Mellin Barnes for massive bubble diagrams

work in progrees, to appear soon in arXiv

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Introduction to Mellin Barnes

Given a function $f(x)$:

- Mellin transform:

$$[\mathcal{M}f](h) \equiv \hat{f}(h) \equiv \int_0^\infty \mathrm{d}x\, x^{h-1} f(x)$$

$*$ $\hat{f}(h)$ convergent for $a < \mathrm{Re}(h) < b$.

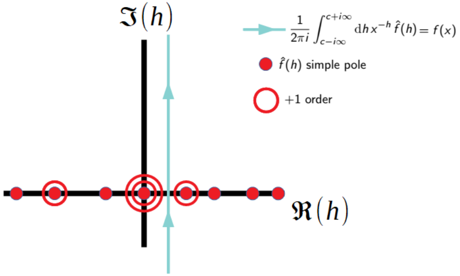
- Inverse Mellin transform

$$[\mathcal{M}^{-1}\hat{f}](x) \equiv \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}h\, x^{-h}\, \hat{f}(h) = f(x)\,; \qquad c \in (a,b)$$

Useful identity for loop integrals

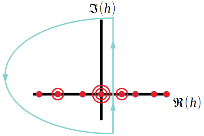
$$\frac{1}{(1+X)^\nu} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}h\, X^{-h}\, \frac{\Gamma(h)\Gamma(\nu-h)}{\Gamma(\nu)}\,; \qquad c \in (0,\nu)$$

Introduction to Mellin Barnes



Pole of $\hat{f}(h)$ at $h = \xi \in$ Reals $\Rightarrow$ $\hat{f}(h) \sim \sum_k \frac{a_{\xi,k}}{(h-\xi)^{k+1}}$

If $\lim_{h\to-\infty} h\, x^{-h}\, \hat{f}(h) = 0$:

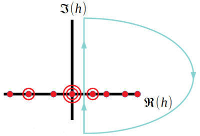


Residue theorem: Small x expansion

$$f(x) = \sum_{\xi,k} \frac{(-1)^k}{k!}\, a_{\xi,k}\, x^{-\xi}\, \boxed{\log^k(x)}$$

Order $k+1$ poles

If $\lim_{h\to\infty} h\, x^{-h}\, \hat{f}(h) = 0$:

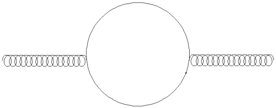


Residue theorem: Large x expansion

$$f(x) = \sum_{\xi,k} \frac{(-1)^{k+1}}{k!}\, a_{\xi,k}\, x^{-\xi}\, \boxed{\log^k(x)}$$

Order $k+1$ poles

Massive quark bubble



Gluon self-energy:

$$\frac{-i}{p^2} g^{\mu\alpha} \underbrace{i\Pi(p^2, m^2)(p^2 g_{\alpha\beta} - p_\alpha p_\beta)}_{\text{Massive Bubble}} \frac{-i}{p^2} g^{\beta\nu} = -\frac{i}{p^2}\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\Pi(p^2, m^2);$$

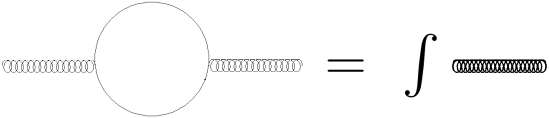
$$\Pi(p^2, m^2) = -\frac{4 n_f T_F g^2}{3 - 2\varepsilon}\frac{\Gamma(\varepsilon)}{(4\pi)^2}\left(\frac{4\pi\tilde{\mu}^2}{m^2}\right)^{\varepsilon}\left[\left(1 - \varepsilon + \frac{2m^2}{p^2}\right) {}_2F_1\left(1, \varepsilon; \frac{3}{2}; \frac{p^2}{4m^2}\right) - \frac{2m^2}{p^2}\right]$$

On-Shell renormalization:

$$\Pi(p^2, m^2) \to \Pi(p^2, m^2) - \Pi(0, m^2)$$

$$\Pi(0, m^2) = -\frac{n_f T_F g^2 \Gamma(\varepsilon)}{12\pi^2}\left(\frac{4\pi\tilde{\mu}^2}{m^2}\right)^{\varepsilon}$$

Dispersive integral method



1) Write massive bubble diagram as an integral of an effective (massive) gluon propagator
2) Perform computations at previous loop order with the modified (massive) gluon propagator
3) Carry out dispersive integral

Dispersive integral method

Derivation

(∗ The classical derivation is based on unitarity and analycity)

