Anisotropic jet substructure observables for flowing matter

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Ongoing work in collaboration with
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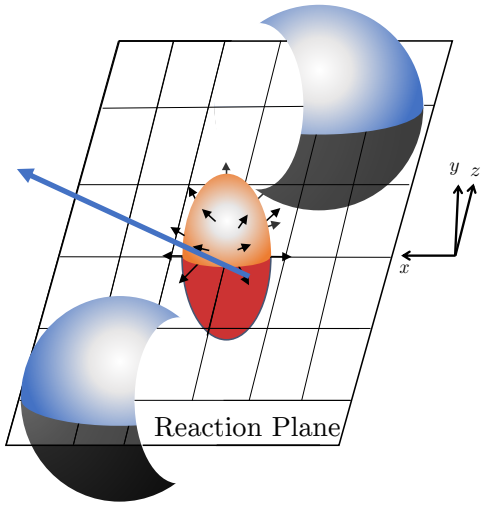
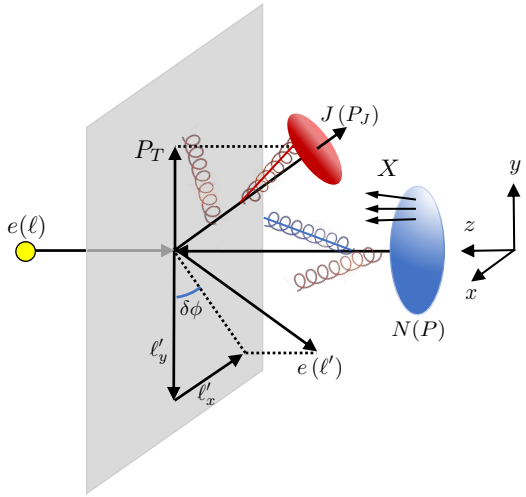
Background on nuclear matter

Example processes with hot and cold nuclear matter (RHIC, LHC, JLab, EIC)

Single-inclusive jet production in A-A



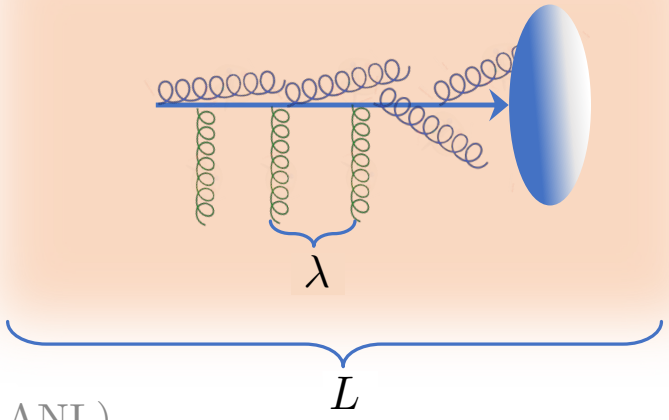
Lepton-jet correlations in e-A



Many more

- *Dijet decorrelations*
- *Boson tagged jets*
- *Back-to-back EECs*
- *Single inclusive jet production*

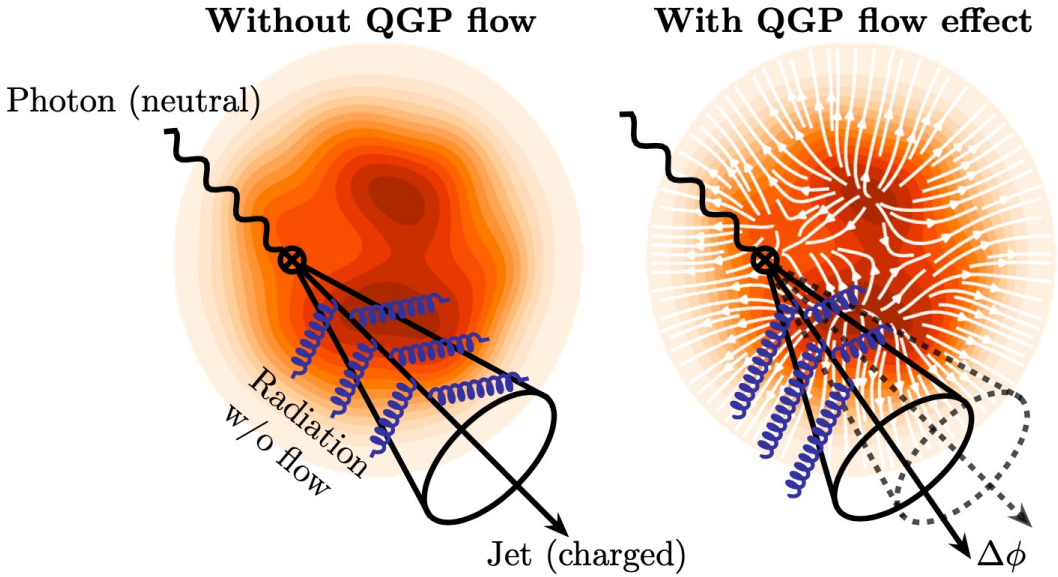
How do we use QCD and EFT to probe the properties of hot and cold nuclear matter?



The size of the corrections is directly proportional to the opacity parameter $\xi = L/\lambda$. QCD and SCET calculations and experimental data allow us to address the question as to the scattering length (and other) properties of the medium.

Why anisotropic jet substructure?

Medium-induced emissions are frequent and wide-angled. Emissions are enhanced along medium velocity



$$E\frac{dN^{(1)}}{d^2k_\perp\,dx\,d^2p_\perp\,dE} = \frac{\alpha_s\,N_c}{\pi^2 x}\left(E\frac{dN^{(0)}}{d^2p_\perp\,dE}\right)\int_0^L dz\,\rho\int d^2q_\perp\bar{\sigma}(q_\perp^2)$$
$$\times\left\{\frac{2\boldsymbol{k}_\perp\cdot\boldsymbol{q}_\perp}{k_\perp^2(k-q)_\perp^2}\left(1-\cos\left(\frac{(k-q)_\perp^2}{2xE(1-u_z)}z\right)\right)+\frac{q_\perp^2}{k_\perp^2(q_\perp^2+\mu^2)}\frac{\boldsymbol{u}_\perp\cdot\boldsymbol{k}_\perp}{2(1-u_z)xE}\right\}$$

q^μ *Medium gluon*

v^μ *Medium velocity*

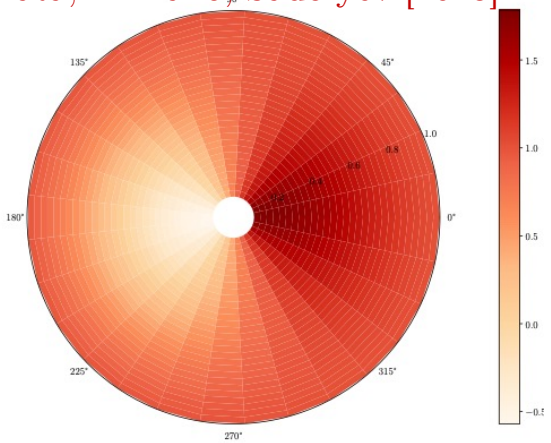
k^μ *Stimulated emission*

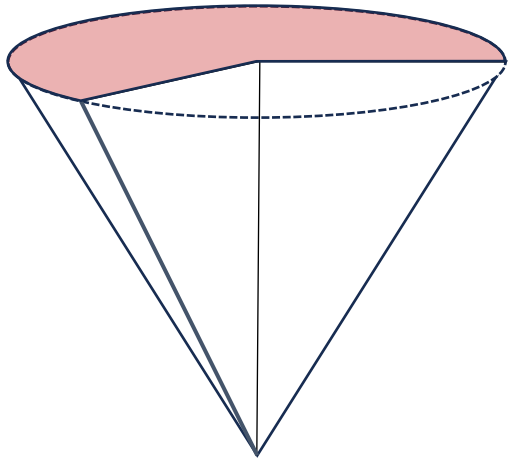
Computation performed at NLO in QCD and NLO in opacity

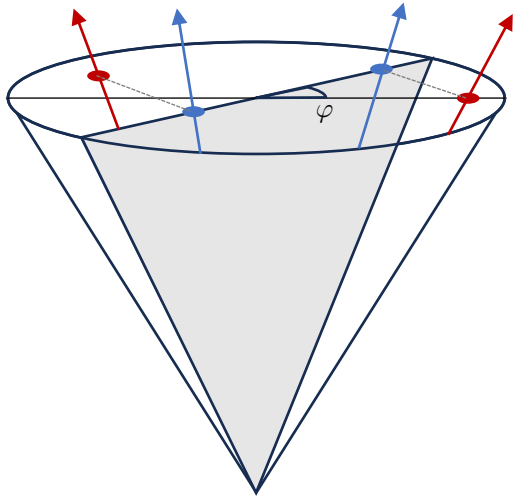
Sadofyev, Sievert, Vitev [2021]

Traditional jet substructure observables (shape and angularities) can tell us whether the jet is altered but they cannot tell us if it is altered more in one direction due to azimuthal integration.

