Position-space renormalization of subleading-power soft-quark functions

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SCET 2024
Salamanca, Spain, April 15 – 18, 2024

Outline

- Introduction
- $\gamma\gamma \to H$ abelian
- $gg \to H$ non-abelian
- Anomalous dimension / RG kernel beyond one loop

MB, Yao Ji, Xing Wang, 2403.17738

Factorization at NLP

$$d\sigma = \sum_{a,b} C_a C_b^\star \otimes \prod_{i=1}^{N} J_a^{(i)} J_b^{(i)} \otimes S_{ab}$$

At LP soft functions are matrix elements built from semi-infinite Wilson lines.
At NLP, the soft operator contains soft fields on the light-cone — $q_s(x_-)$, $F_s^{\mu\nu}(x_-)$ — from subleading-power soft-collinear Lagrangian terms.

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Examples for the DY threshold

$$\text{LP} \quad \text{Tr}\,\bar{\mathbf{T}}([Y_{n_-}^\dagger Y_{n_+}](x^0))\,\mathbf{T}([Y_{n_+}^\dagger Y_{n_-}](0)) \quad \text{[Korchemsky, Marchesini, 1993]}$$

NLP soft gluon / quark

$$\text{Tr}\,\bar{\mathbf{T}}\left([Y_{n_-}^\dagger Y_{n_+}](x^0)\right)\mathbf{T}\left(([Y_{n_+}^\dagger Y_{n_-}](0)\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}_{\nu_\perp}^+(z_-)\right)$$

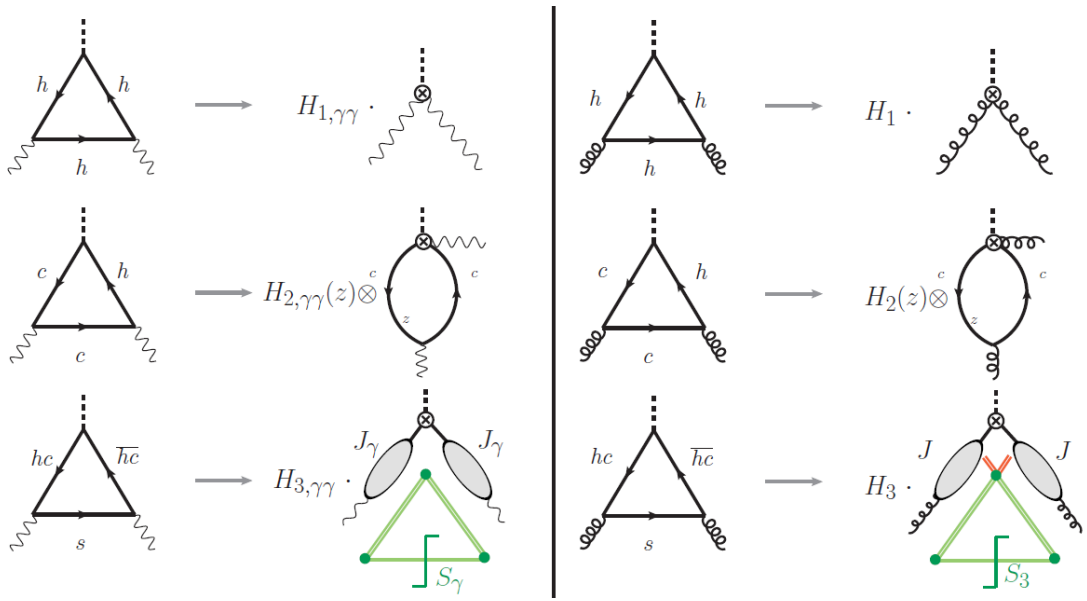
$$\text{Tr}\,\bar{\mathbf{T}}\left([\bar{q}_s Y_{n_-}](x^0+z_{2-})[Y_{n_-}^\dagger Y_{n_+}](x^0)\right)\mathbf{T}\left(([Y_{n_+}^\dagger Y_{n_-}](0)[Y_{n_-}^\dagger q_s](z_{1-})\right)$$

[MB, Broggio, Garny, Jaskiewicz, Szafron Wang, Vernazza, 1809.10631; MB, AB, SJ, LV, 1912.01585]

One-loop ($\mathcal{O}(\alpha_s^2)$) correction known [Broggio, Jaskiewicz, Vernazza, 2107.07353, 2306.06037], but not the RG kernels / anomalous dimensions.

