Progress in Understanding Heavy Quark Fragmentation in the Transverse Plane

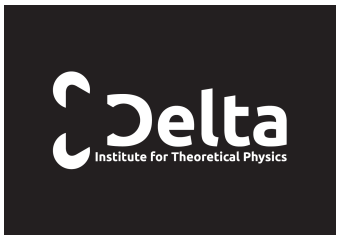
Johannes K. L. Michel

D-ITP Fellow UvA/Nikhef

[2305.15461, 2404.08622, and work in progress]

XXIth Workshop on Soft-Collinear Effective Theory

Salamanca, April 15, 2024



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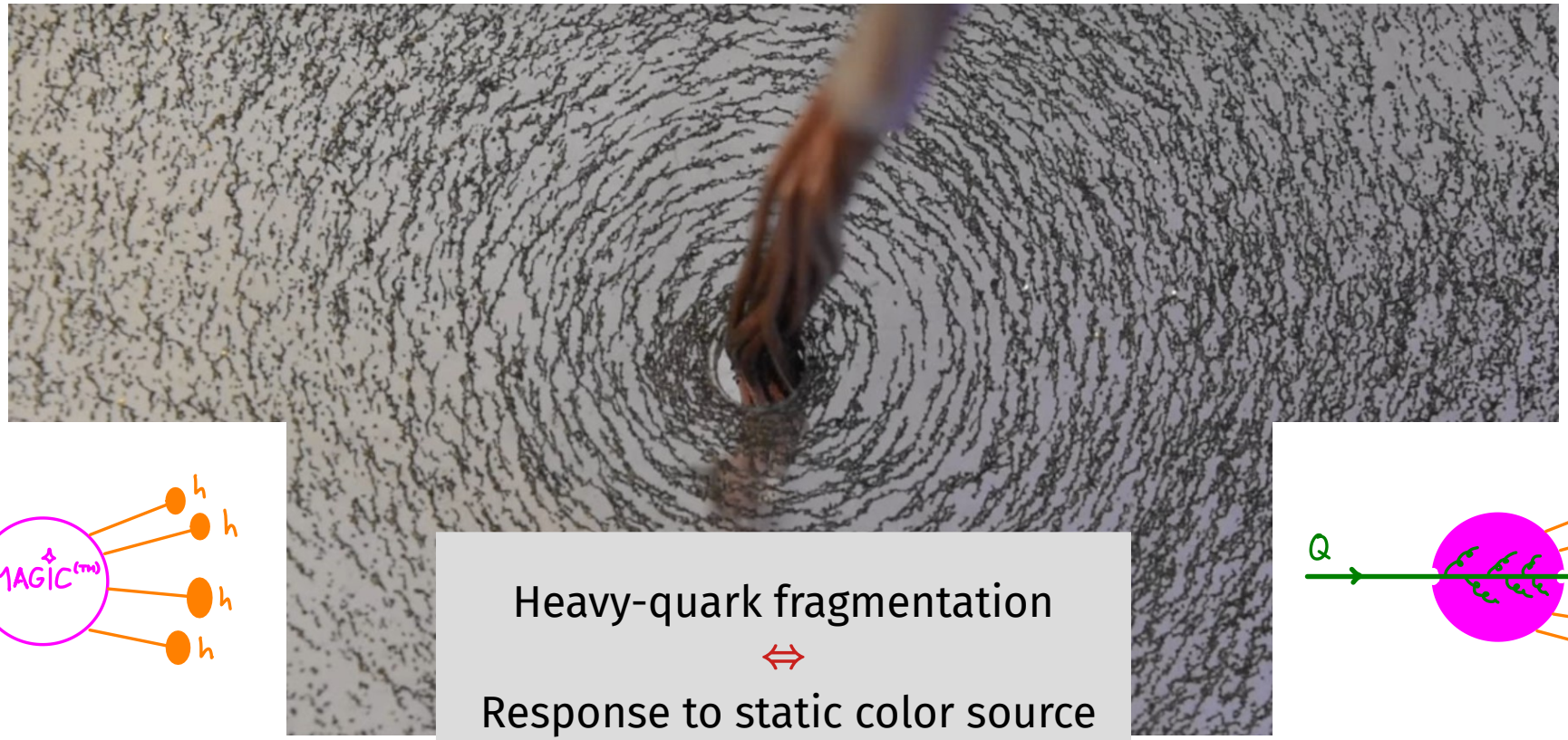
Motivation: Probe Hadronization in Depth



Heavy-quark fragmentation

$\Leftrightarrow$

Response to static color source

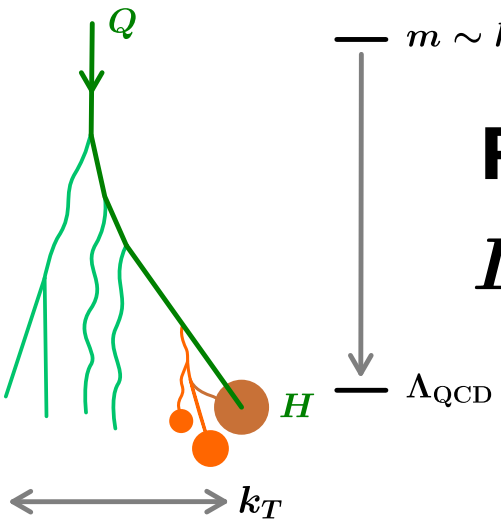
Heavy-quark TMD FFs $D_{1\,H/Q}(z_H, k_T)$ { provide a 3D picture of this / are a universal building block

[→ See talk by D. Shao in next session for a concrete application!]