Barata, Milhano, Sadofyev [2023]





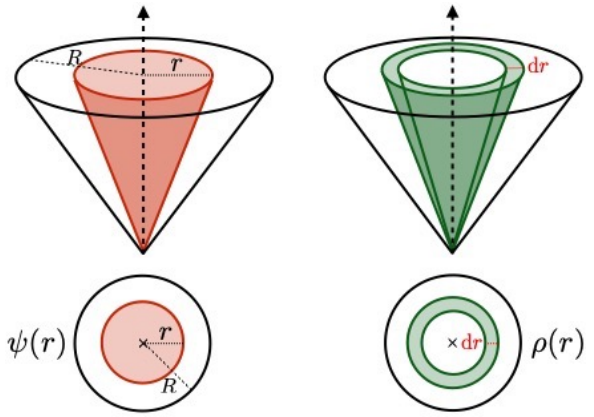


Jet shape 101

The integrated and differential jet shape can be defined by

$$\psi(r,R) = \sum_{i\in j} \bar{n}_J \cdot p_i \Big/ \sum_{i\in J} \bar{n}_J \cdot p_i \,,$$

$$\rho(r,R) = \frac{\partial}{\partial r}\psi(r,R)$$



Ellis, Kunszt, and Soper [1992]
Seymour [1997]
Li, Li, and Yuan [2011]
Chien and Vitev [2014]
Kang, Ringer and Waalewijn [2017]
Cal, Ringer, Waalewijn [2019]

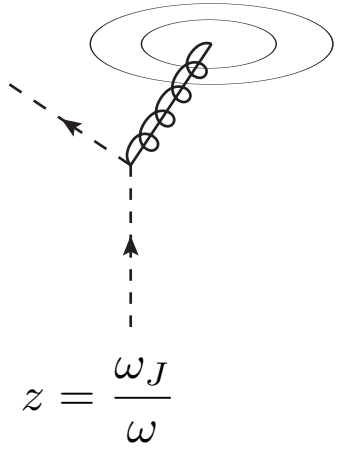
The integrated and differential jet shape can be defined in terms of the partonic cross sections as

$$\psi(r,R) = \int dz_r\, z_r \frac{d\sigma_c}{dz\, d\mathcal{PS}} \otimes \boxed{j_c\left(z, z_r, r, R\right)} \Big/ \frac{d\sigma_c}{dz\, d\mathcal{PS}} \otimes \boxed{J_c\left(z, R\right)}$$

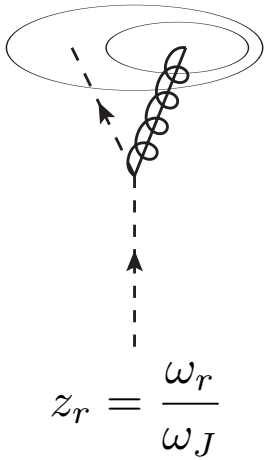
Kang, Ringer and Waalewijn [2017]

Semi-inclusive sub-jet function

Semi-inclusive jet function



$$z = \frac{\omega_J}{\omega}$$



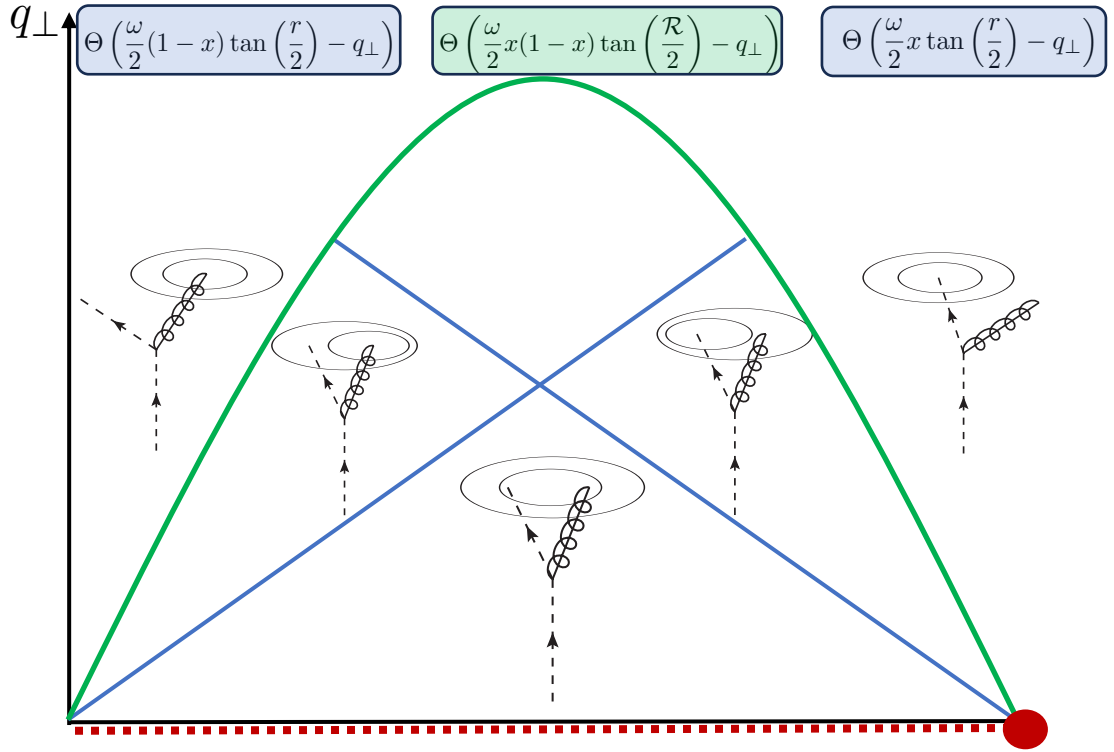
$$z_r = \frac{\omega_r}{\omega_J}$$

Jet shape 102

In the region where $r \sim R$*, there is only a single momentum scaling for the jet mode*

$$p_c^\mu \sim \omega_J \left(r^2, 1, r\right)$$

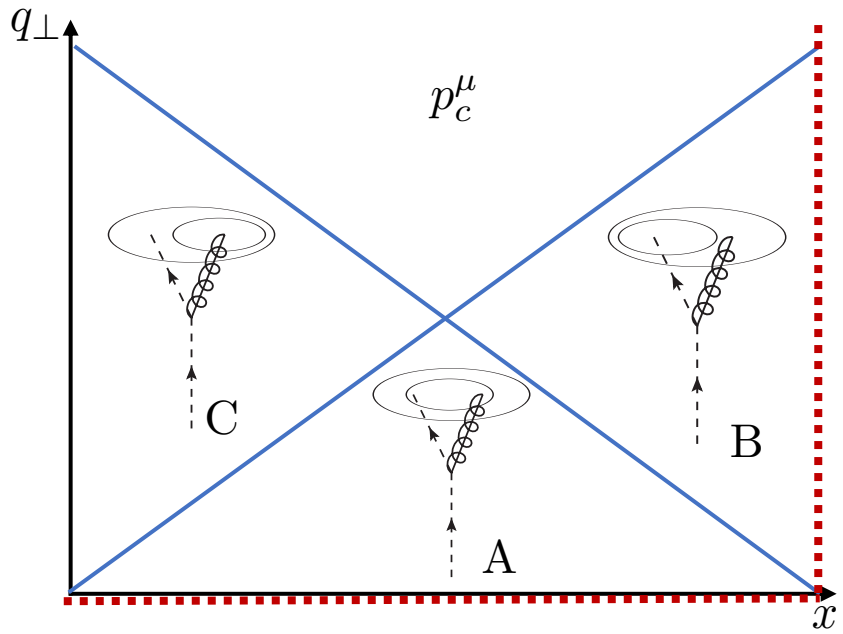
Chien and Vitev [2014]
Kang, Ringer and Waalewijn [2017]



