Subleading power corrections to the color-singlet transverse momentum spectra

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Why the subleading power corrections?

- The **computation of the non-singular contributions** to the spectrum of the given resolution variable (i.e. the transverse momentum of the color singlet q_{T}) is often the main **computational bottleneck** in state-of-the-art QCD NNLO calculations
- The analytic knowledge of the spectrum beyond leading power would allow us to
  - $\rightarrow$ **approximate the non-singular contributions** in the small q_{T} limit **eliminating the need for a numeric subtraction** up to very low q_{T} values
  - $\rightarrow$ Get an insight into the **factorization structure** of the next-to-leading power corrections

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Subleading power corrections

- We consider the process of production of a **color singlet** (inclusive Drell-Yan for now) from a **hadronic scattering**

$$h_a\, h_b \to \mathrm{CS} + X$$

- The differential cross section for the process with respect to the color-singlet **mass** Q, **rapidity** Y and **transverse momentum** q_{T} can be written as

$$\frac{d\sigma_{h_a h_b \to \mathrm{CS}+X}}{dQ^2\, dY\, dq_{\mathrm{T}}^2} = K_\delta\left(Q^2, Y\right) \delta\left(q_{\mathrm{T}}^2\right) + \frac{K\left(Q^2, Y, q_{\mathrm{T}}^2\right)}{q_{\mathrm{T}}^2}$$

- The QCD **logarithmic structure** of K is well known and given by

$$K\left(Q^2, Y, q_{\mathrm{T}}^2\right) = \sum_{n=1}^{\infty} \alpha_{\mathrm{S}}^n \sum_{\ell=0}^{2n-1} \log^\ell\left(q_{\mathrm{T}}^2\right) K_{n\ell}\left(Q^2, Y, q_{\mathrm{T}}^2\right)$$

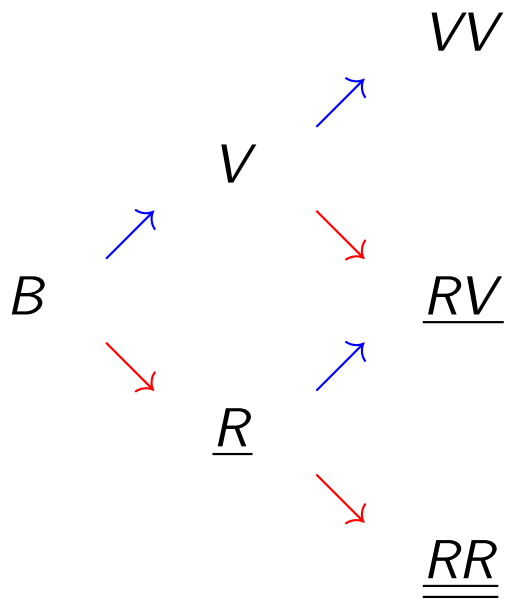
Subleading power corrections

- The **power structure** of the $K_{n\ell}$ terms is obtained by expanding them with respect to q_{T}^2 and is given by

$$K_{n\ell}\left(Q^2, Y, q_{\mathrm{T}}^2\right) = \sum_{p=0}^{\infty} K_{n\ell p}\left(Q^2, Y\right) \left(q_{\mathrm{T}}^2\right)^p$$

- The full **NLO next-to-leading power** result is currently available for the Drell-Yan and gluon-fusion Higgs boson production processes for both the $\mathcal{T}_0$ and q_{T} resolution variables [Boughezal, Isgrò, Petriello '18] [Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18] [Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18]
- The **leading logarithmic** terms for $\mathcal{T}_0$ are also available at NNLO and N^{3}LO [Moult, Rothen, Stewart, Tackmann, Zhu '16] [Boughezal, Liu, Petriello '16] [Vita '24]
- I will present a systematic way of computing the **subleading power corrections** at **NNLO**, i.e. the $K_{2\ell p}$ terms for $0 \leq \ell \leq 3$ and $p \geq 1$

General picture



↗ α_{S} renormalization

↘ PDFs renormalization

- B, V and VV: No partons in the final state, proportional to $\delta\left(q_{\mathrm{T}}^2\right)$
  - → They **do not contribute at subleading power**
- R, RV: One parton in the final state, 4 degrees of freedom: Q^2, Y, q_{T}^2 and z
  - → One PDF convolution, **no additional integrals to be done**
- RR: Two partons in the final state, 7 degrees of freedom
  - → **Up to three integrals to be done**

