Hamiltonian light-front dynamics, holographic QCD and entanglement in high-energy scattering

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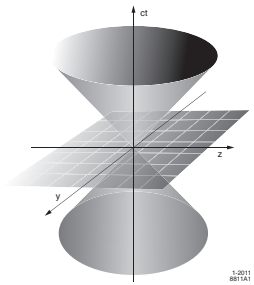
Hamiltonian Field Theory for QCD and Hadron Physics
Granada, 15-18 May 2023

In collaboration with Hans G. Dosch and Stanley J. Brodsky,
[arXiv: 2304.14207 [hep-ph]]

According to one anecdote von Neumann suggested to Shannon to use the term entropy also because *"nobody knows exactly what entropy is and this is a big advantage in any discussion"*

Outline

Hamiltonian formulation and entanglement entropy



The Hamiltonian is the generator of time evolution **and** of Hilbert space of state vectors. It is an ideal formalism to describe entanglement and von Neumann quantum entropy

Instant-form QM

Initial surface is at $t = 0$ and time evolution is given by the Hamiltonian (Schrödinger) equation

$$i\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle, \qquad H = \sum_i \frac{\mathbf{p}_i^2}{2m_i} + V$$

The eigenvectors of H provide an orthonormal basis $|a\rangle$ for Hilbert space

$$H|a\rangle = E_a|a\rangle, \qquad |\psi\rangle = \sum_a c_a|a\rangle$$

where the E_a are the possible energy levels

Measurement probes a single state $|a\rangle$ and the density operator ρ becomes

$$\rho = |\psi\rangle\langle\psi| \quad \longrightarrow \quad \sum_a p_a|a\rangle\langle a|$$

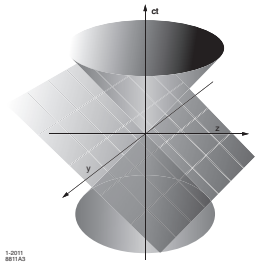
where p_a, $\sum_a p_a = 1$, is the probability of finding the value E_a

The quantum entropy S_Q is given by the trace of the density operator

$$S_Q = -Tr[\rho \ln \rho] = -\sum_a p_a \ln p_a \ \geq 0$$

and is a measure of entanglement. For a pure state, $\rho^2 = \rho$ and the entropy is zero

Front-form QCD



Light-front quantization uses the null plane $x^+ = x^0 + x^3 = 0$ tangent to the light cone as the initial surface (Dirac 1949), thus without reference to a specific Lorentz frame

Evolution in LF time x^+ is given by the Hamiltonian equation

$$i\frac{\partial}{\partial x^+}|\psi\rangle = P^-|\psi\rangle, \qquad P^-|\psi\rangle = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}|\psi\rangle,$$

for a hadron with 4-momentum $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, where P^- is a dynamical generator and P^+ and $\mathbf{P}_\perp$ are kinematical

Hadron mass spectra from LF invariant Hamiltonian $P^2 = P_\mu P^\mu = P^+P^- - \mathbf{P}_\perp^2$

$$P^2|\psi(P)\rangle = M^2|\psi(P)\rangle, \qquad |\psi\rangle = \bigoplus_n^\infty \psi_n|n\rangle$$

The LF Fock expansion is the direct sum of the direct product of N-parton states above the valence state, thus, for a proton

$$|\psi\rangle_p = \psi_{uud/p}|uud\rangle \oplus \psi_{uudg/p}|uudg\rangle \oplus \psi_{uudq\bar{q}/p}|uudq\bar{q}\rangle \oplus \psi_{uudggg}|uudggg\rangle \oplus \cdots$$

$$M_p^2 = \sum_{n,n'} \int \left[dx_i\right]\left[d^2\mathbf{k}_{\perp i}\right]\psi^*_{n/p}(x_i, \mathbf{k}_{\perp i})\Big[\sum_{j=q,\bar{q},g}\Big(\frac{k_{\perp j} + m_j^2}{x_j}\Big)\delta_{nn'} + U_{nn'}\Big]\psi_{n'/p}(x_i, \mathbf{k}_{\perp i})$$