Double pole at $q_\perp = 0\,, x = 1$*, regulated by the jet algorithms*

Power counting and refactorization

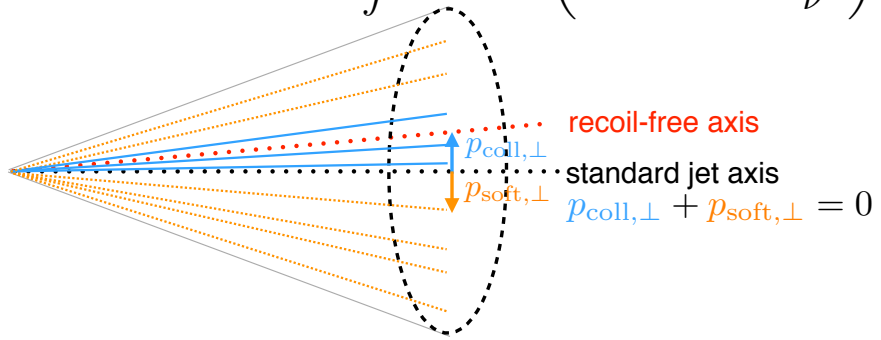
In the region where $r \ll R$, the jet algorithm no longer regulates the rapidity divergences (SCET II). There are three modes that contribute to the observable



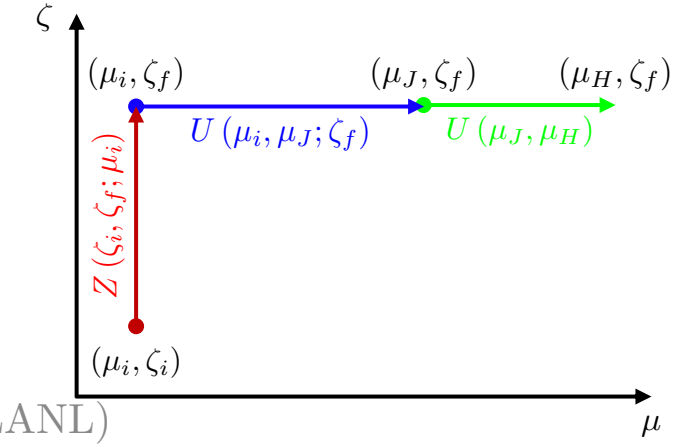
$$p_J^\mu \sim \omega_J \left(R^2, 1, R\right) \qquad p_c^\mu \sim \omega_J \left(r^2, 1, r\right) \qquad p_s^\mu \sim \omega_J \frac{r}{R} \left(R^2, 1, R\right)$$

$$\mu_J \sim \omega_J R \qquad \mu_c \sim \mu_s \sim \omega_J r \qquad \nu_s \sim \omega_J \frac{r}{R} \qquad \nu_c \sim \omega_J$$

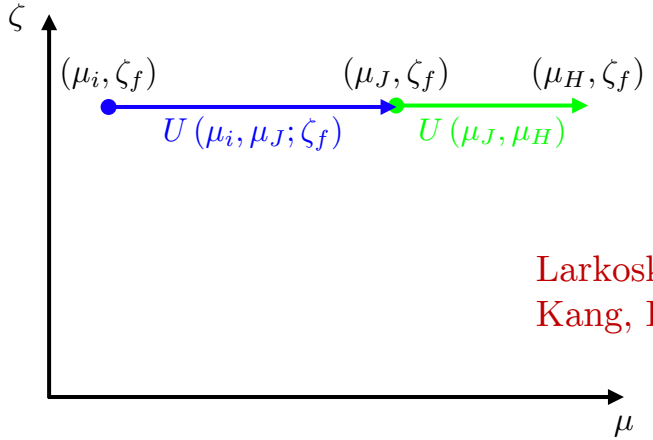
$$j_c\left(z, z_r, \omega_J, r, R, \mu\right) = H_{cd}\left(z, \omega_J, \mu\right) \int d^2 k_\perp C_d \left(z_r, k_\perp, \omega_j, \mu, \frac{\zeta}{\nu^2}\right) S_d\left(k_\perp, R, \omega_j, \mu, \nu\right)$$



Evolution occurs in three parts



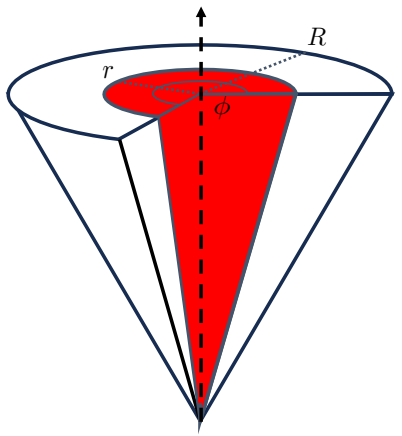
Recoil issue can be removed using a WTA axis



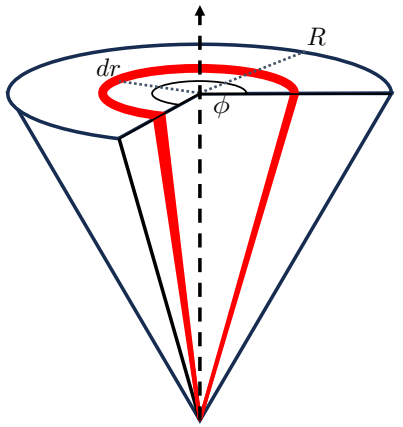
Larkoski, Neill, Thaler [2014]
Kang, Ringer and Waalewijn [2017]

The azimuthal dependent jet shape

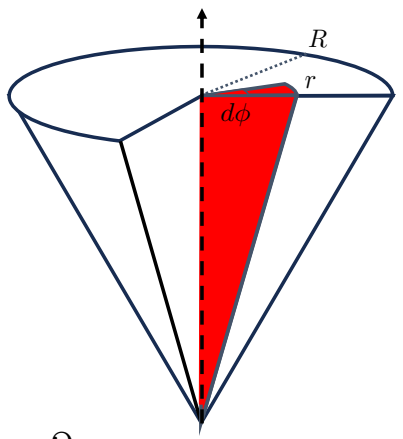
We generalize the jet shape to also contain azimuthal angle dependence



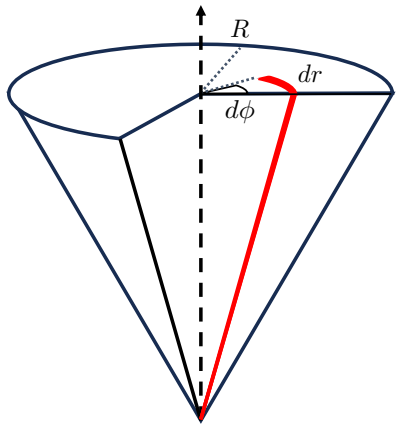
$$\psi(r, R, \varphi_i, \varphi_f)$$



$$\frac{\partial}{\partial r}\psi(r, R, \varphi_i, \varphi_f)\,,$$



$$\frac{\partial}{\partial \varphi_f}\psi(r, R, \varphi_i, \varphi_f)\,,$$



$$\frac{\partial}{\partial r}\frac{\partial}{\partial \varphi_f}\psi(r, R, \varphi_i, \varphi_f)$$

Factorization is unchanged but the ingredients are modified

$$j_c\left(z, z_r, \varphi_i, \varphi_f, \omega_J, r, R, \mu\right) = H_{cd}\left(z, \omega_J, \mu\right) \int d^2k_\perp C_d\left(z_r, \mathbf{k}_\perp, \varphi_i, \varphi_f, \omega_j, \mu, \frac{\zeta}{\nu^2}\right) S_d\left(\mathbf{k}_\perp, R, \mu, \nu\right)$$

Same as integrated at NLO

Tree level modified trivially

Non-trivial NLO modification

Same as integrated at NLO

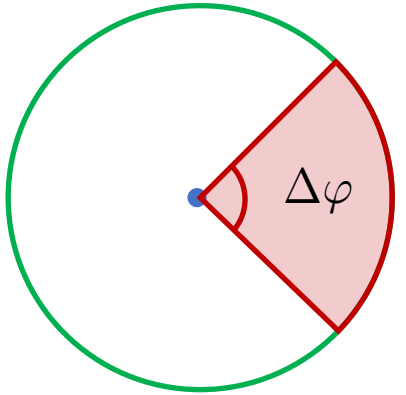
Anomalous dimension of jet shape unchanged

Collinear computation (Graphs B and C)

