Factorization of J/ψ leptoproduction at small p_T

Soft-Collinear Effective Theory

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Work in collaboration with Miguel G. Echevarria and Pieter Taels (Available on arXiv this or next week)

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Gluon TMDs and Quarkonium

- We are interested in TMDs.

- The quark TMDs have been extensively studied by theoretical approaches as well as experimental measurements. On the contrary, the gluon TMDs are almost unknown from experimental aspect.

  ATLAS Collab., https://arxiv.org/abs/2202.00487

- Quarkonium production is a good tool to extract gluon TMDs:

  - Heavy quarks are sensitive to the gluon content of hadrons.

    W. J. den Dunnen, J.-P. Lansberg, C. Pisano, M. Schlegel, https://arxiv.org/abs/1401.7611 *Dominantly produced by g-g fusion*
    J.-P. Lansberg, C. Pisano, F. Scarpa, M. Schlegel, https://arxiv.org/abs/1710.01684 *Gluon TMD fit in Di-J/ψ production*

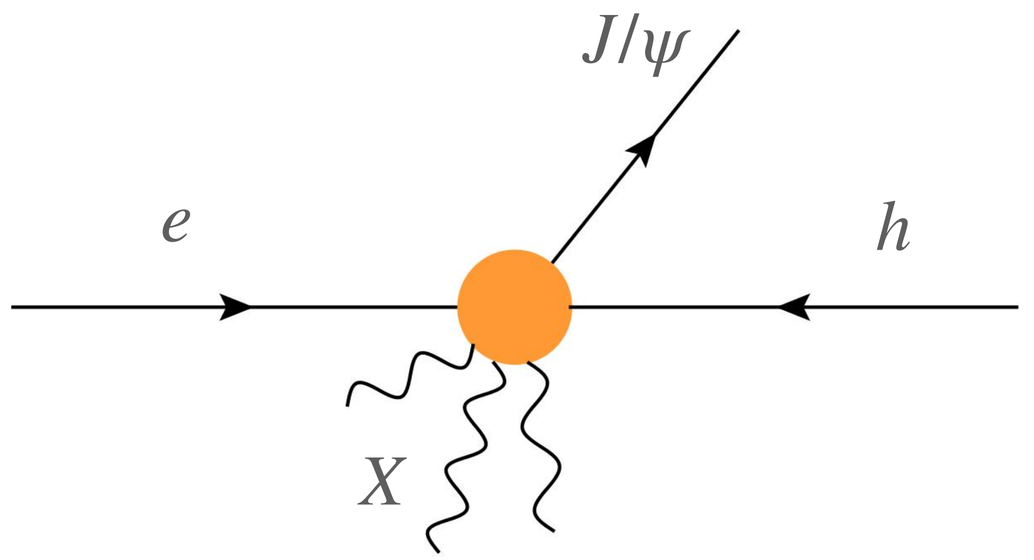
  - J/ψ is straightforward to detect and there are numerous events.

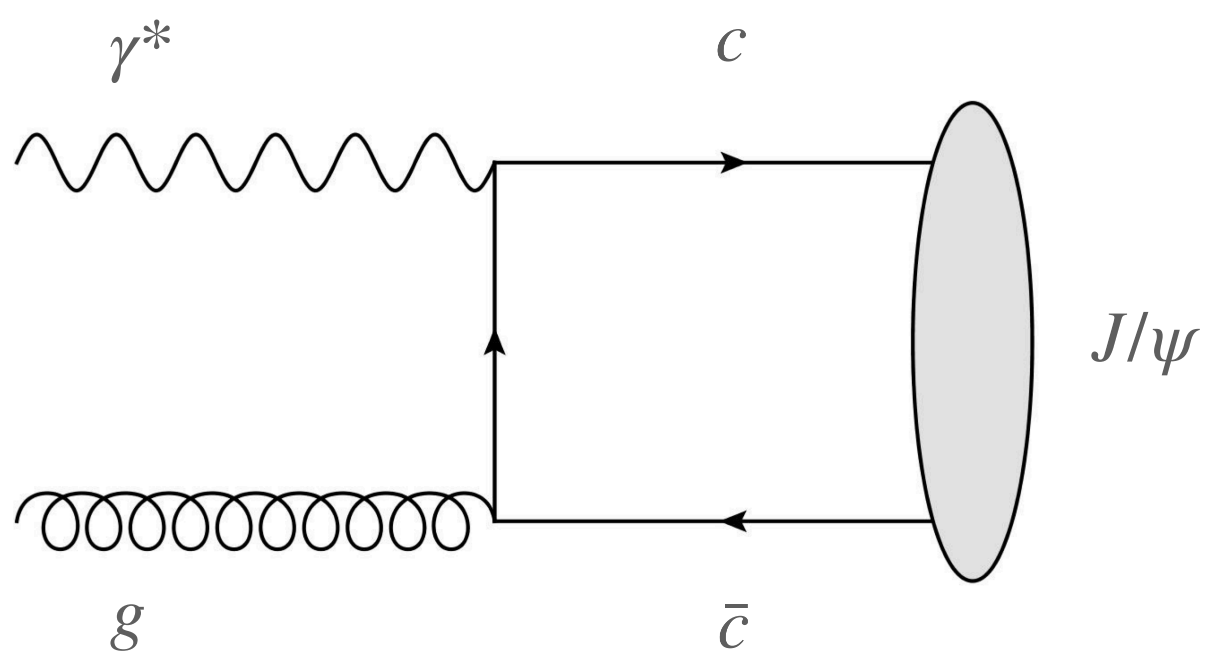
- Challenges: no universality of the LDMEs in the NRQCD approach, quarkonium production mechanism?, theoretical framework, phenomenological analysis, etc.