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The SCET power expansion

- We know from the **SCET** that the contributions to the spectrum will come from the regions of the phase space where the partons are either **soft** or **collinear to the beam**
- Given the two **strongly separated scales**

$$Q \sim \sqrt{q^- q^+} \qquad q_{\mathrm{T}} \sim \lambda Q$$

we will get a non-zero contribution from the regions where the partons of momentum k have a

$\rightarrow$ Soft scaling: $k \sim (\lambda, \lambda, \lambda)\, Q$

$\rightarrow$ n-collinear scaling: $k \sim \left(1, \lambda^2, \lambda\right) Q$

$\rightarrow$ $\bar{n}$-collinear scaling: $k \sim \left(\lambda^2, 1, \lambda\right) Q$

One parton in the final state

- The phase space for the contributions with **one parton** in the final state can be parametrized as

$$d\Phi_{\mathrm{CS}+1j} = \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{q_{\mathrm{T}}^2}\right)^{\epsilon} \frac{dq^- \, dq^+ \, dq_{\mathrm{T}}^2}{S} \frac{\boldsymbol{dy}}{(4\pi)^2} \, d\Phi_{\mathrm{CS}} \times \theta\left(\sqrt{S} - q^- - e^{y} q_{\mathrm{T}}\right) \theta\left(\sqrt{S} - q^+ - e^{-y} q_{\mathrm{T}}\right)$$

- The **3 relevant regions** are those where the parton has a
  - → Soft scaling: $e^{y} \sim 1$
  - → n-collinear scaling: $e^{y} \sim 1/\lambda$
  - → $\bar{n}$-collinear scaling: $e^{y} \sim \lambda$

Two partons in the final state

- The phase space with **two partons** in the final state reads

$$d\Phi_{\mathrm{CS}+2j} = \frac{dq^- \, dq^+ \, dq_{\mathrm{T}}^2}{S} \, \frac{\boldsymbol{dk_1^-} \, \boldsymbol{dk_1^+}}{(4\pi)^2} \, \frac{\boldsymbol{dk_2^-} \, \boldsymbol{dk_2^+}}{(4\pi)^2} \, d\Phi_{\mathrm{CS}} \, J(\Phi_{\mathrm{CS}+2j})$$

- The **9 relevant regions** are those where the two partons have a
  - → Double soft scaling

$$k_1^- \sim k_1^+ \sim k_2^- \sim k_2^+ \sim \lambda \, Q$$

  - → Mixed soft and (n- or $\bar{n}$-) collinear scaling (4 possibilities)

$$k_1^- \sim k_1^+ \sim \lambda \, Q \qquad k_2^- \sim Q \qquad k_2^+ \sim \lambda^2 \, Q$$

  - → Double (n- or $\bar{n}$-) collinear scaling (2 possibilities)

$$k_1^- \sim k_2^- \sim Q \qquad k_1^+ \sim k_2^+ \sim \lambda^2 \, Q$$

  - → Mixed n-collinear and $\bar{n}$-collinear scaling (2 possibilities)

$$k_1^- \sim k_2^+ \sim Q \qquad k_1^+ \sim k_2^- \sim \lambda^2 \, Q$$

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The analytic regulator

- The rapidity divergences are regulated through a **pure rapidity regulator** [Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18] that multiplies the phase space whenever there is at least one parton in the final state
- If k is the **total momentum of the final-state partons**, the regulator is defined as

$$R_Y = \left(\frac{k^-}{k^+}\right)^\alpha$$

- This regulator has **two main advantages**
  - $\rightarrow$ It makes the **soft contributions zero** beyond leading order
  - $\rightarrow$ If there are two partons in the final state, it **does not depend separately on k_1 and k_2 but only on their sum $k = k_1 + k_2$**

The single soft limit

- The real-emission amplitude in the limit where **one parton is soft** can be written as a sum over n of terms proportional to $e^{(n+2\alpha)y}$
- Since the integral

$$\int_{-\infty}^{\infty} dy\, e^{Ay} \qquad x = \frac{1}{1+e^{y}}$$

can be analytically continued to

$$\int_{0}^{1} dx\, x^{-1-A}\,(1-x)^{-1+A} = \frac{\Gamma(-A)\,\Gamma(A)}{\Gamma(0)} = 0$$

all **the single-soft contributions beyond leading order are 0**

- The 1-loop diagrams can also contain terms proportional to

$$\int_{-\infty}^{\infty} dy\, y^{m} e^{Ay} = \frac{d^{m}}{dA^{m}} \int_{-\infty}^{\infty} dy\, e^{Ay} = 0$$