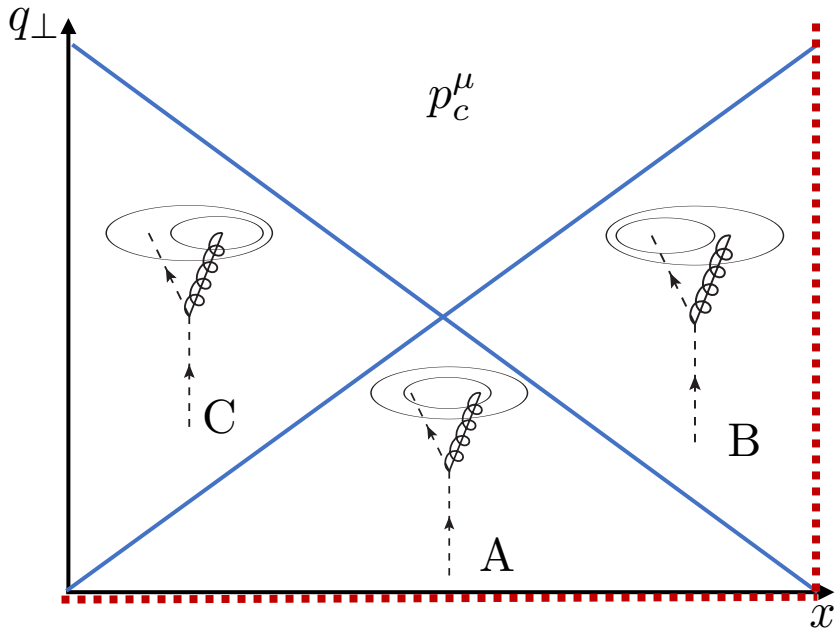
At tree level, we normalize the sub-jet function contains some azimuthal angle dependence

$$C_i^{(0)}\left(z_r, \mathbf{0}_\perp, \varphi_i, \varphi_f, \omega_j, R, \mu, \frac{\zeta}{\nu^2}\right) = \frac{\Delta\varphi}{2\pi}\delta\left(1-z_r\right)$$

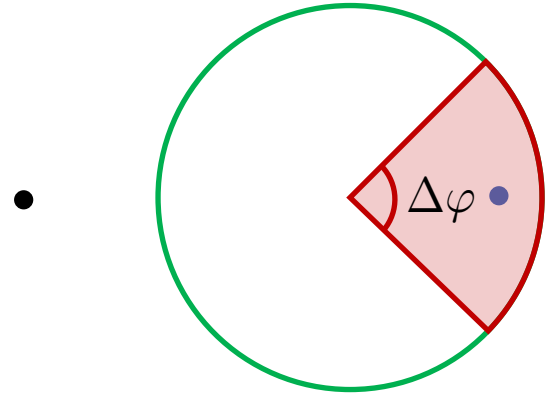
$$j_c^{(0)}\left(z, z_r, \varphi_i, \varphi_f, \omega_J, r, R, \mu\right) = \frac{\Delta\varphi}{2\pi}\delta\left(1-z\right)\delta\left(1-z_r\right)$$



At one loop, we need to consider the collinear mode



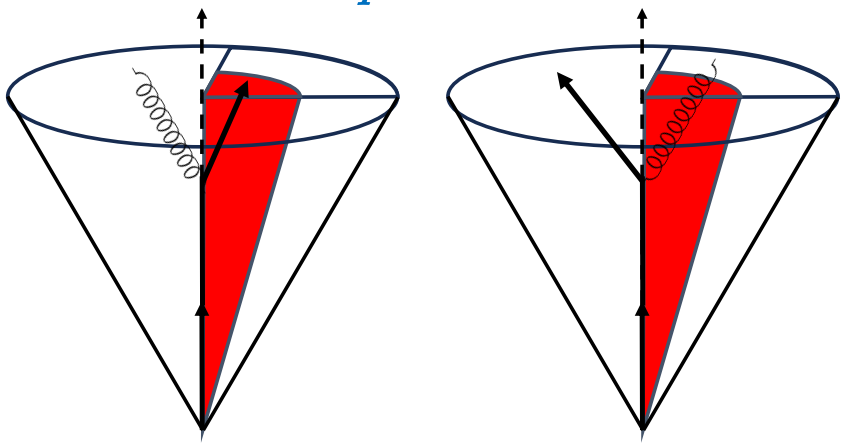
Graphs B and C are trivial without recoil

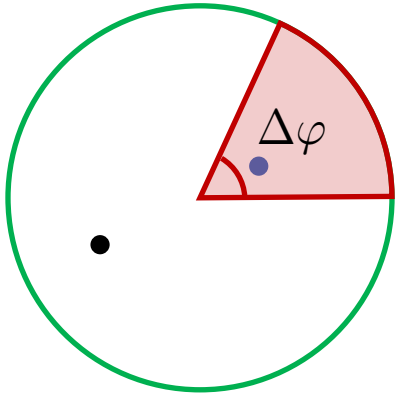


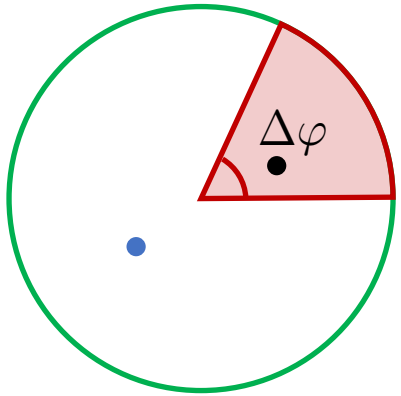
$$C_{i\,\mathrm{B/C}}^{(1)}\left(z_r, \mathbf{0}_\perp, \varphi_i, \varphi_f, \omega_j, R, \mu, \frac{\zeta}{\nu^2}\right) = \frac{\Delta\varphi}{2\pi}C_{i\,\mathrm{B/C}}^{(1)}\left(z_r, \mathbf{0}_\perp, \omega_j, R, \mu, \frac{\zeta}{\nu^2}\right)$$

Collinear computation (Graphs A)

Graph A is more complicated but can be computed $\varphi < \pi$







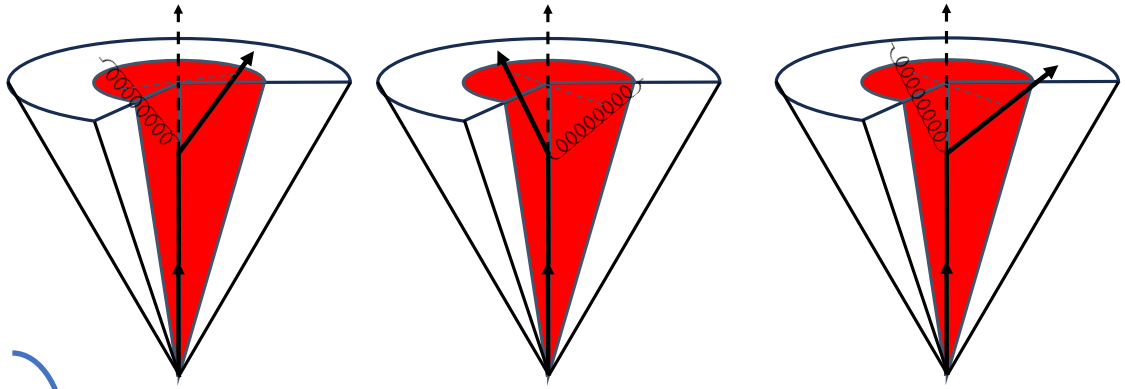
Computation is new

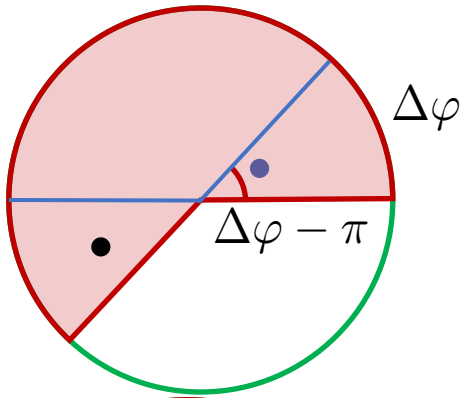
$$C_q^{(A_{\rm I})}(\ldots,\varphi_i,\varphi_f) = \frac{\alpha_s C_F \Delta\varphi}{4\pi^2}\Bigg\{ -P_{qq}(z_r)\left(\frac{1}{\epsilon}+L_r\right) + 2\Theta\left(\frac{1}{2}-z_r\right)P_{qq}(z_r)\ln z_r + \Theta\left(z_r-\frac{1}{2}\right)$$
$$\left[\delta(1-z_r)\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\left(L_r+\frac{3}{2}\right)+\frac{1}{2}L_r^2+\frac{3}{2}L_r-\frac{\pi^2}{12}\right]+1-z_r+2P_{qq}(z_r)\ln(1-z_r)_+\right]\Bigg\}$$

$$C_q^{(A_{\rm II})}(\ldots,\varphi_i,\varphi_f) = \frac{\alpha_s C_F \Delta\varphi}{4\pi^2}\Bigg\{ -P_{gq}(z_r)\left(\frac{1}{\epsilon}+L_r\right) + z_r$$
$$-2P_{gq}(z_r)\left[\Theta\left(\frac{1}{2}-z_r\right)\ln z_r + \Theta\left(z_r-\frac{1}{2}\right)\ln(1-z_r)\right]\Bigg\}$$

