RG-Improved Resummation of Super-Leading Logarithms

Michel Stillger

PRISMA$^+$ Cluster of Excellence & Mainz Institute for Theoretical Physics,
Johannes Gutenberg University Mainz

work in progress with Philipp Böer, Patrick Hager, Matthias Neubert and Xiaofeng Xu

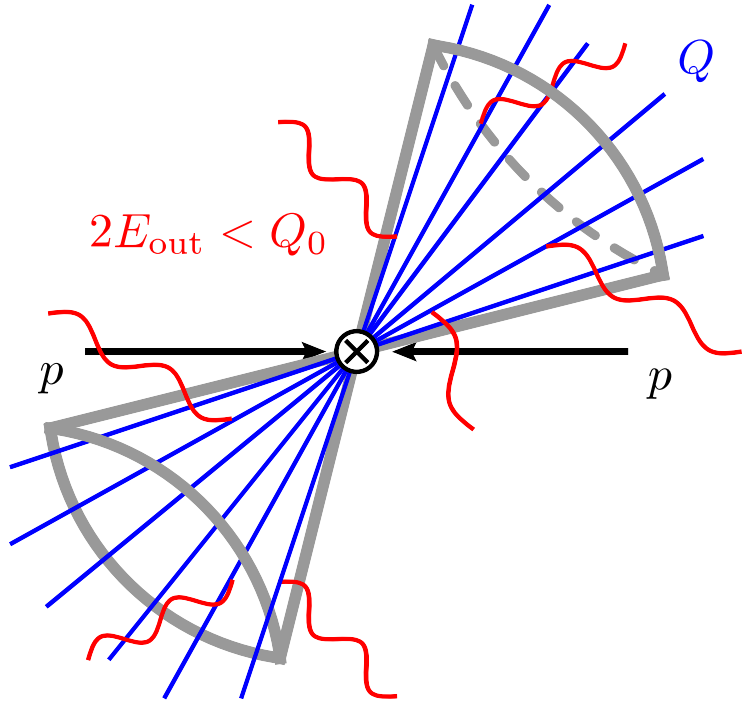
and arXiv:2311.18811

SCET 2024
Salamanca, Spain
April 15 – 18, 2024

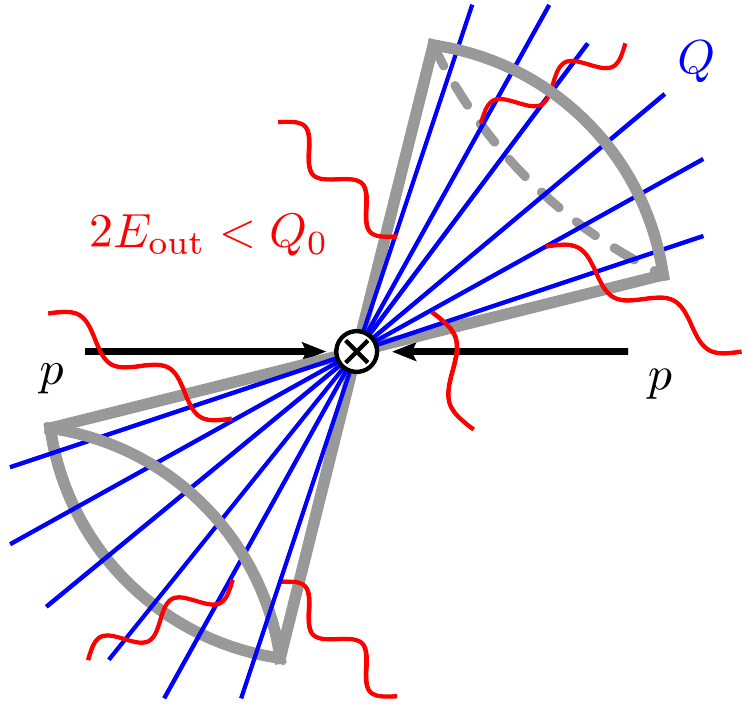
Mainz Institute for Theoretical Physics

GAP-BETWEEN-JET CROSS SECTION

- veto radiation above a low scale Q_0 outside jets
- particles inside jet carry large energy $Q \sim \sqrt{\hat{s}}$
  - non-global logarithms $L = \ln(Q/Q_0)$



GAP-BETWEEN-JET CROSS SECTION

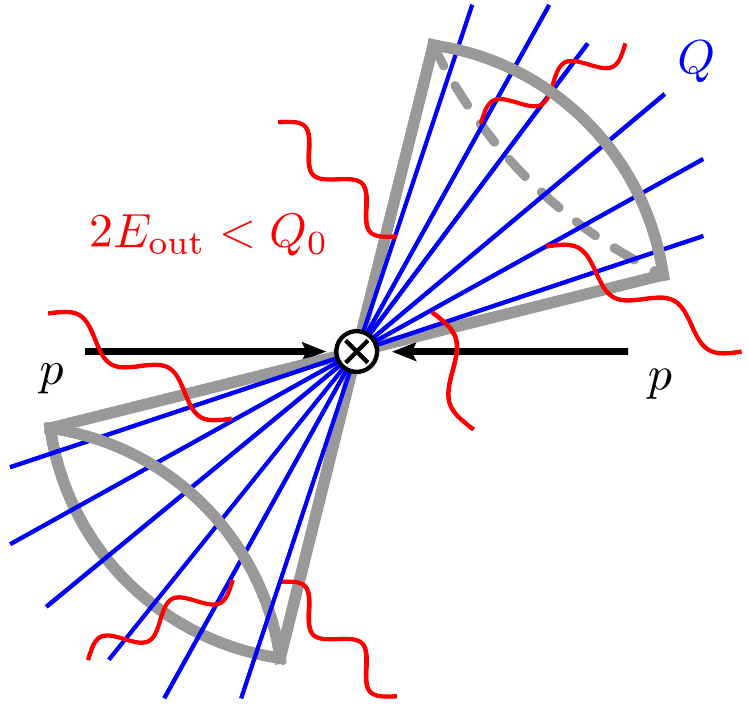


- veto radiation above a low scale Q_0 outside jets
- particles inside jet carry large energy $Q \sim \sqrt{\hat{s}}$
  - non-global logarithms $L = \ln(Q/Q_0)$
- Glauber gluon exchange between initial-state partons breaks color coherence
  - complicated structure at higher order
  - super-leading logarithms (SLLs)

$$\sigma \sim \hat{\sigma}\left\{1 + \alpha_s L + \alpha_s^2 L^2 + (\alpha_s \pi^2)\left(\alpha_s^2 L^3 + \alpha_s^3 L^5 + \alpha_s^4 L^7 + \dots\right)\right\}$$

