Systematic one-loop matching at subleading power on $\mathrm{SCET_{II}}$

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work in progress in collaboration with

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SCET at NLP

- Next-to-leading power is receiving more attention.
- At NLP new interesting features appear and new problems arise:
  - Endpoint divergences, violation of KSZ, soft-quark emissions, ...
  - Factorisation more complicated: Ideas of "refactorisation".
- For exclusive decays, on-shell particles are collinear (not hard-collinear) and the relevant theory is $\mathrm{SCET_{II}}$.
- The matching $\mathrm{SCET_I} \to \mathrm{SCET_{II}}$ is universal, can be done once and for all.
- At tree-level: Integrate out the hard-collinear field using equations of motion [Beneke, Feldmann hep-ph/0311335]
- At one-loop: Perform diagrammatic matching.
- It is convenient to define the intermediate Lagrangian containing hard-collinear, collinear and soft modes, $\mathrm{SCET_{I+}}$ or $\mathrm{SCET(hc, c, s)}$.

Construction of $\mathrm{SCET_I}$

- Power-counting parameter $\lambda = |\frac{p_\perp}{n_+ p}| \ll 1$.
  [Bauer et al. hep-ph/0011336, hep-ph/0109045, hep-ph/0202088;]

- Introduce the mode split $\psi \sim \xi + \eta + q$, $A_\mu = A_{c\mu} + A_{s\mu}$

- Power counting:

$$\xi \sim \lambda\,, \quad \eta \sim \lambda^2\,, \quad q \sim \lambda^3$$
$$(n_+ A_c, A_{c\perp}, n_- A_c) \sim (1, \lambda, \lambda^2)\,, \quad A_s \sim \lambda^2$$

- Control large field component $n_+ A_c$ through the Wilson line W_c.

- Multipole-expand the soft fields in soft-collinear products
  [Beneke, Chapovsky, Diehl, Feldmann hep-ph/0206152; Beneke, Feldmann hep-ph/0211358]

$$\phi_s(x) = \phi_s(x_-) + (x - x_-) \cdot \partial \phi_s + \ldots$$

The subleading $\mathrm{SCET_I}$ Lagrangian

- Start from the QCD Lagrangian in (collinear) light-cone gauge.
- Insert the soft and collinear modes

$$\mathcal{L}^{(1)}_{\xi q} = \bar{q} i \not{\partial}_\perp \xi + \bar{q} g_s \not{A}_{c\perp} \xi + \text{h.c.}$$

- Enforce "momentum-conservation":
  - Lagrangian contains terms of the form $\phi_s^n(x)\phi_c(x)$
  - To remove these terms: employ soft (background) equations of motion
  - This is a background field construction, collinear fluctuation
- Introduce the soft Wilson line R

$$R = \mathbf{P} \exp\left[i g_s \int_0^1 ds\, (x - x_-)^\mu A_{s\mu}(x_- + s(x - x_-))\right]$$

and redefine $A_c \to R A_c R^\dagger$, $\xi \to R\xi$ to move the soft field to fixed-line gauge.

The subleading $\mathrm{SCET_I}$ Lagrangian

- Result: Lagrangian in collinear light-cone gauge, without unphysical terms

$$\mathcal{L}^{(1)}_{\xi q} = \bar{q} g_s \mathcal{A}_{c\perp} \xi + \text{h.c.}$$

$$\mathcal{L}^{(2)}_{\xi q} = \bar{q} g_s n_- A_c \frac{\not{n}_+}{2} \xi + \bar{q} g_s \mathcal{A}_{c\perp} \frac{1}{i n_+ \partial} (i \partial\!\!\!/_\perp + g_s \mathcal{A}_{c\perp}) \frac{\not{n}_+}{2} \xi + \left[\bar{q} \overleftarrow{D}^\mu_s\right] x_{\perp\mu} g_s \mathcal{A}_{c\perp} \xi + \text{h.c.}$$