Soft-quark functions

Begin with the (slightly) simpler case of soft-quark functions for Higgs production via light quark loops.



[Liu, Neubert, Schnubel, Wang, 2212.10447]

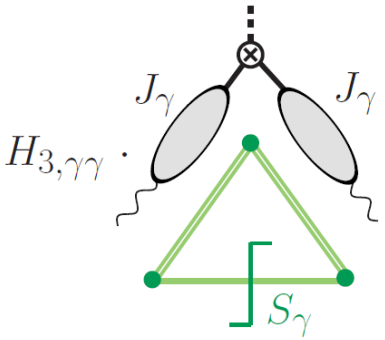
At NLP soft quark functions always arise from the double insertion of

$$\mathcal{L}^{(1)}_{\xi q}(x) = \bar{q}_s(x_-)[\not{A}_{c\perp}\chi_c](x)$$

Abelian $\gamma\gamma \to H$

$$
\begin{aligned}
O_\gamma(s,t) &= \mathbf{T}\left\{\bar{q}(tn_-)Y_{n_-}(t)Y_{n_-}^\dagger(0)\frac{\not{n}_-\not{n}_+}{4}Y_{n_+}(0)Y_{n_+}^\dagger(s)q(sn_+)\right\} \\
&= \mathbf{T}\left\{\bar{q}(tn_-)[tn_-,0]\frac{\not{n}_-\not{n}_+}{4}[0,sn_+]q(sn_+)\right\}
\end{aligned}
$$

$$
[z_1 n_\pm, z_2 n_\pm] \equiv \widehat{\mathrm{P}}\exp\left[ig_s T^a \int_{z_2}^{z_1} \mathrm{d}\lambda\, n_\pm \cdot A^a(\lambda n_\pm)\right]
$$



- Relevant is

$$
S_{\gamma,g}(w) = \frac{1}{2\pi i}\,\mathrm{Disc}_{\ell_-\ell_+}\int_{-\infty}^{\infty}\mathrm{d}s\int_{-\infty}^{\infty}\mathrm{d}t\,\mathrm{e}^{is\ell_+ - it\ell_-}\,\langle 0|O_{\gamma,g}(s,t)|0\rangle
$$

- Anomalous dimension / RG kernel originally [Liu, Mecaj, Neubert, X. Wang, Fleming, 2005.03013] inferred from RG consistency of the factorization formula.

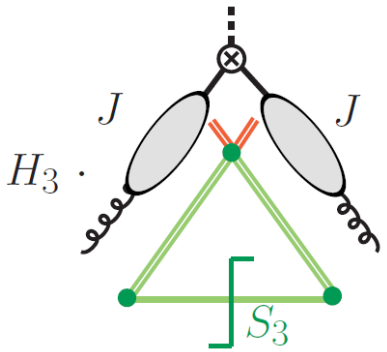
- Direct computation by [Bodwin, Ee, Lee, X.P.Wang, 2101.04972] by a complicated excursion into transverse-momentum dependent soft functions.

Non-Abelian $gg \to H$

$$O_g^{\text{uns}}(s,t) = \mathbf{T}\left\{\bar{q}(tn_-)[tn_-,0]\left(\mathcal{Y}_{n_-}(0)\right)^{ac} T^c \frac{\not{n}_-\not{n}_+}{4}\left(\mathcal{Y}_{n_+}(0)\right)^{bd} T^d [0,sn_+] q(sn_+)\right\}$$

$$= \mathbf{T}\left\{\bar{q}(tn_-)\left(\mathcal{Y}_{n_-}(tn_-)\right)^{ac} T^c [tn_-,0]\frac{\not{n}_-\not{n}_+}{4}[0,sn_+]\left(\mathcal{Y}_{n_+}(sn_+)\right)^{bd} T^d q(sn_+)\right\}$$

$$Y_{n_\pm}(x) T^a Y_{n_\pm}^\dagger(x) = \left(\mathcal{Y}_{n_\pm}(x)\right)^{ab} T^b$$



- Anomalous dimension / RG kernel known only from RG consistency of the factorization formula [Liu, Neubert, Schnubel, X. Wang, 2212.10447]