[Forshaw, Kyrieleis, Seymour: JHEP **08** (2006) 059]
[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]

GAP-BETWEEN-JET CROSS SECTION



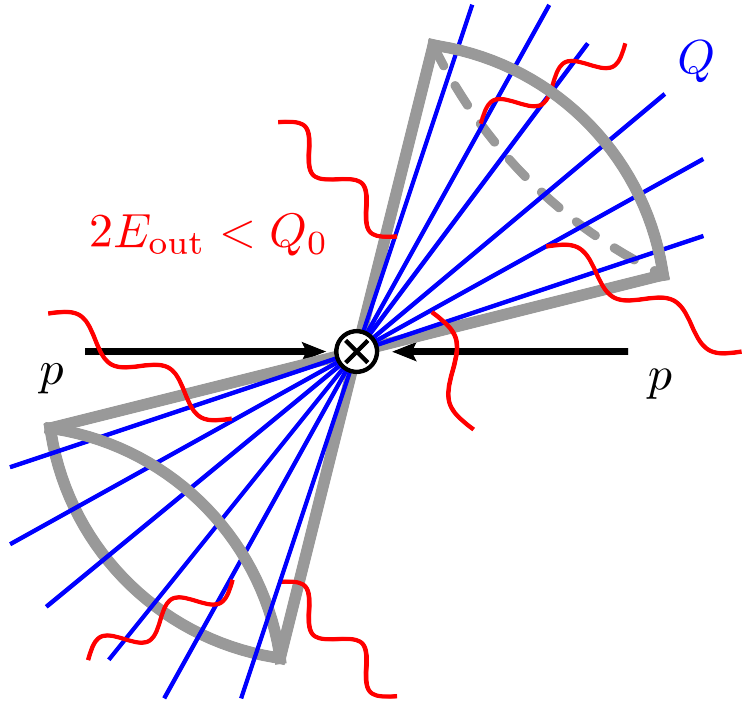
- real and imaginary part of large logarithms

$$\ln\left(-Q^2/Q_0^2\right) = 2L - i\pi$$

- realistic values, e.g. $Q = 1\ \text{TeV}$ and $Q_0 = 40\ \text{GeV}$

$$L \sim \pi$$

GAP-BETWEEN-JET CROSS SECTION



- real and imaginary part of large logarithms

$$\ln\left(-Q^2/Q_0^2\right) = 2L - i\pi$$

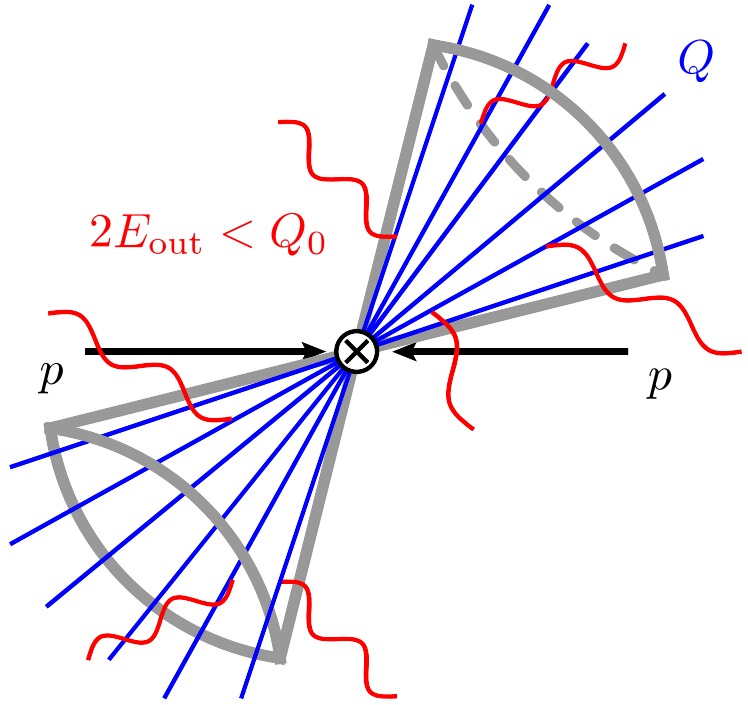
- realistic values, e.g. $Q = 1\ \text{TeV}$ and $Q_0 = 40\ \text{GeV}$

$$L \sim \pi$$

- expansion in two $\mathcal{O}(1)$ parameters

$$\sigma_{2\to M}^{\text{SLL+G}} \sim \sum_{\ell=1}^{\infty}\sum_{n=0}^{\infty} c_{\ell,n}\, w^{n+\ell}\, w_\pi^{\ell} \qquad w = \frac{N_c\,\alpha_s}{\pi} L^2 \qquad w_\pi = \frac{N_c\,\alpha_s}{\pi}\pi^2$$

GAP-BETWEEN-JET CROSS SECTION



- real and imaginary part of large logarithms

$$\ln\left(-Q^2/Q_0^2\right) = 2L - i\pi$$

- realistic values, e.g. $Q = 1\ \text{TeV}$ and $Q_0 = 40\ \text{GeV}$

$$L \sim \pi$$

- expansion in two $\mathcal{O}(1)$ parameters

$$\sigma_{2\to M}^{\text{SLL+G}} \sim \sum_{\ell=1}^{\infty}\sum_{n=0}^{\infty} c_{\ell,n}\, w^{n+\ell}\, w_\pi^{\ell} \qquad w = \frac{N_c\,\alpha_s}{\pi} L^2 \qquad w_\pi = \frac{N_c\,\alpha_s}{\pi}\pi^2$$

- resummation of SLL ($\ell = 1$) for fixed coupling done

[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]
[Becher, Neubert, Shao, MS: JHEP **12** (2023) 116]

FACTORIZATION THEOREM

[Becher, Neubert, Rothen, Shao: Phys. Rev. Lett. **116** (2016) 192001]
[Becher, Neubert, Rothen, Shao: JHEP **11** (2016) 019]
[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]
[Becher, Neubert, Shao, MS: JHEP **12** (2023) 116]

- Factorization theorem for $pp \to M$ jets:

$$\sigma_{2\to M}(Q_0) = \int d\xi_1 \int d\xi_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, s, \xi_1, \xi_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu) \rangle$$

FACTORIZATION THEOREM

[Becher, Neubert, Rothen, Shao: Phys. Rev. Lett. **116** (2016) 192001]
[Becher, Neubert, Rothen, Shao: JHEP **11** (2016) 019]
[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]
[Becher, Neubert, Shao, MS: JHEP **12** (2023) 116]