Overview of 2305.15461


JHEP 09 (2023) 205 with Rebecca von Kuk (DESY) & Zhiquan Sun (MIT)

[→ Talk by Z. Sun at SCET '23!]



Regime (a)

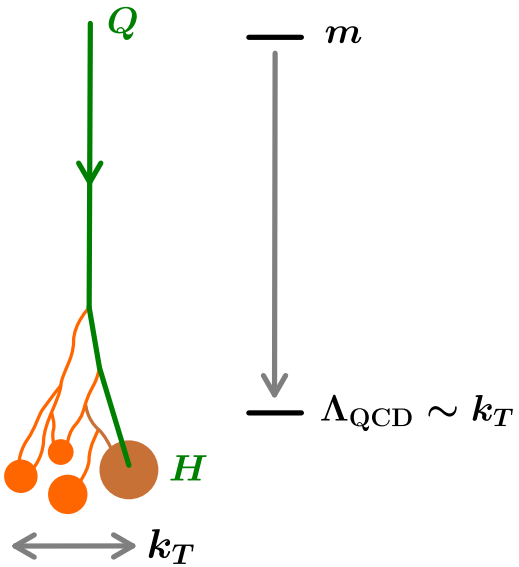
$$D_{1\,H/Q}(z_H, k_T) = d_{1\,Q/Q}(z_H, k_T)\,\chi_H$$

Regime (b)

$$D_{1\,H/Q} = \delta(\bar{z}_H)\,C_m(m)\,N_{H/\ell}\,\chi_{1,\ell}(k_T)$$

$$H_{1\,H/Q}^{\perp} = \delta(\bar{z}_H)\,C_m(m)\,N_{H/\ell}\,\chi_{1,\ell}^{\perp}(k_T)$$

$$\bar{z}_H \equiv 1 - z_H$$



Overview of today's talk

1. Transverse Momentum-Dependent Heavy-Quark Fragmentation at Next-to-Leading Order

2. How Many Structure Functions Characterize Heavy Quark Fragmentation in the Transverse Plane?

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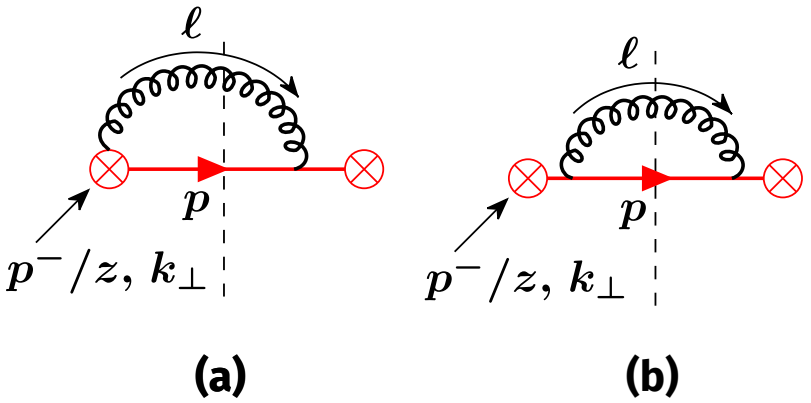
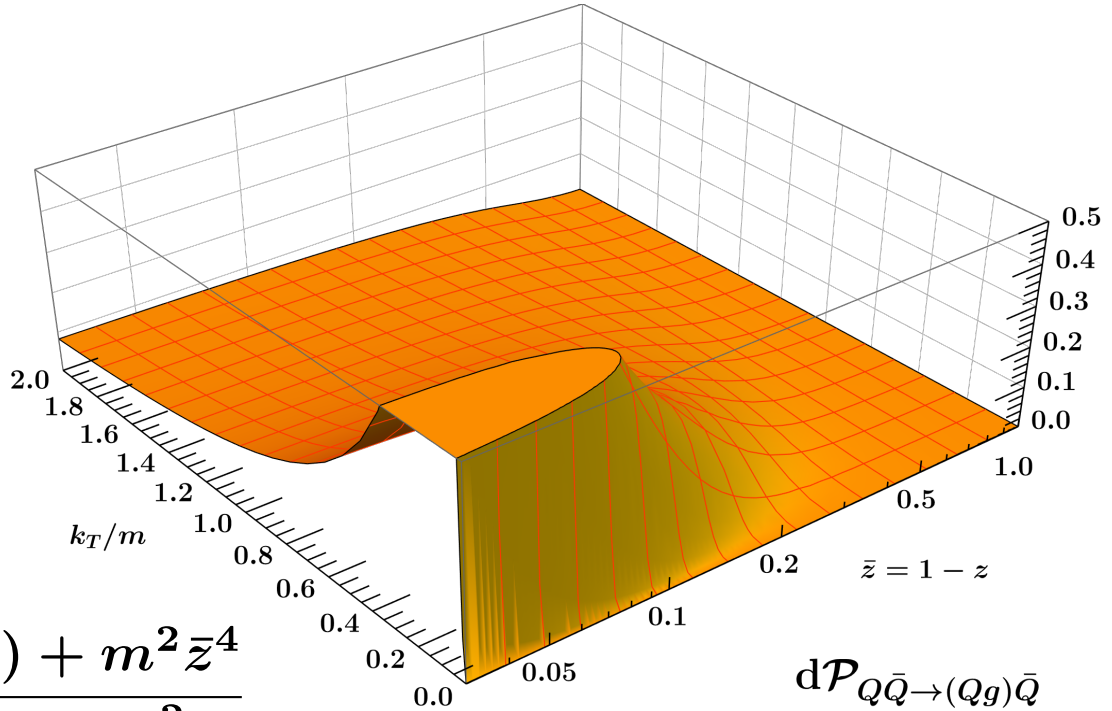
   2404.08622 with Rebecca von Kuk (DESY) & Zhiquan Sun (MIT)