The double soft limit

- In the limit where **two partons are soft** it is convenient to parametrize the phase space as

$$d\Phi_{\mathrm{CS}+2j} = \frac{dq^- \, dq^+ \, dq_{\mathrm{T}}^2}{S} \, \frac{dk_{1\mathrm{T}}^2 \, \boldsymbol{dy_1}}{(4\pi)^2} \, \frac{dk_{2\mathrm{T}}^2 \, \boldsymbol{dy_2}}{(4\pi)^2} \, d\Phi_{\mathrm{CS}} \times \left(\frac{k_{1\mathrm{T}} e^{\boldsymbol{y_1}} + k_{2\mathrm{T}} e^{\boldsymbol{y_2}}}{k_{1\mathrm{T}} e^{-\boldsymbol{y_1}} + k_{2\mathrm{T}} e^{-\boldsymbol{y_2}}} \right)^{\alpha} J\left(q_{\mathrm{T}}^2, k_{1\mathrm{T}}^2, k_{2\mathrm{T}}^2\right)$$

- If we perform the change of variables

$$\tilde{y} = \frac{y_1 - y_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

the integral over y will be 0 as in the previous slide

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The phase space for the collinear limit

- In the limit where **all the partons are (n- or $\bar{n}$-) collinear**, it is convenient to write the N-parton phase space as

$$d\Phi_{\mathrm{CS}+Nj} = \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{q_{\mathrm{T}}^2}\right)^{\epsilon} \frac{dq^- \, dq^+ \, dq_{\mathrm{T}}^2}{S} \frac{dk^- \, dk^+}{(4\pi)^2} \frac{d\Phi_{Nj}}{2\pi} \, d\Phi_{\mathrm{CS}}$$
$$\times \left(\frac{k^-}{k^+}\right)^{\alpha} \theta\left(\sqrt{S} - q^- - k^-\right) \theta\left(\sqrt{S} - q^+ - k^+\right)$$

- The phase space of the N final-state partons is parametrized by $d\Phi_{Nj}$
  - → If there is **one parton** in the final state

$$d\Phi_{1j} = 2\pi \, \delta\left(k^2\right) \theta\left(k^0\right)$$

  - → If there are **two partons** in the final state

$$d\Phi_{2j} = \left(\frac{\mu^2}{k^2}\right)^{\epsilon} \frac{d\Omega^{(3-2\epsilon)}}{2\,(4\pi)^{2-2\epsilon}} \, \theta\left(k^2\right) \theta\left(k^0\right)$$

The double collinear limit

- In the limit where **the two partons are n-collinear** it is convenient to introduce

$$y = \frac{q_{\mathrm{T}}^2}{k^- k^+} \qquad z = \frac{q^-}{q^- + k^-}$$

- The phase space can be rewritten as

$$d\Phi_{\mathrm{CS}+2j} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{q_{\mathrm{T}}^2}\right)^\epsilon \frac{dq^-\, dq^+\, dq_{\mathrm{T}}^2\, q_{\mathrm{T}}^2}{(4\pi)^2\, S} \frac{dz}{z\,(1-z)} \frac{dy}{y^2} \frac{d\Phi_{2j}}{2\pi}\, d\Phi_{\mathrm{CS}}$$
$$\times \left(\frac{(q^-)^2}{q_{\mathrm{T}}^2}\right)^\alpha \left(\frac{z}{1-z}\right)^{-2\alpha} y^\alpha\, \theta\left(\sqrt{S} - \frac{q^-}{z}\right) \theta\left(\sqrt{S} - q^+\right)$$

- We are now left with integrating over Φ_{2j} and y, the integration over z can only be performed numerically since z will appear in the argument of one of the PDFs

The angular integrals

- **Since the analytic regulator does not depend on k_1 and k_2 but only on their sum $k = k_1 + k_2$**, we can always partial fraction the integrand using the equation

$$\frac{1}{\left(A + k_1^{\pm}\right)\left(B + k_2^{\pm}\right)} = \frac{1}{A + B + k^{\pm}} \left(\frac{1}{A + k_1^{\pm}} + \frac{1}{B + k_2^{\pm}} \right)$$