Computational method

Background-field method, R_ξ gauge [Abbott, 1980; Balitsky, Braun 1988/89]

$$q(x) = q_s(x,\mu) + q_q(x,\mu)\,, \qquad A^\mu(x) = A^\mu_s(x,\mu) + A^\mu_q(x,\mu)$$

$$\mathcal{O}_{\text{eff}}(q_s, A_s) = \int \mathcal{D}[q_q, \bar{q}_q, A_q]\, \mathcal{O}(q_s + q_q, A_s + A_q) \mathrm{e}^{iS[q_s+q_q, A_s+A_q] - iS[q_s, A_s]}$$

$$D^{ac}_\mu[A_s] A^{c,\mu}_q(x) = (\partial_\mu \delta^{ac} + g_s f^{abc} A^b_{s,\mu}) A^{c,\mu}_q = 0$$

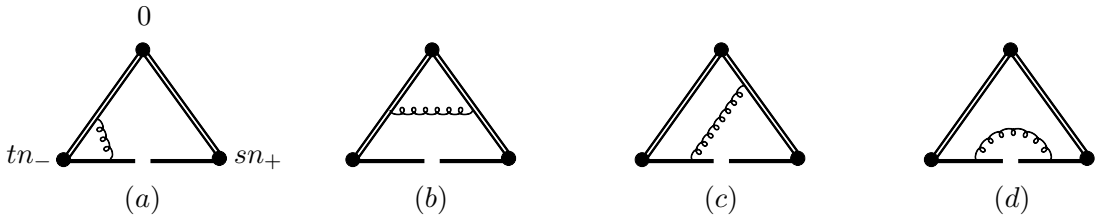
$$[tn_-, 0] = [tn_-, 0]_s + i g_s\, t \int_0^1 \mathrm{d}u\, [tn_-, utn_-]_s (n_- \cdot A^b_q)(utn_-) T^b [utn_-, 0]_s + \cdots$$

Adopt light-cone gauge $n_- \cdot A_s(x) = 0$ for the soft background fields $\rightarrow [*,*]_s = 1$

Position-space

RG kernel is computed first for the operator $O(s,t)$ in position-space.

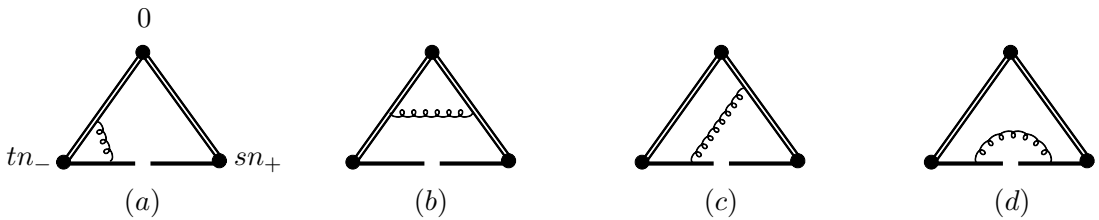
Abelian $\gamma\gamma \to H$



$$
\begin{aligned}
I_a &= (ig_s)^2 \int \mathrm{d}^D z\, \bar{q}(z) \not{A}^a(z) T^a q(z)\, \bar{q}(tn_-) \int_0^1 \mathrm{d}u\, t\, (n_- \cdot A^b)(utn_-) T^b \frac{\not{n}_- \not{n}_+}{4} q(sn_+) \\
&= -g_s^2 C_F \mu^{2\varepsilon} \frac{\mathrm{e}^{\varepsilon\gamma_E}}{(4\pi)^\varepsilon} \int_0^1 \mathrm{d}u\, t \int \frac{\mathrm{d}^D p}{(2\pi)^D} \int \frac{\mathrm{d}^D l}{(2\pi)^D} \mathrm{e}^{it(\bar{u}n_- \cdot l - un_- \cdot p)} \frac{\bar{q}(p) \not{n}_- \not{l}}{l^2 (p+l)^2} \frac{\not{n}_- \not{n}_+}{4} q(sn_+) \\
&= \frac{\alpha_s}{4\pi} \frac{2C_F}{\varepsilon} \int_0^1 \mathrm{d}u\, \frac{u}{\bar{u}} \left[\bar{q}(utn_-) - \bar{q}(tn_-) \right] \frac{\not{n}_- \not{n}_+}{4} q(sn_+) + \mathcal{O}(\varepsilon^0) \\
&= \frac{\alpha_s}{4\pi} \frac{2C_F}{\varepsilon} \int_0^1 \mathrm{d}u \left[\frac{u}{\bar{u}} \right]_+ O_\gamma(s, ut) + \mathcal{O}(\varepsilon^0)
\end{aligned}
$$

- For the computation of the pole part, can set the mass of the soft quark to zero. IR divergences regularized by $p^2 \neq 0$.