2. How Many Structure Functions Characterize Heavy Quark Fragmentation in the Transverse Plane?

Regulating the quasi-collinear splitting probability

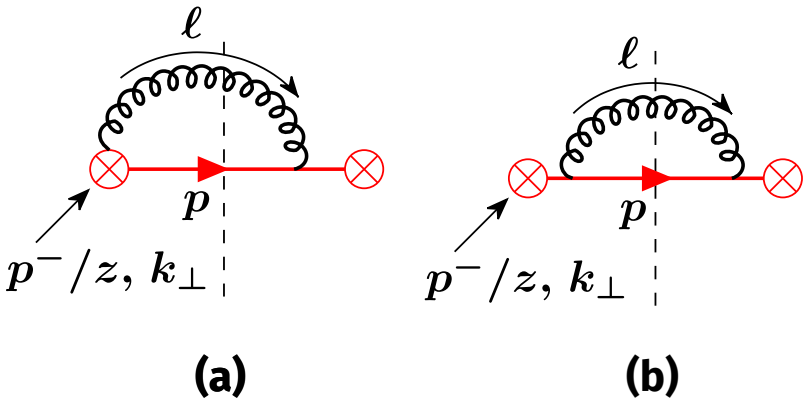


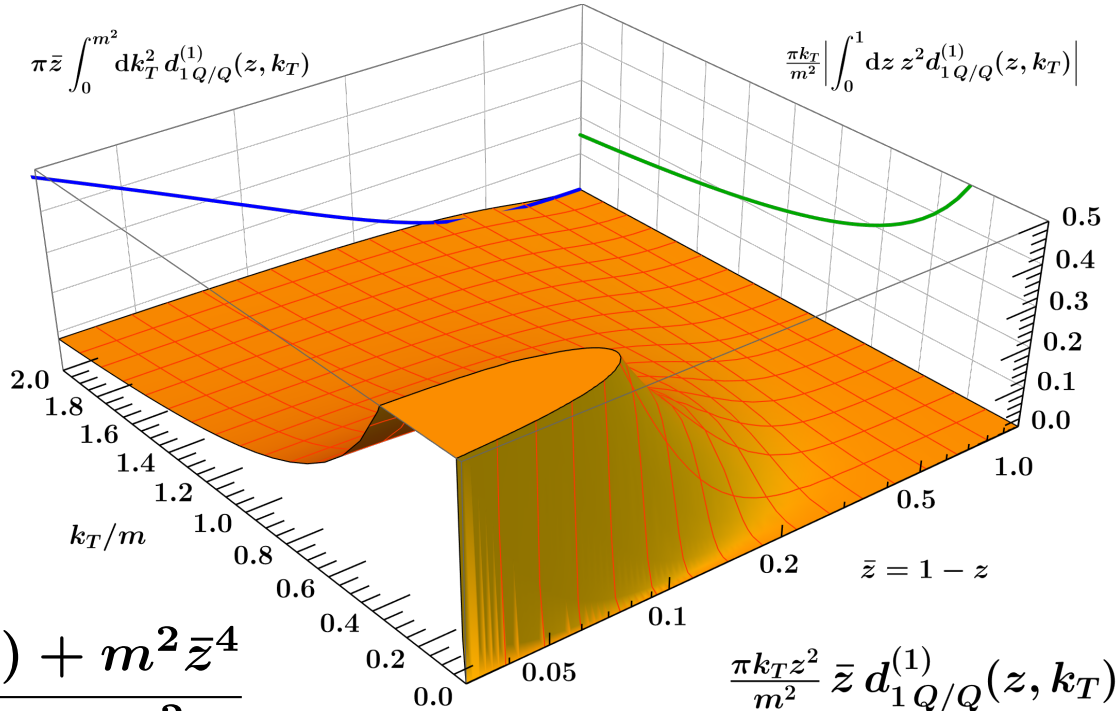


$\bar{z} \equiv z_g = 1 - z > 0$ and $k_T > 0$:

$$\frac{\mathrm{d}\mathcal{P}_{Q\bar{Q}\to(Qg)\bar{Q}}}{\mathrm{d}z\,\mathrm{d}(k_T^2)} = \frac{\alpha_s C_F}{2\pi} \frac{k_T^2 z^2(1+z^2) + m^2\bar{z}^4}{\bar{z}\left[k_T^2 z^2 + m^2\bar{z}^2\right]^2}$$

Regulating the quasi-collinear splitting probability





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$$\iint = \frac{\alpha_s C_F}{4\pi}\left(5-\frac{\pi}{2}-\frac{2\pi^2}{3}\right)$$