Collinear computation continued

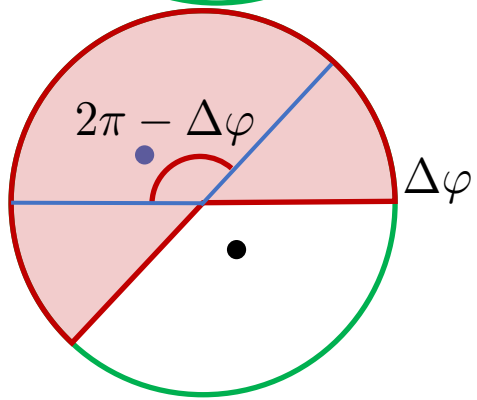
Graph A is more complicated but can be computed $\varphi > \pi$





Same as the isotropic case

$$C_i^{\mathrm{A_{III}}}\left(\ldots,\varphi_i,\varphi_f\right) = \frac{2\left(\Delta\varphi-\pi\right)}{2\pi} C_{i\,\mathrm{A}}\left(\ldots\right) + \frac{2\pi-\Delta\varphi}{2\pi}\left[C_i^{\mathrm{A_I}} + C_i^{\mathrm{A_{II}}}\right]$$



Same as the previous computation

RG consistency and discussion on the fixed order region

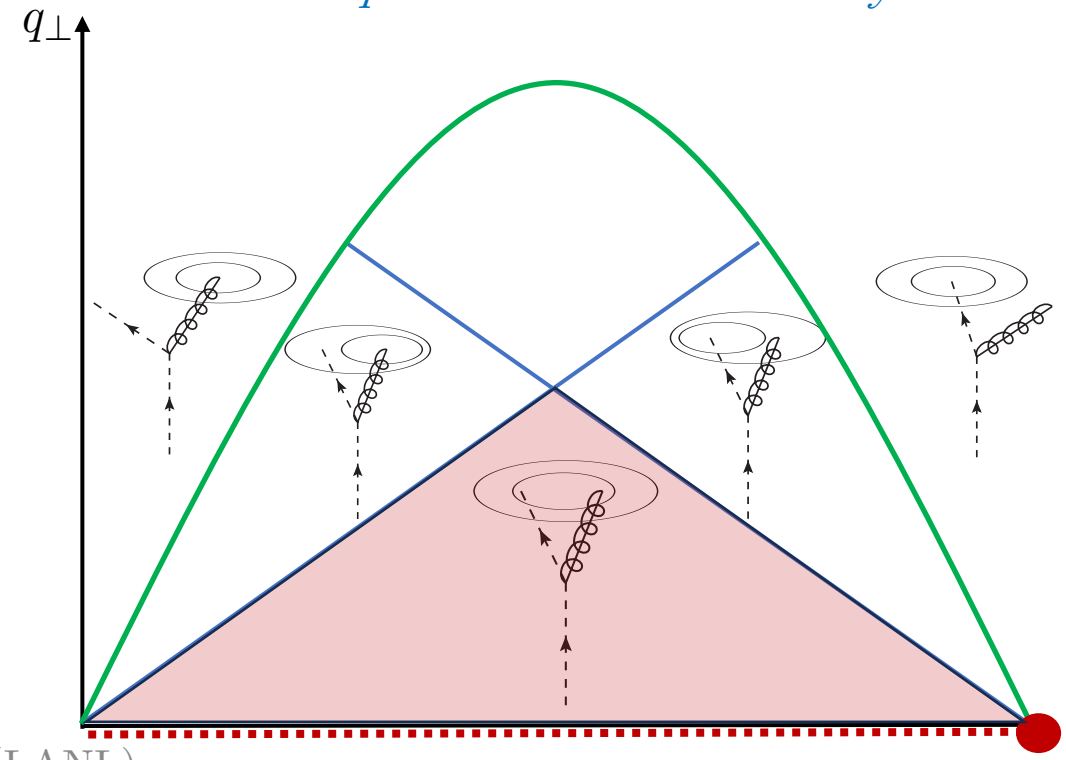
The computation of the jet shape involves integration over the variable

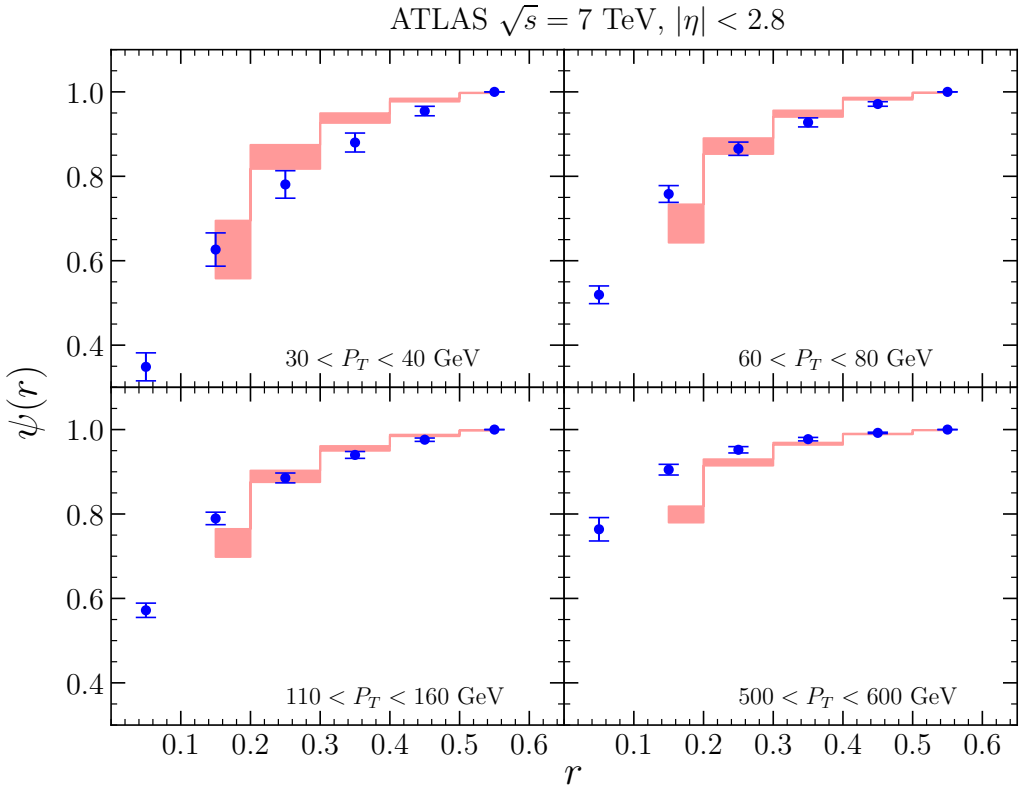
$$\psi\left(r, R, \varphi_i, \varphi_f\right) = \int dz_r\, z_r \frac{d\sigma_c}{dz\, d\mathcal{PS}} \otimes j_c\left(z, z_r, \varphi_i, \varphi_f\right) / \frac{d\sigma_c}{dz\, d\mathcal{PS}} \otimes J_c\left(z, R\right)$$

We find the simple relation

$$\int dz_r\, z_r \left[j_c^{\mathrm{A_I}} + j_c^{\mathrm{A_{II}}}\right]\left(z, z_r, \varphi_i, \varphi_f, \omega_J, r, R, \mu\right) = \frac{\Delta\varphi}{2\pi} \int dz_r\, z_r j_c^{\mathrm{A}}\left(z, z_r, \omega_J, r, R, \mu\right)$$

Required for RG consistency!