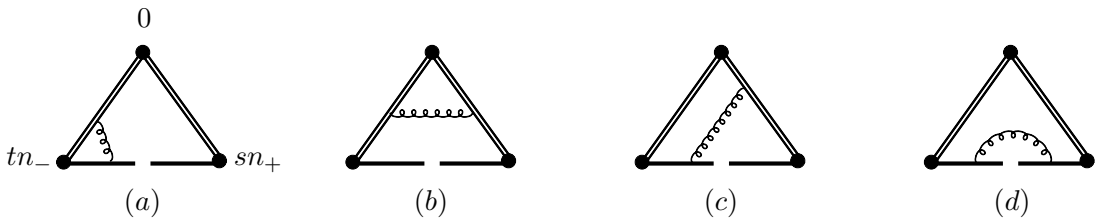
Abelian $\gamma\gamma \to H$



$$O_\gamma(s,t;\mu) = O_\gamma^{\text{bare}}(s,t) + \frac{\alpha_s C_F}{4\pi}\int_0^1 \mathrm{d}u \Bigg[\Bigg(\underbrace{\frac{1}{\varepsilon^2} + \frac{2\ln\left(st\mu^2 \mathrm{e}^{2\gamma_E}\right) - (1-\xi)}{2\varepsilon}}_{(b)} + \underbrace{\frac{(1-\xi)}{\varepsilon}}_{(c)} + \underbrace{\frac{\xi}{2\varepsilon}}_{\text{field ren.}} \Bigg) \delta(1-u)$$

$$\underbrace{-\frac{2}{\varepsilon}\left[\frac{u}{\bar{u}}\right]_+}_{(a)} \Bigg] \left(O_\gamma^{\text{bare}}(us,t) + O_\gamma^{\text{bare}}(s,ut)\right)$$

Abelian $\gamma\gamma \to H$



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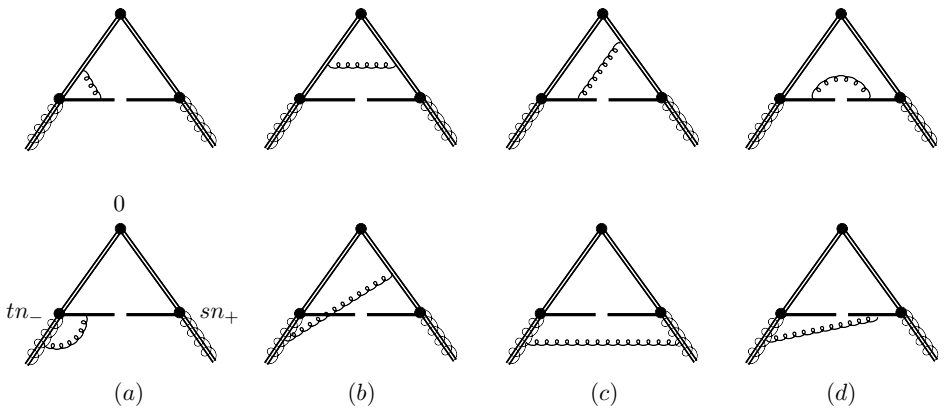
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} O_\gamma(s,t,\mu) = -[\gamma_\gamma O_\gamma](s,t;\mu)$$
$$= \frac{\alpha_s C_F}{\pi}\left\{ -\left(\ln\left(st\mu^2\mathrm{e}^{2\gamma_E}\right) + \frac{1}{2}\right) O_\gamma(s,t;\mu) + \int_0^1 \mathrm{d}u \left[\frac{u}{\bar{u}}\right]_+ \left(O_\gamma(us,t;\mu) + O_\gamma(s,ut;\mu)\right)\right\}$$

- Confirms previous results and gauge-independence of the RG kernel.

Non-Abelian $gg \to H$



$$O_g^{\text{uns}}(s,t) = \mathbf{T}\left\{\bar{q}(tn_-)\left(\mathcal{Y}_{n_-}(tn_-)\right)^{ac} T^c [tn_-, 0] \frac{\not{n}_- \not{n}_+}{4} [0, sn_+] \left(\mathcal{Y}_{n_+}(sn_+)\right)^{bd} T^d q(sn_+)\right\}$$

- First-row diagrams by substitution of colour factors. The sum is ξ-dependent ($\propto C_A$), arising from the 3rd diagram in the 1st row.