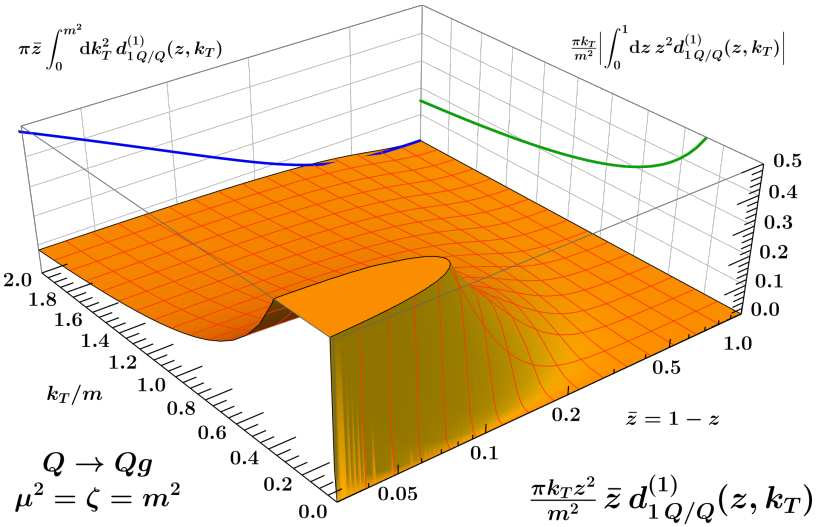
$$d^{(a)}_{1\,Q/Q} = \frac{\alpha_s C_F}{4\pi}\,\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left(\frac{\mu}{k_T}\right)^{2\epsilon}\left(\frac{\sqrt{\zeta}}{\nu}\right)^{-\eta}\frac{1}{\pi z^{2-2\epsilon}}\,\frac{z^\eta}{\bar{z}^{1+\eta}}\,\frac{4z^3}{k_T^2 z^2+m^2\bar{z}^2}$$

$$d^{(b)}_{1\,Q/Q} = \frac{\alpha_s C_F}{4\pi}\,\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left(\frac{\mu}{k_T}\right)^{2\epsilon}\frac{1}{\pi z^{2-2\epsilon}}\,2z^2\bar{z}\left[\frac{k_T^2 z^2+m^2(1-4z+z^2)-\epsilon(k_T^2 z^2+m^2\bar{z}^2)}{(k_T^2 z^2+m^2\bar{z}^2)^2}\right]$$

Expanding in 2D plus distributions

- $\dfrac{z^\eta}{\bar{z}^{1+\eta}} = -\dfrac{\delta(\bar{z})}{\eta} + \mathcal{O}(\eta^0)$ cancels as for massless

$\Rightarrow$ Left to expand: $f(x, z, \epsilon)$ $\qquad x \equiv k_T^2/m^2$



Recall: $f(x, \epsilon) = \big[f(x, 0) + \mathcal{O}(\epsilon)\big]_+ + \delta(x)\, F(\epsilon)$

$$F(\epsilon) = \int_0^1 \mathrm{d}x'\, f(x', \epsilon) = \frac{1}{\epsilon} + \dots$$

↓ Generalize to 2D

$$f(x, z, \epsilon) = \big[f(x, z, 0) + \mathcal{O}(\epsilon)\big]_{+,+} + \delta(x)\big[F_x(z, \epsilon)\big]_+ + \delta(\bar{z})\big[F_z(x, \epsilon)\big]_+ + \delta(x)\,\delta(\bar{z})\, F_{xz}(\epsilon)$$

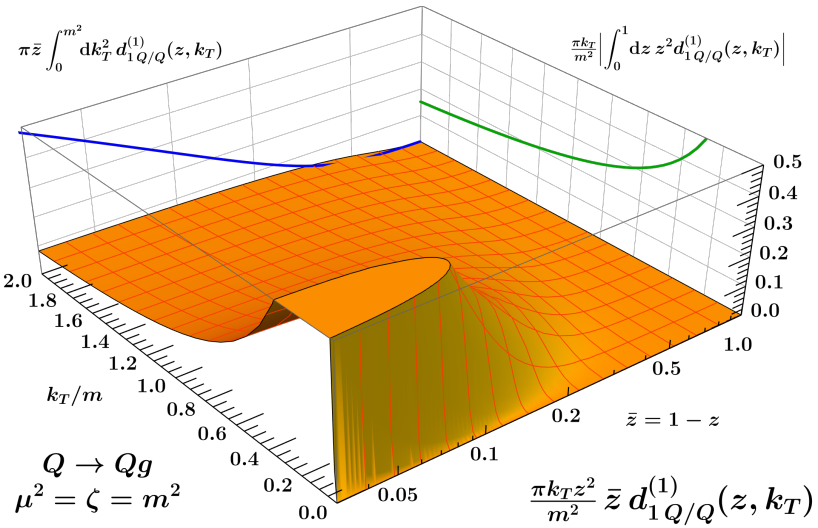
$$F_x(z, \epsilon) \equiv \int_0^1 \mathrm{d}z'\, f(x, z', \epsilon) \qquad F_z(x, \epsilon) \equiv \int_0^1 \mathrm{d}x'\, f(x', z, \epsilon) \qquad F_{xz}(\epsilon) \equiv \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z'\, f(x, z', \epsilon)$$

$$\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z \big[f(x, z)\big]_{+,+} g(x, z) \equiv \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}z\, f(x, z)\big[g(x, z) - g(0, z) - g(x, 0) + g(0, 0)\big]$$