I Integral representation of the hypergeometrc function:

$$\frac{2m^2}{p^2}\,{}_2F_1\left(1,\varepsilon;\frac{3}{2};\frac{p^2}{4m^2}\right) = -\frac{m^2}{\Gamma(\varepsilon)}\frac{4^{1-\varepsilon}\Gamma(1-\varepsilon)}{\Gamma(2-2\varepsilon)}\int_0^1 \mathrm{d}x\,\frac{2m^2}{p^2}\frac{x^{-2+\varepsilon}(1-x)^{\frac{1}{2}-\varepsilon}}{p^2-\frac{4m^2}{x}}$$

II Partial fractions:

$$\frac{2m^2}{p^2}\frac{1}{p^2-\frac{4m^2}{x}} = \frac{x}{2}\left(\frac{1}{p^2-\frac{4m^2}{x}} - \frac{1}{p^2}\right)$$

III Change of variables $x \to \lambda = 4m^2/x$:

$$\frac{\Pi(p^2,m^2)-\Pi(0,m^2)}{p^2} = \frac{n_f T_F g^2}{2\pi^2}\left(\frac{\pi\tilde{\mu}^2}{m^2}\right)^{\varepsilon}\frac{\Gamma(2-\varepsilon)}{\Gamma(4-2\varepsilon)}\int_0^1 \mathrm{d}x\left(1-\varepsilon+\frac{x}{2}\right)\frac{x^{-1+\varepsilon}(1-x)^{\frac{1}{2}-\varepsilon}}{p^2-\frac{4m^2}{x}}$$

$$= \frac{n_f T_F g^2}{2\pi^2}\frac{\Gamma(2-\varepsilon)}{\Gamma(4-2\varepsilon)}\int_{4m^2}^{\infty}\frac{\mathrm{d}\lambda}{\lambda}\left(1-\varepsilon+\frac{2m^2}{\lambda}\right)\left(\frac{4\pi\tilde{\mu}^2}{\lambda}\right)^{\varepsilon}\frac{\left(1-\frac{4m^2}{\lambda}\right)^{\frac{1}{2}-\varepsilon}}{p^2-\lambda+i\epsilon}$$

$$= T_F\frac{\alpha_s}{4\pi}\int_{4m^2}^{\infty}\mathrm{d}\tilde{m}^2\,\frac{\mathcal{V}^{(d=4-2\varepsilon)}(\tilde{m},\tilde{\mu})}{p^2-\tilde{m}^2+i\epsilon}$$

Mellin Barnes for Massive quark bubble

- Integral representation:

$$\frac{\Pi(p^2,m^2)-\Pi(0,m^2)}{p^2} = \frac{n_f T_F g^2}{2\pi^2}\left(\frac{\pi\tilde{\mu}^2}{m^2}\right)^{\varepsilon}\frac{\Gamma(2-\varepsilon)}{\Gamma(4-2\varepsilon)}\int_0^1 \mathrm{d}x\left(1-\varepsilon+\frac{x}{2}\right)\frac{x^{-1+\varepsilon}(1-x)^{\frac{1}{2}-\varepsilon}}{p^2-\frac{4m^2}{x}}$$

- Identity:

$$\frac{1}{1-\frac{p^2}{4m^2}x} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\mathrm{d}h\left(-\frac{p^2}{4m^2}x\right)^{-h}\Gamma(h)\,\Gamma(1-h)$$

- Carry out x integration. Converges for $\epsilon < 3/2$

Mellin Barnes representation

$$\frac{\Pi(p^2,m^2)-\Pi(0,m^2)}{p^2} = \frac{T_F\,\alpha_s}{2\pi^2 i p^2}\left(\frac{4\pi\tilde{\mu}^2}{m^2}\right)^{\varepsilon}\int_{c-i\infty}^{c+i\infty}\mathrm{d}h\left(-\frac{m^2}{p^2}\right)^{-h}\frac{1+h}{3+2h}\,\frac{h\Gamma^2(h)\Gamma(1-h)\Gamma(h+\varepsilon)}{\Gamma(2h+2)}$$

- $\varepsilon < 0 \Rightarrow -\varepsilon < c < 1$
- $\varepsilon > 0 \Rightarrow 0 < c < 1$

$\Pi(p^2,m^2)-\Pi(0,m^2)$ is UV finite $\Longrightarrow \varepsilon\to 0$ & $0<c<1 \Longrightarrow \boxed{\varepsilon > 0}$

Mellin Barnes for Massive quark bubble

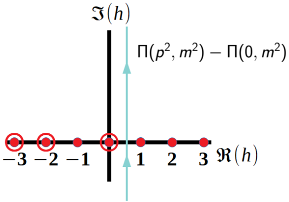
Example

$$\frac{\Pi(p^2, m^2) - \Pi(0, m^2)}{p^2} = \frac{T_F\, \alpha_s}{2\pi^2 i p^2} \left(\frac{4\pi\tilde{\mu}^2}{m^2}\right)^{\varepsilon} \int_{c-i\infty}^{c+i\infty} \mathrm{d}h \left(-\frac{m^2}{p^2}\right)^{-h} G(h,\varepsilon)$$

$$G(h,\varepsilon) \equiv \frac{\Gamma(h)\Gamma(2+h)\Gamma(1-h)\Gamma(h+\varepsilon)}{(3+2h)\Gamma(2h+2)}$$