RG consistency and discussion on the fixed order region

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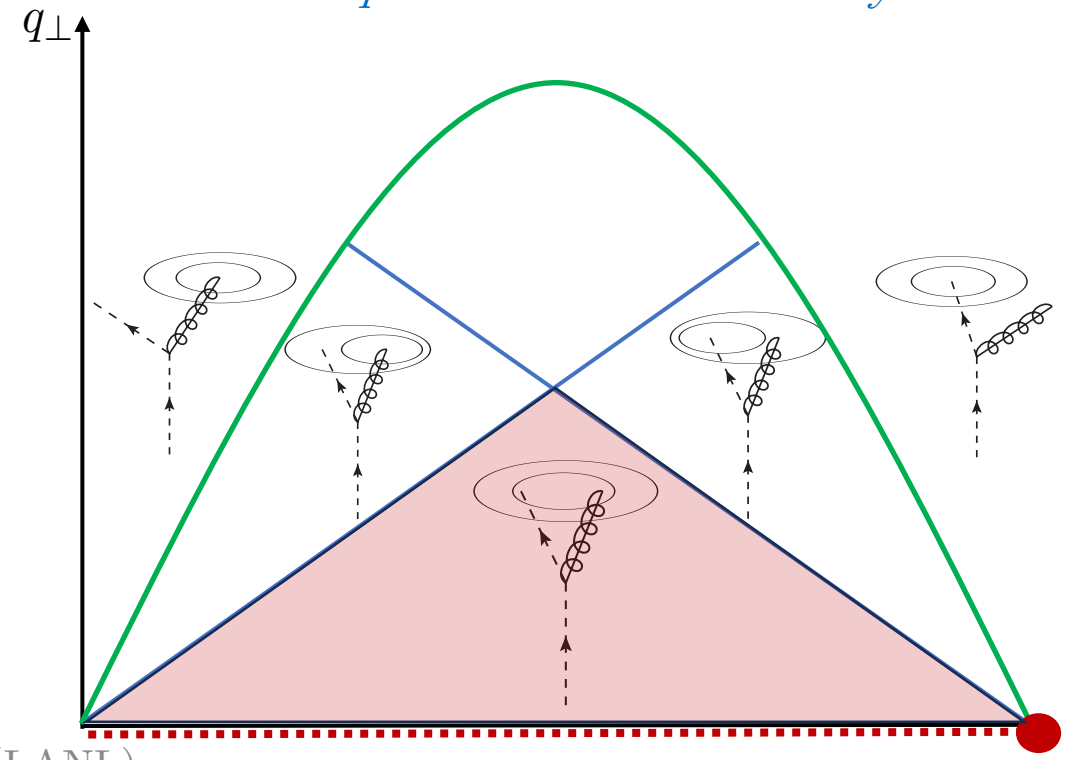
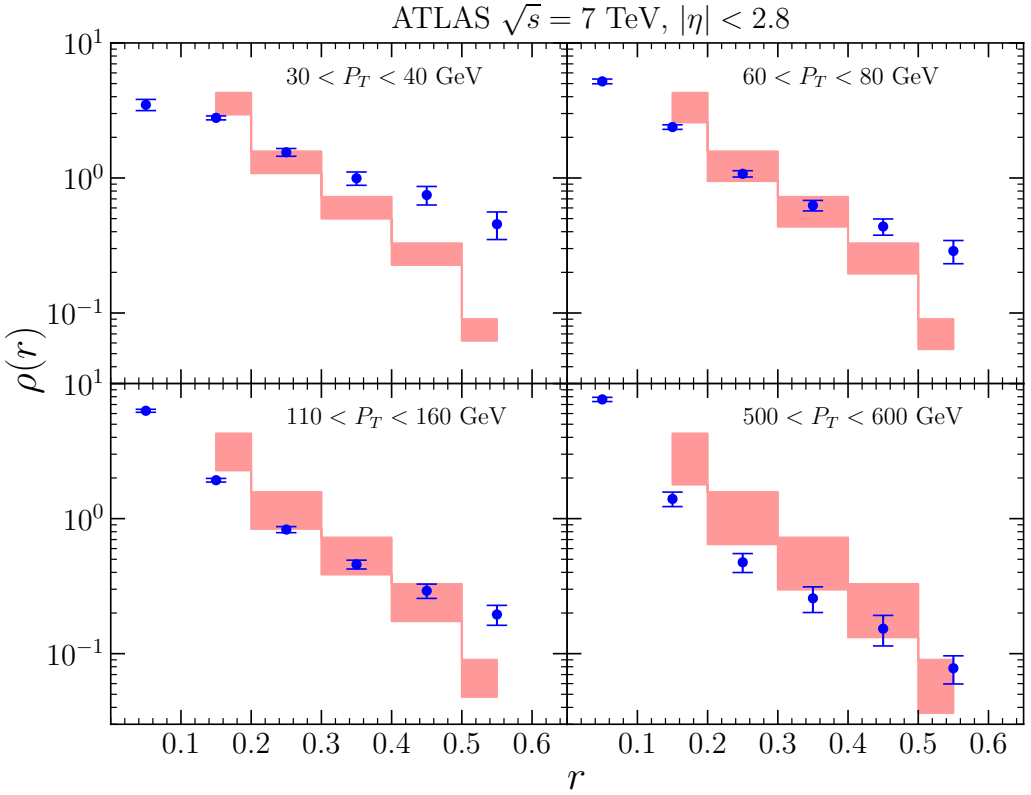
Required for RG consistency!





Azimuthal dependent jet broadening

Traditional jet angularities are defined in hadronic colliders

$$\tilde{\tau}_a = \frac{1}{p_T} \sum_{i \in J} p_T^i \left| \Delta \mathcal{R}_{iJ} \right|^{2-a}$$

Berger, Kucs, and Sterman [2003]

Almeida [2008]

Ellis, Hornig, Lee, Vermilion, Walsh [2008]

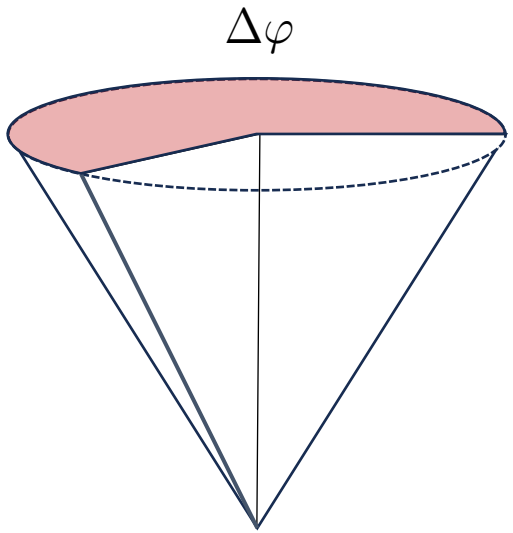
Thomas Becher and Guido Bell [2012]

Focus on $a = 1$ (jet broadening)

Two options on the table for defining azimuthal anisotopic jet broadening

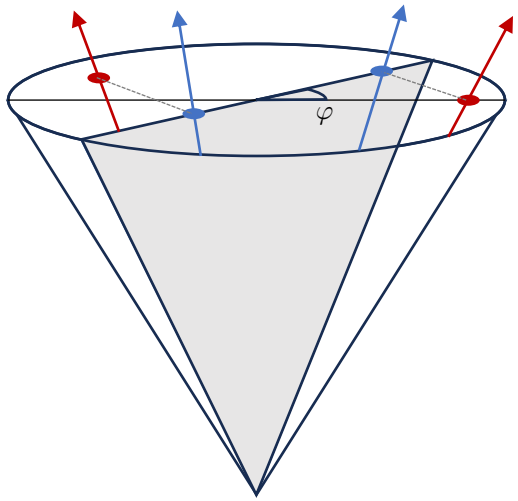
$$\tilde{\tau}_a = \frac{1}{p_T} \sum_{i \in J} p_T^i \left| \Delta \mathcal{R}_{iJ} \right|^{2-a}$$

Slice the jet



$$\tau_a^{\varphi} = \frac{1}{p_T} \sum_{i \in J(\varphi)} p_T^i \left| \Delta \mathcal{R}_{iJ} \right|^{2-a}$$

Measure a single transverse momentum component



$$\tau_a = \frac{1}{p_T} \sum_{i \in J} p_T^i \left| \Delta \mathcal{R}_{iJ}^{\varphi} \right|^{2-a}$$