The coefficients ψ_n in the Fock expansion, the LFWFs, $\psi_n = \langle n|\psi\rangle$, are written in terms of the quark and gluon degrees of freedom and incorporate the off-shellness of the bound state: partons are entangled

$$\rho = \frac{1}{N}\begin{pmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & 0 & & 1 & \\ & & & & 1 \end{pmatrix}$$

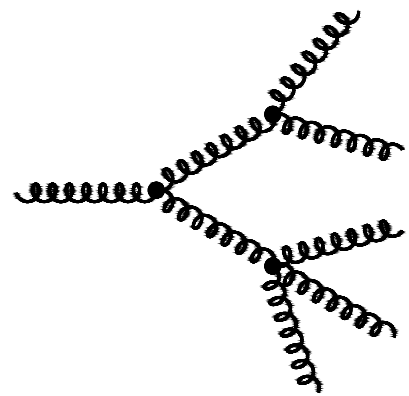
Maximal entropy

A DIS measurement projects one of the parton states in a Fock component and, after the measurement, the state is described by the density operator

$$\rho = \sum_{j=1}^{N} p_j |j\rangle\langle j|, \quad \sum_j p_j = 1$$

where p_j is the probability of hitting the parton $|j\rangle$, thus the entropy

$$S_{DIS} = -\sum_j p_j \ln p_j$$



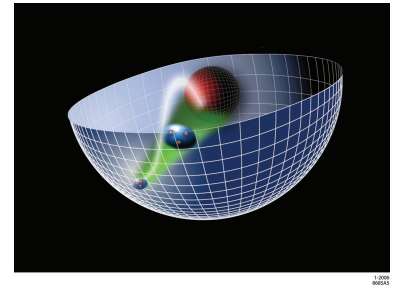
For very slow partons the gluons are dominant and all partons are on equal footing: $p_j = 1/N(x)$. Thus

$$S_{DIS} = \ln N(x) = \ln(x\, g(x))$$

the entropy of a maximally entangled state between the N-partons in a Fock component

$N(x)$ is the number of gluons with longitudinal momentum fraction x, $x\,g(x)$), with $g(x)$ the gluon distribution, the result of Kharzeev and Levin [PRD 95, 114008 (2017)]

Holographic light-front QCD (HLFQCD)



$$ds^2 = \frac{R^2}{z^2}(dx_\mu^2 - dz^2)$$

HLFQCD originatesd from the precise mapping of the AdS and LF expressions of FFs for an arbitrary number of partons

Nonperturbative analytic approach follows from a semiclassical approximation to light-front QCD and its holographic embedding in AdS space: It leads to relativistic wave equations similar to the Schrödinger equation in atomic physics

Underlying superconformal algebraic structure introduces a mass scale and fix the effective confinement potential: The zero mass eigenmode is identified with the pion and is massless in the chiral limit

Hadron supersymmetry leads to relations between the Regge trajectories of mesons, baryons, and tetraquarks

Holographic QCD also incorporates features of the Veneziano model as emerging properties

Further extensions incorporate the exclusive-inclusive connection in QCD and provide nontrivial relations between hadron form factors and quark and gluon distributions

Review: S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, Phys. Rept. **584**, 1 (2015)

Recent review: 50 Years of Quantum Chromodynamics [arXiv:2212.11107 [hep-ph]]