- "Unfix" light-cone gauge: introduce gauge-invariant building blocks

$$\chi = W_c^\dagger \xi\,, \quad \mathcal{A}^\mu_\perp = W_c^\dagger i D^\mu_\perp W_c - i\partial^\mu_\perp\,, \quad n_-\mathcal{A} = W_c^\dagger i n_- D W_c - i n_- D_s$$

  and simply replace $\xi \to \chi$, $g_s A_c \to \mathcal{A}$

- Yields the manifestly gauge-invariant "building-block" Lagrangian

$$\mathcal{L}^{(1)}_{\xi q} = \bar{q} \mathcal{A}\!\!\!/_\perp \chi + \text{h.c.}$$

$$\mathcal{L}^{(2)}_{\xi q} = \bar{q} n_- \mathcal{A} \frac{\not{n}_+}{2} \chi + \bar{q} \mathcal{A}\!\!\!/_\perp \frac{1}{i n_+ \partial} (i \partial\!\!\!/_\perp + \mathcal{A}\!\!\!/_\perp) \frac{\not{n}_+}{2} \chi + \left[\bar{q} \overleftarrow{D}^\mu_s\right] x_{\perp\mu} \mathcal{A}\!\!\!/_\perp \chi + \text{h.c.}$$

The covariant Lagrangian

[Beneke, Chapovsky, Diehl, Feldmann hep-ph/0206152, Beneke, Feldmann hep-ph/0211358]

- Re-express the Lagrangian through the fundamental fields ξ, A_c

$$\mathcal{L}^{(1)}_{\xi q} = \bar{q} W_c^\dagger i \not{D}_\perp \xi - \bar{q} i \not{\partial}_\perp W_c^\dagger \xi + \text{h.c.}\,,$$

$$\mathcal{L}^{(2)}_{\xi q} = \bar{q} W_c^\dagger \Big(i n_- D + i \not{D}_\perp \frac{1}{i n_+ D} i \not{D}_\perp \Big) \frac{\not{n}_+}{2} \xi + \big[\bar{q} \overleftarrow{D}^\mu_s \big] x_{\perp\mu} W_c^\dagger i \not{D}_\perp \xi$$

$$- \Big(\bar{q} i n_- D_s \frac{\not{n}_+}{2} + \big[\bar{q} \overleftarrow{D}^\mu_{s\perp} \big] x_{\perp\mu} i \not{\partial}_\perp \Big) W_c^\dagger \xi + \text{h.c.}$$

- The additional terms vanish either by total derivatives or through the soft-quark equations of motion.

- Gauge-invariance in the covariant Lagrangian is now subtle, as this Lagrangian contains unphysical interaction terms.

Gauge-invariance of the subleading Lagrangian

[Böer, PH, 2306.12412]

- Consider the first term in the Lagrangian

$$\bar{q} W_c^\dagger i n_- D \frac{\not{n}_+}{2} \xi = \bar{q} i n_- D_s \frac{\not{n}_+}{2} \xi + \bar{q} g_s n_- A_c \frac{\not{n}_+}{2} \xi + \bar{q} (W_c^\dagger - 1) i n_- D \frac{\not{n}_+}{2} \xi \,.$$

- All three pieces are required for invariance, but the first is unphysical.

- "Standard prescription": Drop unphysical terms from the Lagrangian in any computation.

- This procedure is not gauge-invariant.

  [Bodwin, Ee, Kang, Wang 2302.05856]

- Does this matter?

A formal justification

- Dropping unphysical terms is again justified by the background field method
- "Standard prescription": Fluctuation ξ on top of soft background.

$$\mathcal{L}^{(2)}_{\xi q,\text{unphys}} = -\left(\bar{q} i n_- \overleftarrow{D}_s \frac{\not{n}_+}{2} + \left[\bar{q} i \overleftarrow{\not{D}}_{s\perp}\right] + m_q \bar{q}\right)\xi\,,$$

- "Gauge-invariant prescription": Fluctuation $\chi = W_c^\dagger \xi$ instead

$$\mathcal{L}^{(2)}_{\xi q,\text{unphys}} = -\left(\bar{q} i n_- \overleftarrow{D}_s \frac{\not{n}_+}{2} + \left[\bar{q} i \overleftarrow{\not{D}}_{s\perp}\right] + m_q \bar{q}\right) W_c^\dagger \xi\,,$$

  This results in the building block Lagrangian.

- For any sensible computation, both prescriptions agree, as they differ by soft-quark equations of motion.
- To avoid this problem entirely: Define radiative jet functions through a matching equation or use building blocks.