Results in k_T space

$$
\begin{aligned}
&d_{1\,Q/Q}^{(1)}(z, k_T, \mu, \zeta) \\
&= \frac{\alpha_s C_F}{4\pi} \frac{1}{\pi z^2} \left\{ \delta(\bar{z}) \left[2 \ln \frac{\zeta}{\mu^2} \mathcal{L}_0(k_T^2, \mu^2) \right.\right. \\
&\quad + 2 \ln \frac{\mu^2}{m^2} \mathcal{L}_0(k_T^2, m^2) \\
&\quad \left.\left. - \ln^2 \frac{\mu^2}{m^2} \delta(k_T^2) + 3 \ln \frac{\mu^2}{m^2} \delta(k_T^2) \right] \right. \\
&\quad + \frac{1}{m^2} \left[\frac{2x z^4 (1+z^2) + 2 z^2 \bar{z}^4}{\bar{z}(x z^2 + \bar{z}^2)^2} \right]_{+,+} + \delta(k_T^2) \left[\frac{2(1+z^2)}{\bar{z}} \ln\left(1 + \frac{z^2}{\bar{z}^2}\right) - \frac{4z^3}{\bar{z}(1-2\bar{z}z)} \right]_+ \\
&\quad + \delta(\bar{z}) \frac{1}{m^2} \left[- \frac{2 + 3x + 4x^2 + 3x^3 + \pi \sqrt{x}(2 + 7x + x^2) + x(1 + 7x + 2x^2) \ln x}{x(1+x)^3} \right]_+ \\
&\quad \left. + \delta(\bar{z})\, \delta(k_T^2) \left(5 - \frac{\pi}{2} - \frac{2\pi^2}{3} \right) \right\}
\end{aligned}
$$

$x \equiv k_T^2/m^2$



→ Confirms expected renormalization by $\gamma_\mu(\zeta, \cancel{m}, \mu)$ and $\gamma_\zeta(k_T, m, \mu)$

[Secondary quark mass effects would only show up at NNLO → see talk by P. Plößl on Thursday.]

Results in b_T space

Plus distributions domains align with primary variables $\Rightarrow$ easy to take integral transforms!

$$\int_0^\infty \mathrm{d}(k_T^2)\, J_0(b_T k_T)\, \big[f(k_T^2, z)\big]_{+,+} = \Big[\int_0^\infty \big[J_0(b_T k_T) - 1\big] f(k_T^2, z)\Big]_+$$

only z

$$\Downarrow$$

$$d_{1\,Q/Q}^{(1)}(z, b_T, \mu, \zeta) = \frac{\alpha_s C_F}{4\pi}\frac{1}{z^2}\Big\{\delta(1-z)\Big[-2L_b \ln\frac{\zeta}{m^2} + 4\ln^2\frac{\mu}{m} + 6\ln\frac{\mu}{m} - L_y^2 + 4 - \frac{\pi^2}{6}\Big]$$
$$- 4(1+L_y)\,\mathcal{L}_0(1-z) - 8\mathcal{L}_1(1-z) + \tilde{\mathcal{R}}(z, b_T m)\Big\}$$

$$\tilde{\mathcal{R}}(z,y) \equiv y\int_0^\infty \mathrm{d}t\, J_1(ty)\,\mathcal{R}(z,t^2) \qquad L_y \equiv \frac{m^2 b_T^2}{(2e^{-\gamma_E})^2} \qquad L_b \equiv \frac{\mu^2 b_T^2}{(2e^{-\gamma_E})^2}$$

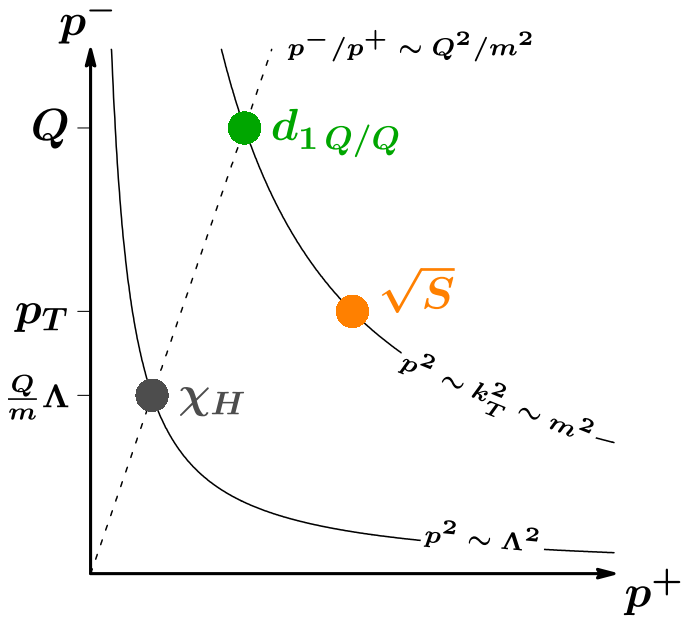
$$= \frac{4}{\bar{z}}\Big[1 + L_y + (1+z^2)K_0\Big(\frac{y\bar{z}}{z}\Big) - y\bar{z}K_1\Big(\frac{y\bar{z}}{z}\Big) + 2\ln\bar{z}\Big] = \mathcal{O}(\bar{z}^0)$$

Full agreement with Dai, Kim Leibovich, 2310.19207, who used a completely orthogonal organization of k_T singularities. ✓

