Factorization and resummation of heavy quark pair angular correlations

Ding-Yu Shao
Fudan University

SCET2024 Salamanca Apr 15, 2023

Gluon TMDs

- **The gluon transverse momentum dependent distributions (TMDs) are important towards understanding the transverse structure of the proton as well as QCD factorization**
- **Gauge link dependent gluon TMDs** (TMD handbook '23)

$$\tilde{f}_{g/h}^{\alpha\beta\,0(\mathrm{u})}(x,\mathbf{b}_T,\epsilon,\tau,xP^+) = \frac{1}{xP^+}\int \frac{\mathrm{d}b^-}{2\pi} e^{-ib^-(xP^+)}\langle h(P,S)\big|G^{+\alpha}(b^\mu)\mathscr{W}_{\sqsubset}(b^\mu,0)G^{+\beta}(0)\big|h(P,S)\rangle$$

- **E.g. gluon Sivers function: T-odd object; two gauge links; process dependence more involved**

$$f_{1T}^{\perp\,g[e\,p^\uparrow \to e'\,Q\bar{Q}\,X]}\left(x,p_T^2\right) = -f_{1T}^{\perp\,g[p^\uparrow\,p\to\gamma\,\gamma\,X]}\left(x,p_T^2\right)$$

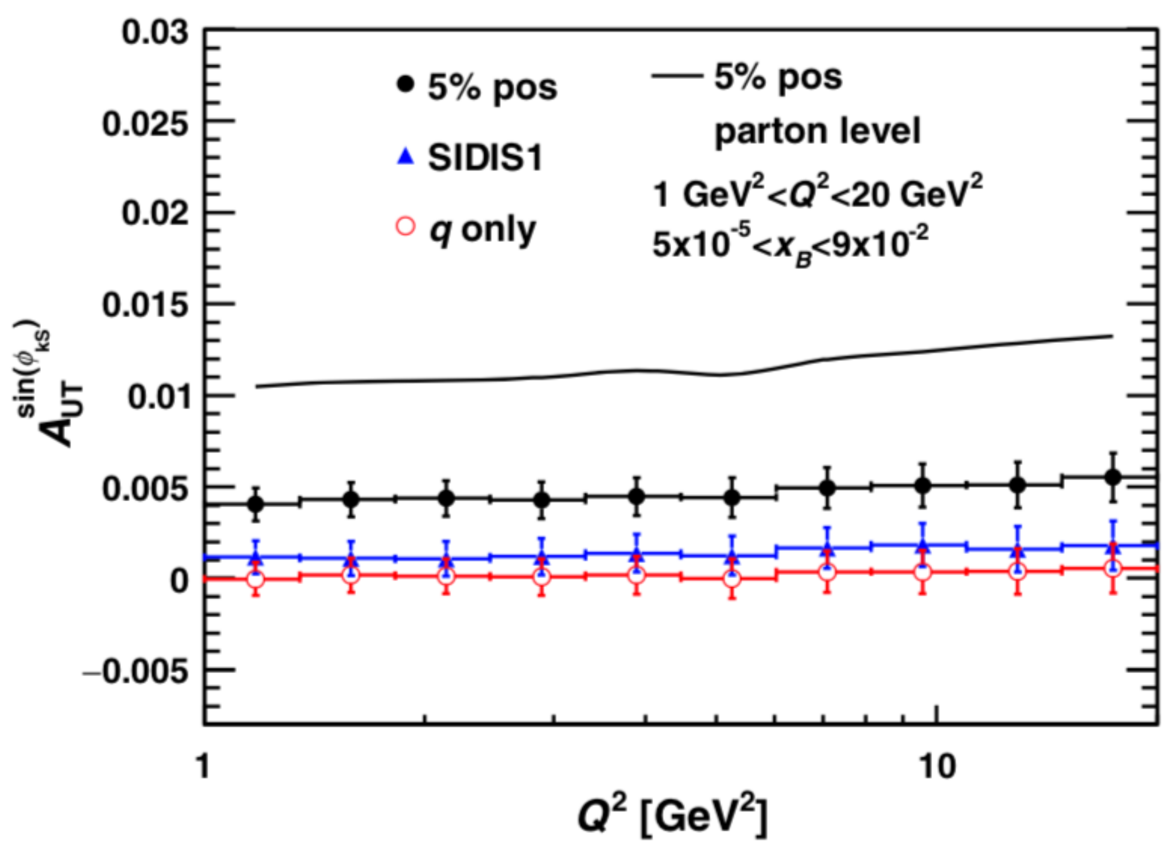
Gluon TMDs and spin asymmetry at the EIC

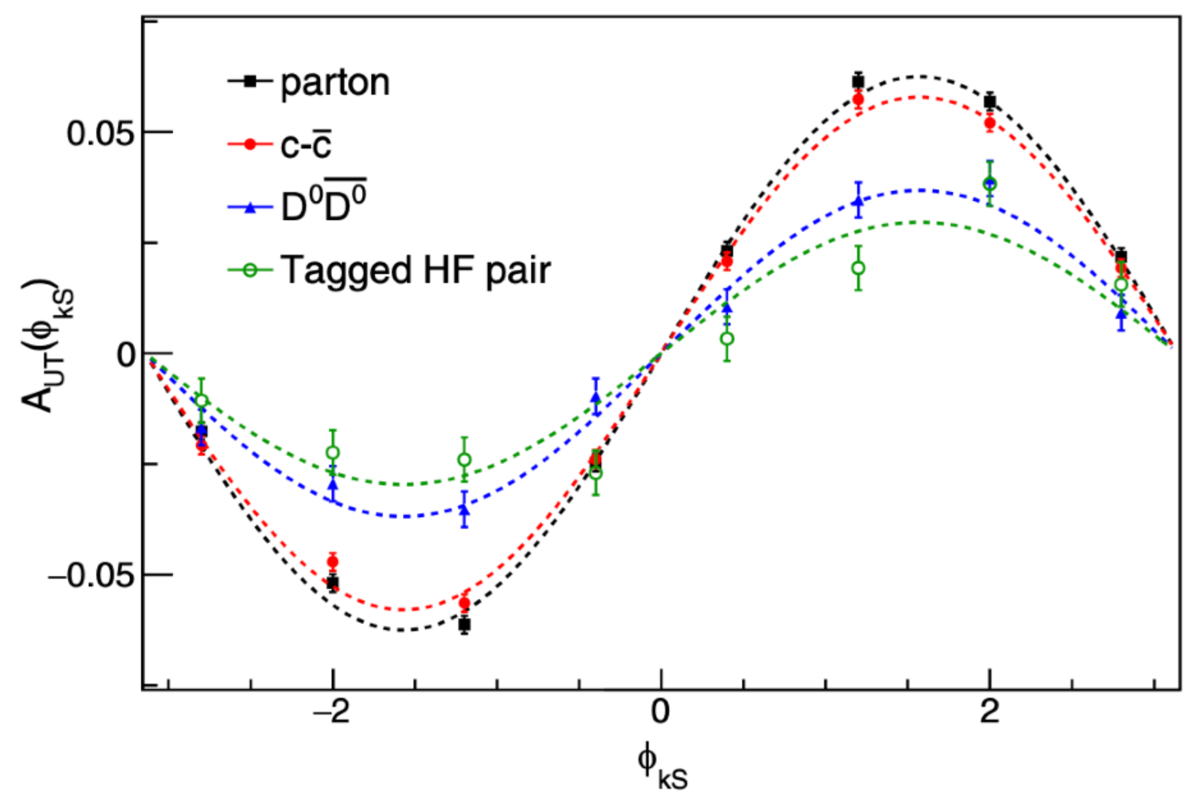
At the EIC , gluon TMDs can be probed via dihadron, open di-charm, di-D-meson and dijet

- **In the small x dijet process is the most promising channel** Zheng, Aschenauer, Lee, Xiao, Yin '18

  At the LO di-jet production in DIS involves two processes: $\gamma^* q \to qg \qquad \gamma^* g \to q\bar{q}$

- **The heavy quark pair production is dominated by the gluon channel at all x and gets only minor contribution from the quark channel** Dong, Ji, Kelsey, Radhakrishnan, Sichtermann, Zhao '23





TMD factorization of open heavy quark ($p_T >> m_Q$) in DIS

- **q_T factorization for heavy meson pair** (del Castillo, Echevarria, Makris, Scimemi ’20 ’22)
- **q_T factorization for heavy flavor dijet production, and study gluon Sivers asymmetry** (Kang, Reiten, DYS, Terry ’20)

also see Michel’s talk

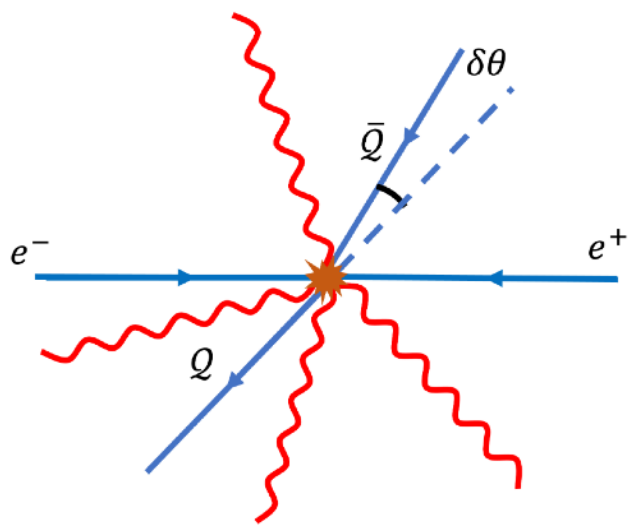
- **heavy quark q_T fragmentations** (Dai, Kim, Leibovich ’23; Kuk, Michel, Sun ’23 ’24)
- **The anomalous dimensions of the q_T soft function are divergent as** $\phi_x = \pi/2$

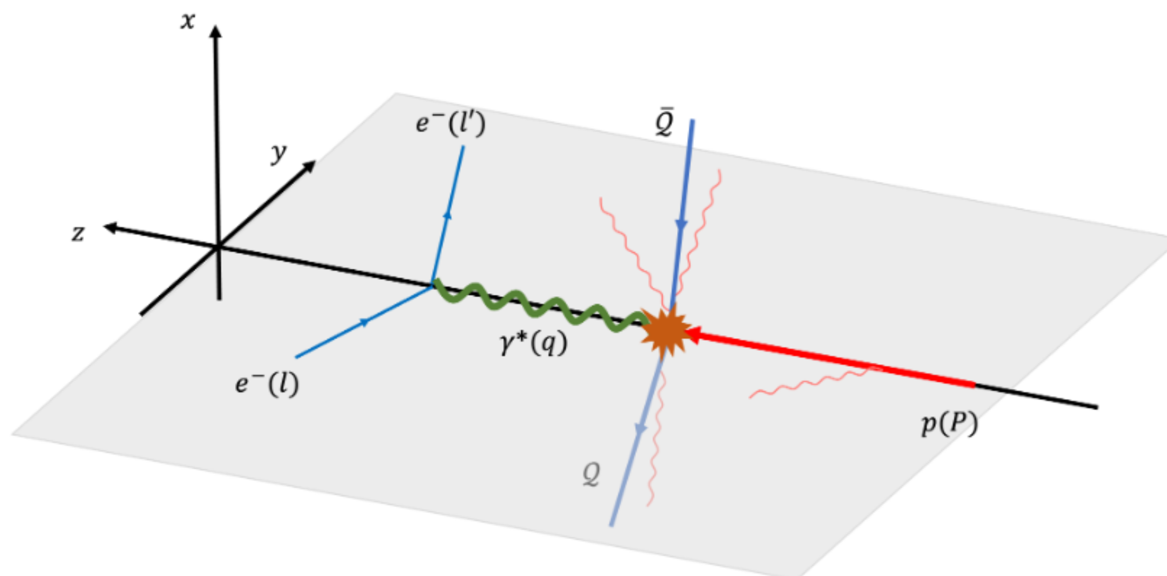
$$\gamma^{S_{\rm global}} \sim \frac{\alpha_s C_F}{\pi}\left[\ln\left(4\cos^2\phi_x\right) - i\pi\,{\rm sign}\left(\cos\phi_x\right)\right]$$