$\Pi(p^2, m^2) - \Pi(0, m^2)$ is UV finite $\Longrightarrow \varepsilon \to 0 \Longrightarrow G(h, 0)$:

- Simple poles at: (-1) and $(n;\ n \in \mathcal{N};\ n \geq 1)$
- Double poles at: (0) and $(-n;\ n \in \mathcal{N};\ n \geq 2)$



Mellin Barnes for Massive quark bubble

Example

Convergence radius:

$$G(h,0)\left(\frac{m^2}{|p^2|}\right)^{-h} \underset{|h|\to\infty}{\sim} \frac{\pi^{3/2}\csc(\pi h)}{4h^{3/2}}\left(\frac{\boxed{4}\,m^2}{|p^2|}\right)^{-h} \Longrightarrow \begin{cases} h>0 \Rightarrow |p^2| \leq 4m^2 \\ h<0 \Rightarrow |p^2| \geq 4m^2 \end{cases}$$

- L.h.s. expansion: $|p^2| \geq 4m^2$:

$$\frac{\Pi(p^2,m^2)-\Pi(0,m^2)}{p^2} = \frac{n_f T_F \alpha_s}{\pi\, p^2}\left\{\frac{1}{3}\log\left(-\frac{p^2}{m^2}\right) - \frac{5}{9} - \frac{2m^2}{p^2} + 2\sum_{n=2}\left(\frac{m^2}{p^2}\right)^n \frac{(2n-2)!}{(3-2n)^2(n!)^2}\right.$$
$$\left.\times\left[(n-1)(2n-3)n\left[\log\left(-\frac{p^2}{m^2}\right)+2(H_{2n-2}-H_{n-2})\right]+2(5-3n)n-3\right]\right\}$$

$$*|S_{n\to\infty}| \sim \frac{1}{4\sqrt{\pi}n^{3/2}}\left(\frac{4m^2}{p^2}\right)^n \log\left(-\frac{4m^2}{p^2}\right) \overset{\text{root test}}{\Rightarrow} |p^2| \geq 4m^2 \ \checkmark$$

- R.h.s. expansion: $|p^2| \leq 4m^2$:

$$\frac{\Pi(p^2,m^2)-\Pi(0,m^2)}{p^2} = -\frac{n_f T_F \alpha_s}{\pi\, p^2}\sum_{n=1}^{\infty}\frac{(n-1)!(n+1)!}{(2n+3)(2n+1)!}\left(\frac{p^2}{m^2}\right)^n$$

$$*|S_{n\to\infty}| \sim \frac{\sqrt{\pi}}{4n^{3/2}}\left(\frac{p^2}{4m^2}\right)^n \overset{\text{root test}}{\Rightarrow} |p^2| \leq 4m^2 \ \checkmark$$

Mellin Barnes Dispersive integral method

two-loop contribution

For IR safe at one loop matrix element M:

Massive bubble two loops contribution:

$$M_{\text{bubble}}^{\text{2-loops}}(m) = \frac{n_f T_F \alpha_s e^{\varepsilon\gamma_E}}{\pi} \left(\frac{\mu^2}{m^2}\right)^{\varepsilon} \left[\int_{c-i\infty}^{c+i\infty} \frac{\mathrm{d}h}{2\pi i} m^{-2h} G(h,\varepsilon) M_h^{\text{1-loop}} - \frac{(\Gamma(\varepsilon) - \frac{1}{\varepsilon})}{3} M_{h=0}^{\text{1-loop}}\right]$$

$$= M^{\text{2-loops, OS-bubble}}(m) + \Pi^{\text{ren}}(0, m^2) M_{h=0}^{\text{1-loop}}$$

with:

$$G(h,\varepsilon) \equiv \frac{\Gamma(h)\Gamma(2+h)\Gamma(1-h)\Gamma(h+\varepsilon)}{(3+2h)\Gamma(2h+2)} ,$$