  U. D'Alesio, A. Mukherjee, F. Murgia, C. Pisano and S. Rajesh, https://arxiv.org/abs/2203.03299 *NRQCD for quarkonium production mechanism*
  H.S. Chung, http://arxiv.org/abs/2211.10201v1 *Review on universality*
  and many others

Setup (of the talk)

Leptoproduction, e/h





e+e-, pp, photoproduction…

S. Fleming, A.K. Leibovich, T. Mehen, arXiv:hep-ph/0607121

S. Fleming, A.K. Leibovich, T. Mehen, arXiv:hep-ph/0306139

M.G. Echevarria, arXiv:1907.06494

S. Fleming, Y. Makris, T. Mehen, arXiv:1910.03586

- Quarkonium production mechanism: NRQCD factorization

G.T. Bodwin, E. Braaten, G.P. Lepage, arXiv:hep-ph/9407339

NRQCD: m, mv, mv^2 (potential, soft, ultra-soft)

$$d\sigma(\gamma^* + g \to J/\psi + X) = \sum_N \underbrace{d\sigma(\gamma^* + g \to c\bar{c}(N) + X)}_{\text{SDCs}} \underbrace{\left\langle \mathcal{O}_{N \to J/\psi} \right\rangle}_{\text{LDMEs}}$$

$$\mathcal{O}_{N \to J/\psi} = \chi^\dagger \mathcal{K}_N^\dagger \psi \sum_X \Big(\big| J/\psi + X \rangle + \langle J/\psi + X \big| \Big) \times \psi^\dagger \mathcal{K}_N \chi \longleftrightarrow$$

$$N = {}^{2S+1}L_J^{[col.]}$$

S: spin
L: angular momentum
J: total angular momentum
col.: color

In the early 2000…

$p_T \gg M$ *Collinear factorization + LDMEs*

$p_T \ll M$ *TMD factorization + TMDShFs*

$p_T \sim m_c v$ $\lambda = p_T/M$

About this talk…

- Factorization:
  - Definition of operators in NRQCD+TMD framework
  - Matching onto QCD
  - TMD shape function (TMDShF)
- LDMEs and TMDShFs at NLO:
  - Calculation at NLO
  - Renormalization group equations
  - Matching onto LDMEs
- Discussion on Hard function

N-operators: scaling

In the equations: $\psi \equiv J/\psi$

$$z = \frac{P \cdot P_\psi}{P \cdot q}$$

J/ψ production in SIDIS: $\ell(k) + N(P) \to \ell'(k') + J/\psi(P_\psi) + X(P_X)$

Defined in NRQCD+TMD framework

$$W^{\mu\nu} = \frac{1}{z} \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{iq\cdot\xi} < PS \,|\, \mathcal{O}^{\mu\dagger}(\xi) \,|\, X; P_\psi S_\psi > < X; P_\psi S_\psi \,|\, \mathcal{O}^{\nu}(0) \,|\, PS >$$

Effective operator: $\mathcal{O}^\mu = \sum_N \underbrace{\mathcal{O}^\mu_N}_{\text{N-operators}}, \qquad N = {}^{2S+1}L_J^{[col.]}$

Which N-states are in the summation?

- QCD coupling constant: color-octet configuration *The color-singlet state contribution starts at $\mathcal{O}(\alpha_s^2)$*
- Relative velocity of the heavy-quark pair: $\underbrace{\left\langle {}^3S_1^{[1]} \right\rangle}_{v^3}, \underbrace{\left\langle {}^1S_0^{[8]} \right\rangle, \left\langle {}^3S_1^{[8]} \right\rangle, \left\langle {}^3P_J^{[8]} \right\rangle}_{v^7}$ $\quad \left\langle \mathcal{O}({}^{2S+1}L_J) \right\rangle \sim v^{3+2L+2E+4M}$

Dominant Fock state

Considering that the color-singlet state is suppressed in the double power counting expansion:

$$N = {}^1S_0^{[8]},\ {}^3S_1^{[8]},\ {}^3P_J^{[8]}$$

$$\longrightarrow \quad \mathcal{O}^\mu = \mathcal{O}^\mu_{{}^1S_0^{[8]}} + \mathcal{O}^\mu_{{}^3S_1^{[8]}} + \sum_{J=0}^{2} \mathcal{O}^\mu_{{}^3P_J^{[8]}}$$

N-operators: definition

- Definition of the operators:

$$\mathcal{O}^{\mu}_{{}^1S_0^{[8]}} = \Gamma^{\mu\alpha}_{{}^1S_0^{[8]}} \Big(\underbrace{\mathcal{S}_v^{cd}\, \psi^\dagger_{\mathbf{p}_c}\, T^d\, \chi_{\mathbf{p}_{\bar c}}}_{\text{pair formation}} \Big) \times \Big(\underbrace{\mathcal{S}_n^{ce}\, B^e_{n\perp\alpha}}_{\text{incoming gluon}} \Big)$$

$$\mathcal{O}^{\mu}_{{}^3S_1^{[8]}} = \Gamma^{\mu\alpha\rho}_{{}^3S_1^{[8]}} \Big(\mathcal{S}_v^{cd}\, \psi^\dagger_{\mathbf{p}_c}\, T^d\, \underbrace{(\mathbf{\Lambda}\cdot\boldsymbol{\sigma})_\rho}_{\text{spin-triplet}}\, \chi_{\mathbf{p}_{\bar c}} \Big) \times \Big(\mathcal{S}_n^{ce}\, B^e_{n\perp\alpha} \Big)$$