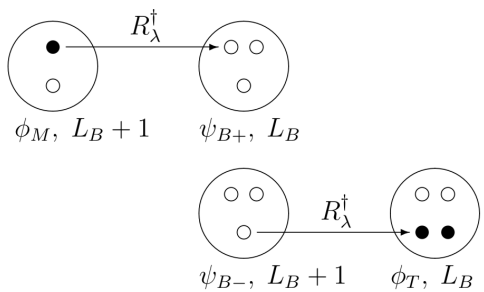
$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L_M^2}{4\zeta^2}+\lambda^2\,\zeta^2+2\lambda(L_M-1)\right)\phi_M=M^2\,\phi_M$$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L_B^2}{4\zeta^2}+\lambda^2\,\zeta^2+2\lambda(L_N+1)\right)\phi_B^+=M^2\,\phi_B^+$$

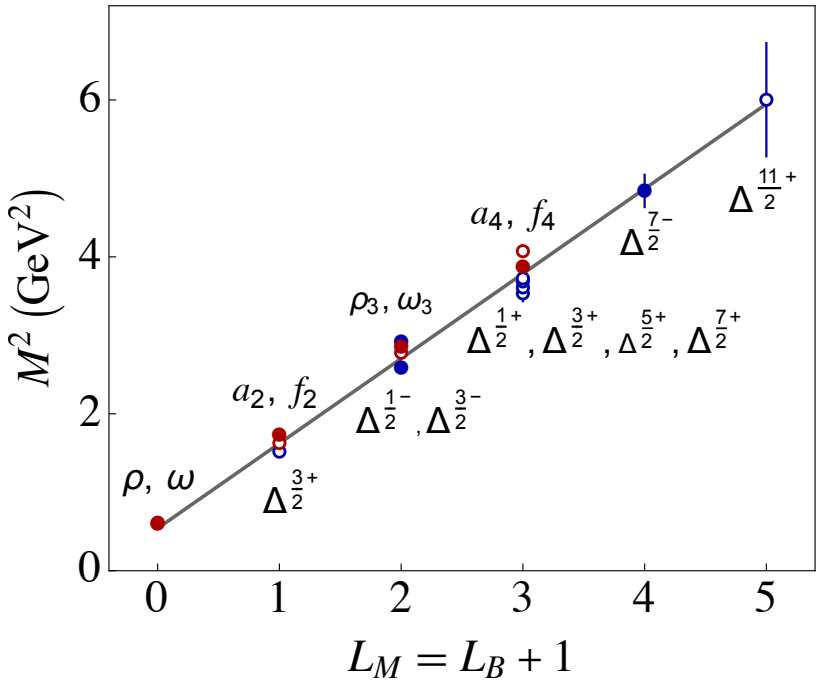
$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4(L_B+1)^2}{4\zeta^2}+\lambda^2\,\zeta^2+2\lambda L_N\right)\phi_B^-=M^2\,\phi_B^-$$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L_T^2}{4\zeta^2}+\lambda^2\,\zeta^2+2\lambda(L_T+1)\right)\phi_T=M^2\,\phi_T$$

$$\Phi=\begin{pmatrix}\phi_M & \phi_B^- \\ \phi_B^+ & \phi_T\end{pmatrix},\qquad L_M=L_B+1=L_T$$



$\bar{3}\to 3\otimes 3\quad 3\to\bar{3}\otimes\bar{3}$



Example of meson-baryon hadron supersymmetry

Light hadrons (Tetraquark mixes with conventional hadrons)
Example: $2^{++}, \frac{3}{2}^+, 1^+$ 4-plet $f_2(1270)$, $\Delta(1232)$, $a_1(1260)$

Double-heavy hadrons (Tetraquarks do not mix with conventional hadrons)
Example: $2^{++}, \frac{3}{2}^+, 1^+$ 4-plet $\chi_{c2}(3565)$, $\Xi_{cc}(3770)$, $T_{cc}(3875)$

From: H. G. Dosch, S. J. Brodsky, GdT, M. Nielsen and L. Zou (2020)

Gravitational form factors and gluon distribution functions

GdT, H. G. Dosch, T. Liu, R. S. Sufian, S. J. Brodsky, A. Deur (HLFHS) PRD **104**, 114005 (2021)