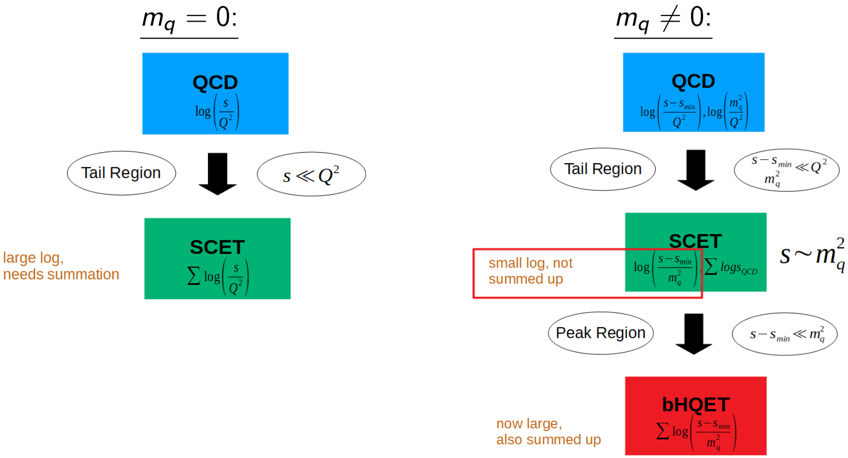
$M^{\text{2-loops, OS-bubble}}(m)$ the two loops contribution from the on-shell renormalized bubble and $M_h^{\text{1-loop}}$ the one loop matrix element with an effective gluon propagator:

$$\frac{-ig^{\mu\nu}}{p^2} \longrightarrow \frac{ig^{\mu\nu}}{(-p^2)^{1-h}}$$

* Massless gluon propagator with modified exponent. Same as in large-β_0 computations

Dijets' primary quarks production

(s = jet invariant mass)

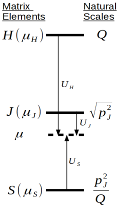


* Q center-of-mass energy

Factorization theorems

$e^+e^- \to$ Hadrons. 2-jettiness. $\left(\tau \approx \frac{s_{h+}+s_{h-}}{Q^2}\right.$ in the peak region$\left.\right)$

- **SCET:** [0801.4569], [hp-ph/0703207], [0711.2079]



($\hat{\sigma} \to$ partonic cross section)

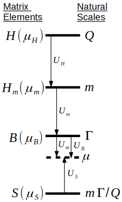
$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_{\rm SCET}}{d\tau} = Q^2 \overbrace{H(Q,\mu)}^{\text{matching coefficient}} \int_0^{Q(\tau-\tau_{\rm min})} d\ell\, J_\tau(Q^2\tau - Q\ell,\mu)\, \overbrace{S_\tau(\ell,\mu)}^{\text{soft radiation}} + \text{p.c.}$$

$$J_\tau(s,\mu) \equiv \int_{s_{\rm min}}^{s-s_{\rm min}} ds'\, J_n(s-s',\mu) J_{\bar{n}}(s',\mu) \quad \text{collinear radiation}$$

each element evaluated at its natural scale. Running to a common scale sums up large logs

- **bHQET (Boosted Heavy Quark):** [0801.4569], [hp-ph/0703207], [0711.2079]

1) Integrate out heavy quark mass
2) Boost back to c.o.m. frame
3) Match onto SCET in order to account for global soft radiation.



$$\frac{1}{\sigma_0}\frac{d\hat{\sigma}_{\rm bHQET}}{d\tau} = Q^2 H(Q,\mu_m)\, \overbrace{H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right)}^{\text{new matching coefficient}} \int d\ell\, \overbrace{B_\tau\left(\frac{Q^2(\tau-\tau_{\rm min}) - Q\ell}{m}, \mu\right)}^{\text{new jet function}} S_\tau(\ell,\mu) + \text{p.c.}$$

$$B_\tau(\hat{s},\mu) = m\int_0^{\hat{s}} d\hat{s}'\, B_n(\hat{s}-\hat{s}',\mu) B_n(\hat{s}',\mu)$$

∗ for more details see talks by M.Benitez and V.Mateu

Applications
SCET Hard function

Definition:

$$\mathcal{J}_{\text{QCD}} = C\,\mathcal{J}_{\text{SCET}}$$
$$H = |C|^2$$

$$\mathcal{J}_{\text{QCD}} = \bar{q}\Gamma_C^\mu q \qquad\qquad \mathcal{J}_{\text{SCET}} = \bar{\xi}_n\, W_n \Gamma_C^\mu W_{\bar{n}}^+ \xi_{\bar{n}}$$

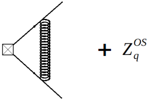
* ξ_n and W_n are the collinear quark field and Wilson line respectively

Computation:

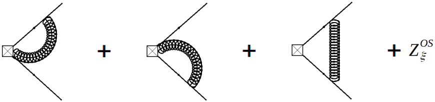
$$C = \frac{\langle q, \bar{q}|\mathcal{J}_{\text{QCD}}|0\rangle}{\langle q, \bar{q}|\mathcal{J}_{\text{SCET}}|0\rangle} = \frac{F_{\text{QCD}}}{F_{\text{SCET}}}$$

Feynman Diagrams:

QCD



SCET



Applications

SCET Hard function: Results

Only QCD Vertex contributes with M.B.