* $\langle c\bar c |\, \psi^\dagger \chi\, | 0\rangle = M\, \xi^\dagger \eta$

$M = 2\, m_c$

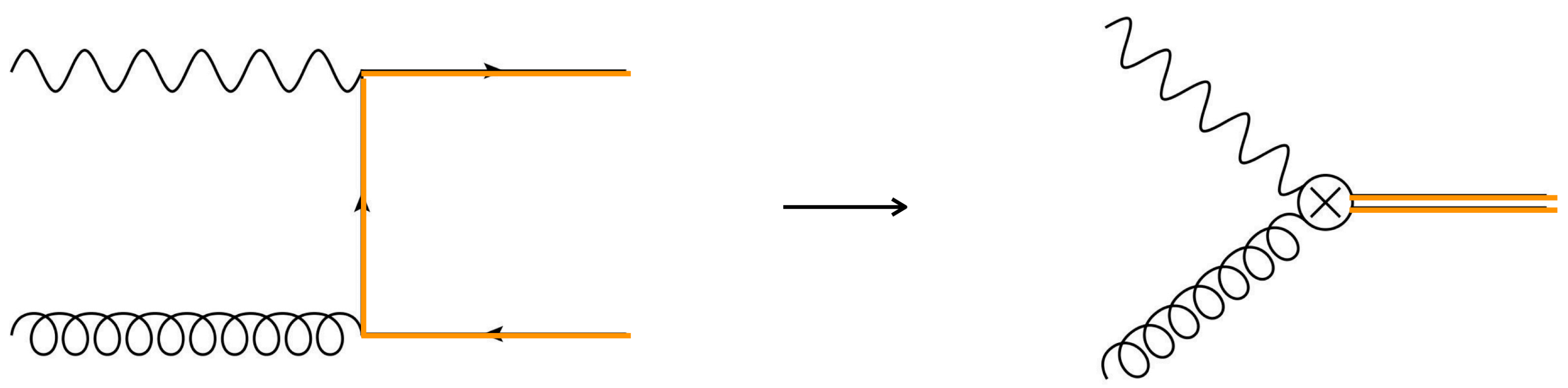
$$\mathcal{O}^{\mu}_{{}^3P_J^{[8]}} = \Gamma^{\mu\alpha\sigma\rho}_{{}^3P_J^{[8]}} \left(\mathcal{S}_v^{cd}\, \underbrace{\frac{(\mathbf{\Lambda}\cdot\mathbf{q})_\sigma}{M^*}}_{L=1}\, \psi^\dagger_{\mathbf{p}_c}\, T^d\, (\mathbf{\Lambda}\cdot\boldsymbol{\sigma})_\rho\, \chi_{\mathbf{p}_{\bar c}} \right) \times \Big(\mathcal{S}_n^{ce}\, B^e_{n\perp\alpha} \Big)$$

Properties of the boost matrix

$$(\Lambda\cdot P)_i = 0\,, \qquad \underbrace{(\Lambda\cdot k_1)_i}_{\text{photon}} = -\underbrace{(\Lambda\cdot k_2)_i}_{\text{gluon}}$$

See also: S. Fleming, Y. Makris, T. Mehen, arXiv:1910.03586

N-operators: definition



+
crossed diagram

- Definition of the operators:

$$\mathcal{O}^{\mu}_{{}^1S_0^{[8]}} = \Gamma^{\mu\alpha}_{{}^1S_0^{[8]}} \Big(\underbrace{\mathcal{S}_v^{cd}\,\psi^\dagger_{\mathbf{p}_c}\,T^d\,\chi_{\mathbf{p}_{\bar c}}}_{\text{pair formation}}\Big) \times \Big(\underbrace{\mathcal{S}_n^{ce}\,B^e_{n\perp\alpha}}_{\text{incoming gluon}}\Big)$$

$$\mathcal{O}^{\mu}_{{}^3S_1^{[8]}} = \Gamma^{\mu\alpha\rho}_{{}^3S_1^{[8]}} \Big(\mathcal{S}_v^{cd}\,\psi^\dagger_{\mathbf{p}_c}\,T^d\,\underbrace{(\mathbf{\Lambda}\cdot\boldsymbol{\sigma})_\rho}_{\text{spin-triplet}}\,\chi_{\mathbf{p}_{\bar c}}\Big) \times \Big(\mathcal{S}_n^{ce}\,B^e_{n\perp\alpha}\Big)$$

* $\langle c\bar c|\,\psi^\dagger\chi\,|0\rangle = M\,\xi^\dagger\eta$

$M = 2\,m_c$

$$\mathcal{O}^{\mu}_{{}^3P_J^{[8]}} = \Gamma^{\mu\alpha\sigma\rho}_{{}^3P_J^{[8]}} \left(\mathcal{S}_v^{cd}\,\underbrace{\frac{(\mathbf{\Lambda}\cdot\mathbf{q})_\sigma}{M\,*}}_{L=1}\,\psi^\dagger_{\mathbf{p}_c}\,T^d\,(\mathbf{\Lambda}\cdot\boldsymbol{\sigma})_\rho\,\chi_{\mathbf{p}_{\bar c}}\right) \times \Big(\mathcal{S}_n^{ce}\,B^e_{n\perp\alpha}\Big)$$

Properties of the boost matrix

$$(\Lambda\cdot P)_i = 0\,, \qquad \underbrace{(\Lambda\cdot k_1)_i}_{\text{photon}} = -\underbrace{(\Lambda\cdot k_2)_i}_{\text{gluon}}$$

- Matching onto QCD (tree level)

$$\Gamma^{\mu\alpha}_{{}^1S_0^{[8]}} = \frac{4\,g_s e}{\mathcal{N}}\,\frac{\epsilon_\perp^{\mu\nu}}{M}$$

$$\Gamma^{\mu\alpha\rho}_{{}^3S_1^{[8]}} = 0$$

Depends on the production mechanism

$$\Gamma^{\mu\nu\sigma\rho}_{{}^3P_J^{[8]}} = \frac{i4\,g_s e}{\mathcal{N}}\,\frac{1}{M}\left\{ g_\perp^{\nu\sigma}\left[g^{\mu\rho}\left(\frac{M^2-Q^2}{M^2+Q^2}\right) - 2\,\bar n^\mu \bar n^\rho\,\frac{M^2+Q^2}{P^{+2}} - n^\mu \bar n^\rho\right] + g_\perp^{\nu\rho}\left[g^{\mu\sigma} + 2\,\bar n^\mu \bar n^\sigma\,\frac{M^2+Q^2}{P^{+2}} - n^\mu \bar n^\sigma\left(\frac{M^2-Q^2}{M^2+Q^2}\right)\right] - 4\,g_\perp^{\nu\mu}\,\bar n^\sigma \bar n^\rho\,\frac{M^2}{P^{+2}}\right\}$$