In AdS the gravitational FF $A(t)$ is computed from the coupling of the energy momentum tensor T_{MN} with the metric fluctuations h_{MN} induced by the Pomeron

$$\int d^4x\, dz \sqrt{g} h_{MN} T^{MN}$$

It allows us to extract the gluon GFF $A^g(t)$ from the hadron matrix element of the EMT

$$A^g_\tau(t) \sim B(\tau - 1, 2 - \alpha_P(t))$$

It has the form of a 3-point Veneziano amplitude for a spin-2 current with Pomeron Regge trajectory

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t$$

where $\alpha_P(0) \simeq 1.08$ and $\alpha' = 0.25\ \mathrm{GeV}^{-2}$ (Value of $\alpha_P(0)$ is an input)

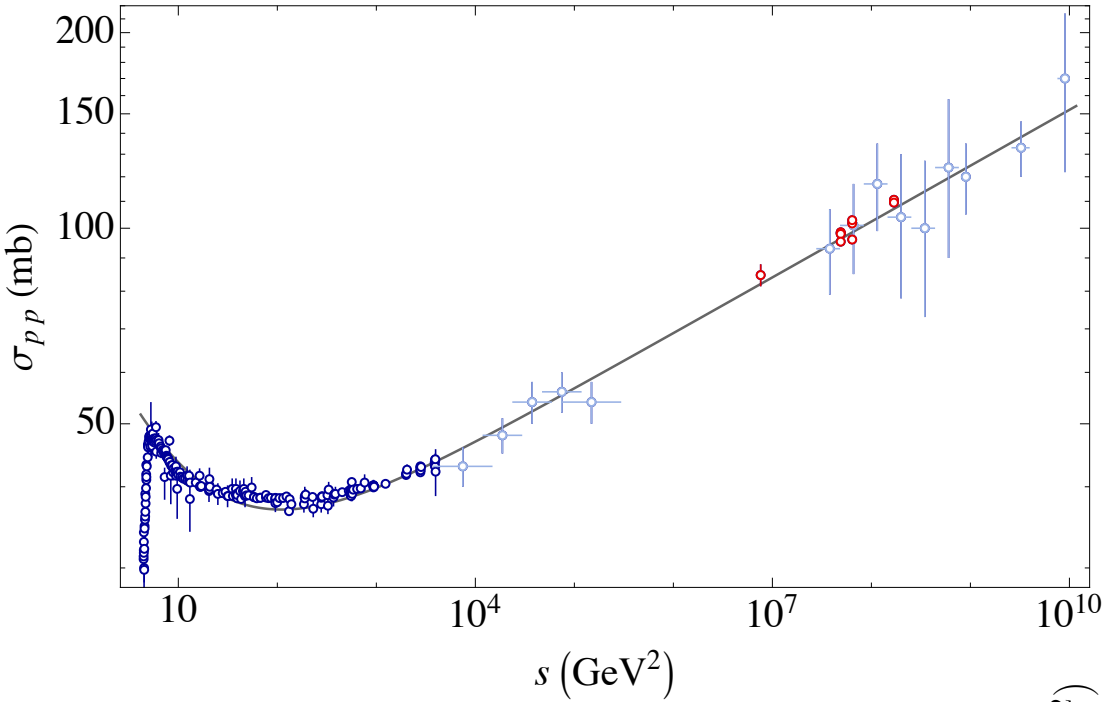
Radial spectrum from t-channel poles of Euler Beta-function in the 2^{++} trajectory

$$-Q^2 = M_n^2 = \frac{1}{\alpha'}\big(n + 2 - \alpha(0)\big), \quad M_0 \simeq 2\ \mathrm{GeV}$$