One loop: (massless gluon propagator with modified exponent)

$$H_h^{\text{1-loop}} = \frac{g^2 C_F}{4\pi^2} Q^{2h} \left(\frac{\mu^2 e^{\gamma_E}}{Q^2} \right)^{\varepsilon} H_{1,h}$$

$$H_{1,h} \equiv -\frac{\Gamma^2(h-\varepsilon)\Gamma(1-h+\varepsilon)\cos[\pi(h-\varepsilon)]}{\Gamma(3+h-2\varepsilon)} \{2 - \varepsilon[3 + h^2 + h(2-3\varepsilon) - \varepsilon(3-2\varepsilon)]\}$$

Two loops:

$$H^{\text{2-loops, OS-bubble}} = \left(\frac{\alpha_s}{\pi} \right)^2 C_F n_f T_F \left(\frac{\mu^2}{m^2} \right)^{2\varepsilon} \left(\frac{Q^2}{m^2} \right)^{-\varepsilon} H_2$$

$$H_2 \equiv e^{2\varepsilon\gamma_E} \int_{c-i\infty}^{c+i\infty} \frac{\mathrm{d}h}{2\pi i} \left(\frac{Q^2}{m^2} \right)^{h} G(h,\varepsilon) H_{1,h}$$

Applications

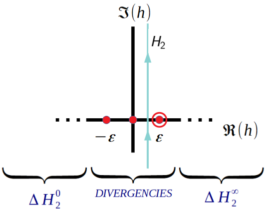
SCET Hard function: Comments

$\rightarrow$ $H_2(\varepsilon = 0) \underset{|h|\to\infty}{\sim} \frac{\csc(\pi h)\cot(\pi h)}{2h^{9/2}} \left(\frac{|Q|^2}{4\, m^2}\right)^h$

$\rightarrow$ Same UV divergencies on the right and left sides (in all carried out calculations):

$$\lim_{\varepsilon\to 0}\Bigg\{\overbrace{\underset{h=0}{\text{Res}}\left[G(h,\varepsilon)H_{1,h}(\varepsilon)\right] + \underset{h=-\varepsilon}{\text{Res}}\left[\left(\frac{Q^2}{m^2}\right)^h G(h,\varepsilon)H_{1,h}(\varepsilon)\right]}^{\text{massless bubble } [H_2]_{m\to 0}} + \overbrace{\underset{h=\varepsilon}{\text{Res}}\left[\left(\frac{Q^2}{m^2}\right)^h G(h,\varepsilon)H_{1,h}(\varepsilon)\right]}^{[H_2]_{m\to\infty}}\Bigg\} =$$

$$\underset{h=0}{\text{Res}}\left[\left(\frac{Q^2}{m^2}\right)^h G(h,0)H_{1,h}(0)\right] = \mathcal{M}(\text{SCET}^{n_f}/\text{SCET}^{n_l})$$



$$\Delta^0 H_2 \equiv H_2 - [H_2]_{m\to 0}\,; \quad \Delta^\infty H_2 \equiv H_2 - [H_2]_{m\to\infty}$$

$\rightarrow$ [P. Pietrulewicz, S. Gritschacher, A. H. Hoang, I. Jemos, V. Mateu, 2014] ✓

Applications

SCET Hard function: Expansion Series

$$\Delta^\infty H_2 = 8\sum_{n=1} \frac{(n+1)\Gamma(n)^2}{\Gamma(2n+5)} \left(-\frac{Q^2}{m^2}\right)^n \left[2(\mathrm{H}_{n-1} - \mathrm{H}_{2n+1}) + L - \frac{1}{n+1}\right]$$

$$\begin{aligned}\Delta^0 H_2 = {} & \frac{Q^4}{3m^4}L^3 + \frac{2Q^4}{m^4}L^2 + \frac{2Q^2}{m^2}L^2 + \frac{Q^4}{m^4}\frac{33-4\pi^2}{6}L - \frac{4Q^2}{m^2}L + \frac{Q^4}{m^4}\frac{24\zeta_3 + 39 - 8\pi^2}{6} \\ & - \frac{Q^2}{m^2}\frac{4(\pi^2-12)}{3} - 16\sum_{n=3}^{\infty}\frac{(n-1)(2n-5)!}{(n!)^2}\left(-\frac{m^2}{Q^2}\right)^n \left\{\frac{2}{n-1}\left(\mathrm{H}_{2n-5} - \mathrm{H}_n - \frac{1}{2}L\right)\right. \\ & \left. +2\left(\mathrm{H}_{2n-5} - \mathrm{H}_n - \frac{1}{2}L\right)^2 + \mathrm{H}_n^{(2)} - 2\mathrm{H}_{2n-5}^{(2)} - \frac{\pi^2}{3}\right\}\end{aligned}$$

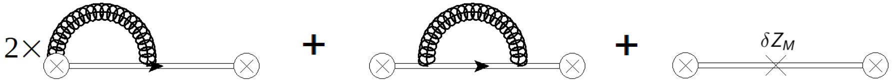
* $L \equiv \log\left(\frac{Q^2}{m^2}\right)$
* $\mathrm{H}_n^{(i)}$ n-th harmonic number of order i.