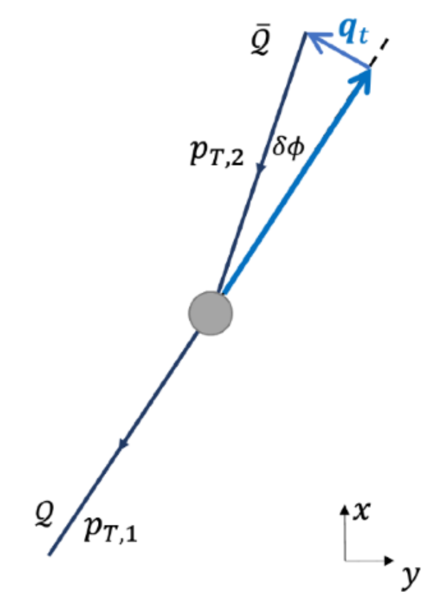
- **In addition to q_T, one can also study angle correlation to probe TMDs**
  - **A better angle on SIDIS** (Gao, Michel, Stewart, Sun ’22)
  - **Lepton jet correction in DIS** (Liu, Ringer, Vogelsang, Yuan ’19, Arratia, Kang, Prokudin, Ringer ’19, Fang, Ke, DYS, Terry ‘23)
- **Discussions on azimuthal and radial correlation can be found in** (Chien, Rahn, DYS, Waalewijn & Wu ’22 + in progress)

Factorization and resummation of heavy quark pair angular correlations

Dai, Jiang, DYS in progress



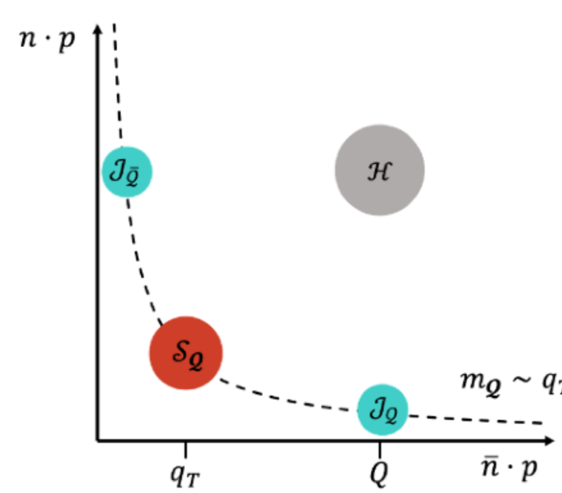


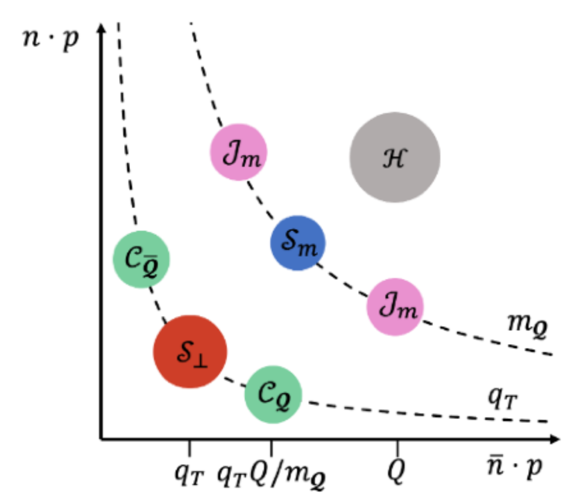


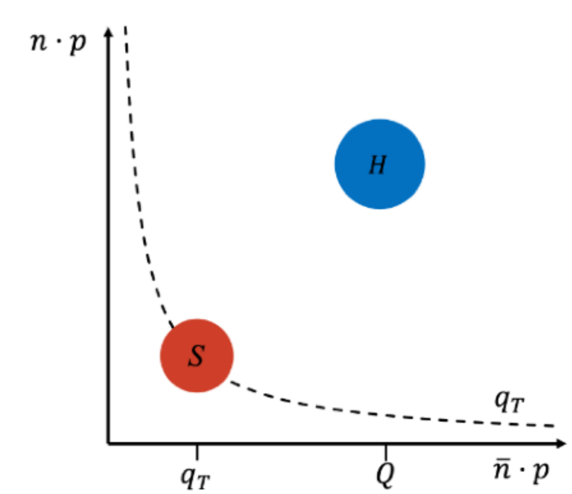
$$\text{Region 1}: \; Q \gg m_Q \sim q_T,$$

$$\text{Region 2}: \; Q \gg m_Q \gg q_T,$$

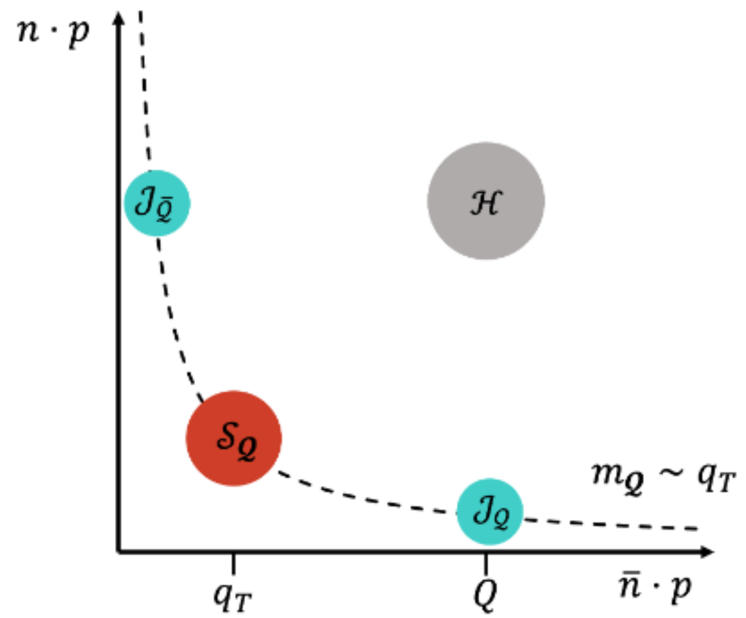
$$\text{Region 3}: \; Q \sim m_Q \gg q_T.$$







Factorization in region 1 $Q \gg m_Q \sim q_T$



$$
\begin{aligned}
\text{hard:} \quad & p_h^\mu \sim Q\,(1,1,1) \\
\text{collinear:} \quad & p_c^\mu \sim Q\,(1,\lambda^2,\lambda) \\
\text{soft:} \quad & p_s^\mu \sim q_T\,(1,1,1)
\end{aligned}
$$

Match the quark current operator onto the SCET operator

$$\bar{\psi}\gamma^\mu\psi \to \mathcal{J}_{\rm SCET} = \bar{\chi}_{\bar{n}} S_{\bar{n}}^\dagger \gamma_\perp^\mu S_n \chi_n$$

Factorization formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_T} = \sigma_0 \mathcal{H}(Q,\mu) \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T} \sum_f e_f^2 \mathcal{J}_{Q/f}\left(b_T, m_Q, \mu, \zeta/\nu^2\right) \mathcal{J}_{\bar{Q}/\bar{f}}\left(b_T, m_Q, \mu, \zeta/\nu^2\right) \mathcal{S}_Q\left(b_T, m_Q, \mu, \nu\right)$$

Since *Q* >> *m*$_Q$, quark mass will not contribute to the hard function

Jet and soft function depend on two scales: *q*$_T$ and *m*$_Q$

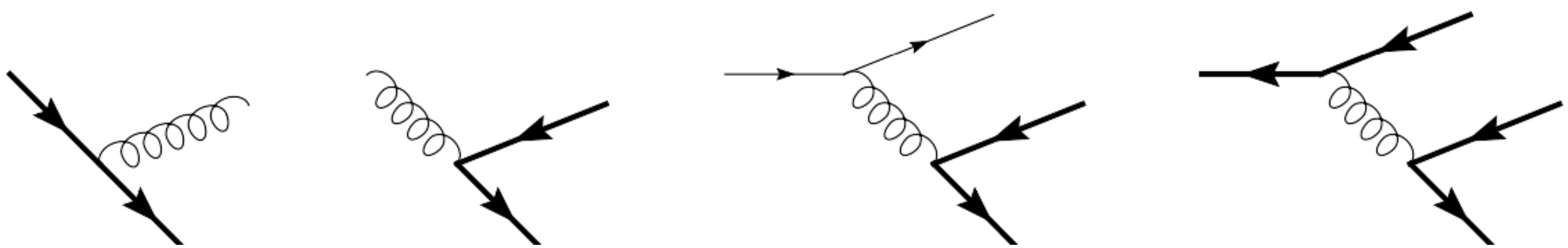
Factorization in region 1 $Q \gg m_{\mathcal{Q}} \sim q_T$

The Jet function can be expressed in terms of heavy quark TMD fragmentation function Dai, Kim, Leibovich ’23; Kuk, Michel, Sun ’24; also see Michel’s talk

$$\mathcal{F}_{\mathcal{Q}/f}(z_{\mathcal{Q}}, P_{\mathcal{Q},\perp}) = \sum_X \frac{1}{z_{\mathcal{Q}}} \int \frac{\mathrm{d}x_+}{4\pi} \frac{\mathrm{d}^{d-2}x_\perp}{(2\pi)^{d-2}} e^{ix_+ P_{\mathcal{Q},-}/(2z_{\mathcal{Q}})} \, \mathrm{Tr} \langle 0 | \frac{\not{\bar{n}}}{2} \chi^f_{\bar{n}}(x_+, 0, x_\perp) \, |\mathcal{Q}(P_{\mathcal{Q}}), X\rangle \langle \mathcal{Q}(P_{\mathcal{Q}}), X| \, \bar{\chi}^f_{\bar{n}}(0) \, |0\rangle$$

quark mass m_Q provides an infrared cutoff

Examples of fragmentation processes converting a parton a into a heavy quark

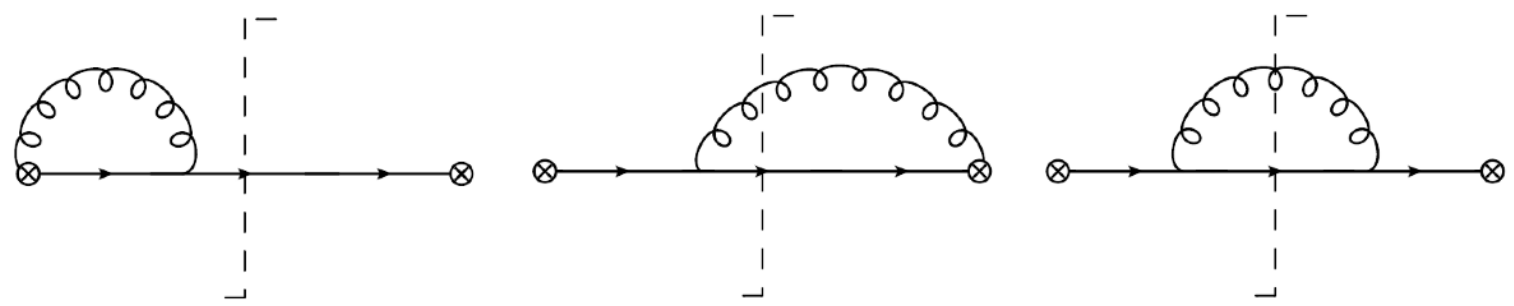


Define jet function (del Castillo, Echevarria, Makris, Scimemi ’20 ’22)

$$\mathcal{J}_{\mathcal{Q}/f}\,(b_\perp, m_{\mathcal{Q}}, \mu, \nu) = \int_0^1 \mathrm{d}z_{\mathcal{Q}}\, z_{\mathcal{Q}}^{2-d} \int \mathrm{d}^{d-2} P_{\mathcal{Q},\perp} e^{ib_\perp \cdot P_{\mathcal{Q},\perp}/z_{\mathcal{Q}}} \mathcal{F}_{\mathcal{Q}/f}(z_{\mathcal{Q}}, P_{\mathcal{Q},\perp})$$

Factorization in region 1 $Q \gg m_\mathcal{Q} \sim q_T$

NLO Jet function



$$\mathcal{J}_{\mathcal{Q}/\mathcal{Q}} = 1 + \frac{\alpha_s^{(n_f)} C_F}{4\pi}\left[\ln\frac{\mu^2 b_T^2}{b_0^2}\left(3 + 2\ln\frac{\nu^2}{Q^2}\right) + 4\ln^2\left(\frac{b_T\, m_\mathcal{Q}}{b_0}\right) - 2\ln\left(\frac{b_T\, m_\mathcal{Q}}{b_0}\right) + 4 + \frac{\pi^2}{3} + j_0(b_T\, m_\mathcal{Q})\right]$$