For large $Q^2 = -t > 0$ we find

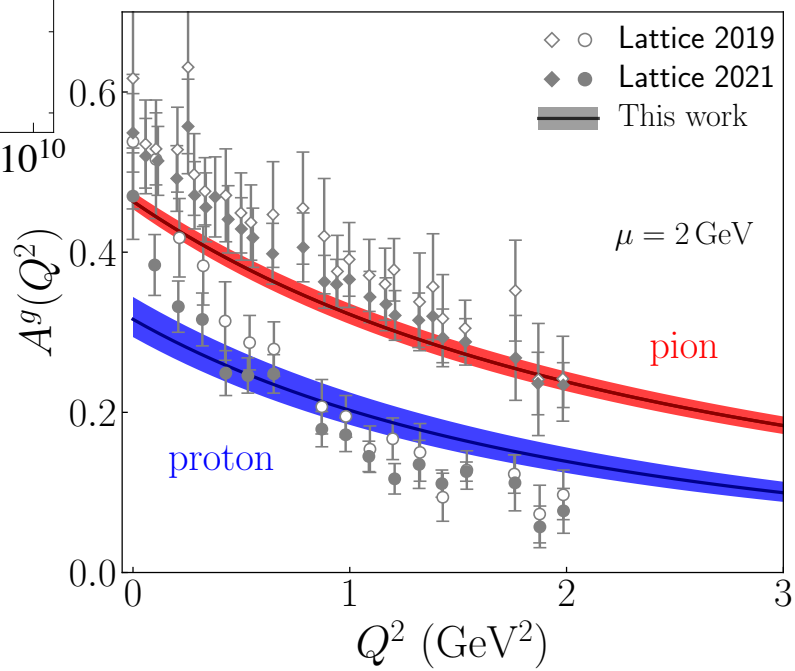
$$A^g_\tau(t) \to \left(\frac{1}{Q^2}\right)^{\tau-1}$$

the Brodsky-Farrar counting rules (1973) with $\tau = N$, the number of constituents



Total p-p scattering cross section: Red data points are recent LHC data

$$\sigma_{pp} = a(s/s_0)^{0.086} + b(s/s_0)^{-0.05}$$



Lattice data: Shanahan *et al.* (2018), Pefkou *et al.* (2021)

Intrinsic gluon distribution function

Using the integral representation of the Beta function the gGFF is expressed in a reparametrization invariant form

$$A_\tau^g(t) = \frac{1}{N_\tau}\int_0^1 dx\, w'(x) w(x)^{1-\alpha_P(t)}\,[1-w(x)]^{\tau-2}$$

provided that $w(0)=0, \quad w(1)=1, \quad w'(x)\geq 0$

gFF is the first moment of the gGPD $H_\tau^g(x,t)\equiv H_\tau^g(x,\xi=0,t)$ at zero skewness

$$A_\tau^g(t) = \int_0^1 x\, dx\, H_\tau^g(x,t) = \int_0^1 x\, dx\, g_\tau(x)\exp[tf(x)]$$

with the profile function $f(x)$ and the collinear PDF $g(x)$ determined by $w(x)$