Definition:

$$\mathcal{J}_n(s,\mu) = \frac{-i}{4\pi N_C Q}\ \mathrm{Tr}\left[\int \mathrm{d}^d x\, e^{ikx} \left\langle 0 \right| T\left\{ \bar{\xi}_n(0) W_n(0)\, \bar{\not{n}}\, W_n^{+}(x)\, \xi_n(x) \right\} \left| 0 \right\rangle \right]$$

$$J_n(\hat{s},\mu) = \mathrm{Im}\left[\mathcal{J}_n(\hat{s},\mu)\right]$$

Diagrams:



δZ_M contribution vanishes with M.B.

One loop: (massless gluon propagator with modified exponent)

$$J_h^{\text{1-loop}} = \frac{g^2 C_F}{4\pi^2} s^h \left(\frac{\mu^2 \varepsilon^{\gamma_E}}{s} \right)^{\varepsilon} J_{1,h}$$

$$J_{1,h} \equiv \frac{1}{2s} \frac{\Gamma(2-\varepsilon)}{\Gamma(1-h)\Gamma(3+h-2\varepsilon)} \left[5 - \varepsilon + \frac{2(2-h)}{h-\varepsilon} \right]$$

Two loops:

$$J^{\text{2-loops, OS-bubble}} = \left(\frac{\alpha_s}{\pi} \right)^2 C_F n_f T_F \left(\frac{\mu^2}{m^2} \right)^{2\varepsilon} \left(\frac{s}{m^2} \right)^{-\varepsilon} J_2$$

$$J_2 \equiv e^{2\varepsilon\gamma_E} \int_{c-i\infty}^{c+i\infty} \frac{\mathrm{d}h}{2\pi i} \left(\frac{s}{m^2} \right)^h G(h,\varepsilon) J_{1,h}$$

→ [P. Pietrulewicz, S. Gritschacher, A. H. Hoang, I. Jemos, V. Mateu, 2014] ✓

Fast convergent expansion series for the RG-evolved jet function

$$J(s,\tilde{\omega}) = \int_0^s \mathrm{d}s'(s-s')^{-1-\tilde{\omega}}\, J^{\mathrm{ren}}(s')$$

1) Renormalize Jet function
2) Move to a band that has no pole crossing when $\epsilon \to 0$
3) Set $\varepsilon = 0$ in Mellin Barnes representation
4) Carry out convolution with evolution kernel
5) Inverse Mellin transform

Evolution of the mass corrections $\Delta^0 J_2 \equiv J_2 - [J_2]_{m\to 0}$

$$\Delta^0 J_2(s,\tilde\omega) = \frac{C_F}{2}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \mathrm{d}h \frac{\Gamma^3(h)}{(2h+3)(h+2)\Gamma(2h+2)\Gamma(h-\tilde\omega)}\left[\frac{2(2-h)}{h}+5\right]\left(\frac{m^2}{s}\right)^{-h}$$

$$= -\frac{2[L+\psi^{(0)}(-\tilde\omega-1)-4]}{\Gamma(-\tilde\omega-1)}\left(\frac{m^2}{s}\right) + \left(\frac{m^2}{s}\right)^2 \frac{1}{24\Gamma(-\tilde\omega-2)}$$

$$\times \{\pi^2 - 6\psi^{(0)}(-\tilde\omega-2)[2L+\psi^{(0)}(-\tilde\omega-2)-5] - 6[(L-5)L+7] + 6\psi^{(1)}(-\tilde\omega-2)\}$$

$$+ 2L\sum_{n=3}^{\infty}\frac{(3n-4)\Gamma(2n-1)}{(2n-3)(n-2)n(n!)^3\Gamma(-n-\tilde\omega)}\left(\frac{m^2}{s}\right)^n + 2\sum_{n=3}^{\infty}\left(\frac{m^2}{s}\right)^n \frac{\Gamma(2n-1)}{(3-2n)^2(n-2)n(n!)^4\Gamma(-n-\tilde\omega)}$$

$$\times \{(2n-3)(3n-4)\Gamma(n+1)[2H_{2n-2} - 3H_{n-3} + \psi^{(0)}(-n-\tilde\omega)]$$