Onwards to $\mathrm{SCET_{I+}}$

[Beneke, Feldmann hep-ph/0311335]

- In addition to hard-collinear and soft modes, introduce collinear modes with

$$\xi_c \sim \lambda^2\,, \quad A_c \sim (1, \lambda^2, \lambda^4)\,.$$

- Start from the all-order $\mathrm{SCET_I}$ building block Lagrangian, employ soft and collinear light-cone gauge $n_- A_s = 0$, $n_+ A_{hc} = n_+ A_c = 0$, and insert

$$A_c \to A_{hc} + A_c\,, \quad \xi \to \xi_{hc} + \xi_c\,.$$

- Use momentum conservation to eliminate unphysical interactions

$$\overline{\xi}_{hc} \not{A}_{hc\perp} q \to \overline{\xi}_{hc} (\not{A}_{hc\perp} + \not{A}_{c\perp}) q + \overline{\xi}_c \not{A}_{hc\perp} q\,.$$

- This can be formally justified by the background field method.

Multipole expansion

- Interaction between hard-collinear and soft: expand around $x_-^\mu = n_+ x \frac{n_-^\mu}{2}$

$$\phi_{hc}(x)\phi_s(x) = \phi_{hc}(x)\Big(\phi_s(x_-) + (x - x_-)^\alpha [\partial_\alpha \phi_s](x_-) + \dots\Big).$$

- Between hard-collinear and collinear: expand around $x_+^\mu = n_- x \frac{n_+^\mu}{2}$ as

$$\phi_{hc}(x)\phi_c(x) = \phi_{hc}(x)\Big(\phi_c(x_+) + (x - x_+)^\alpha [\partial_\alpha \phi_c](x_+) + \dots\Big).$$

- If hard-collinear, collinear and soft fields are present, one performs *both* multipole expansions for the collinear and soft fields.
- If *no* hard-collinear fields are present: $x_\perp^\mu$ is of the same order for both soft and collinear fields, expand in remaining coordinate

$$\phi_c(x)\phi_s(x) = \Big(\phi_c(\tilde{x}_+) + \dots\Big)\Big(\phi_s(\tilde{x}_-) + \dots\Big),$$

where $\tilde{x}_+^\mu = n_- x \frac{n_+^\mu}{2} + x_\perp^\mu$ and $\tilde{x}_-^\mu = n_+ x \frac{n_-^\mu}{2} + x_\perp^\mu$.

The extended gauge symmetry

- Can realise the collinear gluon as a background field to the hard-collinear one

hard-collinear:
$$\hat{A}_{hc} \to U_{hc}\hat{A}_{hc}U_{hc}^\dagger + \frac{1}{g_s}\,U_{hc}\left[iD_{s+c}, U_{hc}^\dagger\right]\,,$$
$$A_c \to A_c\,,$$
$$A_s \to A_s\,,$$

collinear:
$$\hat{A}_{hc} \to U_c(x_+)\hat{A}_{hc}U_c^\dagger(x_+) + U_c(x_+)\left[n_- A_s(x_-)\frac{n_+}{2}, U_c^\dagger(x_+)\right]$$
$$A_c \to U_c A_c U_c^\dagger + \frac{1}{g_s}\,U_c\left[i\partial, U_c^\dagger\right]\,,$$
$$A_s \to A_s\,,$$

soft:
$$\hat{A}_{hc} \to U_s(x_-)\hat{A}_{hc}U_s^\dagger(x_-) + U_s(x_-)\left[n_+ A_c(x_+)\frac{n_-}{2}, U_s^\dagger(x_-)\right]$$
$$A_c \to A_c\,,$$
$$A_s \to U_s A_s U_s^\dagger + \frac{1}{g_s}\,U_s\left[i\partial, U_s^\dagger\right]\,,$$

with $iD_{s+c} = i\partial + g_s n_- A_s(x_-)\frac{n_+}{2} + g_s n_+ A_c(x_+)\frac{n_-}{2}$

A selection of Wilson lines

- To reinstate the gauge symmetry, "unfix" light-cone gauge as in $\mathrm{SCET_I}$.
- The hard-collinear Wilson line

$$W_{hc}(x) = \mathbf{P}\exp\left[ig\int_{-\infty}^{0} ds\,\left(n_+\hat{A}_{hc}(x+sn_+) + n_+A_c(x_+ + sn_+)\right)\right]$$

  fixes $n_+A_{hc} = 0$ and $n_+A_c(x_+) = 0$, transforms as

$$W_{hc} \to U_{hc}W_{hc}\,, \quad W_{hc} \to U_c(x_+)W_{hc}\,, \quad W_{hc} \to U_s(x_-)W_{hc}U_s^\dagger(x_-)\,.$$