$$f(x) = \alpha'_P \log\left(\frac{1}{w(x)}\right)$$

$$x\, g_\tau(x) = \frac{1}{N_\tau}[1-w(x)]^{\tau-2} w(x)^{1-\alpha_P(0)} w'(x)$$

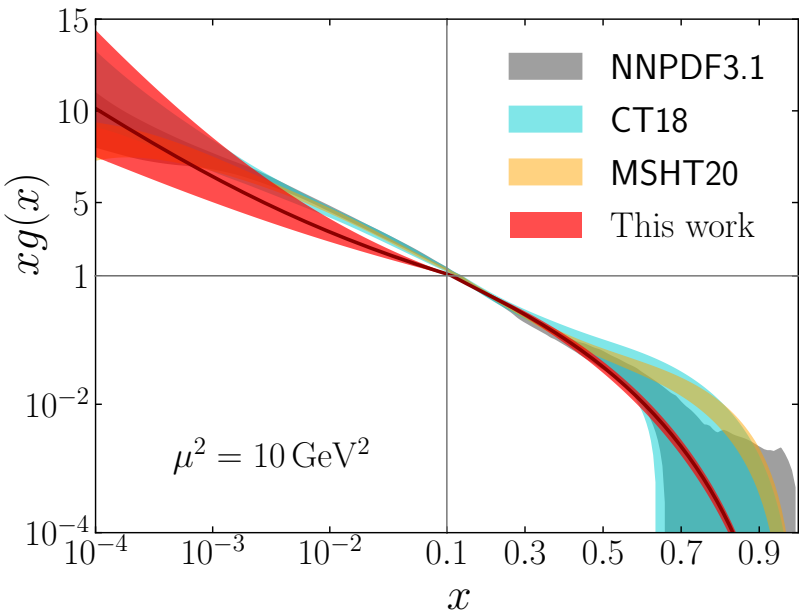
End point physical constraints: At $x\sim 0$, $w(x)\sim x$ from Regge behavior, and at $x\sim 1$, $w'(1)=0$ from the inclusive-exclusive connection

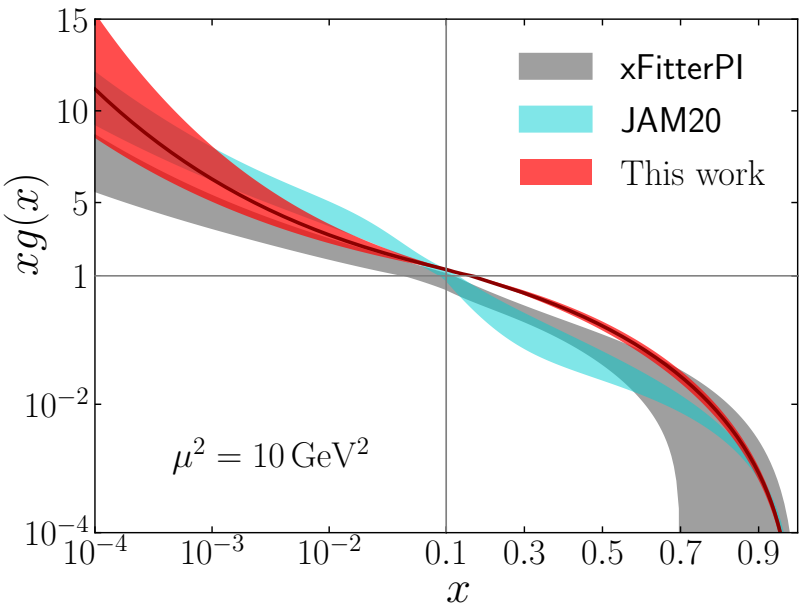
$$x\, g(x) \sim \begin{cases} \left(\frac{1}{x}\right)^{\alpha_P-1}, & \text{if } x\to 0 \\ (1-x)^{2\tau-3}, & \text{if } x\to 1 \end{cases}$$

Normalization determined by the momentum the sum rule:

$$\int_0^1 dx\, x\big[g(x) + \sum_q q(x)\big] = 1, \qquad x\, g(x) = \sum_\tau C_\tau x\, g_\tau(x)$$

Basic parameters and $w(x)$ fixed in the quark sector: No adjustable parameters at leading twist





Gluon PDFs at leading twist: Proton left figure and pion right.
PDFs at scale μ from DGLAP evolution from initial scale $\mu_0 \simeq 1$ GeV

DIS entropy and the Pomeron intercept

A key ingredient for the determination of the DIS entropy is the relation between the Pomeron intercept and the intrinsic gluon distribution at small x