$$+(n(n(6(53-11n)n-551)+398)-96)\Gamma(n-2)\}$$

- $c \in (-1/2, 0)$
- $L \equiv \log\left(\frac{m^2 e^{\gamma_E}}{s}\right)$

Applications

bHQET Jet function

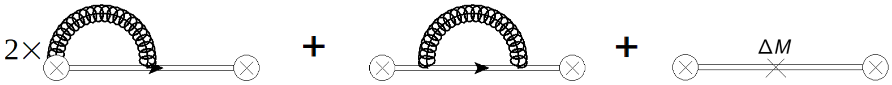
Definition:

$$\mathcal{B}_n(\hat{s},\mu) = \frac{i}{4\pi N_C M}\ \mathrm{Tr}\left[\int \mathrm{d}^d x\, e^{ikx} \left\langle 0\right| T\left\{ W_n^{+}(x)\, h_v(x)\ \bar{h}_v(0)\, W_n(0)\right\} \left|0\right\rangle\right]$$

$$B_n(\hat{s},\mu) = \mathrm{Im}\left[\mathcal{B}_n(\hat{s},\mu)\right]$$

* W_n Wilson lines with u-collinear gluons
* h_v Heavy quark field

Contributing diagrams:



Applications

bHQET Jet function: Comments

- $\rightarrow$ $1/2 < \varepsilon < 1$ to regulate loop UV divergencies
- $\rightarrow$ In Mellin Barnes representation $1/2 < \varepsilon < 1$ to split UV from IR poles in $\mathcal{B}$
- $\rightarrow$ Divergent cases $\Longrightarrow$ analytic continuation ($\epsilon \rightarrow 0$) after inverse Mellin tranform
- $\rightarrow$ ΔM contribution comes from SCET matching at two loops:

$$\mathcal{L}_{\mathrm{bHQET}} = \bar{h}_v \left[i\, v \cdot \mathrm{D} - \Delta M \right] h_v$$

$$\Delta M = M_{m=0,\,\mathrm{pole}} - M_{\mathrm{pole}}$$

- $\rightarrow$ Agrees with computation through dispersive integral method with massive gluon ✓(To be shown elsewhere)

Applications

bHQET Jet function: NEW Results

One loop (massless gluon propagator with modified exponent)

$$MB_h^{\text{1-loop}} = -C_F \frac{\alpha_s}{\pi} \frac{\Gamma(2+h-\varepsilon)\,\hat{s}^{-1+2h} e^{\varepsilon\gamma}}{(\varepsilon-h)\Gamma(1-h)\Gamma(2+2h-2\varepsilon)} \left(\frac{\mu}{\hat{s}}\right)^{2\varepsilon}$$

Evolution of the mass corrections $\Delta^0 M\, B_2 \equiv M\, B_2 - [M\, B_2]_{m\to 0}$

$$\begin{aligned}
\Delta^0 M\, B_2(s,\tilde{\omega}) = &- \frac{1}{2\Gamma(-\tilde{\omega}-2)} \left(\frac{m}{\hat{s}}\right)^2 + 2\log^2\left(\frac{\hat{s}}{m}\right) \sum_{n=2} \left(\frac{m}{\hat{s}}\right)^{2n} \frac{(n-1)^3\Gamma(2n-3)}{(2n-1)(n!)^4\Gamma(-2n-\tilde{\omega})} \\
&+ \frac{\pi^2}{4\Gamma(-\tilde{\omega}-1)} \left(\frac{m}{\hat{s}}\right) + 2\log\left(\frac{\hat{s}}{m}\right) \sum_{n=2} \left(\frac{m}{\hat{s}}\right)^{2n} \frac{(n-1)^2\Gamma(2n-3)}{(2n-1)^2(n!)^4\Gamma(-2n-\tilde{\omega})} \\
&\times [(6n-4n^2-2)H_{2n-4} - 2(n-1)(2n-1)H_{-2n-\tilde{\omega}-1} + 4(n-1)(2n-1)H_n - 4n + 1] \\
&+ \sum_{n=2} \left(\frac{m}{\hat{s}}\right)^{2n} \frac{(n-1)\Gamma(2n-3)}{(2n-1)^3(n!)^4\Gamma(-2n-\tilde{\omega})} \Big\{ 2(2n^2-3n+1)^2 H^2_{-2n-\tilde{\omega}-1} + 4n^2 - 2n + 1 \\
&- 4H_n[2(2n^2-3n+1)^2 H_{-2n-\tilde{\omega}-1} + 2(2n^2-3n+1)^2 H_{2n-4} + (n-1)(2n-1)(4n-1)] \\
&+ H_{2n-4}[4(2n^2-3n+1)^2 H_{-2n-\tilde{\omega}-1} + 2(n-1)(2n-1)(4n-1)] + 8(2n^2-3n+1)^2 H_n^2 \\
&+ 2(2n^2-3n+1)^2 H^2_{2n-4} + 2(n-1)(2n-1)(4n-1)H_{-2n-\tilde{\omega}-1} + 2(2n^2-3n+1)^2 H_n^{(2)} \\
&+ 2(2n^2-3n+1)^2 H^{(2)}_{-2n-\tilde{\omega}-1} - 2(2n^2-3n+1)^2 H^{(2)}_{2n-4} - \frac{\pi^2}{3}(2n^2-3n+1)^2 \Big\}
\end{aligned}$$