- The collinear and soft Wilson lines as usual

$$W_c(x) = \mathbf{P}\exp\left[ig\int_{-\infty}^{0} ds\, n_+A_c(x+sn_+)\right],$$
$$Y_s(x) = \mathbf{P}\exp\left[ig\int_{-\infty}^{0} dt\, n_-A_s(x+tn_-)\right].$$

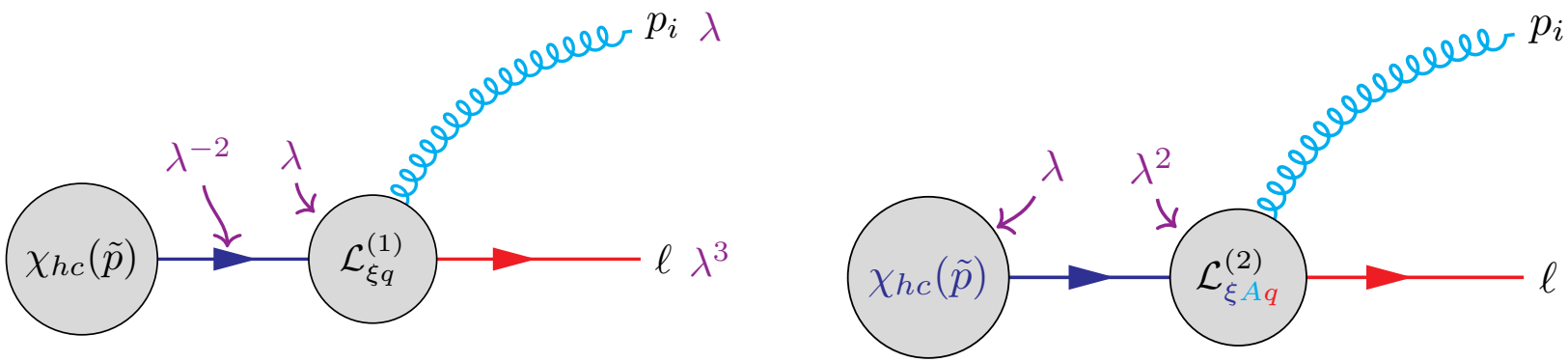
- In the hc-c sector: Can use R_c, the analogue of R_s and fixed-line gauge.

An example matching

- Consider the splitting $\chi_{hc} \to A_c q_s$ starting at $\mathcal{O}(\lambda^3)$.
- From equations of motion: [Beneke, Feldmann hep-ph/0311335]

$$\psi^{(3)} \supset \frac{1}{in_- \partial} g_s \mathcal{A}_{c\perp} \frac{\not{n}_-}{2} q$$

- Diagrammatically in both $\mathrm{SCET_I}$ and $\mathrm{SCET_{I+}}$:

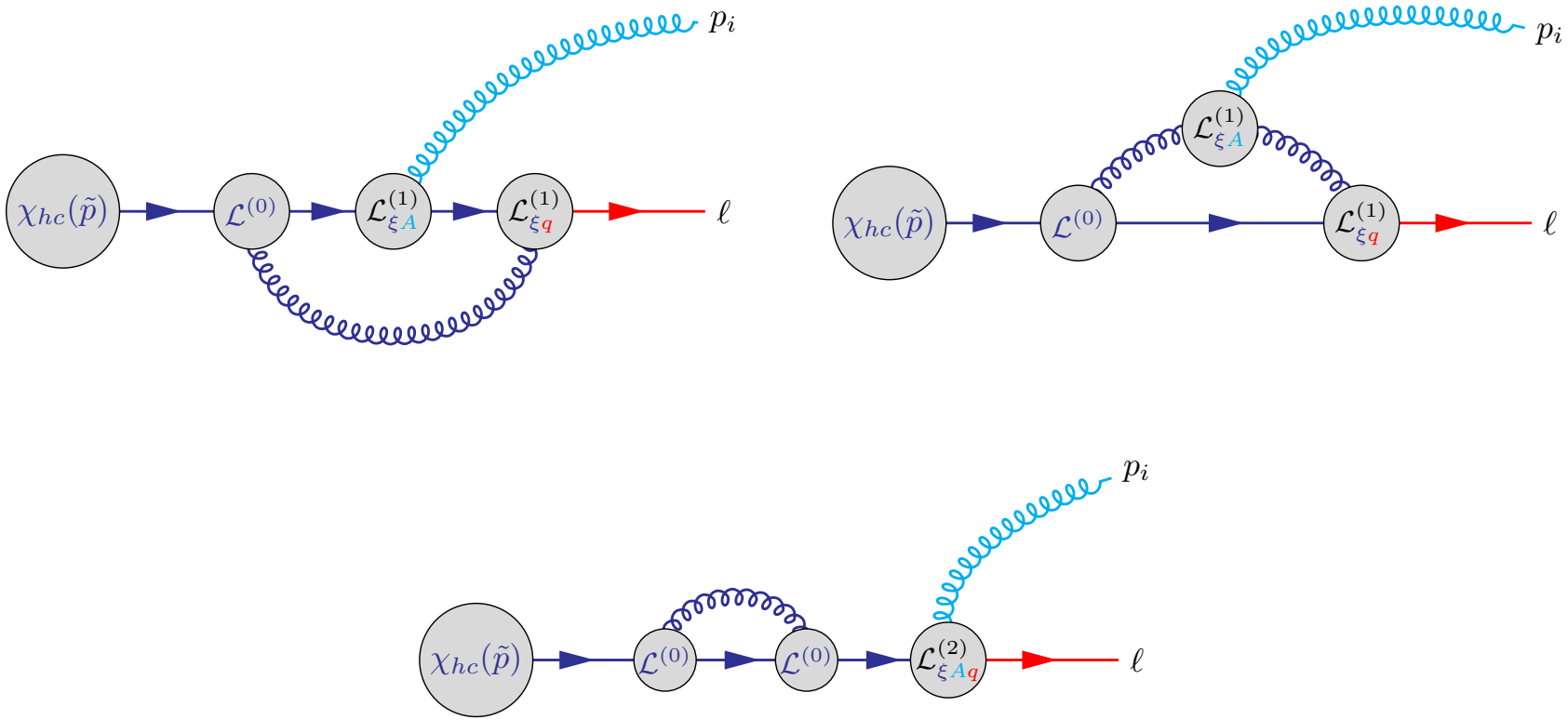


- Relevant $\mathrm{SCET_{I+}}$ Lagrangian

$$\mathcal{L}^{(2)}_{\xi A q} = \bar{q} g_s \mathcal{A}_{c\perp} \xi_{hc} + \text{h.c.}$$

Loop corrections

- Splitting of hard-collinear field receives loop corrections
- For $\chi_{hc} \to A_c q$ diagrammatically:



All other diagrams vanish in light-cone gauge.

- For $\psi^{(3)}$, this is equivalent to computing the one-loop radiative jet function.

Loop corrections

- Relevant pieces of the $\mathrm{SCET_{I+}}$ Lagrangian:

$$\mathcal{L}^{(1)}_{\xi A} = \bar{\xi}_{hc}\left(i\partial\!\!\!/_\perp + g_s \mathcal{A}\!\!\!/_{c\perp}\right)\frac{1}{in_+\partial}\left(i\partial\!\!\!/_\perp + g_s \mathcal{A}\!\!\!/_{c\perp}\right)\frac{n\!\!\!/_+}{2}\xi_{hc}$$

$$\mathcal{L}^{(1)}_{\xi q} = \bar{q} g_s \mathcal{A}\!\!\!/_{hc\perp}\xi_{\mathrm{hc}} + \mathrm{h.c.}$$

$$\mathcal{L}^{(2)}_{\xi A q} = \bar{q} g_s \mathcal{A}\!\!\!/_{c\perp}\xi_{hc} + \mathrm{h.c.}$$

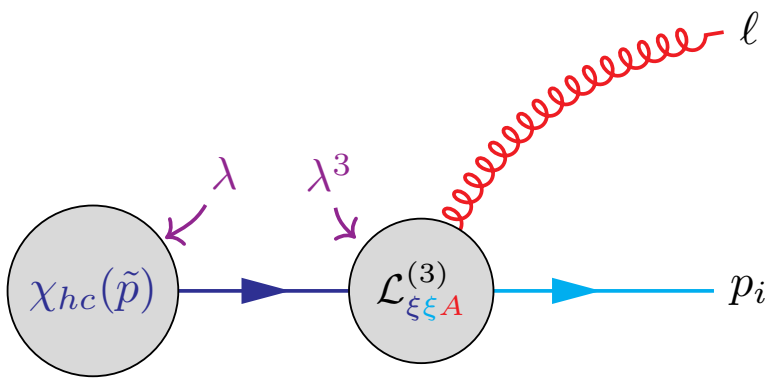
- All that remains is a straightforward evaluation of the above diagrams.
- Result: [Liu, Neubert, Schnubel, Wang 2112.0018]