There are other results for the matching tensor

S. Fleming, A. K. Leibovich, T. Mehen, hep-ph/0306139

See also: S. Fleming, Y. Makris, T. Mehen, arXiv:1910.03586

TMD Shape function

In the equations: $\psi \equiv J/\psi$

$$W^{\mu\nu} = \frac{1}{z}\sum_N \int \frac{d^4\xi}{(2\pi)^4} e^{iq\cdot\xi} \sum_{X_n} < PS|\mathscr{B}^{e'}_{n\perp\beta}(\xi)|X_n> < X_n|\mathscr{B}^{e}_{n\perp\alpha}(0)|PS>$$

$$\times \sum_{X_s} <0|\left[\mathscr{S}_n^{c'e'}\mathscr{S}_v^{c'd'}\chi^\dagger_{\mathbf{p}_{\bar{c}}}\left(\Gamma\cdot\mathscr{K}\right)^{\nu\beta\dagger}_N T^{d'}\psi_{\mathbf{p}_c}\right](\xi)|P_\psi S_\psi, X_s> < P_\psi S_\psi, X_s|\left[\mathscr{S}_v^{cd}\mathscr{S}_n^{ce}\psi^\dagger_{\mathbf{p}_c}\left(\Gamma\cdot\mathscr{K}\right)^{\mu\alpha}_N T^d\chi_{\mathbf{p}_{\bar{c}}}\right](0)|0>$$

- Disentanglement of Hilbert space
- Taylor expansion

$q \sim 1/Q_h(1,1,1/\lambda)$

$$\left|P_\psi S_\psi, X\right\rangle = \left|X_n\right\rangle \otimes \left|P_\psi S_\psi, X_s\right\rangle$$

TMDShF encodes soft and non-perturbative effects

$$\mathscr{L} = -g_s \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}}\left(\frac{\mathbf{B}_{us}\cdot\mathbf{P}}{m_c}\right)\psi_{\mathbf{p}}(x) + (\psi\to\chi)$$

$$W^{\mu\nu} = \frac{1}{z}\sum_N \Gamma^{\nu\beta\,\dagger}_N\Gamma^{\mu\alpha}_N \int d^2\mathbf{k}_{n\perp}\, d^2\mathbf{k}_{s\perp}\, \delta^{(2)}(\mathbf{q}_\perp + \mathbf{k}_{n\perp} - \mathbf{k}_{s\perp})\, G^{\alpha\beta}_{g/P}(x,\mathbf{k}_{n\perp})\, S_{N\to J/\psi}(\mathbf{k}_{s\perp})$$

root of soft function

$$G^{\alpha\beta}_{g/P}(x,\mathbf{k}_{n\perp}) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3} e^{-i\left(\frac{1}{2}xP^+\xi^- - \boldsymbol{\xi}_\perp\cdot\mathbf{k}_{n\perp}\right)} J^{\alpha\beta(0)}_n(0^+,0^-,\boldsymbol{\xi}_\perp)\sqrt{S(0^+,0^-,\boldsymbol{\xi}_\perp)}$$

$$S_{N\to J/\psi}(\mathbf{k}_{s\perp}) = \int \frac{d^2\boldsymbol{\xi}_\perp}{(2\pi)^2} e^{i\boldsymbol{\xi}_\perp\cdot\mathbf{k}_{s\perp}} \frac{S^{(0)}_{N\to J/\psi}(0^+,0^-,\boldsymbol{\xi}_\perp)}{\sqrt{S(0^+,0^-,\boldsymbol{\xi}_\perp)}}$$

Just enough, the shape function only has RDs in the n-collinear sector

$$J^{\alpha\beta}_n(\xi^-,\boldsymbol{\xi}_\perp) = \langle PS|\, \mathscr{B}^{\beta,e}_{n\perp}(\xi^-,\boldsymbol{\xi}_\perp)\, \mathscr{B}^{\alpha,e}_{n\perp}(0)\,|PS\rangle$$

Gluon TMDPDF

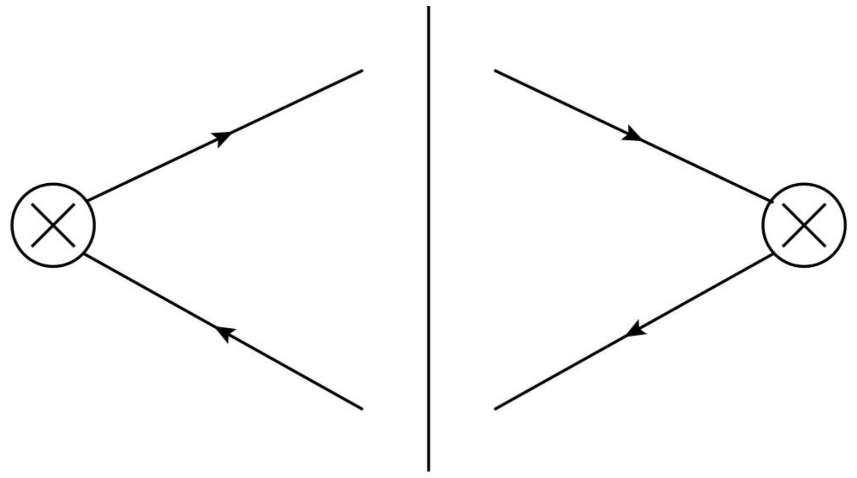
$$S^{(0)}_{N\to J/\psi}(\boldsymbol{\xi}_\perp) = \langle 0|\left[\mathscr{S}_n^{c'e'}\mathscr{S}_u^{c'd'}\chi^\dagger_{\mathbf{p}_{\bar{c}}}\mathscr{K}_N T^{d'}\psi_{\mathbf{p}_c}\right](\boldsymbol{\xi}_\perp)\mathscr{P}_\psi\left[\mathscr{S}_u^{cd}\mathscr{S}_n^{ce}\psi^\dagger_{\mathbf{p}_c}\mathscr{K}_N T^d\chi_{\mathbf{p}_{\bar{c}}}\right](0)|0\rangle$$

No RDs

Calculation at NLO

The calculation for the P-wave is analogous

- LO:



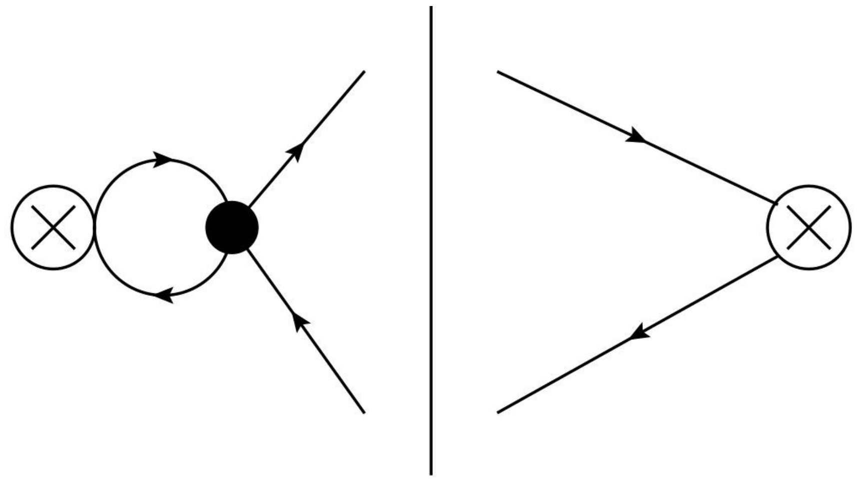
$$\left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} = M^2 \eta^\dagger T^a \xi \times \xi^\dagger T^a \eta$$

$$S^{\text{LO}}_{{}^1S_0^{[8]} \to J/\psi}(\mathbf{k}_\perp) = \frac{d-3}{d-2} \delta^{(2)}(\mathbf{k}_\perp) \left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}}$$

- LDME:

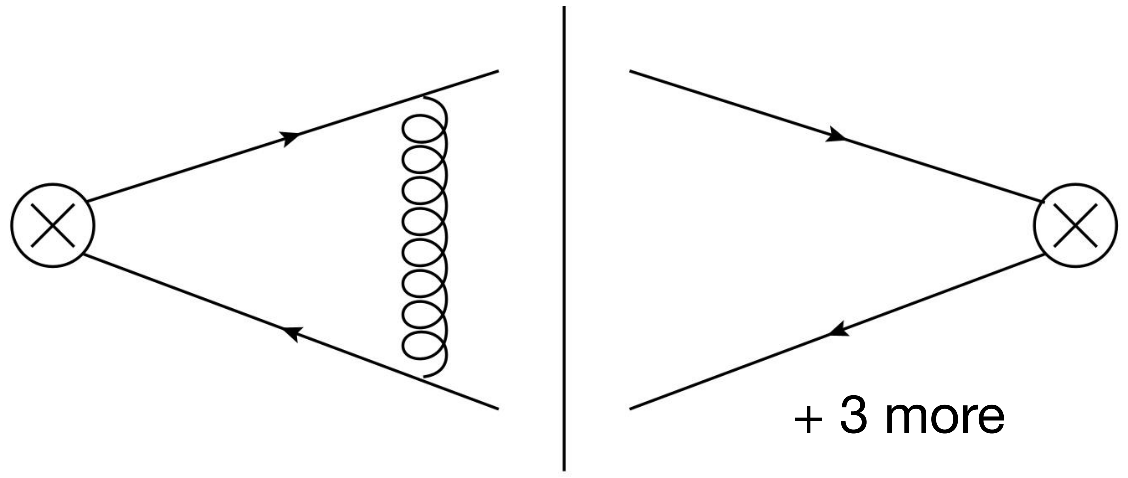
$$\left\langle {}^1S_0^{[8]} \right\rangle = \Bigg(1 + \underbrace{(C_F - C_A/2) \frac{\pi \alpha_s}{2v}}_{\text{Coulomb singularity}} \Bigg) \left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} + \frac{4\alpha_s}{3\pi m_c^2} \underbrace{\left(C_F \left\langle {}^1P_1^{[1]} \right\rangle^{\text{LO}} + B_F \left\langle {}^1P_1^{[8]} \right\rangle^{\text{LO}} \right)}_{\text{Mixing between channels}} \left(\frac{1}{\varepsilon_{\text{UV}}} - \frac{1}{\varepsilon_{\text{IR}}} \right)$$

$$\mathscr{L}^{\text{Coul.}} = \sum_{\mathbf{p},\mathbf{p}'} \frac{4\pi\alpha_s}{(\mathbf{p} - \mathbf{p}')^2} \psi^\dagger_{\mathbf{p}} T^a \psi_{\mathbf{p}'} \chi^\dagger_{-\mathbf{p}} \bar{T}^a \chi_{-\mathbf{p}'}$$



$$\mathscr{L}^{\text{Chro.}} = - g_s \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}} \left(\frac{\mathbf{B}_{us} \cdot \mathbf{P}}{m_c} \right) \psi_{\mathbf{p}}(x) + (\psi \to \chi)$$