- Factorization theorem for $pp \to M$ jets:

$$\sigma_{2\to M}(Q_0) = \int d\xi_1 \int d\xi_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, s, \xi_1, \xi_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu)\rangle$$

- Hard function in color space

energy integrals including phase space constrains

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}) = \int d\mathcal{E}_m \, |\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})|$$

FACTORIZATION THEOREM

[Becher, Neubert, Rothen, Shao: Phys. Rev. Lett. **116** (2016) 192001]
[Becher, Neubert, Rothen, Shao: JHEP **11** (2016) 019]
[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]
[Becher, Neubert, Shao, MS: JHEP **12** (2023) 116]

- Factorization theorem for $pp \to M$ jets:

$$\sigma_{2\to M}(Q_0) = \int d\xi_1 \int d\xi_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, s, \xi_1, \xi_2, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, \xi_1, \xi_2, \mu)\rangle$$

- Hard function in color space

energy integrals including phase space constrains

$$\boldsymbol{\mathcal{H}}_m(\{\underline{n}\}) = \int d\mathcal{E}_m \, |\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})|$$

- Soft and collinear dynamics in lowest order at $\mu_s \sim Q_0$

$$\boldsymbol{\mathcal{W}}_m(\mu_s) = f_1(\xi_1, \mu_s)\, f_2(\xi_2, \mu_s)\, \mathbf{1}$$

RENORMALIZATION GROUP EVOLUTION

- RG evolution from $\mu_h \sim Q$ down to $\mu_s \sim Q_0$

$$\begin{aligned}\boldsymbol{\mathcal{H}}(\mu_s) &= \boldsymbol{\mathcal{H}}(\mu_h)\,\boldsymbol{U}(\mu_h,\mu_s) \\ &= \boldsymbol{\mathcal{H}}(\mu_h) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \boldsymbol{\mathcal{H}}(\mu_h)\,\boldsymbol{\Gamma}^H(\mu_1) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_s}^{\mu_1} \frac{d\mu_2}{\mu_2} \boldsymbol{\mathcal{H}}(\mu_h)\,\boldsymbol{\Gamma}^H(\mu_1)\,\boldsymbol{\Gamma}^H(\mu_2) + \dots\end{aligned}$$

RENORMALIZATION GROUP EVOLUTION

- RG evolution from $\mu_h \sim Q$ down to $\mu_s \sim Q_0$

$$\boldsymbol{\mathcal{H}}(\mu_s) = \boldsymbol{\mathcal{H}}(\mu_h)\, \boldsymbol{U}(\mu_h, \mu_s)$$
$$= \boldsymbol{\mathcal{H}}(\mu_h) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \boldsymbol{\mathcal{H}}(\mu_h)\, \boldsymbol{\Gamma}^H(\mu_1) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_s}^{\mu_1} \frac{d\mu_2}{\mu_2} \boldsymbol{\mathcal{H}}(\mu_h)\, \boldsymbol{\Gamma}^H(\mu_1)\, \boldsymbol{\Gamma}^H(\mu_2) + \dots$$

- Anomalous dimension matrix $\boldsymbol{\Gamma}^H = \boldsymbol{\Gamma}^C + \boldsymbol{\Gamma}^S$ with

$$\boldsymbol{\Gamma}^S(\mu) = \gamma_{\text{cusp}}\big(\alpha_s(\mu)\big) \left(\boldsymbol{\Gamma}^c \ln \frac{\mu^2}{\mu_h^2} + \boldsymbol{V}^G \right) + \left[\frac{\alpha_s(\mu)}{4\pi} \overline{\boldsymbol{\Gamma}} + \mathcal{O}(\alpha_s^2) \right]$$

- Glauber operator

$$\boldsymbol{V}^G = -2i\pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R} \right)$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

- three important properties

$$[\boldsymbol{\Gamma}^c, \overline{\boldsymbol{\Gamma}}] = 0 \qquad \langle \boldsymbol{\mathcal{H}}\, \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0 \qquad \langle \boldsymbol{\mathcal{H}}\, \boldsymbol{V}^G \otimes \mathbf{1} \rangle = 0$$

[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]

RESUMMATION OF SUPER-LEADING LOGARITHMS

- three important properties

$$[\boldsymbol{\Gamma}^c, \overline{\boldsymbol{\Gamma}}] = 0 \qquad \langle \boldsymbol{\mathcal{H}}\, \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0 \qquad \langle \boldsymbol{\mathcal{H}}\, \boldsymbol{V}^G \otimes \mathbf{1} \rangle = 0$$

[Becher, Neubert, Shao: Phys. Rev. Lett. **127** (2021) 212002]

- all-order structure of SLLs

$$\hat{\sigma}^{\text{SLL}}_{2\to M} = \sum_{n=0}^{\infty} \sum_{r=0}^{n} I_{rn}(\mu_h, \mu_s) \left\langle \boldsymbol{\mathcal{H}}_{2\to M} \left(\boldsymbol{\Gamma}^c\right)^r \boldsymbol{V}^G \left(\boldsymbol{\Gamma}^c\right)^{n-r} \boldsymbol{V}^G \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \right\rangle$$

- nested scale integrals

$$I_{rn}(\mu_h, \mu_s)\Big|_{\text{fixed coupling}} = \left(\frac{\alpha_s(\bar{\mu})}{4\pi}\right)^{n+3} \frac{(-4)^n\, n!}{(2n+3)!} \frac{(2r)!}{4^r\, (r!)^2} L_s^{2n+3} \qquad L_s \equiv \ln \frac{\mu_h}{\mu_s}$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

- SLLs generating evolution operator

$$\boldsymbol{U}_{\rm SLL}(\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$

$$\times\,\boldsymbol{U}^c(\mu_h,\mu_1)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}^c(\mu_1,\mu_2)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

[Böer, Hager, Neubert, MS, Xu: JHEP **02** (2024) 109]

RESUMMATION OF SUPER-LEADING LOGARITHMS

- SLLs generating evolution operator

$$\boldsymbol{U}_{\rm SLL}(\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$
$$\times\,\boldsymbol{U}^c(\mu_h,\mu_1)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}^c(\mu_1,\mu_2)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

[Böer, Hager, Neubert, MS, Xu: JHEP **02** (2024) 109]

- generalized Sudakov operator

$$\boldsymbol{U}^c(\mu_i,\mu_j) = \exp\left[\boldsymbol{\Gamma}^c\int_{\mu_j}^{\mu_i}\frac{d\mu}{\mu}\,\gamma_{\rm cusp}\big(\alpha_s(\mu)\big)\ln\frac{\mu^2}{\mu_h^2}\right]$$

no longer path-ordered

GLAUBER SERIES

[Böer, Neubert, MS: JHEP **10** (2023) 075]
[Böer, Hager, Neubert, MS, Xu: JHEP **02** (2024) 109]