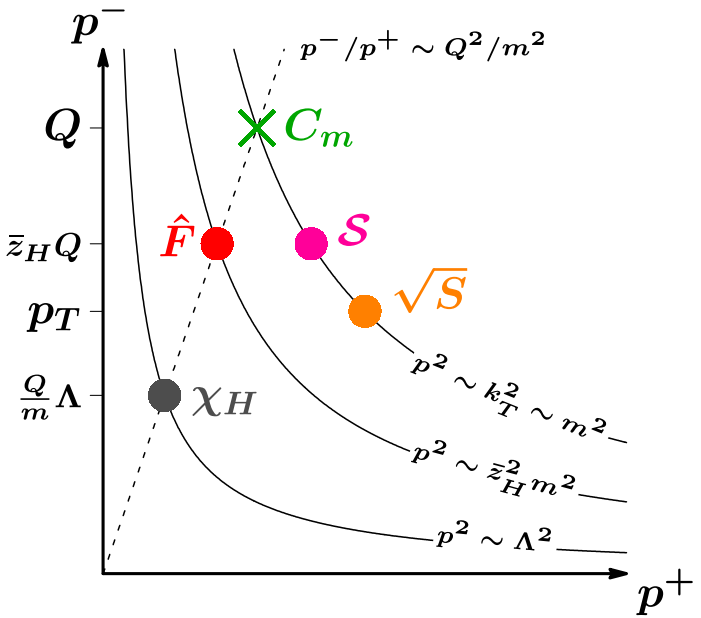
Why so simple as $z \to 1$? Why only logarithms of $m\, b_T$?

Joint resummation for massive quark fragmentation

$k_T \sim m \ll \bar{z}_H\, Q \sim Q$



$k_T \sim m \ll \bar{z}_H\, Q \ll Q$



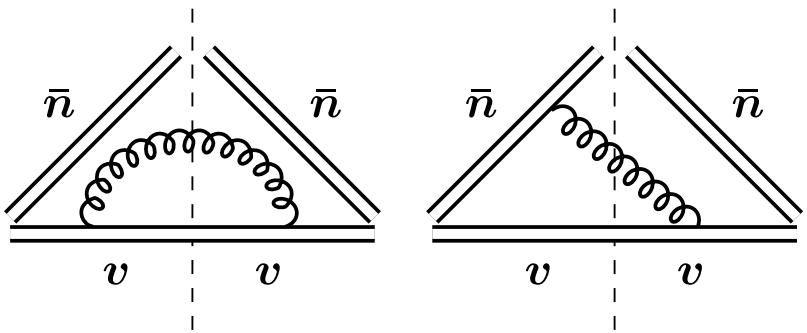
$$d_{1\,Q/Q}\Big(z_H = 1 - \frac{k^-}{\omega}, b_T, \mu, \zeta\Big) = C_m(m,\mu,\zeta)\, S\Big(b_T, m, \mu, \frac{k^-}{\nu}\Big) \otimes_{k^-} \hat{F}\Big(k^-, \bar{n}\cdot v\,\mu\Big)$$
$$\times \sqrt{S}(b_T, m, \mu, \nu) + \mathcal{O}\big[(1-z_H)^0\big]$$

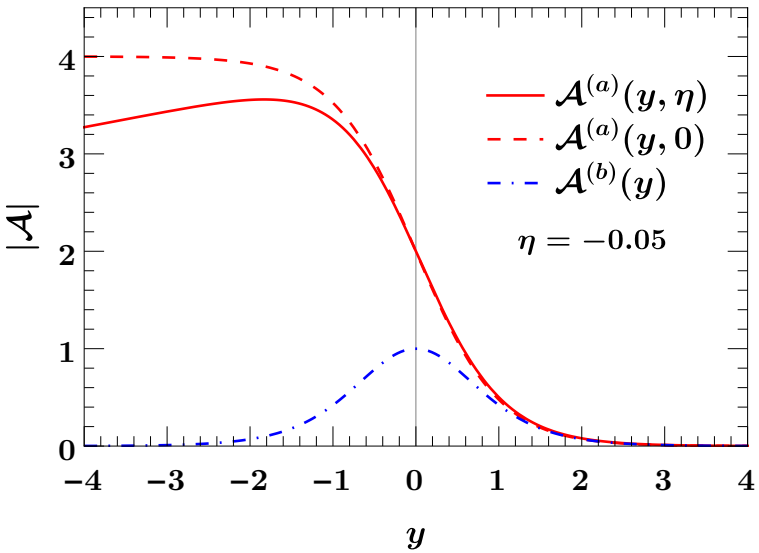
✓

[Full two-loop RHS known: Becher, Neubert '06; Pietrulewicz, Samitz, Spiering, Tackmann '17]

Not discussed today

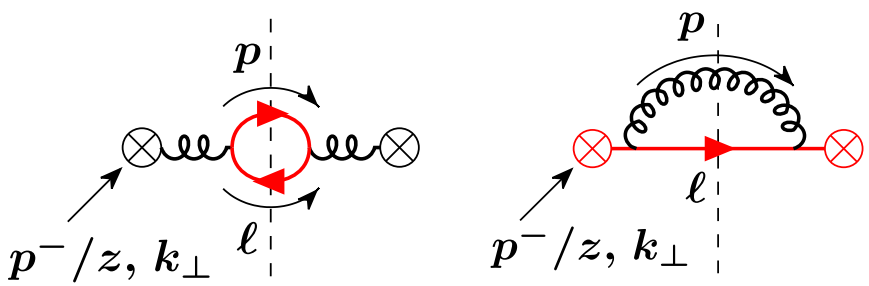
- Large-mass limit ✓





$$\chi_1(b_T,\mu,\bar{n}\cdot v) = 1 + \frac{\alpha_s C_F}{4\pi}(-L_b)\left(4\ln \bar{n}\cdot v - 2\right)$$