Auxiliary function (vanishes in the heavy quark limit *m_Q >> q_T*) $\boxed{j_0(x) \xrightarrow{x\to\infty} 0.}$

$$j_0(x) = 2\int_0^\infty dt\, e^{-xt}\left[\frac{2x^2t^2 \arccos\left(t - i0^+\right)}{\sqrt{1-t^2+i0^+}} - \frac{3t\sqrt{1-t^2+i0^+}(t^2-2) + (8-9t^2+4t^4)\arccos\left(t-i0^+\right)}{(1-t^2+i0^+)^{5/2}}\right]$$

Anomalous dimension reads

$$\gamma_\mu^{\mathcal{J}_\mathcal{Q}} = C_F\gamma^{\text{cusp},(n_f)}\ln\frac{\nu^2}{Q^2} + \gamma^{\mathcal{J},(n_f)} \qquad \gamma_\nu^{\mathcal{J}_\mathcal{Q}} = -2C_F A_\Gamma^{(n_f)}(b_0/b_T,\mu) - \frac{C_F}{2}\gamma_\nu^{(n_f)}$$

Factorization in region 1 $Q \gg m_{\mathcal{Q}} \sim q_T$

Soft function Pietrulewicz, Samitz, Spiering, Tackmann ‘17

$$\mathcal{S}_{\mathcal{Q}}\left(b_{\perp}, m_{\mathcal{Q}}\right)=\frac{1}{N_c} \mathrm{Tr}\left\langle 0\right| \bar{\mathrm{T}}\left[S_n^{\dagger}\left(b_{\perp}\right) S_{\bar{n}}\left(b_{\perp}\right)\right] \mathrm{T}\left[S_{\bar{n}}^{\dagger}(0) S_n(0)\right]|0\rangle$$

Different from the standard TMD soft function due to heavy-quark corrections. The difference arises starting at the two loop

Anomalous dimension reads

$$\gamma_\mu^{\mathcal{S}}=-2 C_F \gamma^{\mathrm{cusp},(n_f)} \ln \frac{\nu^2}{\mu^2}+\gamma^{\mathcal{S}}, \quad \gamma_\nu^{\mathcal{S}}=4 C_F A_{\Gamma}^{(n_f)}\left(b_0 / b_T, \mu\right)+C_F \gamma_\nu$$

RG and RRG consistency relation

$$\gamma_\mu^{\mathcal{H}}+\gamma_\mu^{\mathcal{S}_{\mathcal{Q}}}+2 \gamma_\mu^{\mathcal{J}_{\mathcal{Q}}}=0 \qquad \gamma_\nu^{\mathcal{S}_{\mathcal{Q}}}+2 \gamma_\nu^{\mathcal{J}_{\mathcal{Q}}}=0$$

Resummation in region 1 $Q \gg m_{\mathcal{Q}} \sim q_T$

We apply Collins-Soper-Sterman treatment to resum rapidity logs (TMD handbook ’23)

$$J_{\mathcal{Q}/f}(b_T, m_{\mathcal{Q}}, \mu, \zeta) \equiv \mathcal{J}_{\mathcal{Q}/f}(b_T, m_{\mathcal{Q}}, \mu, \zeta/\nu^2)\sqrt{\mathcal{S}_{\mathcal{Q}}(b_T, m_{\mathcal{Q}}, \mu, \nu)}$$

Evolve the jet function from the pair of initial and final scales $\{\mu_b, \zeta_i\} \to \{\mu, \zeta_f\}$

$$J_{\mathcal{Q}/f}(b_T, m_{\mathcal{Q}}, \mu, \zeta_f) = \exp\left[\int_{\mu_b}^{\mu} \frac{\mathrm{d}\,\bar{\mu}}{\bar{\mu}} \gamma_{\mu}^{J_{\mathcal{Q}}}(\bar{\mu}, \zeta_f)\right] \left(\frac{\zeta_f}{\zeta_i}\right)^{\frac{1}{2}\gamma_{\zeta}^{J_{\mathcal{Q}}}(\mu_b, b_T)} J_{\mathcal{Q}/f}(b_T, m_{\mathcal{Q}}, \mu_b, \zeta_i)$$

We have the resummation formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\delta\theta} = \frac{\sigma_0 Q^2 \delta\theta}{4} \mathcal{H}(Q, \mu_h) \int_0^{\infty} b_T\, \mathrm{d}b_T\, J_0(Q\delta\theta b_T/2) \sum_f e_f^2 J_{\mathcal{Q}/f}(b_T, m_{\mathcal{Q}}, \mu_h, \zeta_f)\, J_{\bar{\mathcal{Q}}/\bar{f}}(b_T, m_{\mathcal{Q}}, \mu_h, \zeta_f)$$

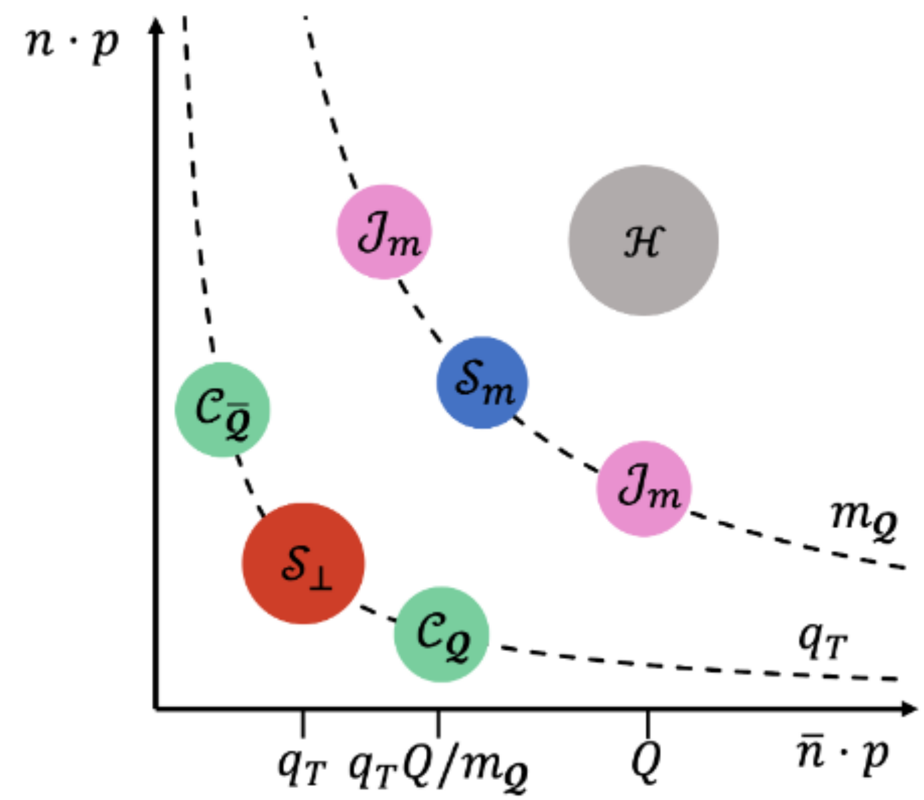
The scale choice $\mu_h = \sqrt{\zeta_f} = Q \qquad \mu_b = \sqrt{\zeta_i} = \dfrac{b_0}{b_T}$

Factorization in region 2 $Q \gg m_Q \gg q_T$

- **In the limit $m_Q \gg q_T$, jet function in region 1 contains** $\alpha_s^n \log^m(m_Q/q_T)$
- **The factorization formula in region 1 with active flavors** $n_f = n_l + 1$ **should be matched onto a theory with** n_l **active flavors**

  E.g. matching relation of α_s **from region 1 to 2**

$$\alpha_s^{(n_f)}(\mu) = \alpha_s^{(n_l)}(\mu)\left[1 + \frac{4}{3}T_F\frac{\alpha_s^{(n_l)}(\mu)}{4\pi}\ln\frac{\mu^2}{m_Q^2} + \mathcal{O}(\alpha_s^2)\right]$$



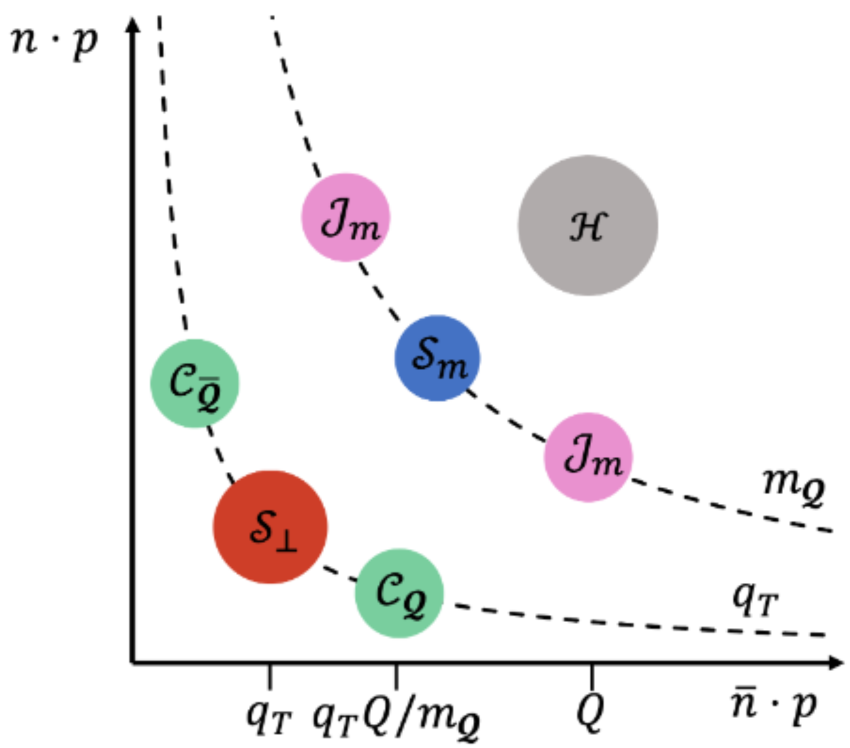
- **Heavy quark momenta** $P_Q^\mu = m_Q v_+^\mu + p^\mu$

$$v_+^\mu = \left(\frac{Q}{m_Q}, \frac{m_Q}{Q}, 0_\perp\right), \quad p_{uc}^\mu \sim q_T\left(\frac{Q}{m_Q}, \frac{m_Q}{Q}, 1\right)$$

- **Decouple the interaction between heavy quark and ultra-collinear modes; Match SCET onto bHQET** Fleming, Hoang, Mantry & Stewart '07

$$h_{v_\pm} \to Y_{v_\pm} h_{v_\pm} \qquad \mathcal{J}_{\mathrm{SCET}} \to \mathcal{J}_{\mathrm{bHQET}} = \bar{h}_{v_-} W_{\bar{n}}^\dagger Y_{v_-}^\dagger \gamma_\perp^\mu Y_{v_+} W_n h_{v_+}$$

Factorization in region 2 $Q \gg m_{\mathcal{Q}} \gg q_T$



hard: $p_h^\mu \sim Q\,(1,1,1)$

collinear: $p_c^\mu \sim Q\,(1,\lambda^2,\lambda)$

massive-soft: $p_{ms}^\mu \sim Q\,(\lambda,\lambda,\lambda)$

soft: $p_s^\mu \sim q_T\,(1,1,1)$

ultra-collinear: $p_{uc}^\mu \sim q_T/\lambda\,(1,\lambda^2,\lambda)$

$$\lambda = m_{\mathcal{Q}}/Q \gg q_T/Q$$

Factorization formula

Matching coefficients form SCET to bHQET

Hoang, Pathak, Pietrulewicz &Stewart, '15

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_T} = \sigma_{\mathcal{Q}} \mathcal{H}(Q,\mu)\, \boxed{\mathcal{J}_m^2(m_{\mathcal{Q}},\mu,\zeta_{\mathcal{J}}/\nu^2)\, \mathcal{S}_m(m_{\mathcal{Q}},\mu,\nu)}$$
$$\times \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\, \mathcal{C}_{\mathcal{Q}}(b_T,\mu,\zeta_{\mathcal{C}}/\nu^2)\, \mathcal{C}_{\bar{\mathcal{Q}}}(b_T,\mu,\zeta_{\mathcal{C}}/\nu^2)\, \boxed{\mathcal{S}_\perp(b_T,\mu,\nu)}$$