- generalize evolution operator

$$\boldsymbol{U}^{(l)}_{\rm SLL}(\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\dots\int_{\mu_s}^{\mu_l}\frac{d\mu_{l+1}}{\mu_{l+1}}\left[\prod_{i=1}^{l}\boldsymbol{U}^c(\mu_{i-1},\mu_i)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_i)\big)\,\boldsymbol{V}^G\right]\frac{\alpha_s(\mu_{l+1})}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

$\mu_0 = \mu_h$

- Glauber series

$$\boldsymbol{U}_{\rm SLL+G}(\mu_h,\mu_s) = \sum_{l=1}^{\infty}\boldsymbol{U}^{(l)}_{\rm SLL}(\mu_h,\mu_s)$$

GLAUBER SERIES

[Böer, Neubert, MS: JHEP **10** (2023) 075]
[Böer, Hager, Neubert, MS, Xu: JHEP **02** (2024) 109]

- generalize evolution operator

$$\boldsymbol{U}^{(l)}_{\rm SLL}(\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\dots\int_{\mu_s}^{\mu_l}\frac{d\mu_{l+1}}{\mu_{l+1}}\left[\prod_{i=1}^{l}\boldsymbol{U}^c(\mu_{i-1},\mu_i)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_i)\big)\,\boldsymbol{V}^G\right]\frac{\alpha_s(\mu_{l+1})}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

$\mu_0 = \mu_h$

- Glauber series

$$\boldsymbol{U}_{\rm SLL+G}(\mu_h,\mu_s) = \sum_{l=1}^{\infty}\boldsymbol{U}^{(l)}_{\rm SLL}(\mu_h,\mu_s)$$

- l is number of Glauber-operator insertions
- odd l also interesting as hard function in general contains non-vanishing imaginary part (e.g. for electroweak processes)

COLOR BASIS

[Böer, Neubert, MS: JHEP **10** (2023) 075]
[Böer, Hager, Neubert, MS, Xu: JHEP **02** (2024) 109]

- express relevant operators in a color basis

$$\langle \boldsymbol{\mathcal{H}}\, \boldsymbol{\Gamma}^c \boldsymbol{X}_i \rangle = \sum_{\tilde{i}} \langle \boldsymbol{\mathcal{H}}\, \boldsymbol{X}_{\tilde{i}} \rangle\, N_c\, (\mathbb{\Gamma}^c)_{\tilde{i}i}$$

$$\langle \boldsymbol{\mathcal{H}}\, \boldsymbol{V}^G \boldsymbol{X}_i \rangle = \sum_{\tilde{i}} \langle \boldsymbol{\mathcal{H}}\, \boldsymbol{X}_{\tilde{i}} \rangle\, i\pi N_c\, (\mathbb{V}^G)_{\tilde{i}i}$$

$qq, q\bar{q}, \bar{q}\bar{q} \to M$	5 structures
$qg, \bar{q}g \to M$	14 structures
$gg \to M$	20 structures

COLOR BASIS

[Böer, Neubert, MS: JHEP **10** (2023) 075]
[Böer, Hager, Neubert, MS, Xu: JHEP **02** (2024) 109]

- express relevant operators in a color basis

$$\langle \boldsymbol{\mathcal{H}}\, \boldsymbol{\Gamma}^c \boldsymbol{X}_i \rangle = \sum_{\tilde{i}} \langle \boldsymbol{\mathcal{H}}\, \boldsymbol{X}_{\tilde{i}} \rangle\, N_c\, (\mathbb{\Gamma}^c)_{\tilde{i}i}$$

$$\langle \boldsymbol{\mathcal{H}}\, \boldsymbol{V}^G \boldsymbol{X}_i \rangle = \sum_{\tilde{i}} \langle \boldsymbol{\mathcal{H}}\, \boldsymbol{X}_{\tilde{i}} \rangle\, i\pi N_c\, (\mathbb{V}^G)_{\tilde{i}i}$$

$qq, q\bar{q}, \bar{q}\bar{q} \to M$	5 structures
$qg, \bar{q}g \to M$	14 structures
$gg \to M$	20 structures

- map most right insertions according to

$$\boldsymbol{V}^G\, \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \to 16 i\pi\, \boldsymbol{X}_1 \equiv 16 i\pi\, \boldsymbol{X}^T \varsigma \qquad \varsigma = (1, 0, \dots)^T$$

COLOR BASIS

[Böer, Neubert, MS: JHEP **10** (2023) 075]
[Böer, Hager, Neubert, MS, Xu: JHEP **02** (2024) 109]

- generalized Sudakov operator for quark-initiated processes

$$\mathbb{U}^c(\mu_i,\mu_j) = \begin{pmatrix} U^c(1;\mu_i,\mu_j) & 0 & 0 & 0 & 0 \\ 0 & U^c(1;\mu_i,\mu_j) & 0 & 0 & 0 \\ 0 & 0 & U^c(\tfrac{1}{2};\mu_i,\mu_j) & 0 & 0 \\ 0 & 0 & 2\left[U^c(\tfrac{1}{2};\mu_i,\mu_j) - U^c(1;\mu_i,\mu_j)\right] & U^c(1;\mu_i,\mu_j) & 0 \\ 0 & 0 & \frac{2C_F}{N_c}\left[1 - U^c(\tfrac{1}{2};\mu_i,\mu_j)\right] & 0 & 1 \end{pmatrix}$$

- with scalar functions

$$U^c(v;\mu_i,\mu_j) = \exp\left[v\, N_c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu}\, \gamma_{\text{cusp}}\big(\alpha_s(\mu)\big) \ln\frac{\mu^2}{\mu_h^2}\right]$$

eigenvalue

RG-IMPROVED RESUMMATION

- scalar evolution function

$$U^c(v;\mu_i,\mu_j)\,U^c(v;\mu_j,\mu_k) = U^c(v;\mu_i,\mu_k) \qquad U^c(0;\mu_i,\mu_j) = 1$$

$$U^c(v_{k_1},\ldots,v_{k_l};\mu_h,\mu_1,\ldots,\mu_l) \equiv U^c(v_{k_1};\mu_h,\mu_1)\,U^c(v_{k_2};\mu_1,\mu_2)\ldots U^c(v_{k_l};\mu_{l-1},\mu_l)$$