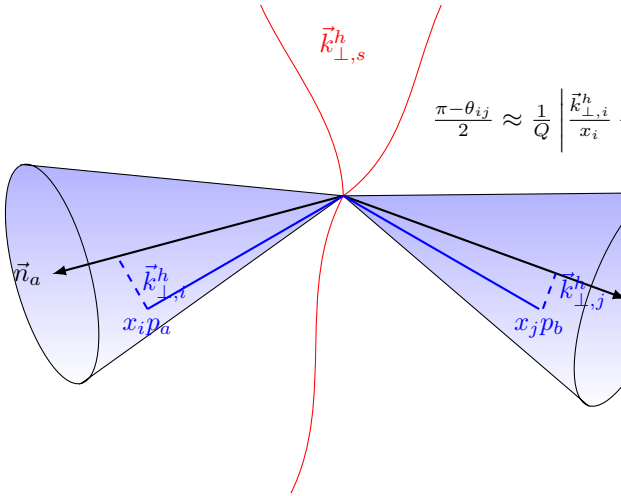
- Nonvalence channels & small-mass limit ✓



Application: Back-to-back EEC with heavy quarks

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}z_\chi} = \sum_{i,j} \mathcal{H}_{ee\to ij}(Q^2,\mu)\; Q^2 \int_0^\infty \mathrm{d}b_T\, b_T\, J_0\big(\sqrt{1-z_\chi}\, b_T Q\big)$$
$$\times\; J_i(b_T, m, \mu, Q/\nu)\; J_j(b_T, m, \mu, Q/\nu)\; S(b_T, m, \mu, \nu)$$

$$i,j = q, Q, g$$



[Figure credit: Moult, Zhu, 1801.02627]

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}z_\chi} = \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(1)}_{ee\to q\bar{q}}}{\mathrm{d}z_\chi} + 2\mathcal{H}^{(0)}_{ee\to Q\bar{Q}}\,\frac{\alpha_s C_F}{4\pi}\Bigg\{8(2\ln\rho - 1)\,\mathcal{L}_0(\bar{z}_\chi) + 8\Big(2\ln^2\rho + \ln\rho + \frac{2\pi^2}{3} - 2\Big)\,\delta(\bar{z}_\chi)$$
$$+ 4\rho^2\,\frac{\pi(-3-18x+x^2) + \sqrt{x}(9+x-9x^2-x^3) - \sqrt{x}(1+24x+9x^2+2x^3)\ln x}{\sqrt{x}(1+x)^4}\Bigg\}$$
$$+ \mathcal{O}(\alpha_s^2) + \bar{z}_\chi^0 + \mathcal{O}\Big(\frac{1}{\rho}\Big)$$

NEW

[Cf. Lepenik, Mateu, 1912.08211 for analytic SCET$_\mathrm{I}$ massive event shapes at one loop.]

Fun Fact: The heavy-quark EEC at nonperturbative values

Sum over heavy hadrons is built into EEC ...

$$\left.\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}z_\chi}\right|_{\text{heavy}} = \mathcal{H}_{ee\to Q\bar{Q}}(Q^2,\mu)\, C_m^2(m,\mu,Q^2) \times Q^2\int_0^\infty \mathrm{d}b_T\, b_T\, J_0\big(\sqrt{1-z_{\chi,H}}\, b_T Q\big) \left[\chi_1\left(b_T,\mu,\frac{Q}{m}\right)\right]^2$$

... which lets us complete the sum over states in the EFT! [2305.15461]

$$\chi_1(b_T) \equiv \frac{1}{N_c}\,\mathrm{Tr}\big\langle 0\big|W^\dagger(b_\perp)\, Y_v(b_\perp)\, Y_v^\dagger(0)\, W(0)\big|0\big\rangle$$

$\Rightarrow$ Plain Wilson line correlators, also nonperturbatively!

[→ See talks by Z. Sun and H. Chen on Wednesday for – presumably – more nontrivial cases 😉]

Overview of today's talk

1. Transverse Momentum-Dependent Heavy-Quark Fragmentation at Next-to-Leading Order

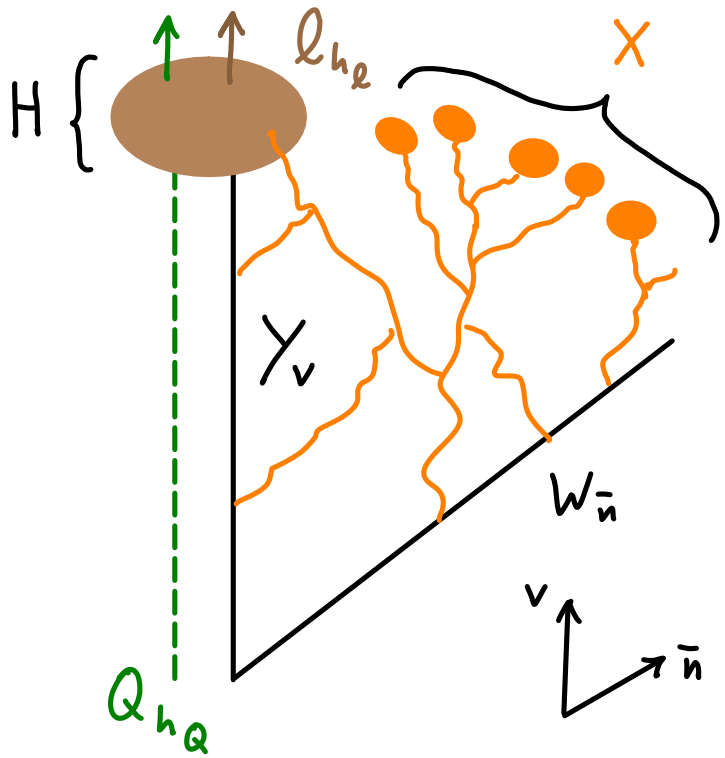
2. How Many Structure Functions Characterize Polarized Heavy Quark Fragmentation in the Transverse Plane?