Refactorization of the jet function in the resummation limit

The semi-inclusive jet angularity functions are given by the SCET matrix elements

$$G_q(\tau, \omega_J, R, \mu) = \frac{1}{2N_c}\mathrm{Tr}\left[\frac{\bar{\not{n}}_J}{2}\langle 0|\delta\left(\omega_J - \bar{n}_J\cdot\mathcal{P}\right)\delta(\tau-\hat{\tau})\chi_{n_J}(0)|J\rangle\langle J|\bar{\chi}_{n_J}(0)|0\rangle\right]$$

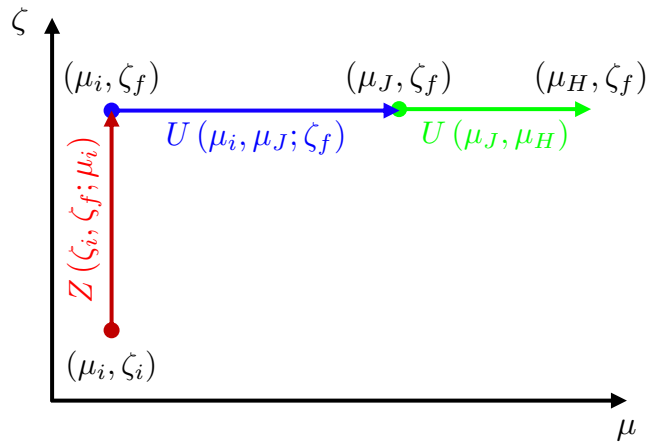
In the region where tau << R, there are two momenta scalings which contribute to the observable

$$p_J^\mu \sim \omega_J\left(R^2, 1, R\right) \qquad p_c^\mu \sim \omega_J\left(\lambda^2, 1, \lambda\right), \qquad p_s^\mu \sim \omega_J\frac{\lambda}{R}\left(R^2, 1, R\right)$$

$$\mu_J \sim \omega_J R \qquad \mu_c \sim \mu_s \sim \omega_J\lambda, \qquad \nu_c \sim \omega_J, \qquad \nu_s \sim \omega_J\frac{\lambda}{R},$$

Rapidity evolution resums logs of the power counting parameters

$$G_i\left(z, \tau, \omega_J, R, \mu, \zeta\right) = H_{ij}\left(z, \omega_J, \mu\right)\int_{c-i\infty}^{c+i\infty}\frac{d\kappa}{2\pi i}\exp\left(\frac{\kappa\tau}{e^{\gamma_E}}\right)\mathcal{C}_j\left(\kappa, \omega_J, R, \mu, \frac{\zeta}{\nu^2}\right)\mathcal{S}_j\left(\kappa, \omega_J, R, \mu, \nu\right)$$



Computation in the resummed limit

Collinear graphs for the traditional and the one-dimensional broadening (no jet algorithm dependence)

$$\tilde{C}_i^{\rm bare}\left(\tilde{\tau},\omega_J,\mu,\frac{\zeta}{\nu^2}\right)=\sum_j\int dx\,d^{d-2}q_\perp\,\hat{P}_{ji}\left(x,q_\perp\right)\,\delta\left[\tilde{\tau}-\frac{2q_\perp}{\omega_J}\right]\left(\frac{\nu^2}{(1-x)^2\zeta}\right)^{\eta/2}$$

$$C_i^{\rm bare}\left(\tau,\omega_J,\mu,\frac{\zeta}{\nu^2}\right)=\sum_j\int dx\,d^{d-2}q_\perp\,\hat{P}_{ji}\left(x,q_\perp\right)\,\delta\left[\tau-\frac{2q_x}{\omega_J}\right]\left(\frac{\nu^2}{(1-x)^2\zeta}\right)^{\eta/2}$$

$$\hat{P}_{qq}\left(x,q_\perp\right)=\frac{\alpha_s C_F}{2\pi^2}\frac{1}{q_\perp^2}\left(\pi\mu^2 e^{\gamma_E}\right)^\epsilon\left[\frac{1+x^2}{1-x}-\epsilon(1-x)\right],$$

$$\hat{P}_{gq}\left(x,q_\perp\right)=0\,,$$

$$\hat{P}_{gg}\left(x,q_\perp\right)=\frac{\alpha_s C_A}{2\pi^2}\frac{1}{q_\perp^2}\left(\pi\mu^2 e^{\gamma_E}\right)^\epsilon\left[\frac{x}{1-x}+\frac{1-x}{x}+x(1-x)\right],$$

$$\hat{P}_{gq}\left(x,q_\perp\right)=\frac{\alpha_s N_f T_R}{2\pi^2}\frac{1}{q_\perp^2}\left(\pi\mu^2 e^{\gamma_E}\right)^\epsilon\left[\frac{2x}{1-\epsilon}(1-x)-1\right].$$

Soft graphs for the traditional and the one-dimensional broadening

$$\tilde{S}_i^{\rm bare}\left(\tilde{\tau},\omega_J,R,\mu,\nu\right)=g^2C_i\left(\frac{\mu^2e^{\gamma_E}}{4\pi}\right)^\epsilon\int\frac{dl_J^+\,dl_J^-\,d^{d-2}l_\perp}{(2\pi)^d}\frac{n_J\cdot\bar{n}_J}{n_J\cdot l\,\bar{n}_J\cdot l}2\pi\delta\left(l^2\right)$$
$$\times\,\delta\left(\tilde{\tau}-2\frac{l_\perp}{\omega_J}\right)\Theta\left(\tan^2\frac{R}{2}-\frac{l^+}{l^-}\right)\left(\frac{2l_0}{\nu}\right)^\eta$$

$$S_i^{\rm bare}\left(\tau,\omega_J,R,\mu,\nu\right)=g^2C_i\left(\frac{\mu^2e^{\gamma_E}}{4\pi}\right)^\epsilon\int\frac{dl_J^+\,dl_J^-\,dl_x\,d^{d-3}l_\perp}{(2\pi)^d}\,2\pi\,\delta\left(l^2\right)\,\frac{n_J\cdot\bar{n}_J}{n_J\cdot l\,\bar{n}_J\cdot l}$$
$$\times\,\delta\left(\tau-2\frac{l_x}{\omega_J}\right)\Theta\left(\tan^2\frac{R}{2}-\frac{l^+}{l^-}\right)\left(\frac{2l_0}{\nu}\right)^\eta$$

Requires additional integration in d-3 dimensions

Factor of 2 associated with collinear recoil. One of the logs is changed but the result is left unchanged for the traditional broadening and the one dimensional case

Fixed order computation

Computation in the fixed order region is slightly more complicated due to the jet algorithm

$$G_i^{\text{bare}}\left(\tau_a, \omega_J, R, \mu\right) = \sum_j \int d\Phi \left[\hat{P}_{ji}\left(x, q_\perp, \epsilon\right)\right] \delta\left[\tau - \frac{2q_\perp}{\omega_J}\right] \Theta_{\text{alg}}\left(x, q_T, \omega, R\right)$$

$$G_i^{\text{bare}}\left(\tilde{\tau}, \omega_J, R, \mu\right) = \sum_j \int d\Phi \left[\hat{P}_{ji}\left(x, q_\perp, \epsilon\right)\right] \delta\left[\tilde{\tau} - \frac{2q_\perp}{\omega_J}\right] \Theta_{\text{alg}}\left(x, q_T, \omega, R\right)$$

We have calculated the traditional jet broadening in the fixed order region

$$G_q\left(\tau, \omega_J, R, \mu\right) = \frac{\alpha_s C_F}{2\pi} \frac{1}{\tau^{\max}} \left[-3 \left(\frac{\sqrt{1-\hat{\tau}}}{\hat{\tau}}\right)_+ + 4 \left(\frac{1}{\hat{\tau}} \ln\left(\frac{\left(\sqrt{1-\hat{\tau}}+1\right)^2}{\hat{\tau}}\right)\right)_+ \right. \qquad \hat{\tau} = \frac{\tau}{\tau^{\max}}$$

$$\left. + \delta(\hat{\tau}) \left(\frac{L_R^2}{2} + \frac{L_R}{\epsilon} + \frac{3L_R}{2} + \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{\pi^2}{12} + \frac{1}{2} - 8\ln^2(2) + 6\ln(2)\right) \right]$$

$$G_g\left(\tau, \omega_J, R, \mu\right) = \frac{\alpha_s n_f T_F}{2\pi} \frac{1}{\tau^{\max}} \left[-\frac{4}{3\epsilon}\delta(\hat{\tau}) - \frac{2}{3}\sqrt{1-\hat{\tau}} + \frac{8}{3}\left(\frac{\sqrt{1-\hat{\tau}}}{\hat{\tau}}\right)_+ \right]$$

$$\frac{\alpha_s C_A}{2\pi} \frac{1}{\tau^{\max}} \left[\frac{1}{3}\sqrt{1-\hat{\tau}} - \frac{22}{3}\left(\frac{\sqrt{1-\hat{\tau}}}{\hat{\tau}}\right)_+ + 8\left(\frac{1}{\hat{\tau}}\ln\left(\frac{\left(\sqrt{1-\hat{\tau}}+1\right)^2}{\hat{\tau}}\right)\right)_+ \right.$$

$$\left. \delta(\hat{\tau}) \left(L_R^2 + \frac{2L_R}{\epsilon} + \frac{11L_R}{3} + \frac{2}{\epsilon^2} + \frac{11}{3\epsilon} - \frac{\pi^2}{6} - 16\ln^2(2) + \frac{44}{3}\ln(2)\right) \right]$$