RG-IMPROVED RESUMMATION

- scalar evolution function

$$U^c(v;\mu_i,\mu_j)\,U^c(v;\mu_j,\mu_k) = U^c(v;\mu_i,\mu_k) \qquad U^c(0;\mu_i,\mu_j) = 1$$

$$U^c(v_{k_1},\ldots,v_{k_l};\mu_h,\mu_1,\ldots,\mu_l) \equiv U^c(v_{k_1};\mu_h,\mu_1)\,U^c(v_{k_2};\mu_1,\mu_2)\ldots U^c(v_{k_l};\mu_{l-1},\mu_l)$$

- 2-loop approximation in exponent ($x_i \equiv \alpha_s(\mu_i)/\alpha_s(\mu_h)$)

$$U^c(v;\mu_i,\mu_j) = \exp\Bigg\{ v\,N_c\,\frac{\gamma_0}{2\beta_0^2}\left[\frac{4\pi}{\alpha_s(\mu_h)}\left(\frac{1}{x_i}-\frac{1}{x_j}-\ln\frac{x_j}{x_i}\right)\right.$$
$$\left.+\left(\frac{\gamma_1}{\gamma_0}-\frac{\beta_1}{\beta_0}\right)\left(x_i-x_j+\ln\frac{x_j}{x_i}\right)+\frac{\beta_1}{2\beta_0}\left(\ln^2 x_j-\ln^2 x_i\right)\right]\Bigg\}$$

RG-IMPROVED RESUMMATION

- SLL coefficients for quark-initiated processes

$$\mathbb{U}^{(2)}_{\rm SLL}(\mu_h,\mu_s)\,\varsigma = -\frac{32\pi^2}{\beta_0^3}\,N_c\int_1^{x_s}\frac{dx_2}{x_2}\,\ln\frac{x_s}{x_2}\int_1^{x_2}\frac{dx_1}{x_1}\begin{pmatrix} 0 \\ -\frac{1}{2}\,U^c(1;\mu_h,\mu_2) \\ U^c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2) \\ 2\left[U^c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)-U^c(1;\mu_h,\mu_2)\right] \\ \frac{2C_F}{N_c}\left[U^c(1;\mu_1,\mu_2)-U^c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)\right] \end{pmatrix}$$

RG-IMPROVED RESUMMATION

- SLL coefficients for quark-initiated processes

$$\mathbb{U}^{(2)}_{\rm SLL}(\mu_h,\mu_s)\,\varsigma = -\frac{32\pi^2}{\beta_0^3}\,N_c\int_1^{x_s}\frac{dx_2}{x_2}\,\ln\frac{x_s}{x_2}\int_1^{x_2}\frac{dx_1}{x_1}\begin{pmatrix}0\\ -\frac{1}{2}\,U^c(1;\mu_h,\mu_2)\\ U^c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)\\ 2\left[U^c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)-U^c(1;\mu_h,\mu_2)\right]\\ \frac{2C_F}{N_c}\left[U^c(1;\mu_1,\mu_2)-U^c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)\right]\end{pmatrix}$$

- coefficients for $l=3$ Glauber-operator insertions

$$\mathbb{U}^{(3)}_{\rm SLL}(\mu_h,\mu_s)\,\varsigma = -\frac{64i\pi^3}{\beta_0^4}\,N_c^2\int_1^{x_s}\frac{dx_3}{x_3}\,\ln\frac{x_s}{x_3}\int_1^{x_3}\frac{dx_2}{x_2}\int_1^{x_2}\frac{dx_1}{x_1}$$
$$\times\left[K_{12}\,U^c(1;\mu_h,\mu_3)+\tfrac{4}{N_c^2}\,U^c(1,\tfrac{1}{2},1;\mu_h,\mu_1,\mu_2,\mu_3)\right]\varsigma$$

FIXED COUPLING AND ASYMPTOTIC BEHAVIOR

- fixed coupling $\alpha_s \equiv \alpha_s(\bar{\mu})$ with $\bar{\mu} \in (\mu_s, \mu_h)$ reference scale ($L_i = \ln(\mu_h/\mu_i)$)

$$U^c(v;\mu_i,\mu_j) \equiv \exp\left[v\, w(z_i^2 - z_j^2)\right] \qquad L_i \equiv z_i\, L_s$$

FIXED COUPLING AND ASYMPTOTIC BEHAVIOR

- fixed coupling $\alpha_s \equiv \alpha_s(\bar{\mu})$ with $\bar{\mu} \in (\mu_s, \mu_h)$ reference scale ($L_i = \ln(\mu_h/\mu_i)$)

$$U^c(v;\mu_i,\mu_j) \equiv \exp\left[v\, w(z_i^2 - z_j^2)\right] \qquad L_i \equiv z_i\, L_s$$

- standard counting scheme $\alpha_s L_s = \mathcal{O}(1)$

$$w \sim 1/\alpha_s \gg 1 \qquad w_\pi \sim \alpha_s$$

FIXED COUPLING AND ASYMPTOTIC BEHAVIOR

- fixed coupling $\alpha_s \equiv \alpha_s(\bar{\mu})$ with $\bar{\mu} \in (\mu_s, \mu_h)$ reference scale ($L_i = \ln(\mu_h/\mu_i)$)

$$U^c(v;\mu_i,\mu_j) \equiv \exp\left[v\, w(z_i^2 - z_j^2)\right] \qquad L_i \equiv z_i\, L_s$$