Standard TMD soft function

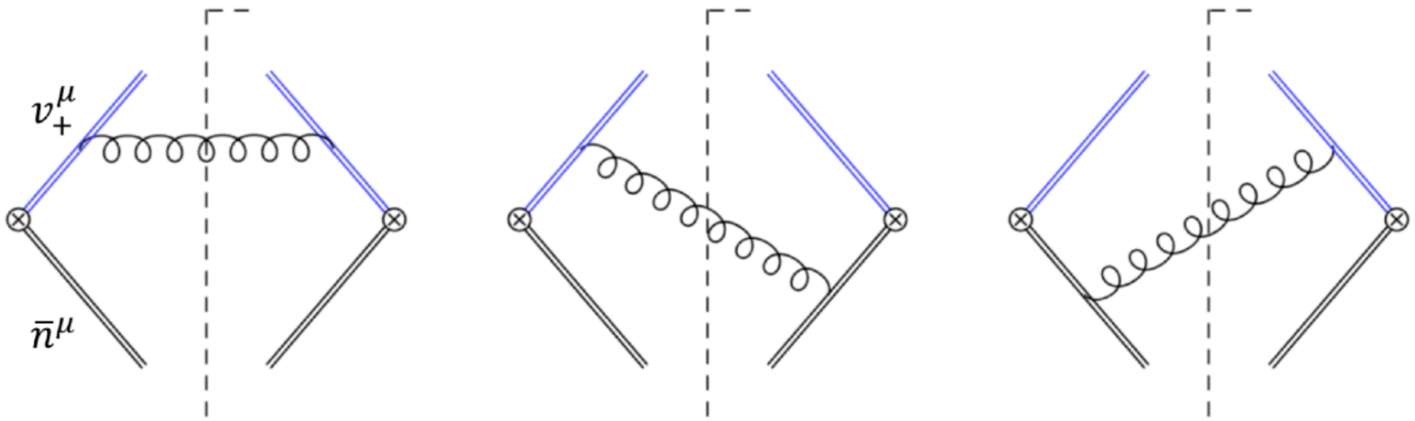
Factorization in region 2 $Q \gg m_{\mathcal{Q}} \gg q_T$

Definition of ultra-collinear function

$$\mathcal{C}_{\mathcal{Q}}\left(b_T, \mu, \zeta_{\mathcal{C}}/\nu^2\right) = \frac{1}{N_c} \mathrm{Tr}\,\langle 0|\bar{\mathrm{T}}[W_n^\dagger\left(b_\perp\right) Y_{v_+}\left(b_\perp\right)]\mathrm{T}[Y_{v_+}^\dagger\left(0\right) W_n\left(0\right)]|0\rangle$$

$$\boxed{v_+^\mu = \frac{Q}{m_{\mathcal{Q}}}\frac{n^\mu}{2} + \frac{m_{\mathcal{Q}}}{Q}\frac{\bar{n}^\mu}{2}}$$

One-loop expression



$$\mathcal{C}_{\mathcal{Q}}^{\mathrm{bare}} = 1 + \frac{\alpha_s^{(n_l)} C_F}{4\pi}\left[\left(\frac{2}{\eta} + \ln\frac{\nu^2 m_{\mathcal{Q}}^2}{Q^2\mu^2}\right)\left(\frac{2}{\epsilon} + 2L_b\right) - \frac{2}{\epsilon^2} + \frac{2}{\epsilon} + L_b^2 + 2L_b + \frac{\pi^2}{6}\right]$$

Anomalous dimensions (n_l flavor)

$$\gamma_\mu^{\mathcal{C}_{\mathcal{Q}}} = -C_F\gamma^{\mathrm{cusp},(n_l)} \ln\frac{\mu^2 Q^2}{\nu^2 m_{\mathcal{Q}}^2} + \gamma^{\mathcal{C},(n_l)}$$

$$\gamma_0^{\mathcal{C}} = 4C_F$$

Refactorization of jet function in region 1

$$\mathcal{J}_{\mathcal{Q}/\mathcal{Q}}(b_T, m_{\mathcal{Q}}) \xrightarrow{m_{\mathcal{Q}} \gg q_T} \mathcal{J}_m(m_{\mathcal{Q}})\mathcal{C}_{\mathcal{Q}}(b_T)$$

Factorization in region 2 $Q \gg m_{\mathcal{Q}} \gg q_T$

RG consistency relation

hard:	$p_h^\mu \sim Q\,(1,1,1)$
collinear:	$p_c^\mu \sim Q\,(1,\lambda^2,\lambda)$
massive-soft:	$p_{ms}^\mu \sim Q\,(\lambda,\lambda,\lambda)$
soft:	$p_s^\mu \sim q_T\,(1,1,1)$
ultra-collinear:	$p_{uc}^\mu \sim q_T/\lambda\,(1,\lambda^2,\lambda)$

$$\gamma_\mu^H + 2\gamma_\mu^{\mathcal{J}_m} + \gamma_\mu^{\mathcal{S}_m} + \gamma_\mu^{\mathcal{S}} + 2\gamma_\mu^{\mathcal{C}_\mathcal{Q}} = 0,$$

$$\gamma_\nu^{\mathcal{S}_m} + 2\gamma_\nu^{\mathcal{J}_m} = 0, \qquad \gamma_\nu^{\mathcal{S}} + 2\gamma_\nu^{\mathcal{C}_\mathcal{Q}} = 0$$

$$\gamma_\mu^{\mathcal{H}} = 2C_F\gamma^{\text{cusp},(n_f)}\ln\frac{Q^2}{\mu^2} + 4\gamma^{q,(n_f)}$$

$$\gamma_\mu^{\mathcal{C}_\mathcal{Q}} = -C_F\gamma^{\text{cusp},(n_l)}\ln\frac{\mu^2 Q^2}{\nu^2 m_\mathcal{Q}^2} + \gamma^{\mathcal{C},(n_l)}$$

$$\gamma_\mu^{\mathcal{J}_m} = -C_F\gamma^{\text{cusp},(n_l)}\ln\frac{m_\mathcal{Q}^2}{\mu^2} + \gamma^{J_m,(n_l)} + \mathcal{O}(\alpha_s^2)$$

$$\gamma_\mu^{\mathcal{S}} = -2C_F\gamma^{\text{cusp},(n_l)}\ln\frac{\nu^2}{\mu^2} + \gamma^{\mathcal{S},(n_l)}$$

$$\gamma_\mu^{\mathcal{S}_m} = \mathcal{O}(\alpha_s^2)$$

Hoang, Pathak, Pietrulewicz & Stewart, ’15

Two-loop anomalous dimension of the ultra-collinear function

$$\gamma_1^{\mathcal{C}} = C_A C_F\left(22\zeta_3 - \frac{184}{27} - \frac{13\pi^2}{18}\right) - C_F T_F n_f\left(\frac{128}{27} + \frac{2\pi^2}{9}\right)$$

NNLL resummation ingredients are known in region 2

Resummation in region 2 $Q \gg m_{\mathcal{Q}} \gg q_T$

Evolve the jet and ultra-collinear function

$$J_m(m_{\mathcal{Q}}, \mu, \zeta_{J,f}) = \exp\left[\int_{\mu_m}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_{\mu}^{J_m}(\bar{\mu}, \zeta_{J,f})\right] \left(\frac{\zeta_{J,f}}{\zeta_{J,i}}\right)^{\frac{1}{2}\gamma_{\zeta}^{J_m}(m_{\mathcal{Q}}, \mu_j)} J_m(m_{\mathcal{Q}}, \mu_m, \zeta_{J,i})$$

$$C_{\mathcal{Q}}(b, \mu, \zeta_{C,f}) = \exp\left[\int_{\mu_b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_{\mu}^{C_{\mathcal{Q}}}(\bar{\mu}, \zeta_{C,f})\right] \left(\frac{\zeta_{C,f}}{\zeta_{C,i}}\right)^{\frac{1}{2}\gamma_{\zeta}^{C_{\mathcal{Q}}}(b, \mu_b)} C_{\mathcal{Q}}(b, \mu_b, \zeta_{C,i}).$$

We have the resummation formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\delta\theta} = \frac{\sigma_{\mathcal{Q}} Q^2 \delta\theta}{4} \mathcal{H}(Q, \mu_h) J_m^2(m_{\mathcal{Q}}, \mu_h, \zeta_{J,f}) \int_0^{\infty} b\, \mathrm{d}b\, J_0(Q\delta\theta b/2)\, C_{\mathcal{Q}}^2(b, \mu_h, \zeta_{S,f})$$

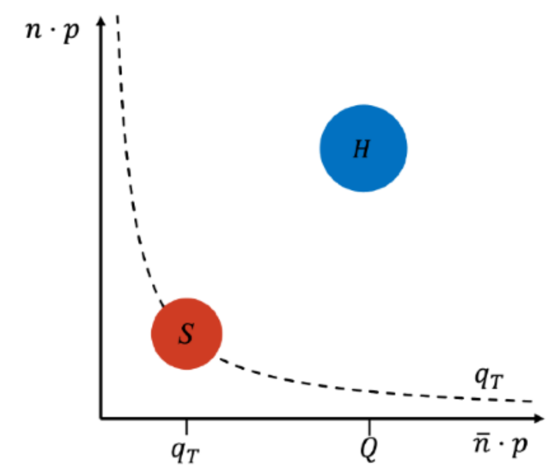
The scale choice

$$\mu_h = Q, \quad \mu_m = m_{\mathcal{Q}}, \quad \mu_b = b_0/b_T,$$

$$\zeta_{J,f} = Q^2, \quad \zeta_{J,i} = m_{\mathcal{Q}}^2, \quad \zeta_{C,f} = \frac{Q^2 \mu_b^2}{m_{\mathcal{Q}}^2}, \quad \zeta_{C,i} = \mu_b^2$$

Factorization in region 3 $Q \sim m_{\mathcal{Q}} \gg q_T$

- **In region 3, TMD factorization of heavy quark pair production are well studied**
  - **Factorization and resummation** (Li, Li, DYS, Yang, Zhu '12 '13 & Catani, Grazzini, Torre '14& Catani, Grazinni & Sargsyan '18)
  - **Two-loop soft function** Angeles-Martinez, M. Czakon, and S. Sapeta'18; Catani, Devoto, Grazzini, Mazzitelli '23
- **Factorization formula**



$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_T} = \sigma_{\mathcal{Q}} H(Q, m_{\mathcal{Q}}, \mu) \int \frac{\mathrm{d}^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{q}_T}\, S(b_T, \beta_Q, \mu)$$