- We define **four types of angular integrals**
  - $\rightarrow$ I_1: proportional to $\left(k_1^-\right)^{-j_1}\left(k_2^+\right)^{-j_2}$ or $\left(k_2^-\right)^{-j_1}\left(k_1^+\right)^{-j_2}$
  - $\rightarrow$ I_2: proportional to $\left(k_1^-\right)^{-j_1}\left(k_1^+\right)^{-j_2}$ or $\left(k_2^-\right)^{-j_1}\left(k_2^+\right)^{-j_2}$
  - $\rightarrow$ I_3: proportional to $\left(q^- + k_1^-\right)^{-j_1}\left(k_2^+\right)^{-j_2}$ or $\left(q^- + k_2^-\right)^{-j_1}\left(k_1^+\right)^{-j_2}$
  - $\rightarrow$ I_4: proportional to $\left(q^- + k_1^-\right)^{-j_1}\left(k_1^+\right)^{-j_2}$ or $\left(q^- + k_2^-\right)^{-j_1}\left(k_2^+\right)^{-j_2}$

The angular integrals

- After expressing $k_1^{\pm}$ and $k_2^{\pm}$ in terms of the variable of Φ_{2j}, the integration over $d\Phi_{2j}$ can be written in terms of the integral [Somogyi '11]

$$I_\Omega = \int \frac{d\Omega^{(3-2\epsilon)}(q)}{(p_1 \cdot q)^{j_1} (p_2 \cdot q)^{j_2}}$$

- p_1 and p_2 are two **auxilary four-vectors** with $p_1^0 = p_2^0 = 1$ and

$$q = (1, ..., \sin\theta_1 \sin\theta_2 \cos\theta_3, \sin\theta_1 \cos\theta_2, \cos\theta_1)$$

- The result of this integral will depend on the **scalar quantities** $p_1 \cdot p_2$, p_1^2 and p_2^2: we are interested in the **2 cases**
  - $\rightarrow$ $p_1^2 = p_2^2 = 0$
  - $\rightarrow$ $p_1^2 \neq 0$ and $p_2^2 = 0$

The angular integrals (massless)

- We define the overall factor

$$\mathcal{F} = \frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{k^- k^+ - q_{\mathrm{T}}^2} \right)^{\epsilon} \theta(k^-)\, \theta(k^+)\, \theta(k^- k^+ - q_{\mathrm{T}}^2)$$

- The two **massless angular integrals** are given by

$$I_1 = \mathcal{F}\, \frac{\Gamma(1-j_1-\epsilon)\,\Gamma(1-j_2-\epsilon)}{\Gamma(2-j_1-j_2-2\epsilon)}\, \frac{(k^-)^{-j_1}\,(k^+)^{-j_2}}{(4\pi)^2} \times {}_2F_1\left(j_1, j_2;\, 1-\epsilon;\, 1-\frac{q_{\mathrm{T}}^2}{k^- k^+} \right)$$

$$I_2 = \mathcal{F}\, \frac{\Gamma(1-j_1-\epsilon)\,\Gamma(1-j_2-\epsilon)}{\Gamma(2-j_1-j_2-2\epsilon)}\, \frac{(k^-)^{-j_1}\,(k^+)^{-j_2}}{(4\pi)^2} \times {}_2F_1\left(j_1, j_2;\, 1-\epsilon;\, \frac{q_{\mathrm{T}}^2}{k^- k^+} \right)$$

The angular integrals (massive)

- The two **massive angular integrals** are given by

$$I_3 = \mathcal{F}\,\frac{\Gamma(1-\epsilon)\,\Gamma(1-j_2-\epsilon)}{\Gamma(2-j_2-2\epsilon)}\,\frac{\left(q^- + k^-\right)^{-j_1}\left(k^+\right)^{-j_2}}{(4\pi)^2}$$
$$\times\, F_1\left(j_1;\, j_2,\, 1-j_2-\epsilon;\, 2-j_2-2\epsilon;\, \frac{k^-k^+ - q_{\mathrm{T}}^2}{k^+\left(q^- + k^-\right)},\, \frac{k^-}{q^- + k^-}\right)$$

$$I_4 = \mathcal{F}\,\frac{\Gamma(1-\epsilon)\,\Gamma(1-j_2-\epsilon)}{\Gamma(2-j_2-2\epsilon)}\,\frac{\left(q^- + k^-\right)^{-j_1}\left(k^+\right)^{-j_2}}{(4\pi)^2}$$
$$\times\, F_1\left(j_1;\, j_2,\, 1-j_2-\epsilon;\, 2-j_2-2\epsilon;\, \frac{q_{\mathrm{T}}^2}{k^+\left(q^- + k^-\right)},\, \frac{k^-}{q^- + k^-}\right)$$

The y integrals

- We are left with solving the integral over y

$$\int_0^1 dy\, y^{\epsilon+\alpha}\,(1-y)^{-\epsilon}\,[\mathcal{C}(y)]^{-j_3}\,\mathcal{S}(y)$$