- The new 2nd row diagrams containing semi-infinite adjoint Wilson lines contain rapidity divergences. We regulate them by

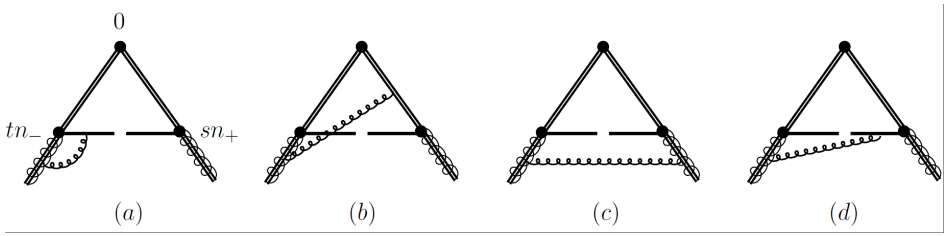
$$\left(\mathcal{Y}_{n_\pm}(x)\right)^{ab} = \widehat{\mathrm{P}} \exp\left[-g_s f^{abc} \int_{-\infty}^{0} \mathrm{d}\lambda\, \mathrm{e}^{-i\lambda(\delta_\pm + i0^+)}\, n_\pm \cdot A^c(x + \lambda n_\pm)\right]$$

- The δ-regulator modifies Wilson lines as

$$\frac{in_+ \cdot p_c}{(p_c+\ell)^2+i0^+} \to \frac{i}{n_- \cdot \ell + \frac{p_c^2}{n_+ \cdot p_c} + i0^+} \equiv \frac{i}{n_- \cdot \ell + \delta_- + i0^+}$$
$$\frac{in_- \cdot p_{\bar{c}}}{(p_{\bar{c}}+\ell)^2+i0^+} \to \frac{i}{n_+ \cdot \ell + \frac{p_{\bar{c}}^2}{n_- \cdot p_{\bar{c}}} + i0^+} \equiv \frac{i}{n_+ \cdot \ell + \delta_+ + i0^+} \,.$$

and is induced by an off-shell regulator in the full theory.

$$\delta_+ = \frac{p_c^2}{n_- \cdot p_c} + i0^+, \qquad \delta_- = \frac{p_{\bar{c}}^2}{n_+ \cdot p_{\bar{c}}} + i0^+$$



- Only diagrams (a), (b) contribute to the anomalous dimension and they are ξ-independent.

Non-Abelian $gg \to H$

Sum of all contributions is rapidity-divergent and gauge-parameter dependent

$$
\begin{aligned}
O_g^{\text{uns},1-\text{loop}}(s,t) \;=\; & O_g^{\text{uns,tree}}(s,t) + \frac{\alpha_s}{4\pi}\left[\frac{C_A}{\varepsilon}\left(2 + \ln\left(st\mu^2 \mathrm{e}^{2\gamma_E}\right) + 2\ln\frac{-\delta_- \delta_+}{\mu^2} - \ln\frac{\partial_s \partial_t}{\mu^2}\right)\right. \\
& \left. -2C_F\left(\frac{1}{\varepsilon^2} + \frac{\ln\left(st\mu^2\mathrm{e}^{2\gamma_E}\right)}{\varepsilon}\right) + \frac{C_A}{\varepsilon}(1-\xi) - \frac{C_F}{\varepsilon}\right] O_g^{\text{uns}}(s,t) \\
& + \frac{\alpha_s}{4\pi}\frac{2}{\varepsilon}\left(C_F - \frac{C_A}{2}\right)\int_0^1 \mathrm{d}u \left[\frac{u}{\bar{u}}\right]_+ \left(O_g^{\text{uns}}(us,t) + O_g^{\text{uns}}(s,ut)\right)
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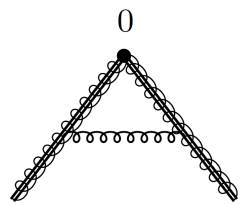
& + \frac{\alpha_s}{4\pi}\frac{2}{\varepsilon}\left(C_F - \frac{C_A}{2}\right)\int_0^1 \mathrm{d}u \left[\frac{u}{\bar{u}}\right]_+ \left(O_g^{\text{uns}}(us,t) + O_g^{\text{uns}}(s,ut)\right)
\end{aligned}
$$