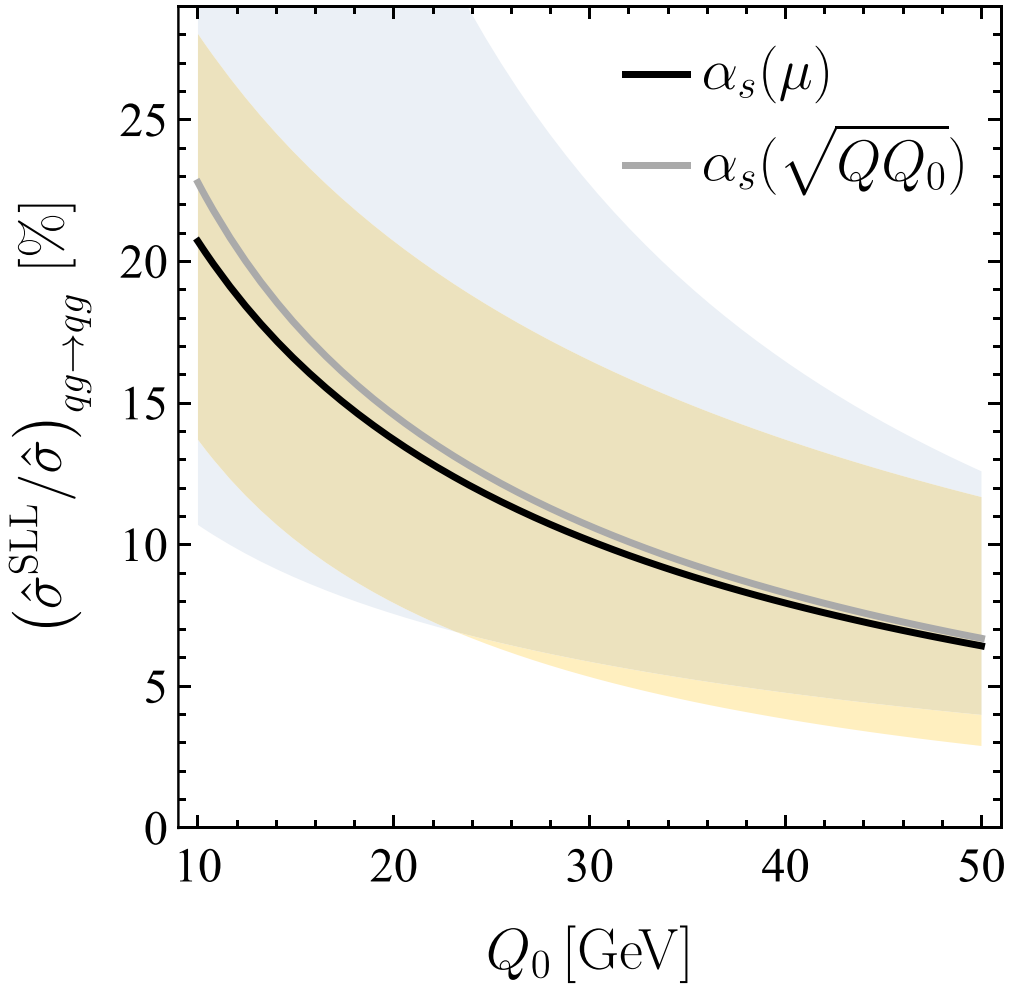
- standard counting scheme $\alpha_s L_s = \mathcal{O}(1)$

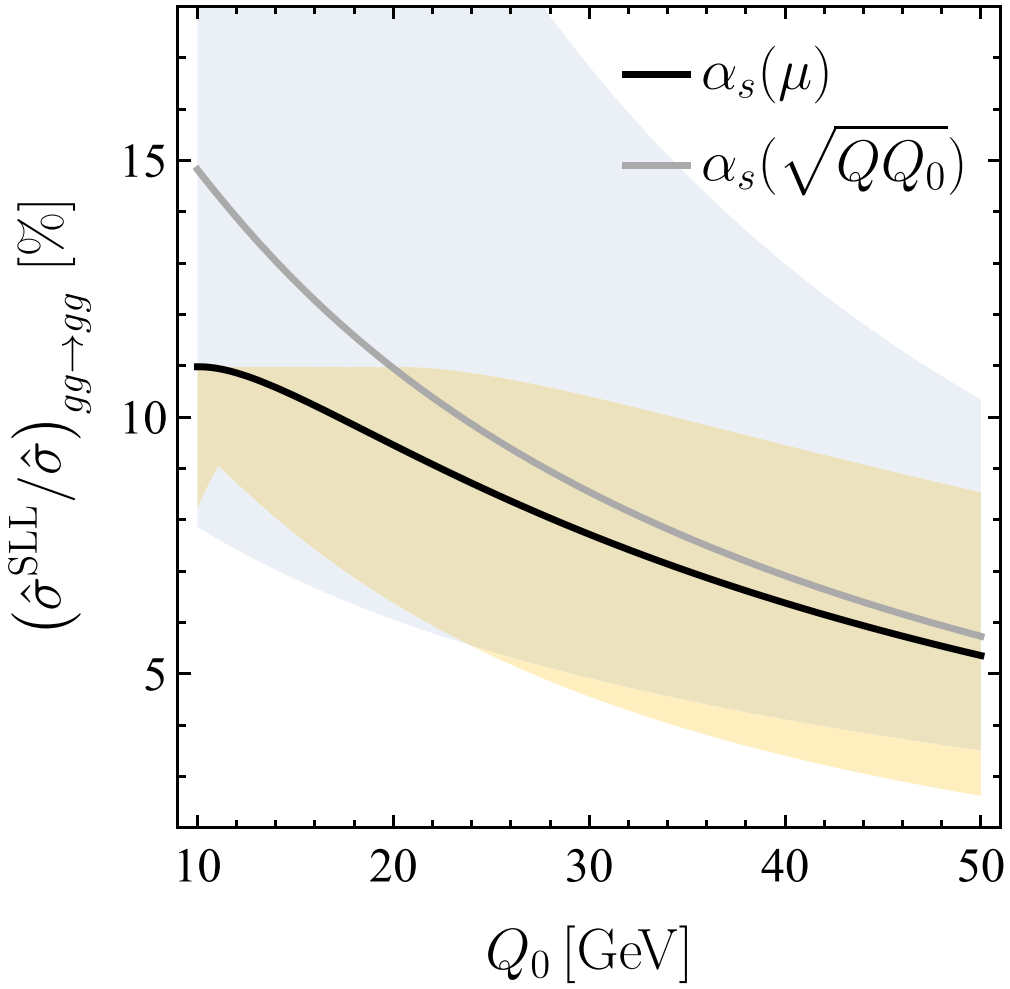
$$w \sim 1/\alpha_s \gg 1 \qquad w_\pi \sim \alpha_s$$

- perform scale integrals in this case

$$\mathbb{U}^{(l)}_{\text{SLL}}(\mu_h,\mu_s)\varsigma \sim \frac{(i\pi)^l}{(l+1)!}\, N_c^{l-1} \left(\frac{\alpha_s L_s}{\pi}\right)^{l+1} \frac{1}{w^{l/2}} = \frac{i^l}{(l+1)!}\, \frac{\alpha_s L_s}{\pi N_c}\, w_\pi^{l/2}$$

NUMERICAL RESULTS





LARGE-N_c RESUMMATION

- coefficients for $l = 3$ Glauber-operator insertions

$$\mathbb{U}^{(3)}_{\mathrm{SLL}}(\mu_h, \mu_s)\varsigma = -\frac{64i\pi^3}{\beta_0^4} N_c^2 \int_1^{x_s} \frac{dx_3}{x_3} \ln\frac{x_s}{x_3} \int_1^{x_3} \frac{dx_2}{x_2} \int_1^{x_2} \frac{dx_1}{x_1}$$
$$\times \left[K_{12} U^c(1; \mu_h, \mu_3) + \frac{4}{N_c^2} U^c(1, \tfrac{1}{2}, 1; \mu_h, \mu_1, \mu_2, \mu_3)\right]\varsigma$$