- The function $\mathcal{S}(y)$ is one among the **4 special functions**
  - $\to {}_2F_1(j_1, j_2; 1-\epsilon; 1-y)$
  - $\to {}_2F_1(j_1, j_2; 1-\epsilon; y)$
  - $\to F_1(j_1; j_2, 1-j_2-\epsilon; 2-j_2-2\epsilon; (1-z)(1-y), 1-z)$
  - $\to F_1(j_1; j_2, 1-j_2-\epsilon; 2-j_2-2\epsilon; (1-z)\,y, 1-z)$
- The function $\mathcal{C}(y)$ is one among the **3 factors**

$$y \qquad 1-y \qquad 1+\frac{1-z}{z}\,y$$

- The integrals can be **solved analytically** in terms of hypergeometric functions that can then be expanded using the **HypExp** [Huber, Maitre '05] and **MultiHypExp** [Bera '23] Mathematica packages

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The mixed collinear and anti-collinear limit

- In the limit where **one parton is n-collinear and the other is $\bar{n}$-collinear** it is convenient to introduce

$$z_a = \frac{q^-}{q^- + k_1^-} \qquad z_b = \frac{q^+}{q^+ + k_2^+}$$

- The phase space can now be parametrized as

$$\begin{aligned} d\Phi_{\mathrm{CS}+2j} = {} & \frac{dq^- dq^+ dq_{\mathrm{T}}^2}{S}\, d\Phi_{\mathrm{CS}}\, \mu^{2\epsilon} \frac{d^{2-2\epsilon} k_{1\mathrm{T}}}{2\,(2\pi)^{3-2\epsilon}} \frac{dz_a}{z_a\,(1-z_a)} \\ & \times \mu^{2\epsilon} \frac{d^{2-2\epsilon} k_{2\mathrm{T}}}{2\,(2\pi)^{3-2\epsilon}} \frac{dz_b}{z_b\,(1-z_b)}\, \delta\left(q_{\mathrm{T}}^2 - \left(\vec{k}_{1\mathrm{T}} + \vec{k}_{2\mathrm{T}}\right)^2\right) \\ & \times R_Y\, \theta\left(\sqrt{S} - \frac{q^-}{z_a}\right) \theta\left(\sqrt{S} - \frac{q^+}{z_b}\right) \end{aligned}$$

- We are left with integrating over $k_{1\mathrm{T}}$ and $k_{2\mathrm{T}}$, the integration over z_a and z_b can only be performed numerically since they appear in the argument of the PDFs

The master integrals

- We define the master integrals as

$$I = \int \frac{d^{2-2\epsilon} k_{1\mathrm{T}}}{(k_{1\mathrm{T}}^2)^{j_1}} \int \frac{d^{2-2\epsilon} k_{2\mathrm{T}}}{(k_{2\mathrm{T}}^2)^{j_2}} \, \delta\left(q_{\mathrm{T}}^2 - \left(\vec{k}_{1\mathrm{T}} + \vec{k}_{2\mathrm{T}}\right)^2\right)$$

- Calling $j = \max(j_1, j_2)$ and $\delta = |j_1 - j_2|$ the master integral reads

$$I = \frac{2^{4j-2\delta-2}\, \pi^{1-2\epsilon}\, \Gamma\left(\frac{1}{2} - \epsilon\right)^2}{(q_{\mathrm{T}}^2)^{2j-1-\delta+2\epsilon}\, \Gamma(1-2\epsilon)}$$

$$\times \sum_{n=0}^{\delta} \frac{\Gamma(1+2\delta)}{\Gamma(1+2n)\,\Gamma(1+2\delta-2n)} \frac{\Gamma(1+\epsilon-n)\,\Gamma\left(\frac{1}{2}+\delta-n\right)}{\Gamma(1-\epsilon+\delta-n)\,\Gamma\left(\frac{3}{2}-n\right)}$$

$$\times {}_3F_2\left(j,\ \frac{1}{2}+\delta-n,\ 1+\epsilon-n;\ 1-\epsilon+\delta-n,\ \frac{3}{2}-n;\ 1\right)$$

Conclusion

Steps of the calculation

- [x] Compute the matrix elements
- [x] Expand the amplitudes in the relevant limits
- [x] Identify the master integrals
- [x] Analytically compute the master integrals
- [x] Check the cancellation of the α and ϵ poles
- [x] Reproduce the leading power result
- [ ] Read off the next-to-leading power corrections

Thanks for your attention!

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