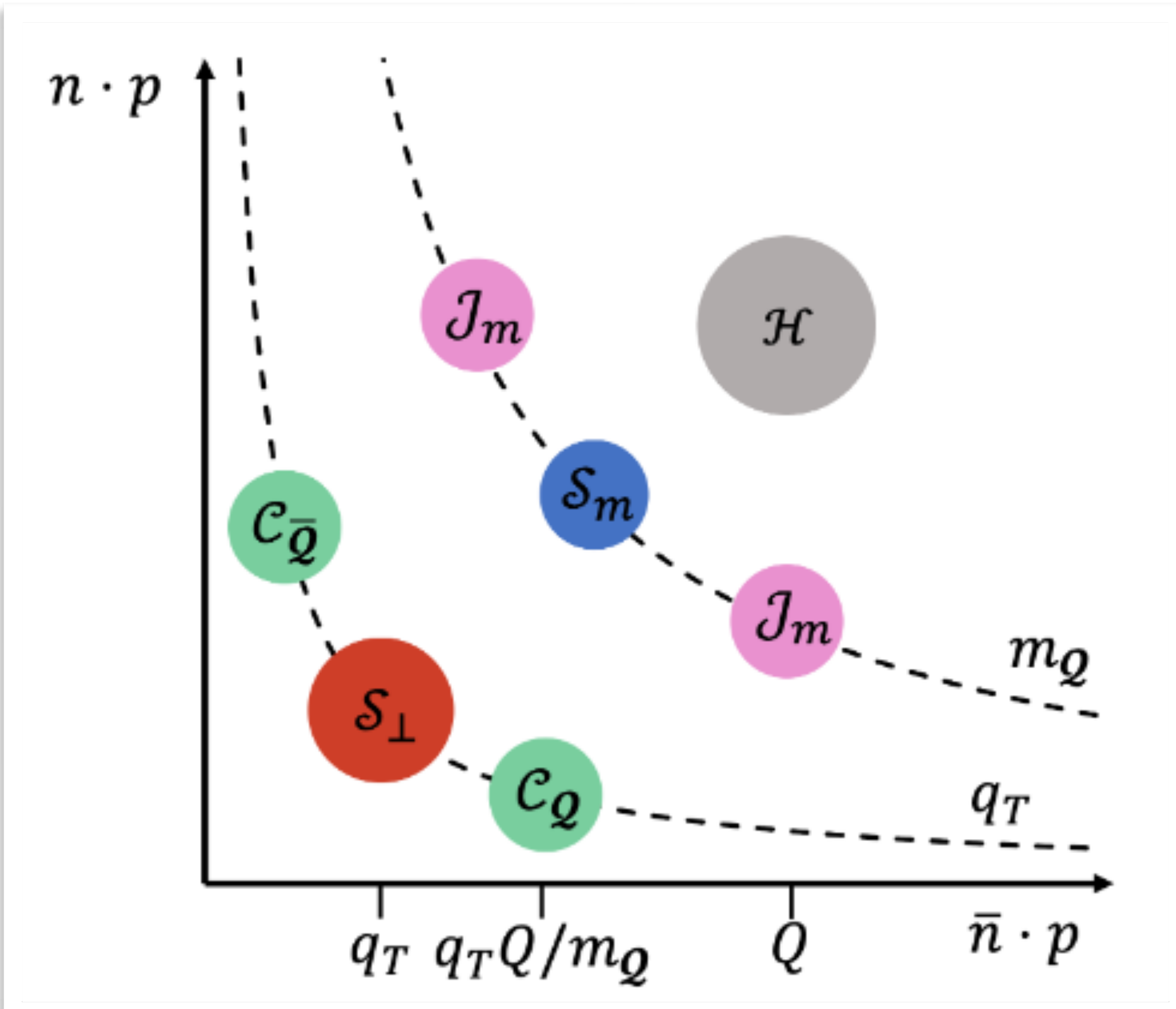
- **We use their expressions to verify refactorization of soft function at two loop**

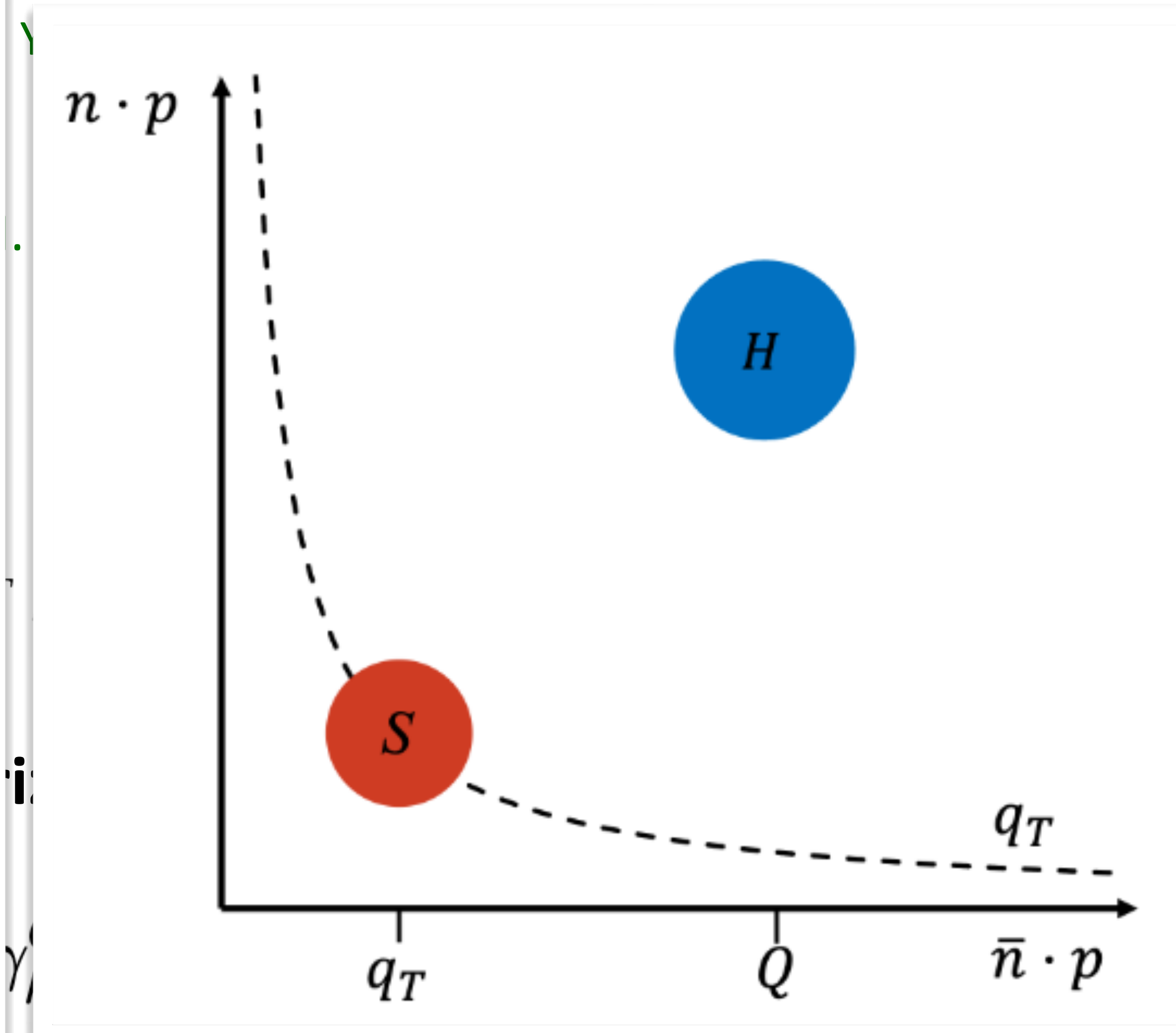
$$\gamma_\mu^S \xrightarrow{Q \gg m_{\mathcal{Q}}} 2\gamma_\mu^{\mathcal{C}_{\mathcal{Q}}} + \gamma_\mu^{\mathcal{S}}$$

- **Two loop ultra-collinear function can be determined based on** $S \xrightarrow{Q \gg m_{\mathcal{Q}}} \mathcal{C}_{\mathcal{Q}}^2 \mathcal{S}_\perp$

Factorization in region 3 $Q \sim m_Q \gg q_T$

- **In region 3, TMD factorization of heavy quark pair production are well studied**