IR rearrangement: Subtract (Divide out) Wilson lines for the "charges from / to infinity", similar to QED case for $B_s \to \ell^+\ell^-$ [MB, Bobeth, Szafron, 1908.0711; MB, Böer, Toelstede, Vos, 2008.10615]

$$
\mathbb{S}_g(0) = \left(\mathcal{Y}_{n_-}(0)\right)^{ac}\left(\mathcal{Y}_{n_+}(0)\right)^{cb} = \left[-\infty n_-, 0n_-\right]^{ac}\left[0n_+, -\infty n_+\right]^{cb}
$$

$$
\langle \mathbb{S}_g(0)\rangle = 1 + \frac{\alpha_s(\mu)}{4\pi} C_A \left[-\frac{2}{\varepsilon^2} + \frac{2\ln(-\delta_-\delta_+/\mu^2) + (1-\xi)}{\varepsilon}\right]
$$



Non-Abelian $gg \to H$

$$O_g(s,t) \equiv \frac{O_g^{\text{uns}}(s,t)}{\langle \mathbb{S}_g(0) \rangle}$$

The subtractions cancels both, the dependnce on the rapidity regulator and gauge-dependence

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$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} O_g(s,t,\mu) = -[\gamma_\gamma O_g](s,t;\mu)$$

$$= \frac{\alpha_s}{2\pi}\left\{ -\left[C_A \ln\left(\frac{\partial_s\partial_t}{\mu^2}\right) + 2\left(C_F - \frac{C_A}{2}\right)\ln\left(st\mu^2 \mathrm{e}^{2\gamma_E}\right) + (C_F - 2C_A)\right] O_g(s,t) \right.$$

$$\left. +2\left(C_F - \frac{C_A}{2}\right)\int_0^1 \mathrm{d}u \left[\frac{u}{\bar{u}}\right]_+ \Big(O_g(us,t) + O_g(s,ut)\Big)\right\}$$

Momentum-space kernel agrees with the one from RGE consistency (which, however, was for the unsubtracted operator)

$$\gamma_g(w,w') = \frac{\alpha_s}{\pi}\left\{\left((C_A - C_F)\ln\frac{w}{\mu^2} - \frac{3}{2}C_F\right)\delta(w-w') + (C_A - 2C_F) w\,\Gamma(w,w')\right\}$$

$$\Gamma(w,w') = \left[\frac{\theta(w-w')}{w(w-w')} + \frac{\theta(w'-w)}{w'(w'-w)}\right]_+$$

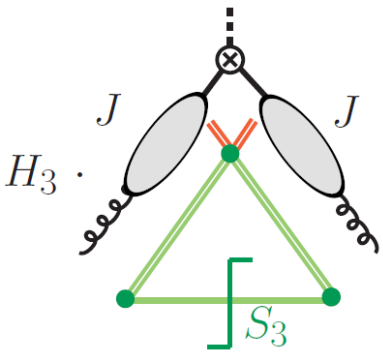
$gg \to H$ factorization formula revisited

$$\mathcal{M}_{gg\to h} \supseteq H_3 \cdot \left[J_g(M_h\ell_-)J_g(-M_h\ell_+)\right] \otimes \underbrace{S_g^{\text{uns}}(\ell_-\ell_+)\,\mathfrak{A}_g^{+,(\xi)}(k_1^2)\,\mathfrak{A}_g^{-,(\xi)}(k_2^2)}_{\text{do not have separately well-defined anomalous dimensions}}$$

Factors $\mathfrak{A}_g^{\pm,(\xi)}(k_i^2)$ from $\langle 0|\mathcal{A}^{\mu(0)}_{\perp,n_\pm}(0)|g(k,a)\rangle \equiv g_s T^a \varepsilon_\perp^\mu(k)\,\mathfrak{A}_g^{\pm,(\xi)}(k^2)$ are omitted in [Liu et al., 2212.10447], because they equal 1 to all orders on-shell.

Off-shell (with $k_1^2 = M_h\delta_-$, $k_2^2 = M_h\delta_-$)

$$\mathfrak{A}_g^{\pm,(\xi)}(k^2) = 1 + \frac{\alpha_s(\mu)C_A}{4\pi}\left(\frac{2}{\varepsilon^2} - \frac{2}{\varepsilon}\ln\frac{-k^2}{\mu^2} - \frac{1-\xi}{2\varepsilon}\right)$$



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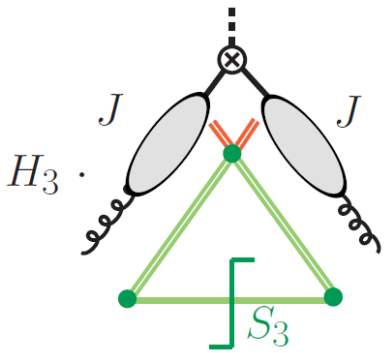