Applications
bHQET Hard function

Definition:

$$\mathcal{J}_{\text{SCET}} = C_M \, \mathcal{J}_{\text{bHQET}}$$
$$H_M = |C_M|^2$$

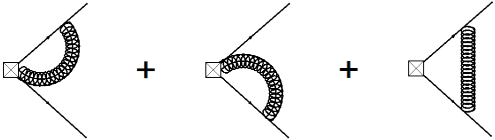
$$\mathcal{J}_{\text{bHQET}} = \bar{h}_{v_+} \, W_n^{v} \Gamma_C^{\mu} (W_{\bar{n}}^{v})^{+} h_{v_-}$$

Computation:

$$C_M = \frac{\langle q, \bar{q} | \mathcal{J}_{\text{SCET}} | 0 \rangle}{\langle q, \bar{q} | \mathcal{J}_{\text{bHQET}} | 0 \rangle} = \frac{F_{\text{SCET}}}{F_{\text{bHQET}}}$$

Contributing diagrams:

SCET & bHQET:



$+ \, Z_{\xi}^{OS}$

Applications

bHQET Hard function: NEW Results

Only $F_{\rm SCET}$ contributes with M.B.

One loop (massless gluon propagator with modified exponent)

$$F_h^{\rm SCET,\ 1\text{-}loop} = \frac{\alpha_s C_F}{\pi}\left(\mu^2 e^{\gamma}\right)^{\varepsilon}(h-1)\left(1+\frac{(3-2\varepsilon)(\varepsilon-h-1)(\varepsilon-h)}{2+h-2\varepsilon}\right)\frac{\Gamma(\varepsilon-h)\Gamma(2h-2\varepsilon)}{\Gamma(2+h-2\varepsilon)}\left(M^2\right)^{h-\varepsilon}$$

Two loops

$$\Delta^0 H_M^{\text{2-loops}} = \left(\frac{\alpha_s^{(n_l+1)}}{4\pi}\right)^2 C_F T_f\Bigg[6\pi^2\frac{m}{M}+\frac{4m^2}{M^2}\left(6+8\hat{L}_m\right)-\frac{110}{9}\pi^2\frac{m^3}{M^3}$$
$$+\frac{m^4}{3M^4}\left(145+12\pi^2-72\left(2-\hat{L}_m\right)\hat{L}_m\right)+\frac{m^6}{M^6}\sum_{n=0}^{\infty}a_n(m/M)\left(\frac{m^2}{M^2}\right)^n\Bigg]$$

$$a_n(m/M) = \frac{8}{(n+1)(n+2)(n+3)^3(2n+3)(2n+5)}\Big[506+750n+413n^2+100n^3+9n^4$$
$$+\left(408+778n+589n^2+221n^3+41n^4+3n^5\right)\log(4)+(n+2)(n+3)(n+4)\left(17+14n+3n^2\right)\times$$
$$\times\left[2\,\hat{L}_m+\psi(-1/2-n)+\psi(1+n)-2\,\psi(7+2n)\right]\Big]$$

* $\hat{L} \equiv \log(m/M)$

→ Agrees with dispersive integral method with massive gluon ✓(To be shown elsewhere)

Conclusions

→ We proposed a dispersive integral method in Mellin plane to compute massive bubble corrections:

- Does not introduce an additional energy scale in previous order computations.
- Many loop integrals become scaleless and vanish.
- MB parameter (h) acts as analytic regulator $\Rightarrow$ no trace of rapidity divergencies in loop integrals. No 0−bin subtractions.
- Inverse Mellin transform by residues yields directly fast converging expansions even for RG evolved matrix elements.
- In some cases it could also provide closed expresions in terms of Meijer G-functions or carring out the sum of the series.

→ We applied it to the SCET Hard and Jet functions, making the comparison with known results

→ We obtained the corresponding contributions for the Hard and Jet bHQET functions

→ Trivially adapted to massive bosons: (∗ not shown due to lack of time)

$$\frac{1}{-p^2+m_g^2} = \frac{1}{-p^2}\frac{1}{1-\frac{m_g^2}{p^2}} = \frac{1}{-p^2}\int_{c-i\infty}^{c+i\infty}\frac{ds}{2\pi i}\left(-\frac{m_g^2}{p^2}\right)^{-s}\Gamma(s)\Gamma(1-s)$$