$$\xi^{(3,1)}_{hc} = \xi^{(3,0)}_{hc}\left(-\frac{Q^2}{\mu^2}\right)^{-\varepsilon}\frac{\alpha_s}{4\pi}(C_F - C_A)e^{\varepsilon\gamma_E}\frac{\Gamma(1+\varepsilon)\Gamma(-\varepsilon)^2}{\Gamma(2-2\varepsilon)}(2-4\varepsilon-\varepsilon^2)$$

$$\xi^{(3,0)}_{hc} = -\frac{n\!\!\!/_-}{2}\frac{1}{n_- l}g_s \mathcal{A}\!\!\!/_{c\perp} q$$

General Strategy

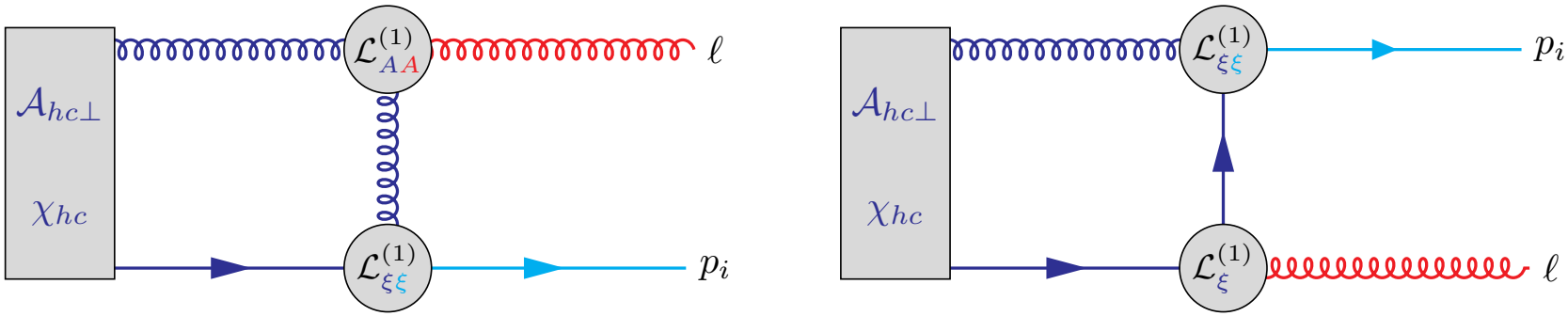
- Work in Progress!
- Organise the computation by initial-state particle content.
- Compute loop-corrections to the tree-level splittings $\psi^{(n)}$, $A^{(n)}_{hc}$.
- In light-cone gauge: each additional splitting costs one power of λ.
- Example contribution to $\psi^{(4)}$: $\chi_{hc} \to A_{\perp s}\chi_c$



receives loop corrections, again a radiative jet function.

Matching at the one-loop level

- Starting at $\mathcal{O}(\lambda^4)$, new structures emerge: Initial state can contain two particles $\chi_{hc}\mathcal{A}_{hc} \to A_{s\perp}\chi_c$



- In light-cone gauge: each additional initial particle costs one power of λ.
- Complexity increases at higher-orders:
  - Generate more splittings at tree-level with leading-power loops.
  - Subleading-power emissions and loops.
  - Contain more particles in the initial state.
- With the $\mathrm{SCET}_{\mathrm{I}+}$-Lagrangian, we have consistent power-counting and the diagrams can be evaluated straightforwardly.

Conclusion

- Gauge-invariance in the covariant form of the SCET Lagrangian is subtle.
- Insertions of individual pieces of the subleading Lagrangian are not necessarily gauge-invariant but can differ by soft equations of motion.
- Individual insertions are not physical, only the sum of all allowed vertices is. For any physical observable, this is therefore not an issue.
- Construct the $\mathrm{SCET}_{\mathrm{I}+}$ Lagrangian featuring hard-collinear, collinear and soft modes to facilitate the matching computation.
- Work in progress: Compute the universal matching $\mathrm{SCET}_{\mathrm{I}} \to \mathrm{SCET}_{\mathrm{II}}$ at one-loop up to λ^4 (potentially λ^5).
- Besides loop corrections, new structures featuring multiple initial hard-collinear particles emerge.