Computation for one-dimensional case is more complicated due to the restriction of the jet algorithm

$$\Theta_{\text{alg}}\left(x, q_\perp, \omega, R\right) = \Theta\left(\frac{\omega}{2} x(1-x) \tan\frac{\mathcal{R}}{2} - q_\perp\right)$$

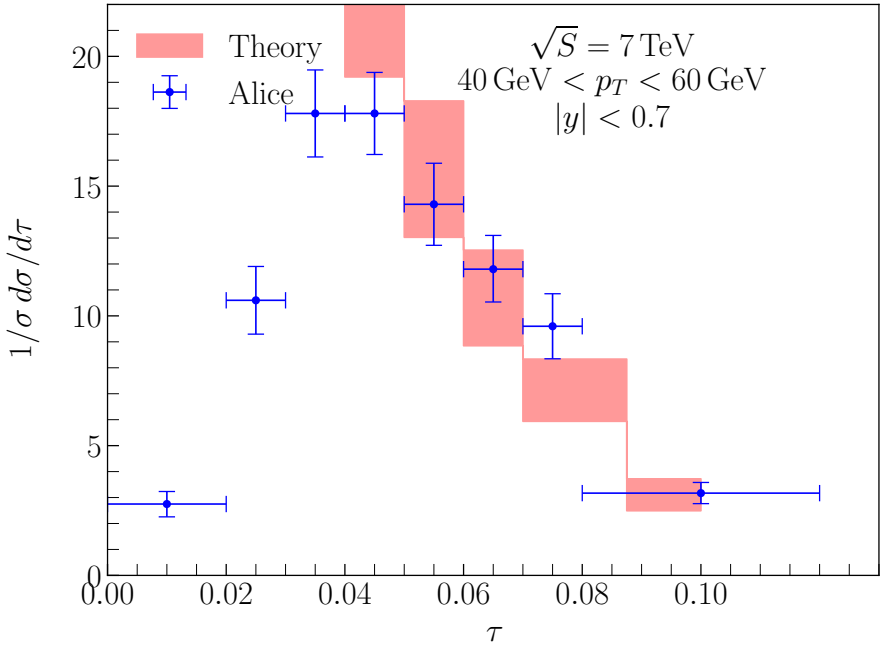
Fixed order computation

Computation in the fixed order region is slightly more complicated due to the jet algorithm

$$G_i^{\text{bare}}(\tau_a, \omega_J, R, \mu) = \sum_j \int d\Phi \left[\hat{P}_{ji}(x, q_\perp, \epsilon)\right] \delta\left[\tau - \frac{2q_\perp}{\omega_J}\right] \Theta_{\text{alg}}(x, q_T, \omega, R)$$

$$G_i^{\text{bare}}(\tilde{\tau}, \omega_J, R, \mu) = \sum_j \int d\Phi \left[\hat{P}_{ji}(x, q_\perp, \epsilon)\right] \delta\left[\tilde{\tau} - \frac{2q_\perp}{\omega_J}\right] \Theta_{\text{alg}}(x, q_T, \omega, R)$$

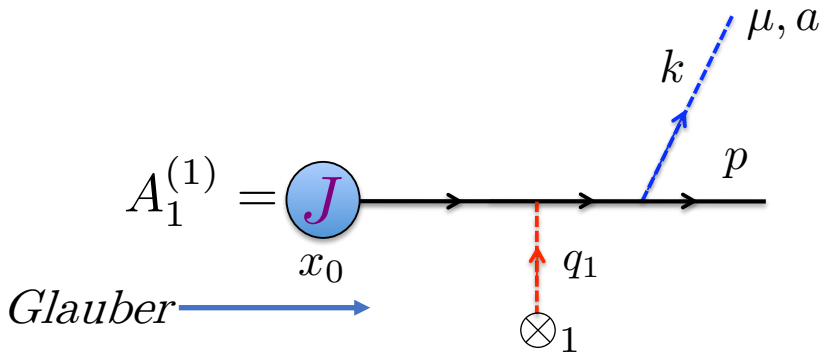
Numerics are sounds, just the plot is ugly



Moving forward (pun intended)

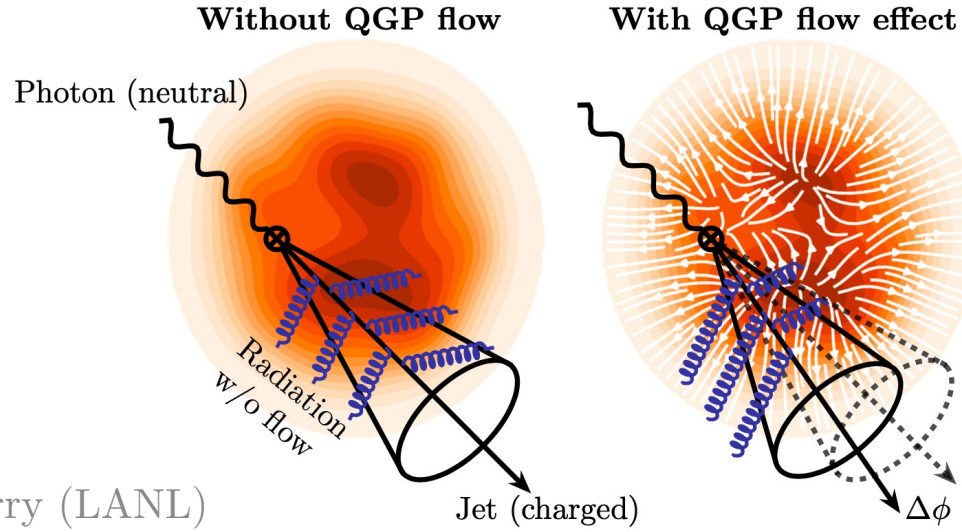
Evolution of the jet function should be modified due to coupling with the plasma. Analogous effects should enter into FFs and EECs

Effect should be similar to Adi's talk. Resummation of emissions should lead to a ladder diagram, modifying the structure of the jet. Effect should be universal for many types of collinear functions



Ke, Vitev, Terry (WIP)

After iso-tropic computation, we should add back the anisotopic flow effects and move back to anisotropic jet sub



$$E\frac{dN^{(1)}}{d^2k_\perp\,dx\,d^2p_\perp\,dE} = \frac{\alpha_s\,N_c}{\pi^2 x}\left(E\frac{dN^{(0)}}{d^2p_\perp\,dE}\right)\int_0^L dz\,\rho\int d^2q_\perp\bar{\sigma}(q_\perp^2)$$
$$\times\left\{\frac{2\boldsymbol{k}_\perp\cdot\boldsymbol{q}_\perp}{k_\perp^2(k-q)_\perp^2}\left(1-\cos\left(\frac{(k-q)_\perp^2}{2xE(1-u_z)}z\right)\right)+\frac{q_\perp^2}{k_\perp^2(q_\perp^2+\mu^2)}\frac{\boldsymbol{u}_\perp\cdot\boldsymbol{k}_\perp}{2(1-u_z)xE}\right\}$$

Can define azimuthal asymmetries to isolate flow term

Conclusion

- *We have generalized jet shape and jet broadening to allow one to define asymmetries which will allow us to address how strongly systems are modified by flowing matter.*

- *Studied the jet shape and demonstrated that the sub-jet function is modified at NLO. Demonstrated renormalization group consistency.*

- *Calculated the one-loop expression for the jet broadening and demonstrated that additional integration leaves the net anomalous dimension unchanged of the jet function is left unchanged.*

- *Future work has many different directions, such as medium-modified jet evolution, heavy flavor contributions, spin physics.*