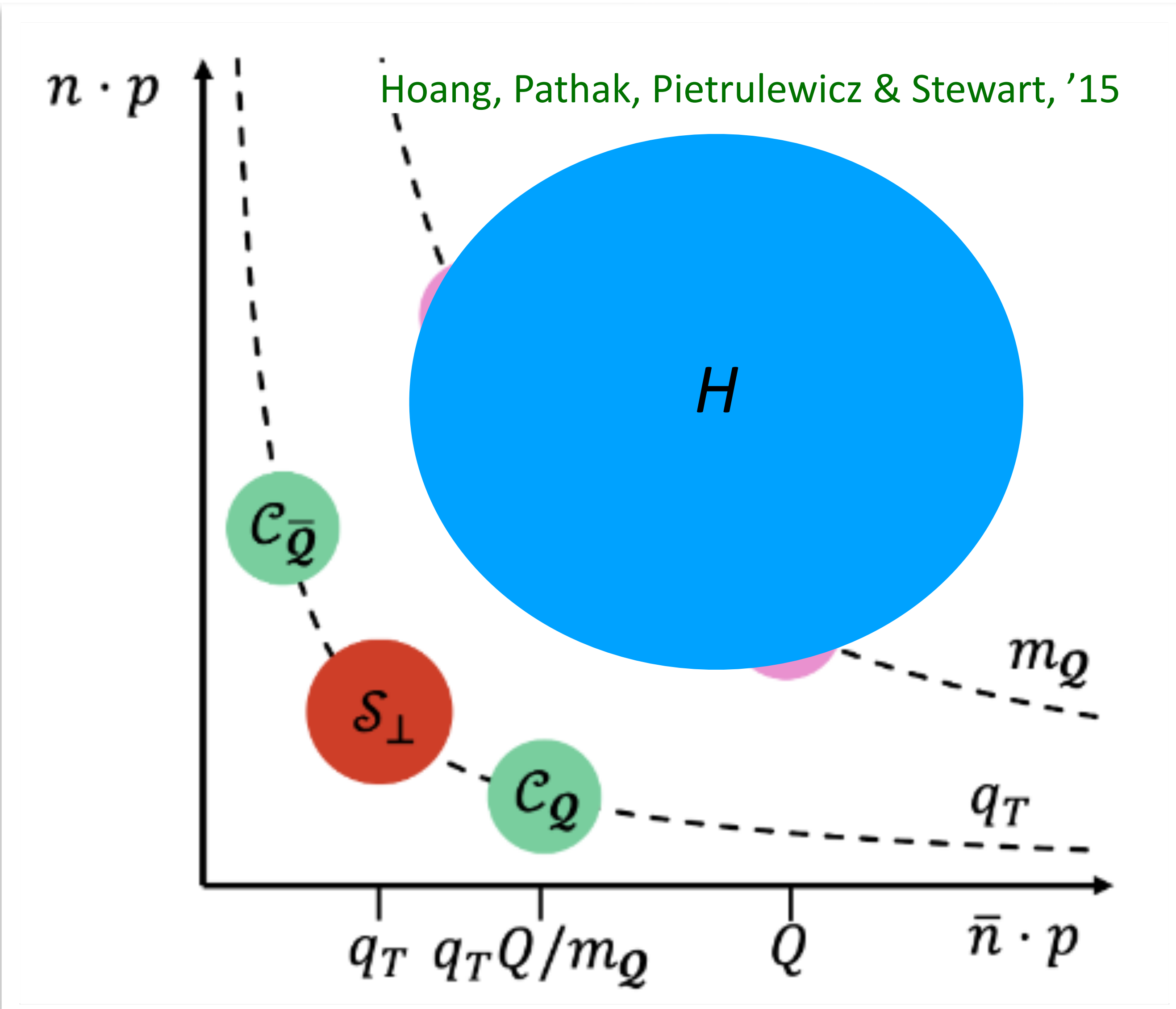


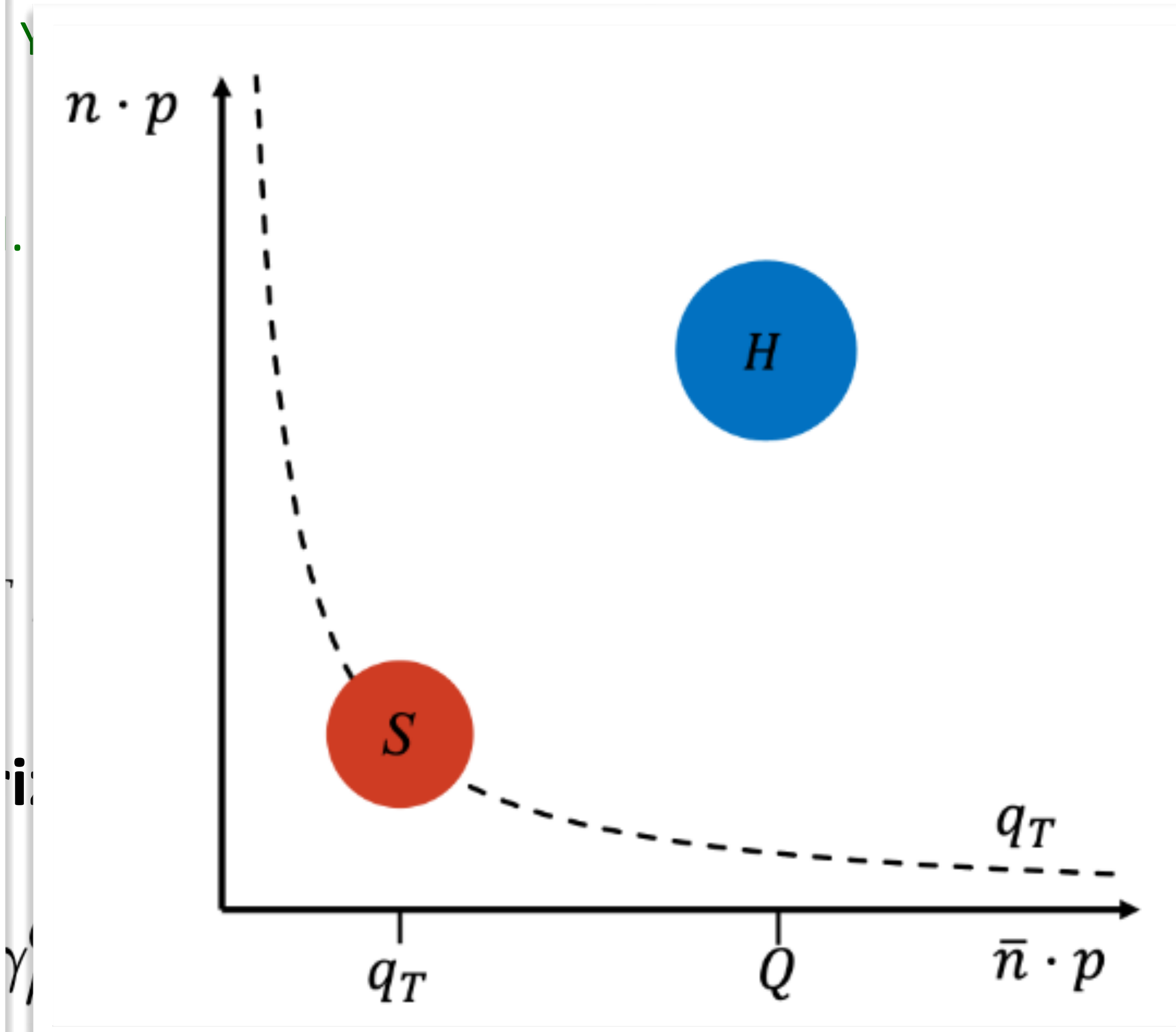


-
-
- **Two loop ultra-collinear function can be determined based on** $S \xrightarrow{Q \gg m_Q} \mathcal{C}_Q^2 \mathcal{S}_\perp$

Factorization in region 3 $Q \sim m_Q \gg q_T$

- **In region 3, TMD factorization of heavy quark pair production are well studied**

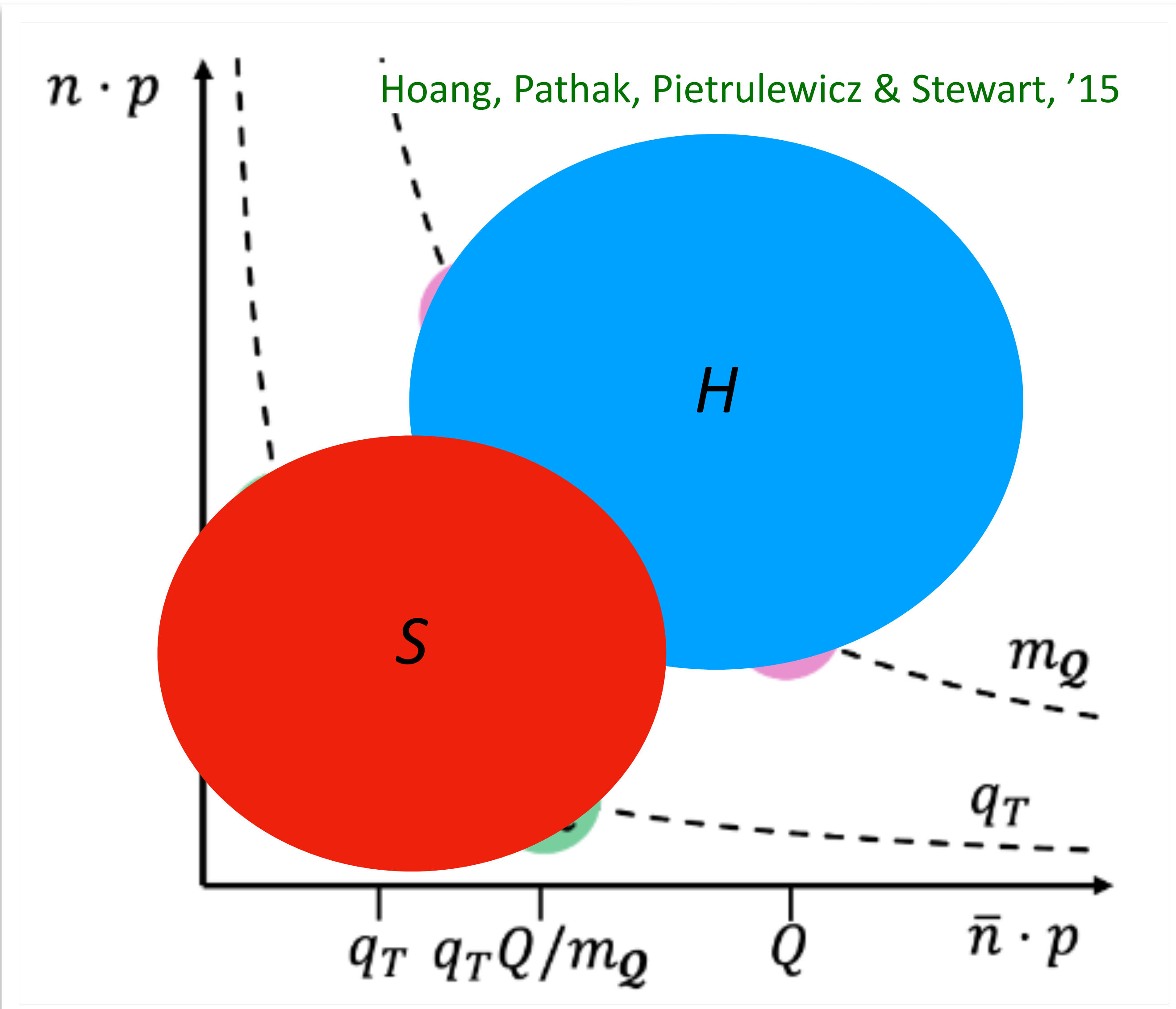


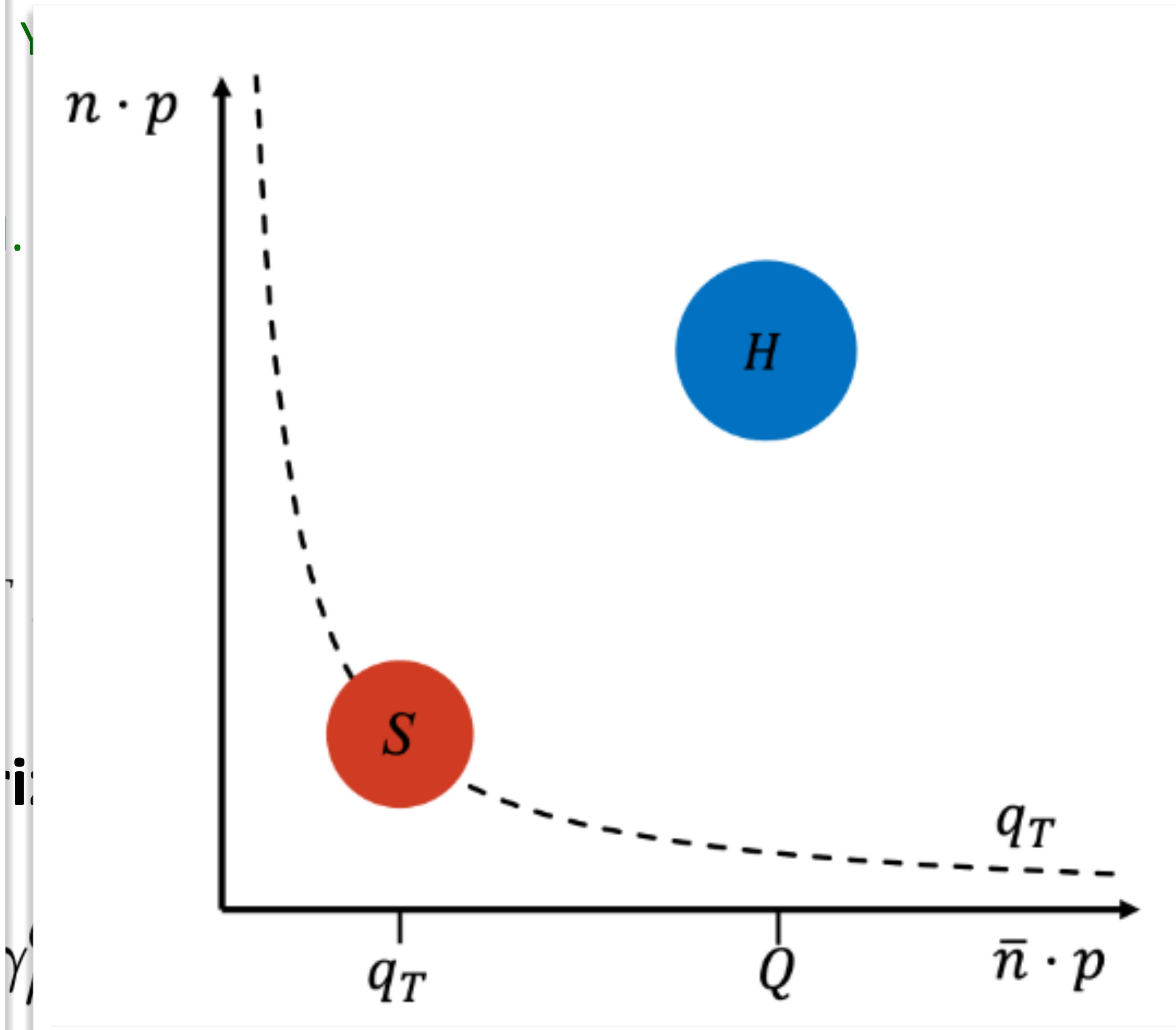


- **Two loop ultra-collinear function can be determined based on** $S \xrightarrow{Q \gg m_Q} \mathcal{C}_Q^2 \mathcal{S}_\perp$

Factorization in region 3 $Q \sim m_Q \gg q_T$

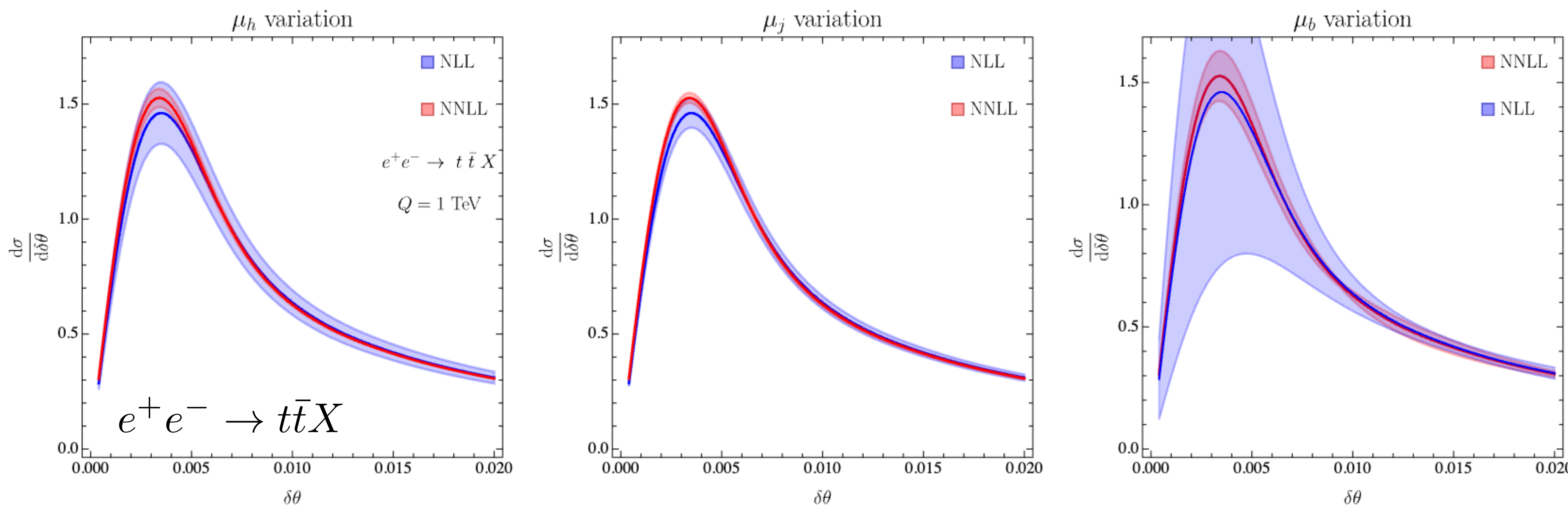
- **In region 3, TMD factorization of heavy quark pair production are well studied**