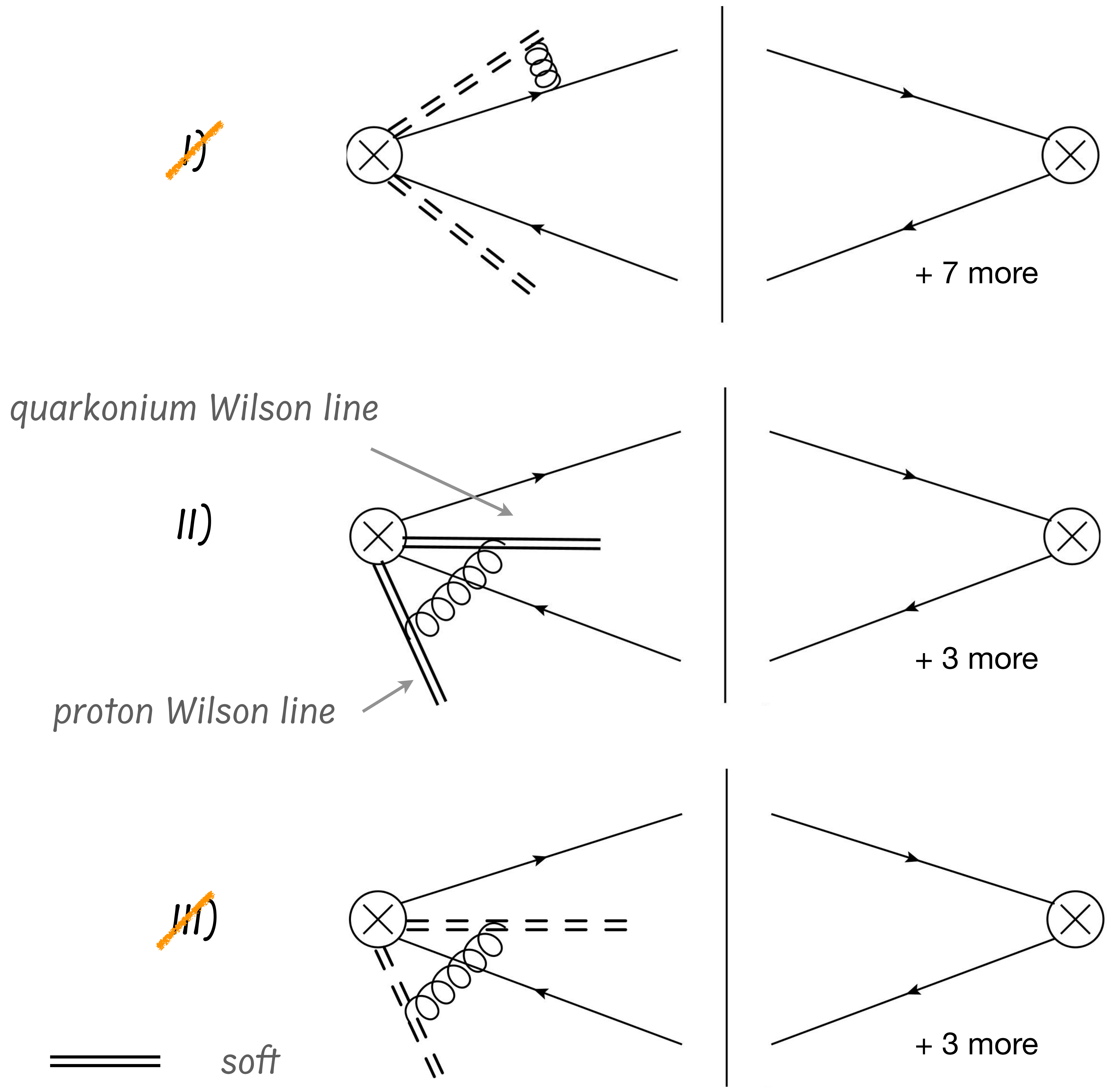
After BPS field redefinition



Calculation at NLO

The calculation for the P-wave is analogous

- TMDShF:



Vanishes in pairs

Only this set of diagrams survives

Double-counting

+ Coulomb + Chromo-electric

δ-regularization

Half of the soft function

$$S^{(0)}_{{}^1S_0^{[8]}\to J/\psi}(\mathbf{k}_\perp;\mu,\delta^+) = \frac{d-3}{d-2}\Bigg\{ \underbrace{\frac{S(\mathbf{k}_\perp;\mu,\delta^+)}{2}\left\langle {}^1S_0^{[8]}\right\rangle^{\mathrm{LO}}}_{II}$$

$$+\frac{\alpha_s}{2\pi}\delta^{(2)}(\mathbf{k}_\perp)\left(\underbrace{(C_F - C_A/2)\frac{\pi^2}{\mathrm{v}}}_{\text{Coulomb}} - \underbrace{\frac{C_A}{\varepsilon_{\mathrm{IR}}}}_{II}\right)\left\langle {}^1S_0^{[8]}\right\rangle^{\mathrm{LO}}$$

$$+\frac{\alpha_s C_A}{2\pi^2}\left(\underbrace{\frac{\pi}{\varepsilon_{\mathrm{UV}}}\delta^{(2)}(\mathbf{k}_\perp)}_{II} - \underbrace{\frac{1}{\mathbf{k}_\perp^2}}_{II}\right)\left\langle {}^1S_0^{[8]}\right\rangle^{\mathrm{LO}}$$

$$+\frac{4\alpha_s}{3\pi^2 m^2}\frac{1}{\mathbf{k}_\perp^2}\underbrace{\left(C_F\left\langle {}^1P_1^{[1]}\right\rangle^{\mathrm{LO}} + B_F\left\langle {}^1P_1^{[8]}\right\rangle^{\mathrm{LO}}\right)}_{\text{Chromo-electric}}\Bigg\}$$

RG evolution: LDMEs

$$\frac{d}{d\ln\mu}\left\langle \mathcal{O}_\psi^n \right\rangle^\mu = \sum_m \gamma_{\mathcal{O}}^{nm} \left\langle \mathcal{O}_\psi^m \right\rangle^\mu$$

$$\left\langle \mathcal{O}_\psi^n \right\rangle^\mu = Z_{\mathcal{O}}^{nm} \left\langle \mathcal{O}_\psi^m \right\rangle^\mu$$

$$Z_{^1S_0^{[8]}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{4\alpha_s(\mu)}{3\pi m^2}\frac{1}{\varepsilon_{\mathrm{UV}}} \begin{pmatrix} 0 & C_F & B_F \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\gamma_{^1S_0^{[8]}} = \frac{8\alpha_s(\mu)}{3\pi m^2} \begin{pmatrix} 0 & C_F & B_F \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\frac{d}{d\ln\mu}\left\langle {}^1S_0^{[8]} \right\rangle = \frac{8\alpha_s(\mu)}{3\pi m^2}\left(C_F \left\langle {}^1P_1^{[1]} \right\rangle + B_F \left\langle {}^1P_1^{[8]} \right\rangle \right)$$

$$\left\langle {}^1S_0^{[8]} \right\rangle^\mu = \mathcal{U}^{1m}(\mu,\mu_f)\langle \mathcal{O}^m \rangle^{\mu_f}$$

$$\left\langle {}^1S_0^{[8]} \right\rangle^\mu = \left\langle {}^1S_0^{[8]} \right\rangle^{\mu_f} - \frac{8}{3m^2\beta_0}\ln\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_f)}\right)\left(C_F \left\langle {}^1P_1^{[1]} \right\rangle^{\mu_f} + B_F \left\langle {}^1P_1^{[8]} \right\rangle^{\mu_f} \right)$$

$$\left\langle {}^3P_J^{[8]} \right\rangle^\mu = \left\langle {}^3P_J^{[8]} \right\rangle^{\mu_f} - \frac{8}{3m^2\beta_0}\ln\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_f)}\right)\left(C_F \left\langle {}^3D_J^{[1]} \right\rangle^{\mu_f} + B_F \left\langle {}^3D_J^{[8]} \right\rangle^{\mu_f} \right)$$

