{\mathrm{d}y_1^-}{2\pi}\frac{\mathrm{d}y_2^-}{2\pi}\, e^{-\mathrm{i}\left(\frac{y_1^-}{z_1}+\frac{y_2^-}{z_2}\right)P_2^+}$$
$$\times \mathrm{tr}\Big[\langle 0|\, U_2(\{y_1^-,y_2^-\},b)\,|P_2,X\rangle\,\langle P_2,X|\,\overline{U}(0^-,0)\,|0\rangle\Big]$$

- Related by $p_2^+ = q^+$ (large jet radius) and

$$\Gamma = b^\mu \epsilon_{T,\mu\alpha}\sigma^{\alpha+}\gamma^5 \qquad z_1 = \frac{1}{\bar{\xi}} \qquad z_2 = \frac{1}{\xi} \qquad z_3 = -\frac{1}{\xi\bar{\xi}}$$

Backup V - Evolution



Backup VI - Kinematics

$$y = \frac{P_2 \cdot \ell}{P_2 \cdot q} \qquad \cos\phi_1 = \frac{\boldsymbol{\ell}_\perp \cdot \mathbf{P}_{1\perp}}{|\boldsymbol{\ell}_\perp||\mathbf{P}_{1\perp}|} \qquad \cos\alpha = \frac{\boldsymbol{\ell}_T \cdot \mathbf{q}_T}{|\boldsymbol{\ell}_T||\mathbf{q}_T|}$$

$$g_T^{\mu\nu} = g^{\mu\nu} - \frac{1}{P_1 \cdot P_2}\left(P_1^\mu P_2^\nu + P_2^\mu P_1^\nu\right),$$

$$g_\perp^{\mu\nu} = g^{\mu\nu} + \frac{Q^2}{(P_2 \cdot q)^2} P_2^\mu P_2^\nu - \frac{1}{P_2 \cdot q}\left(q^\mu P_2^\nu + P_2^\mu q^\nu\right),$$