Introduce the subtracted soft function ...

$$S_g(\ell_+\ell_-) = \frac{S_g^{\text{uns},(\xi)}(\ell_-\ell_+)}{R_+^{(\xi)}R_-^{(\xi)}}\,, \qquad \langle \mathbb{S}_g(0)\rangle \equiv R_+^{(\xi)}\,R_-^{(\xi)}$$

$$R_\pm^{(\xi)} = 1 + \frac{\alpha_s(\mu)}{4\pi}C_A\left[-\frac{1}{\varepsilon^2} + \frac{2}{\varepsilon}\ln\frac{\pm\delta_\pm}{\mu} + \frac{1-\xi}{2\varepsilon}\right]$$

$gg \to H$ factorization formula revisited

... and rearrange

$$\mathcal{M}_{gg\to h} \supseteq H_3 \cdot \left[J_g(M_h\ell_-)J_g(-M_h\ell_+)\right] \otimes \underbrace{S_g(\ell_-\ell_+)(R_+^{(\xi)}\,\mathfrak{A}_g^{+,(\xi)}(k_1^2))\,(R_-^{(\xi)}\,\mathfrak{A}_g^{-,(\xi)}(k_2^2))}_{\text{Every factor can be separately renormalized}}$$

$$R_\pm^{(\xi)}\,\mathfrak{A}_g^{\pm,(\xi)}(k_i^2) = 1 + \frac{\alpha_s(\mu)}{4\pi} C_A \left[\frac{1}{\varepsilon^2} - \frac{2}{\varepsilon}\ln\frac{\mp M_h}{\mu}\right],$$

The hard-collinear function J_g is gauge-independent and independent of the off-shell regulator (explicitly checked).

$gg \to H$ factorization formula revisited

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$$R_\pm^{(\xi)}\,\mathfrak{A}_g^{\pm,(\xi)}(k_i^2) = 1 + \frac{\alpha_s(\mu)}{4\pi} C_A \left[\frac{1}{\varepsilon^2} - \frac{2}{\varepsilon}\ln\frac{\mp M_h}{\mu}\right],$$

The hard-collinear function J_g is gauge-independent and independent of the off-shell regulator (explicitly checked).

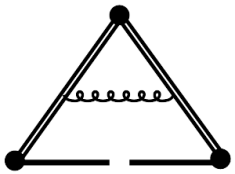
Interpretation of the subtraction: factors the anomalous dimension of the non-decoupled collinear fields, for which off-shellness cancels only in the sum of soft and collinear contributions:

$$(R_+^{(\xi)}\,\mathfrak{A}_g^{+,(\xi)}(k_1^2))\,(R_-^{(\xi)}\,\mathfrak{A}_g^{-,(\xi)}(k_2^2)) = 1 + \frac{\alpha_s(\mu)}{4\pi} C_A \left[\frac{2}{\varepsilon^2} - \frac{2L_h}{\varepsilon}\right] = Z_{gg}^{-1}\,.$$

where Z_{gg}^{-1} is the renormalization factor of $\mathcal{A}^{\mu,a}_{\perp,n_-}\,\mathcal{A}^{a}_{\perp\mu,n_+} = \mathcal{Y}^{ab}_{n_+}\,\mathcal{Y}^{ac}_{n_-}\,\mathcal{A}^{(0)\mu,b}_{\perp,n_-}\,\mathcal{A}^{(0)c}_{\perp\mu,n_+}$ with non-decoupled (anti-) collinear SCET gluon fields.

Beyond one loop

$$O_\gamma(s,t) \;=\; \mathbf{T}\Big\{ \underbrace{\bar q(tn_-)[tn_-,0]\frac{\not n_-}{2}}_{n_-\text{ light-ray}} \underbrace{\frac{\not n_+}{2}[0,sn_+]q(sn_+)}_{n_+\text{ light-ray}} \Big\}$$



$$\downarrow$$

$$\mathbf{T}\Big\{\bar q(tn_-)[tn_-,0]\frac{\not n_-}{2} h_v(0)\Big\} \qquad \text{twist-2 B-LCDA operator}$$