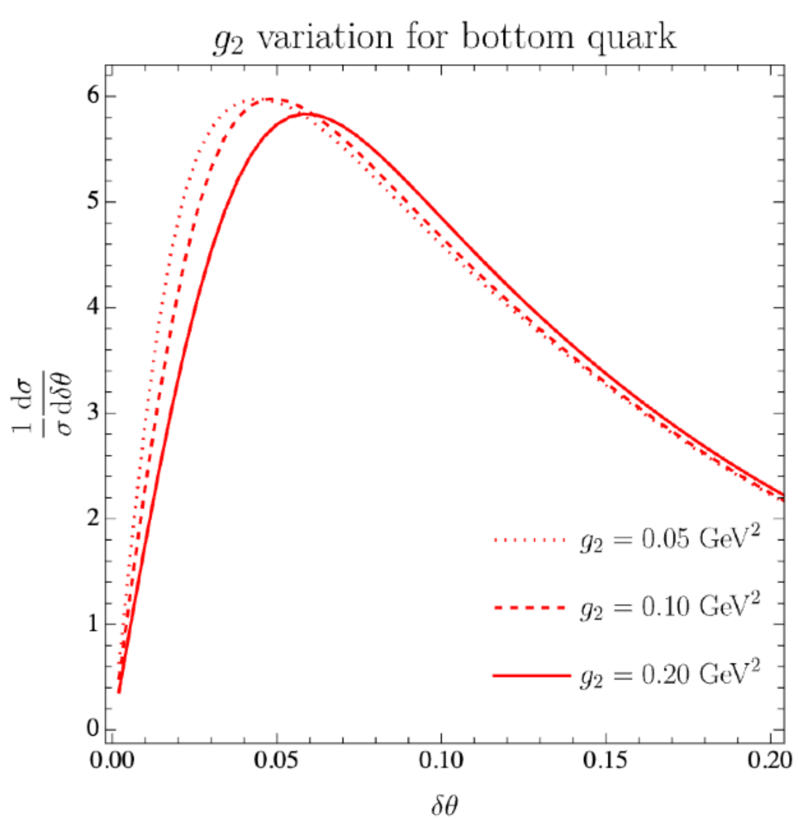




- **Two loop ultra-collinear function can be determined based on** $S \xrightarrow{Q \gg m_Q} C_Q^2 S_\perp$

NNLL resummation in region 2





- **We apply *b**-prescription to avoid Landau pole**

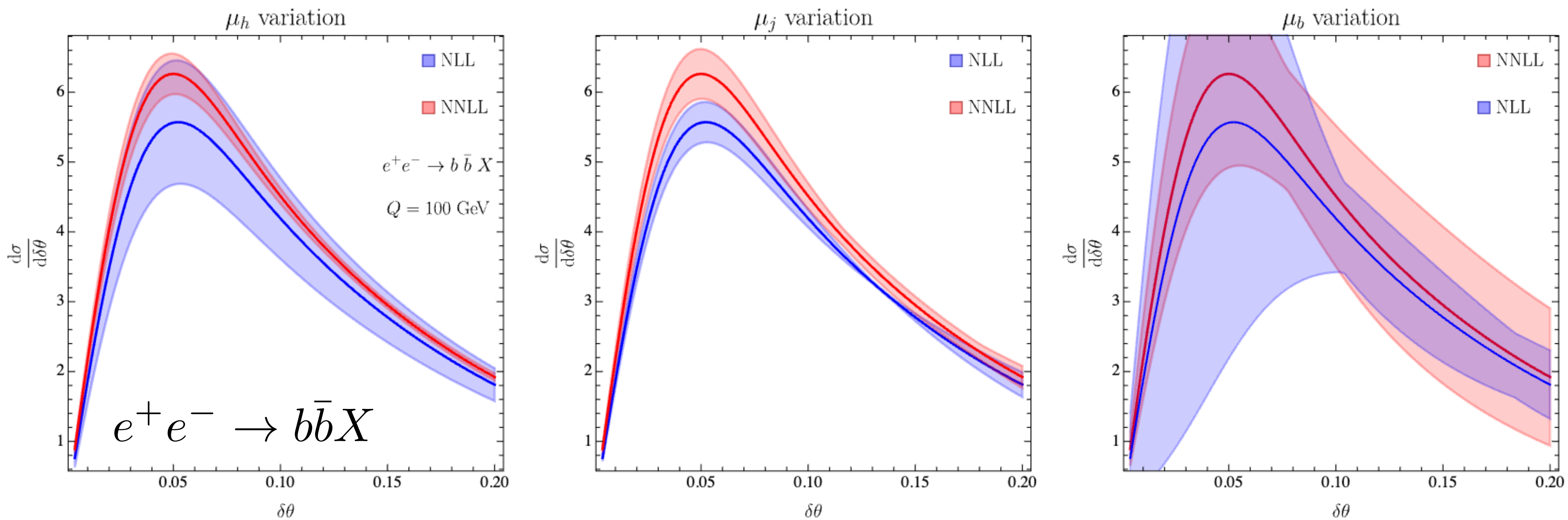
$$\mu_b = \frac{2e^{\gamma_E}}{b^*},\ b^* = \frac{b}{\sqrt{1 + b^2/b_{max}^2}}$$

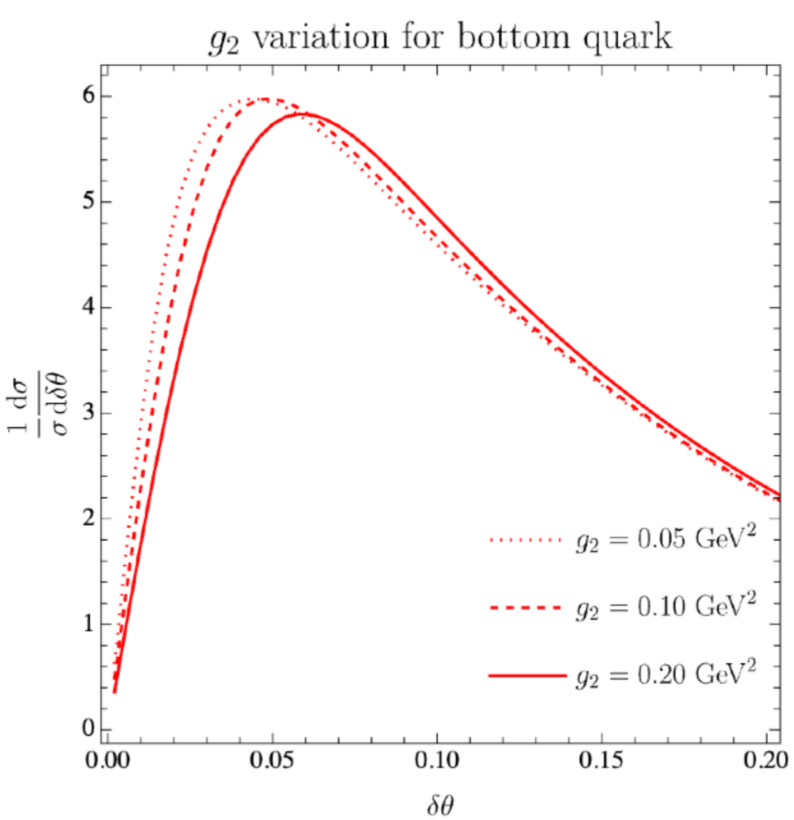
- **Leading non-perturbative effects are estimated based on the modification of CS kernel**

$$\gamma_\zeta^{S_\mathcal{Q}} \to \gamma_\zeta^{S_\mathcal{Q}} - g_2 b^2$$

Becher, Bell '13; Vladimirov '20

NNLL resummation in region 2





- **We apply *b**-prescription to avoid Landau pole**

$$\mu_b = \frac{2e^{\gamma_E}}{b^*},\ b^* = \frac{b}{\sqrt{1+b^2/b_{max}^2}}$$

- **Leading non-perturbative effects are estimated based on the modification of CS kernel**

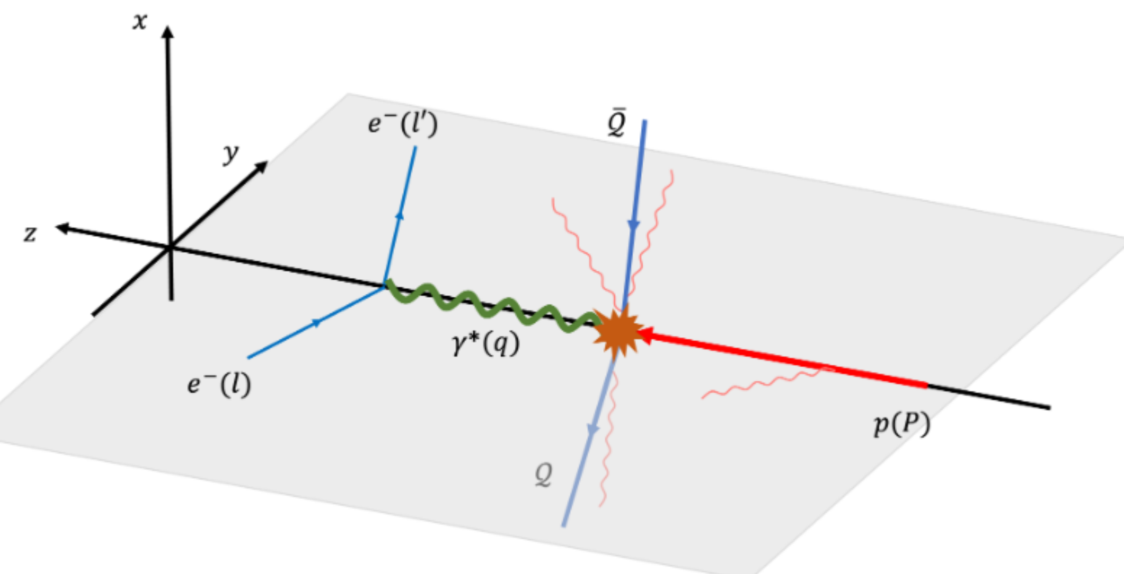
$$\gamma_\zeta^{S_\mathcal{Q}} \to \gamma_\zeta^{S_\mathcal{Q}} - g_2 b^2$$