- large-N_c expansion

$$K_{12} \equiv \frac{N_c^2 - 4}{N_c^2} \delta_{q\bar{q}} \to \begin{cases} 1 & \text{for } q\bar{q} \text{ scattering} \\ 0 & \text{for } qq \text{ scattering} \end{cases}$$

LARGE-N_c RESUMMATION

- major simplification in large-N_c limit for quark-initiated processes

$$\mathbb{V}^G\,\mathbb{U}^c(\mu_i,\mu_j) = \mathbb{V}^G\,U^c(1;\mu_i,\mu_j) + \mathcal{O}(1/N_c^2)$$

LARGE-N_c RESUMMATION

- major simplification in large-N_c limit for quark-initiated processes

$$\mathbb{V}^G\,\mathbb{U}^c(\mu_i,\mu_j) = \mathbb{V}^G\,U^c(1;\mu_i,\mu_j) + \mathcal{O}(1/N_c^2)$$

- coefficient vector

can be performed except for x_1 and x_l

$$\mathbb{U}^{(l)}_{\rm SLL}(\mu_h,\mu_s)\,\varsigma = (i\pi)^l\,N_c^{l-1}\,\frac{2^{l+3}}{\beta_0^{l+1}}\int_1^{x_s}\frac{dx_l}{x_l}\,\ln\frac{x_s}{x_l}\int_1^{x_l}\frac{dx_{l-1}}{x_{l-1}}\,\ldots\int_1^{x_2}\frac{dx_1}{x_1}$$
$$\times\,U^c(1;\mu_1,\mu_l)\,\mathbb{U}^c(\mu_h,\mu_1)\left(\mathbb{V}^G\right)^{l-1}\varsigma$$