Up to NLL

RG evolution: TMDShFs

$$\frac{d}{d\ln\mu}S_{^1S_0^{[8]}\to J/\psi}(b_T;\zeta_B,\mu) = \gamma_{^1S_0^{[8]}}(\alpha_s(\mu),\zeta_B,\mu)$$

$$Z_{Sh} = 1 - \frac{\alpha_s C_A}{2\pi}\frac{1}{\varepsilon_{\rm UV}}\left(1 - \ln\frac{\zeta_B}{Q_h^2}\right)$$

$$\gamma_{Sh} \equiv \frac{d}{d\ln\mu}\ln Z_{Sh} = \frac{\alpha_s C_A}{\pi}\left(1 - \ln\frac{\zeta_B}{Q_h^2}\right)$$

Consistency result confirmed at NLO

$$\frac{d}{d\ln\zeta_B}\ln\, S_{^1S_0^{[8]}\to J/\psi}(b_T;\mu,\zeta_B) = -\,\mathscr{D}_g(b_T;\mu)$$

$$\mathscr{D}_g(b_T;\mu) = \frac{\alpha_s C_A}{2\pi}L_T$$

$$L_T = \ln\left(\mu^2 b_T^2 e^{2\gamma_E}/4\pi\right)$$
$$Q_h^4 = \zeta_A\zeta_B$$

Same as for the TMPDF:

M.G. Echevarria, T. Kasemets, P.J. Mulders, C. Pisano, arXiv:1502.05354
(Higgs production in hadron-hadron collisions)

$$\begin{pmatrix} S_{^1S_0^{[8]}\to J/\psi}(b_T;\mu_f,\zeta_f) = \exp\left[\int_P\left(\gamma_{^1S_0^{[8]}}(\mu,\zeta)\frac{d\mu}{\mu} - \mathscr{D}_g(b_T;\mu)\frac{d\zeta}{\zeta}\right)\right] S_{^1S_0^{[8]}\to J/\psi}(b_T;\mu_i,\zeta_i) \\ S_{^3P_J^{[8]}\to J/\psi}(b_T;\mu_f,\zeta_f) = \exp\left[\int_P\left(\gamma_{^1S_0^{[8]}}(\mu,\zeta)\frac{d\mu}{\mu} - \mathscr{D}_g(b_T;\mu)\frac{d\zeta}{\zeta}\right)\right] S_{^3P_J^{[8]}\to J/\psi}(b_T;\mu_i,\zeta_i) \end{pmatrix}$$

Matching onto LDMEs

- Operator product expansion:

$$S_{N\to J/\psi}(b_T;\mu,\zeta_B) = \sum_n C_n^N(b_T;\mu,\zeta_B) \times \left\langle \mathcal{O}_\psi^n \right\rangle(\mu) + \mathcal{O}(b_T\Lambda_{QCD})$$

- Renormalized LDME and TMDShF:

$$\left\langle {}^1S_0^{[8]} \right\rangle = \left(1 + (C_F - C_A/2)\frac{\pi\alpha_s}{2v}\right)\left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} - \frac{4\alpha_s}{3\pi m^2}\frac{1}{\varepsilon_{\text{IR}}}\left(C_F\left\langle {}^1P_1^{[1]} \right\rangle^{\text{LO}} + B_F\left\langle {}^1P_1^{[8]} \right\rangle^{\text{LO}}\right)$$

$$S_{{}^1S_0^{[8]}\to J/\psi}(b_T;\mu,\zeta_B) = \frac{d-3}{d-2}\left\{\left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} + \frac{\alpha_s}{2\pi}\left[\underbrace{\frac{\pi^2}{v}(C_F - C_A/2)\left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} - \frac{8}{3m^2}\frac{1}{\varepsilon_{\text{IR}}}\left(C_F\left\langle {}^1P_1^{[1]} \right\rangle^{\text{LO}} + B_F\left\langle {}^1P_1^{[8]} \right\rangle^{\text{LO}}\right)}_{\text{Infrared part}}\right.\right.$$

$$\left.\left. + C_A L_T\left(1 - \ln\frac{\zeta_B}{Q^2}\right)\left\langle {}^1S_0^{[8]} \right\rangle^{\text{LO}} - \frac{8}{3m^2}L_T\left(C_F\left\langle {}^1P_1^{[1]} \right\rangle^{\text{LO}} + B_F\left\langle {}^1P_1^{[8]} \right\rangle^{\text{LO}}\right)\right]\right\}$$

- Matching coefficients:

$$C^S_{{}^1S_0^{[8]}}(b_T;\mu,\zeta_B) = \frac{1}{2}\left[1 + \frac{\alpha_s C_A}{2\pi}L_T\left(1 - \ln\frac{\zeta_B}{Q_h^2}\right)\right]$$

$$C^S_{{}^1P_1^{[1]}}(b_T;\mu) = -\frac{\alpha_s}{2\pi}\frac{4\,C_F}{3m^2}L_T$$

$$C^S_{{}^1P_1^{[8]}}(b_T;\mu) = -\frac{\alpha_s}{2\pi}\frac{4\,B_F}{3m^2}L_T$$

It is analogous for the P-waves, the matching coefficients will be the same

Check: Hard function

- Virtual contribution of the cross section in photoproduction (F. Maltoni, M. L. Mangano, A. Petrelli, hep-ph/9708349):
  $Q_h = M_\psi$

$$\sigma_N = \sigma_N^0 \left[1 + \frac{\alpha_s}{2\pi} \left\{ \left[\left(C_F - C_A/2\right) \frac{\pi^2}{\mathrm{v}} - \frac{C_A}{\varepsilon_{\mathrm{IR}}} \right] - \left[\frac{C_A}{\varepsilon_{\mathrm{IR}}^2} + \frac{1}{\varepsilon_{\mathrm{IR}}} \left(b_0 + C_A \ln \frac{\mu^2}{Q_h^2} \right) \right] - C_A \left(\frac{b_0}{C_A} \ln \frac{\mu^2}{Q_h^2} + \ln \frac{\mu^2}{Q_h^2} + \frac{1}{2} \ln^2 \frac{\mu^2}{Q_h^2} + \frac{\pi^2}{12} \right) + \mathscr{D}_N \right\} \right] .$$