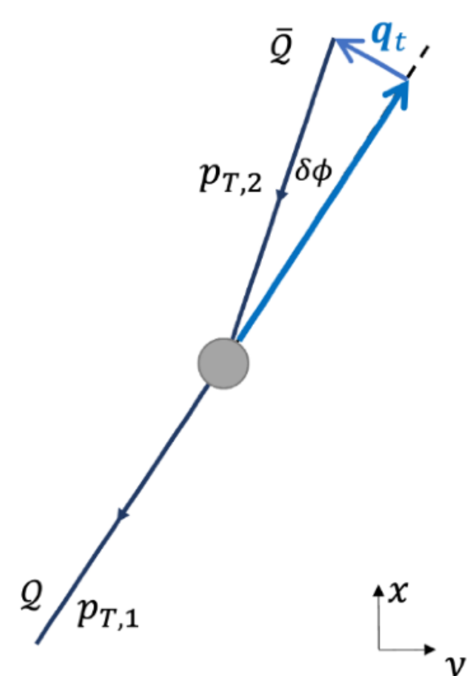
Becher, Bell ’13; Vladimirov ‘20

Heavy quark pair azimuthal angular correlation in DIS

Dai, Jiang, DYS in progress

$$e(l) + p(P) \to e(l') + \mathcal{Q}(p_1) + \bar{\mathcal{Q}}(p_2) + X$$





$$p_T \gg m_Q \gg q_t$$

hard: $p_h \sim p_T\,(1,1,1)$

soft: $p_s \sim q_t\,(1,1,1)$

beam: $p_b \sim p_T\,(1,\delta\phi^2,\delta\phi)$

jet: $p_j \sim p_T\,(1,\lambda^2,\lambda)_{\mathcal{J}}$

ultra-collinear: $p_{uc} \sim q_t/\lambda\,(1,\lambda^2,\lambda)_{\mathcal{J}}$

$$\frac{d\sigma}{dQ^2\ dy\ d\eta_1\ d^2\boldsymbol{p}_T\ d\delta\phi} = \sum_i H^i\left(Q,y,\eta_1,\boldsymbol{p}_T,\mu\right)\mathcal{J}_m^2\left(m_{\mathcal{Q}},\mu,\zeta_{\mathcal{J}}/\nu^2\right)\mathcal{S}_m\left(m_{\mathcal{Q}},\mu,\nu\right)$$

$$\times\int\frac{db_t}{\pi}e^{ib_tp_T\delta\phi}\boxed{\mathcal{B}^i_{g/N}\left(x,b_t,\mu,\zeta_{\mathcal{B}}/\nu^2\right)}\mathcal{C}_{\mathcal{Q}}\left(b_t,\mu,\zeta_{\mathcal{C}}/\nu^2\right)\mathcal{C}_{\overline{\mathcal{Q}}}\left(b_t,\mu,\zeta_{\mathcal{C}}/\nu^2\right)\boxed{\mathcal{S}\left(b_t,\mu,\nu\right)}$$

Gluon TMDs

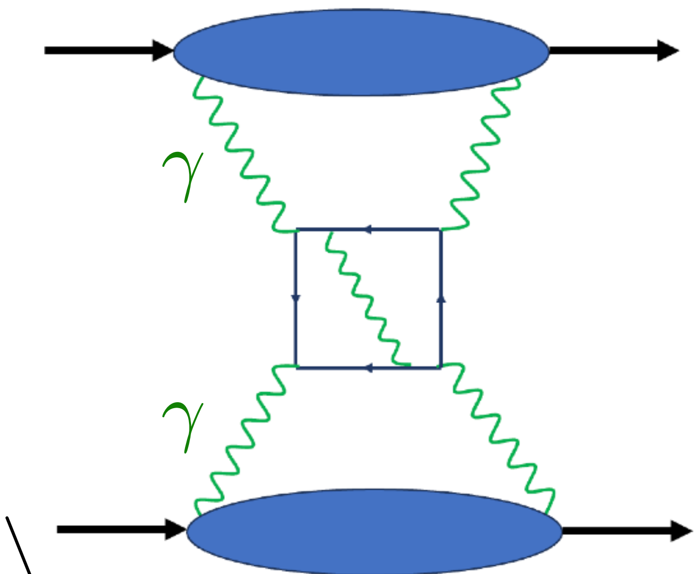
1-dim soft function

Gao, Li, Moult, Zhu ’19 ’23; Chien, Rahn, DYS, Waalewijn & Wu ’22 + Schrignder ’21

QED resummation of lepton pair azimuthal correlation in UPCs

DYS, Zhang, Zhou, Zhou '23

- **Ultra-peripheral heavy ion collisions without nuclear breakup** $AA \to AAl^+l^-$
- **Set a baseline for QGP study**
- **Determine photon flux: standard candle**
  - **Photon Wigner distribution:** (Klein, Mueller, Xiao, Yuan, '20)



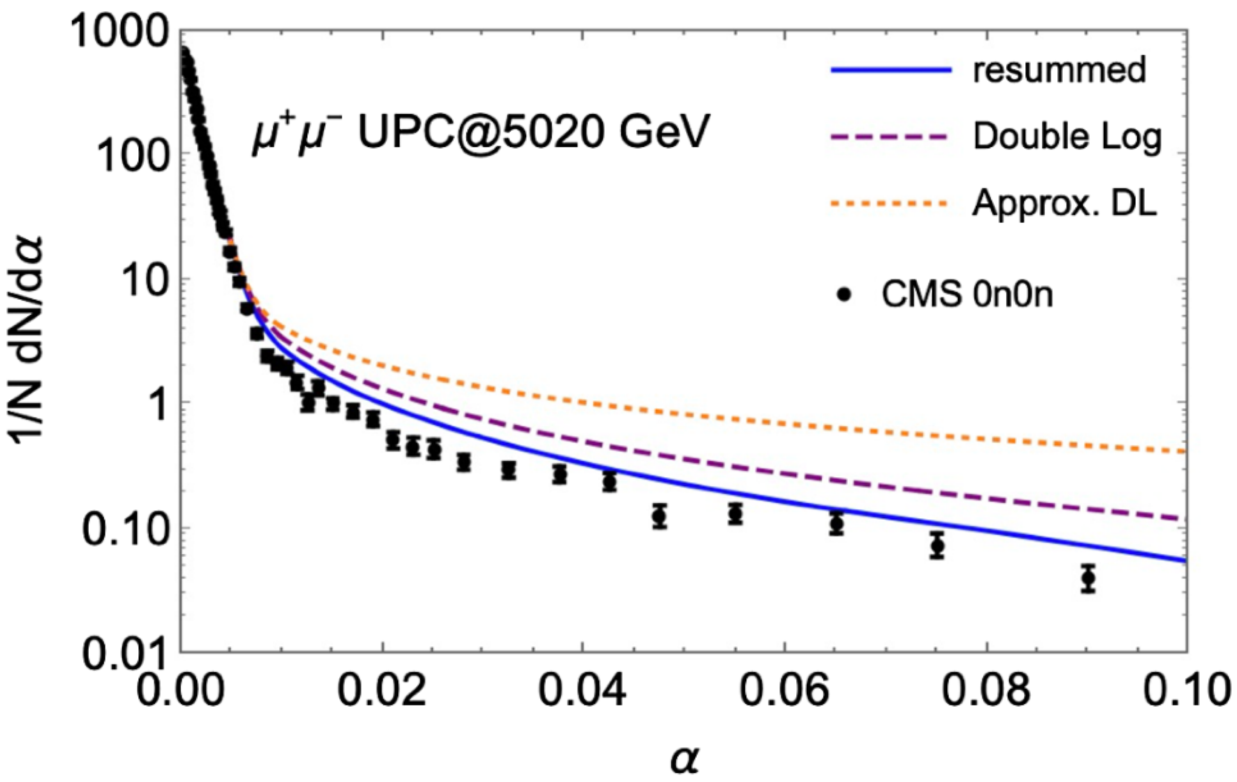
$$xf_\gamma(x, k_T; b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} \int \frac{d\xi^- d^2 r_\perp}{(2\pi)^3} e^{ixP^+\xi^- - ik_T \cdot r_\perp} \times \left\langle A, -\frac{\Delta_\perp}{2} \right| F^{+\perp}\left(0, \frac{r_\perp}{2}\right) F^{+\perp}\left(\xi^-, -\frac{r_\perp}{2}\right) \left| A, \frac{\Delta_\perp}{2} \right\rangle$$

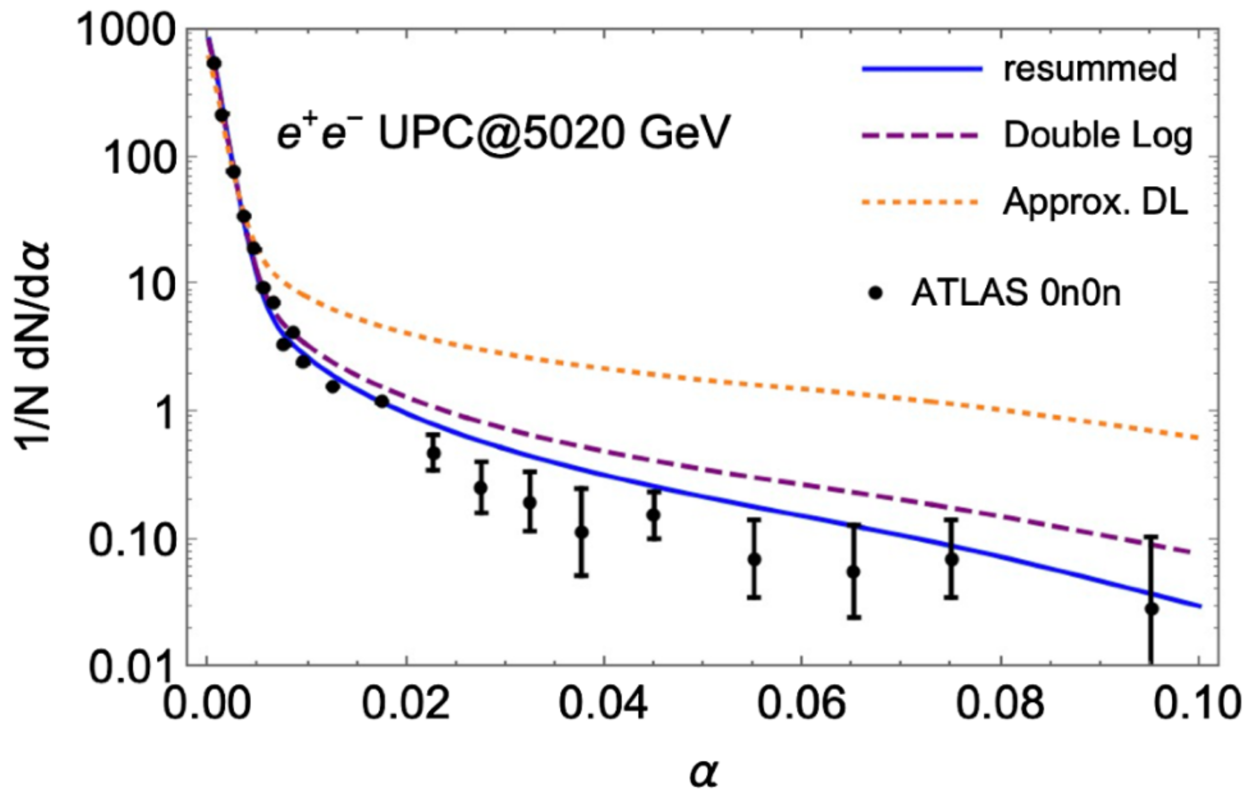
- **BSM search: E.g. tau g-2 and tau EDM** (ATLAS, '23; CMS '23; DYS, Yan, Yuan, Zhang '24)
- **We apply impact parameter dependent formalism and factorization in region 2 to derive the resummation formula**

$$\frac{d\sigma}{dq_x d^2\boldsymbol{P}_\perp dy_1 dy_2 d^2\boldsymbol{b}_\perp} = \int \frac{dr_x}{2\pi} e^{ir_x q_x} e^{-\,\mathrm{Sud}(r_x)} \int dq'_x dq'_y e^{-ir_x q'_x} \frac{d\sigma_0(q'_\perp)}{d\mathcal{P}.\mathcal{S}.},$$

QED resummation of lepton pair azimuthal correlation in UPCs

DYS, Zhang, Zhou, Zhou '23





- **Single log contribution is sizable**

$$\mathrm{Sud_a}\left(r_x\right) = \frac{\alpha_e}{2\pi}\left[\left(\ln^2\frac{M^2}{\mu_{rx}^2} - 3\ln\frac{M^2}{\mu_{rx}^2}\right) - \left(\ln^2\frac{m^2}{\mu_{rx}^2} - \ln\frac{m^2}{\mu_{rx}^2}\right)\right]$$

- **Our findings demonstrate the accessibility of these single log resummation effects through the analysis of angular correlations in lepton pairs.**

Conclusion

- We investigate the factorization and resummation formula for heavy quark pair production in back to back limit
- We analyze factorization in three distinct scale hierarchies within SCET, bHQET, HQET

$$Q \gg m_Q \sim q_T \qquad Q \gg m_Q \gg q_T \qquad Q \sim m_Q \gg q_T$$

- Refactorization jet function in region 1 are verified at one loop
- In region 2 two-loop ultra-collinear function results are given by RG consistency and refactorization of the soft function in region 3
- We perform the NNLL resummation predictions of heavy quark pair angular correlation in region 2
- We also apply our formalism to study QED resummation of lepton pair azimuthal correlation in UPCs, and find sizable single log contribution
- Our findings demonstrate the accessibility of these single log resummation effects through the analysis of angular correlations in lepton pairs.

Thank you