- need to distinguish whether l is even or odd

LARGE-N_c RESUMMATION

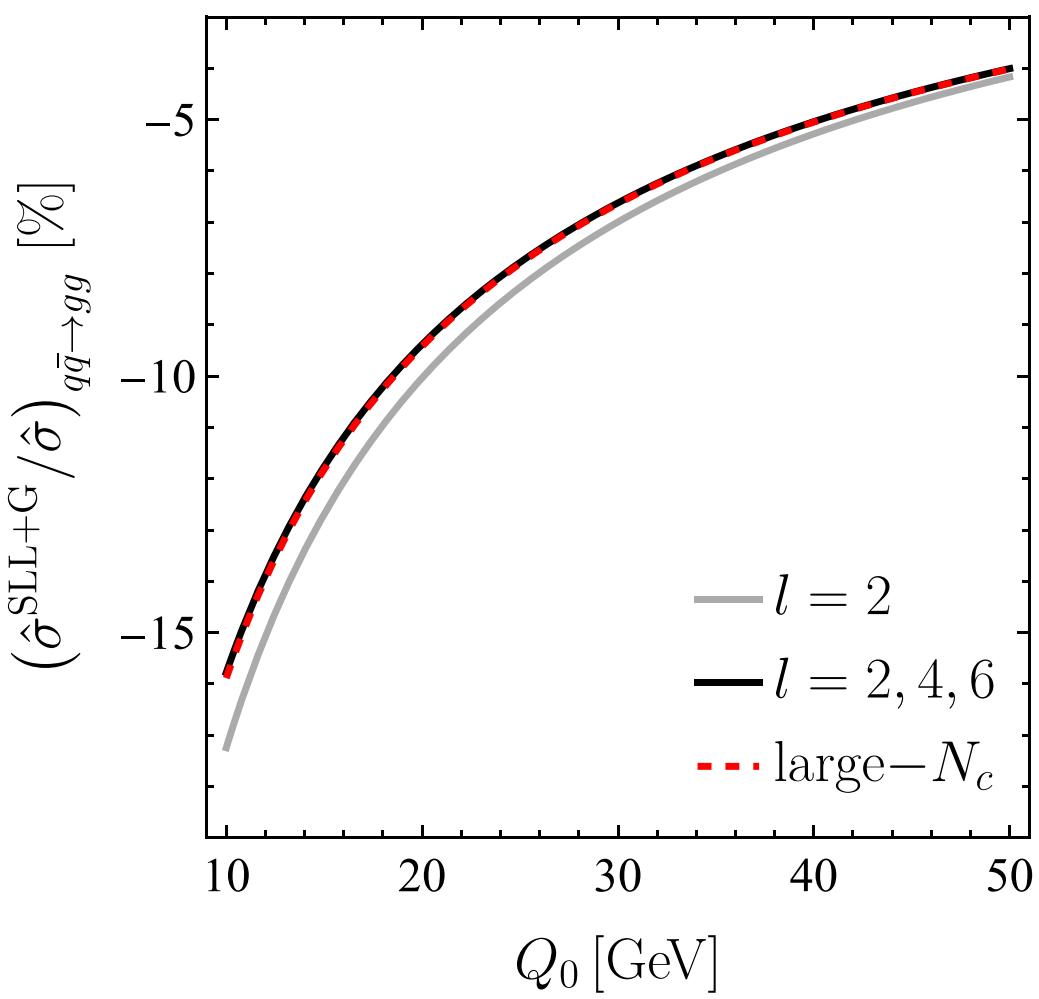
- resumed result for even $l = 2\ell$

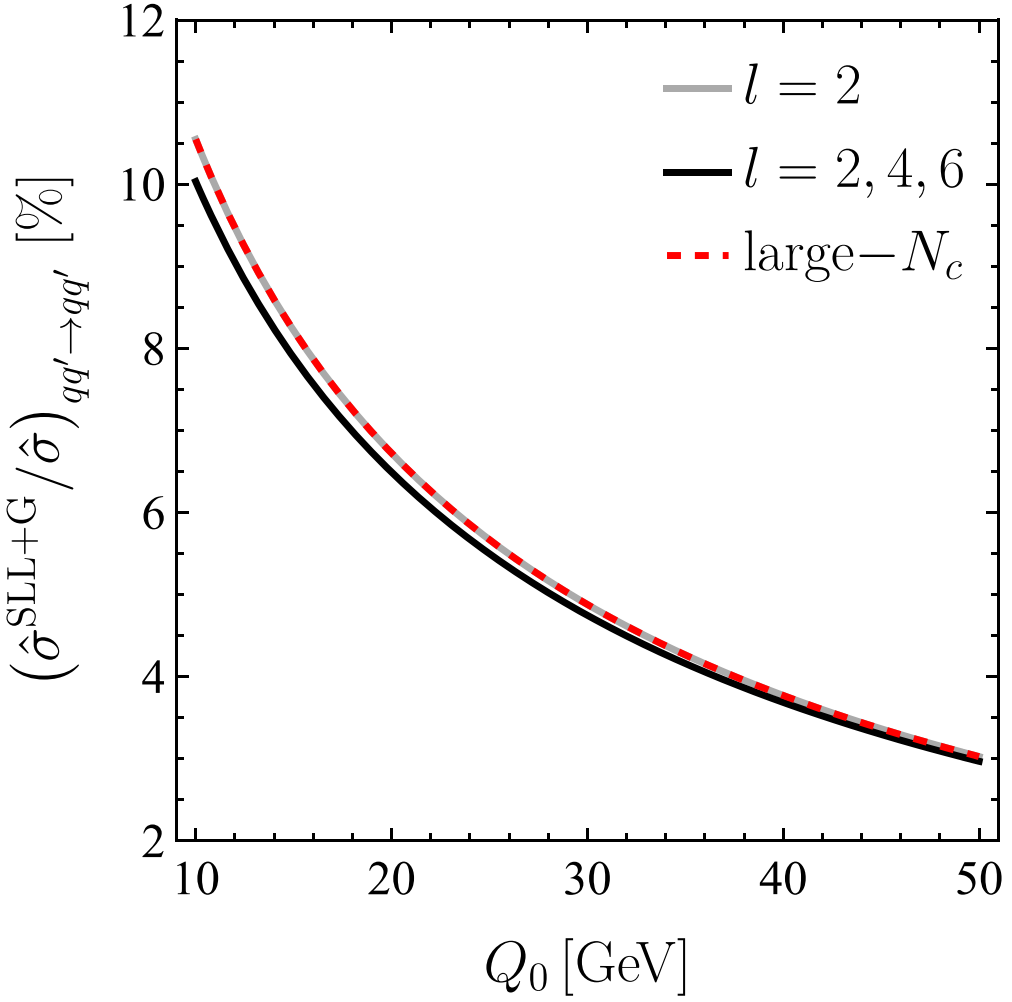
$$\sum_{\ell=1}^{\infty} \mathbb{U}_{\mathrm{SLL}}^{(2\ell)}(\mu_h, \mu_s)\, \varsigma = \frac{16\pi}{\beta_0^2}\, \frac{2\pi N_c}{\beta_0} \int_1^{x_s} \frac{dx_2}{x_2}\, \ln\frac{x_s}{x_2} \int_1^{x_2} \frac{dx_1}{x_1} \left[1 - 2\delta_{q\bar{q}}\, \sin^2\left(\frac{\pi N_c}{\beta_0}\, \ln\frac{x_2}{x_1}\right)\right]$$

$$\times \begin{pmatrix} 0 \\ \frac{1}{2}\, U^c(1; \mu_h, \mu_2) \\ -U^c(\frac{1}{2}, 1; \mu_h, \mu_1, \mu_2) \\ 2\left[U^c(1; \mu_h, \mu_2) - U^c(\frac{1}{2}, 1; \mu_h, \mu_1, \mu_2)\right] \\ \frac{2C_F}{N_c}\left[U^c(\frac{1}{2}, 1; \mu_h, \mu_1, \mu_2) - U^c(1; \mu_1, \mu_2)\right] \end{pmatrix}$$

- SLLs are included exactly

LARGE-N_c RESUMMATION





CONCLUSION

- performed resummation of SLLs in RG-improved perturbation theory
  - reliable estimates for perturbative uncertainties
  - generalizes straightforwardly to whole Glauber series

CONCLUSION

- performed resummation of SLLs in RG-improved perturbation theory
  - reliable estimates for perturbative uncertainties
  - generalizes straightforwardly to whole Glauber series

- demonstrated parametric suppression of higher-Glauber terms $\sim \alpha_s^{l/2}$

CONCLUSION

- performed resummation of SLLs in RG-improved perturbation theory
  - reliable estimates for perturbative uncertainties
  - generalizes straightforwardly to whole Glauber series

- demonstrated parametric suppression of higher-Glauber terms $\sim \alpha_s^{l/2}$

- obtained simple expression for resumed Glauber series in the large-N_c limit
  - contains only twofold scale integral for quark-initiated processes
  - extension to processes with gluons is possible (fourfold integral)

THANK YOU FOR YOUR ATTENTION!

BACKUP

LARGE-N_c RESUMMATION WITH GLUONS

- in large-N_c limit for processes with gluons in the initial state

$$\mathbb{V}^G\,\mathbb{U}^c(\mu_i,\mu_j) = \mathbb{V}_1^G\,U^c(1;\mu_i,\mu_j) + \mathbb{V}_{3/2}^G\,U^c(\tfrac{3}{2};\mu_i,\mu_j) + \mathbb{V}_2^G\,U^c(2;\mu_i,\mu_j) + \mathcal{O}(1/N_c^2)$$

- coefficient matrices are “ordered”

$$\mathbb{V}_1^G\,\mathbb{V}_{3/2}^G = 0 \qquad \mathbb{V}_{3/2}^G\,\mathbb{V}_2^G = 0 \qquad \mathbb{V}_1^G\,\mathbb{V}_2^G = 0 \qquad \mathbb{V}_2^G\,\mathbb{V}_1^G = 0$$

LARGE-N_c RESUMMATION WITH GLUONS

- coefficient vector

$$\mathbb{U}^{(l)}_{\rm SLL}(\mu_h,\mu_s)\,\varsigma = (i\pi)^l\, N_c^{l-1}\,\frac{2^{l+3}}{\beta_0^{l+1}}\int_1^{x_s}\frac{dx_l}{x_l}\,\ln\frac{x_s}{x_l}\int_1^{x_l}\frac{dx_{l-1}}{x_{l-1}}\,\dots\int_1^{x_2}\frac{dx_1}{x_1}\,\mathbb{U}^c(\mu_h,\mu_1)$$
$$\times\Bigg[\left(\mathbb{V}_1^G\right)^{l-1}U^c(1;\mu_1,\mu_l)+\sum_{i=2}^{l-1}\left(\mathbb{V}_{3/2}^G\right)^{i-1}\left(\mathbb{V}_1^G\right)^{l-i}U^c(\tfrac{3}{2},1;\mu_1,\mu_i,\mu_l)$$
$$+\sum_{j=3}^{l-1}\sum_{i=2}^{j-1}\left(\mathbb{V}_2^G\right)^{i-1}\left(\mathbb{V}_{3/2}^G\right)^{j-i}\left(\mathbb{V}_1^G\right)^{l-j}U^c(2,\tfrac{3}{2},1;\mu_1,\mu_i,\mu_j,\mu_l)\Bigg]\,\varsigma$$