- Virtual contribution of the cross section in leptoproduction(M.G. Echevarria, R. Kishore, S.F. Romera, P. Taels, In progress):
  $Q_h = f(Q, M_\psi) \gtrsim M_\psi$

$$\sigma_N = \sigma_N^0 \left[1 + \underbrace{\frac{\alpha_s}{2\pi} \left\{ \left[\left(C_F - C_A/2\right) \frac{\pi^2}{\mathrm{v}} - \frac{C_A}{\varepsilon_{\mathrm{IR}}} \right]\right.}_{\text{TMDShF}} - \underbrace{\left[\frac{C_A}{\varepsilon_{\mathrm{IR}}^2} + \frac{1}{\varepsilon_{\mathrm{IR}}} \left(b_0 + C_A \ln \frac{\mu^2}{Q_h^2} \right) \right]}_{\text{Gluon TMDPDF}} \left. + H_N \right\} \right] .$$

- TMDShF:

$$\begin{aligned} S_{{}^1S_0^{[8]} \to J/\psi}(\mathbf{k}_\perp; \mu, \delta^+) = \frac{d-3}{d-2} \Bigg\{ & \frac{S(\mathbf{k}_\perp^2; \mu, \delta^+)}{2} \left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}} + \underbrace{\frac{\alpha_s}{2\pi} \delta^{(2)}(\mathbf{k}_\perp) \left((C_F - C_A/2) \frac{\pi^2}{\mathrm{v}} - \frac{C_A}{\varepsilon_{\mathrm{IR}}} \right) \left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}}}_{\text{Virtual contribution}} \\ & \frac{\alpha_s C_A}{2\pi^2} \left(\frac{\pi}{\varepsilon_{\mathrm{UV}}} \delta^{(2)}(\mathbf{k}_\perp) - \frac{1}{\mathbf{k}_\perp^2} \right) \left\langle {}^1S_0^{[8]} \right\rangle^{\mathrm{LO}} + \frac{4\alpha_s}{3\pi^2 m^2} \frac{1}{\mathbf{k}_\perp^2} \left(C_F \left\langle {}^1P_1^{[1]} \right\rangle^{\mathrm{LO}} + B_F \left\langle {}^1P_1^{[8]} \right\rangle^{\mathrm{LO}} \right) \Bigg\} \end{aligned}$$

Summary

- Quarkonia production is a promising tool to access gluon TMDs.

- TMDShFs: NP effects unique to quarkonia production.

- A complete and consistent computation at one loop of the TMDShF has been done.

- Future: Phenomenology (predictions for EIC, LHC…) and consider more processes.

Summary

- Quarkonia production is a promising tool to access gluon TMDs.
- TMDShFs: NP effects unique to quarkonia production.
- A complete and consistent computation at one loop of the TMDShF has been done.
- Future: Phenomenology (predictions for EIC, LHC…) and consider more processes.

Thank you!

Backup slides

Matching tensors for J=0,1,2

- Tensorial decomposition:

$$q^i\sigma^j = \frac{\delta^{ij}}{3}\mathbf{q}\cdot\boldsymbol{\sigma} + \frac{e^{ijk}}{2}(\mathbf{q}\times\boldsymbol{\sigma})^k + q^{(i}\sigma^{j)} \ ,$$

- Operators in vNRQCD:

$$\mathscr{O}(^3P_0^{[8]}) = \frac{1}{3}\psi_{\mathbf{q}}^\dagger\left(\frac{\overleftrightarrow{\mathscr{P}}}{2}\cdot\frac{\boldsymbol{\sigma}}{\sqrt{2}}\right)\sqrt{2}T^a\chi_{-\mathbf{q}} = \frac{1}{3}\psi_{\mathbf{q}}^\dagger\left(\mathbf{q}\cdot\boldsymbol{\sigma}\right)T^a\chi_{-\mathbf{q}} \ ,$$

$$\mathscr{O}^k(^3P_1^{[8]}) = \frac{1}{2}\psi_{\mathbf{q}}^\dagger\left(\frac{(\overleftrightarrow{\mathscr{P}}\times\boldsymbol{\sigma})^k}{2\sqrt{2}}\right)\sqrt{2}T^a\chi_{-\mathbf{q}} = \frac{1}{2}\psi_{\mathbf{q}}^\dagger\left(\mathbf{q}\times\boldsymbol{\sigma}\right)^k T^a\chi_{-\mathbf{q}} \ ,$$

$$\mathscr{O}^{ij}(^3P_2^{[8]}) = \psi_{\mathbf{q}}^\dagger\left(\frac{\overleftrightarrow{\mathscr{P}}^{(i}}{2}\frac{\sigma^{j)}}{\sqrt{2}}\right)\sqrt{2}T^a\chi_{-\mathbf{q}} = \psi_{\mathbf{q}}^\dagger\left(q^{(i}\sigma^{j)}\right)T^a\chi_{-\mathbf{q}} \ .$$

- Matching tensors:

$$\Gamma^{\mu\nu}_{^3P_0^{[8]}} = \Gamma^{\mu\nu\sigma\rho}_{^3P_J^{[8]}}\left(-g_{\sigma\rho} + \frac{P_\sigma P_\rho}{s}\right) \ , \qquad \Gamma^{\mu\nu}_{k,^3P_1^{[8]}} = \Gamma^{\mu\nu\sigma\rho}_{^3P_J^{[8]}}\epsilon_{\alpha\beta\sigma\rho}\Lambda^\alpha_k\frac{P^\beta}{M} \ , \qquad \Gamma^{\mu\nu}_{ij,^3P_2^{[8]}} = \Gamma^{\mu\nu\sigma\rho}_{^3P_J^{[8]}}\Lambda_{i\sigma}\Lambda_{j\rho} \ .$$