- The one-loop anomalous dimension factorizes

$$\gamma_i(s,t) \;=\; \gamma_i(s,\mu)+\gamma_i(t,\mu)\,, \qquad i=\gamma,g$$

- The anomalous dimension of the twist-2 heavy-light light-cone operator is determined by conformal symmetry up to a constant [Braun, Ji, Manashov, 1905.04998]

- $SL(2,\mathbb{R})\otimes SL(2,\mathbb{R})$ collinear conformal group

$$\widehat S_+ = s^2\partial_s + 2js = s\theta_s + 2js, \quad \widehat S_0 = s\partial_s + j = \theta_s + j, \quad \widehat S_- = -\partial_s$$

$\gamma\gamma \to H$ kernel as a "double copy" of the leading-twist B-LCDA

$$\begin{aligned}\gamma_\gamma(s,t) &= \frac{\alpha_s(\mu)C_F}{\pi}\left[\ln\left(\mu^2\widehat{S}_+\widehat{T}_+\right)+4\gamma_E-\frac{3}{2}\right]\\ &\overset{\text{all orders}}{\to} \Gamma^{\text{F}}_{\text{cusp}}(\alpha_s)\ln\left(\mathcal{K}(\alpha_s;s)\mu e^{2\gamma_E}\right)+\Gamma_\gamma(\alpha_s)+(s\leftrightarrow t)\end{aligned}$$

where

$$K^\mu(\alpha_s^*;s)=\frac{n_+^\mu}{2}\left(\widehat{S}_+ + \widehat{\Delta K}(\alpha_s^*;s)\right)$$

is the same as for the B-LCDA. The different constant can be determined from the local limit $s,t\to 0$.

All-order relation

$$\gamma_\gamma(s,t,\mu)=\mathcal{H}_B(s,\mu)+\mathcal{H}_B(t,\mu)-2\gamma_Q$$

with the B-LCDA kernel $\mathcal{H}_B(s,\mu)$, where γ_Q is the heavy-quark anomalous dimension [Becher, Neubert, 0904.1021]

$$\gamma_Q=\frac{\alpha_s}{4\pi}(-2C_F)+\left(\frac{\alpha_s}{4\pi}\right)^2 C_F\left[C_A\left(\frac{2\pi^2}{3}-\frac{98}{9}-4\zeta_3\right)+n_f T_F\frac{40}{9}\right]+\mathcal{O}(\alpha_s^3)$$

Non-abelian generalization

$$\gamma_g(s,t) = \frac{\alpha_s(\mu)}{4\pi}\left[4\left(C_F-\frac{C_A}{2}\right)\left(\ln\left(\mu^2\widehat{S}_+\widehat{T}_+\right)+4\gamma_E-2\right)+2C_A\left(\ln\frac{\widehat{S}_-\widehat{T}_-}{\mu^2}-2\right)+2C_F\right]$$

$$\overset{\text{ansatz}}{\to}\left(\Gamma^{\mathrm{F}}_{\mathrm{cusp}}(\alpha_s)-\frac{1}{2}\Gamma^{\mathrm{A}}_{\mathrm{cusp}}(\alpha_s)\right)\ln\left(i\mathcal{K}(\alpha_s;s)\mu e^{2\gamma_E}\right)+\frac{1}{2}\Gamma^{\mathrm{A}}_{\mathrm{cusp}}(\alpha_s)\ln\left(\frac{\widehat{S}_-}{\mu}\right)+\Gamma_g(\alpha_s)+(s\leftrightarrow t)$$

where

$$2\Gamma_g = \underbrace{2\Gamma_\gamma}_{\text{abelian}} - \underbrace{\Gamma_\wedge}_{\text{subtraction operator}} + \underbrace{\Delta}_{\text{mismatch}}$$

From consistency of the factorization theorem find

$$\Delta = \left(\frac{\alpha_s}{4\pi}\right)^2 8C_A(C_A+4C_F)\,\zeta_3+\mathcal{O}(\alpha_s^3)$$

???

Summary

I Direct computation of the 1-loop RG kernel for the $gg \to H$ soft-quark function

II NLP soft functions with "colour charges at infinity" require a soft/rapidity subtraction / rearrangement

III RG kernel of the $\gamma\gamma \to H$ / $H \to \gamma\gamma$ soft-quark function is a "double copy" of the kernel of the leading-twist heavy-light (B-LCDA) light-cone operator.

IV First application of conformal symmetry methods to light-cone operators with semi-infinite Wilson lines

V Similar method can be used for amplitude-squared NLP soft functions that appear in threshold production [work in progress]