Work in progress with Rebecca von Kuk (DESY), Kyle Lee (MIT), Zhiquan Sun (MIT)



TMD light-spin density matrix

$$\rho_{\ell,h_\ell h'_\ell}(b_\perp, z) \equiv \frac{1}{N_c}\,\mathrm{Tr}\sum\!\!\!\!\!\!\int_X \langle 0|W^\dagger(b_\perp)\,Y_v(b_\perp)|s_\ell, h_\ell, f_\ell; X\rangle\langle s_\ell, h'_\ell, f_\ell; X|Y_v^\dagger(0)\,W(0)|0\rangle$$



Useful unit vector choices:

$$x^\mu = \frac{b_\perp^\mu}{b_T}$$

$$z^\mu = v^\mu - \frac{\bar n^\mu}{\bar n \cdot v}$$

$$y^\mu = x_\nu \epsilon_\perp^{\mu\nu}$$

Building a basis

$$\rho_{\ell,h_\ell h'_\ell}(b_\perp, z) \equiv \frac{1}{N_c}\,\mathrm{Tr}\sum\!\!\!\!\!\!\int_X \langle 0|W^\dagger(b_\perp)\,Y_v(b_\perp)|s_\ell, h_\ell, f_\ell; X\rangle\langle s_\ell, h'_\ell, f_\ell; X|Y_v^\dagger(0)\,W(0)|0\rangle$$

- Hermiticity: $\rho_\ell^\dagger(b_\perp, z) = \rho_\ell(b_\perp, z)$
- Parity: $\rho_\ell(b_\perp, z) = \rho_\ell(-b_\perp, -z)$
- Rotations: $\rho_\ell(R_{\vec\alpha} b_\perp, R_{\vec\alpha} z) = e^{+i\alpha\cdot\Sigma_\ell}\rho_\ell(b_\perp, z)e^{-i\alpha\cdot\Sigma_\ell}$

$$x^\mu = \frac{b_\perp^\mu}{b_T}$$
$$z^\mu = v^\mu - \frac{\bar n^\mu}{\bar n\cdot v}$$
$$y^\mu = x_\nu \epsilon_\perp^{\mu\nu}$$

$$\Sigma^{\mu_1\ldots\mu_N} \equiv \left\{\Sigma_1^\mu, \ldots, \Sigma_N^\mu\right\} \text{ — Gram-Schmidt} \qquad N \le 2s_\ell$$

NEW

$$\Rightarrow \quad \rho_\ell(b_\perp, z) = \sum_N \sum_n \chi_{1,\ell}^{(N,n)}(b_T)\,\Sigma^{\mu_1,\ldots,\mu_N} \times \begin{cases} x^{\mu_1}\ldots x^{\mu_n} z^{\mu_{n+1}\ldots z^{\mu_N}}, & N \in 2\mathbb{Z} \\ i x^{\mu_1}\ldots x^{\mu_{n-1}} y^{\mu_n} z^{\mu_{n+1}\ldots z^{\mu_N}}, & N \notin 2\mathbb{Z} \end{cases}$$

So how many structure functions are there?

At light spin s_ℓ, rank $N = 2s_\ell$, add only $2\lfloor s_\ell \rfloor + 1$ structure functions!

Hadron multiplet	s_ℓ	# LP TMD FFs	# $\chi_{1,\ell}^{(N,n)}$'s	# $\chi_{1,\ell}^{(N,0)}$'s ($b_T \to 0$)
Λ_b	0	8	1	1
B, B^*	$1/2$	$2 + 12$	2	1
Σ_b, Σ_b^*	1	$8 + 16$	5	2
B_1, B_2^*	$3/2$	$12 + 20$	8	2

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$\Rightarrow$ Can get complete spin ½ density matrix from unpolarized hadrons!

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⇒ Can get complete spin ½ density matrix from unpolarized hadrons!

⇒ Nice target e.g. for an LHCb polarized hadron-in-jet measurement!

Stay tuned for final results for TMD FFs!

Summary

- Showed complete NLO results for unpolarized heavy-quark TMD FFs, with nontrivial momentum-space singularities.
- Checked vs. large-mass, small-mass, and new joint threshold/transverse momentum resummation regime.
- As application, gave NLO back-to-back EEC for heavy quarks.
- Showed how many structure functions characterize all possible heavy-quark TMD FFs in the bHQET limit (very few!).

Summary

- Showed complete NLO results for unpolarized heavy-quark TMD FFs, with nontrivial momentum-space singularities.
- Checked vs. large-mass, small-mass, and new joint threshold/transverse momentum resummation regime.
- As application, gave NLO back-to-back EEC for heavy quarks.
- Showed how many structure functions characterize all possible heavy-quark TMD FFs in the bHQET limit (very few!).

Thank you for your attention!