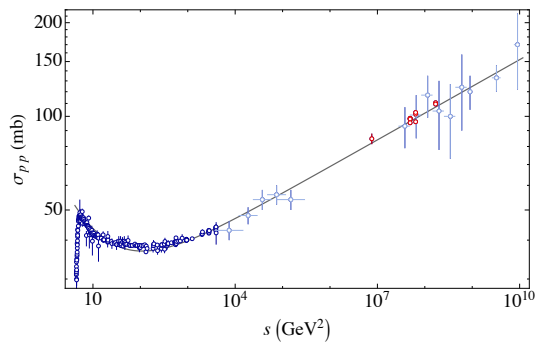
$$x\, g(x,\mu) \sim \left(\frac{1}{x}\right)^{\alpha_P - 1}, \quad x \ll 1, \quad \mu = \mu_0 \simeq 1 \text{ GeV}$$

It follows from the relation of form factors and trajectories in the 3-point Veneziano amplitude and the structure of form factors in the light-front holographic framework

We obtain

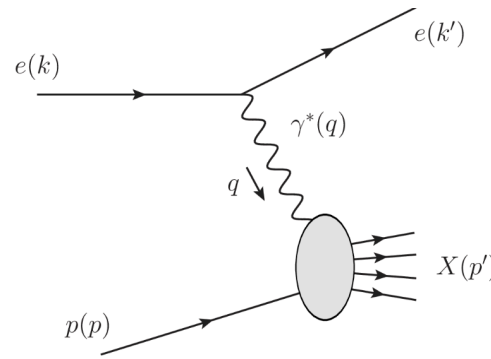
$$S_{DIS} = \ln\left(x\, g(x,\mu_0)\right) \sim (\alpha_P(0) - 1) \ln\left(\frac{1}{x}\right) \qquad x \ll 1$$

Hypercritical Pomeron $\alpha_P(0) - 1 > 1$, which leads to the rising of the total pp cross section at large energies, is uniquely related to a positive von Neumann entropy, a consequence of separation of entangled parton states in a DIS measurement



The LF momentum fraction x can be identified with the DIS Bjorken variable x_{bj}

$$x_{bj} = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{W^2 + Q^2 - m_p^2}$$



where

$$q^2 = (k' - k)^2 = -Q^2$$

is the photon virtuality, and

$$W^2 = (p + Q)^2$$

the total photon-hadron energy squared

In the high-energy kinematic domain, $W^2 \gg Q^2$, $x = \frac{Q^2}{W^2}$ and, for fixed Q^2

$$S_{DIS} \sim (\alpha_P(0) - 1) \ln\left(\frac{W^2}{Q^2}\right), \qquad W^2 \gg Q^2.$$

Therefore, one can identity the value of $\alpha_P(0)$, which determines the gluon distribution $xg(x, \mu)$ at small x with the value of $\alpha_P(0)$, obtained from the Pomeron contribution to the W dependence of the DIS cross section

It also implies that the Pomeron intercept should also be dependent on the virtuality scale μ !

The scale dependent Pomeron

The proton structure function $F_2(x, Q^2)$ in inclusive cross section for the DIS process $\gamma^* + p \to X$

$$\sigma_{\gamma^* + p \to X}(W^2) = \frac{4\pi^2 \alpha_{em}}{Q^2} F_2\left(x, Q^2\right)$$

has been studied at HERA (2001) over a large range of values for x and Q^2

$$F_2(x, Q^2) \sim x\, g\left(x, Q^2\right) \sim x^{-\lambda(Q^2)}, \quad \text{with} \quad \lambda(Q^2) = 0.048 \ln\left(\frac{Q^2}{\Lambda^2}\right)$$

and $\Lambda \simeq 290$ MeV

If one identifies the power $\lambda(Q^2)$ from the DIS experiments with an effective scale-dependent Pomeron intercept $\alpha(0, \mu^2)$

$$x\, g(x, \mu) \sim \left(\frac{1}{x}\right)^{(\alpha_P(0,\mu^2)-1)}$$

namely

$$\lambda(Q^2) = \alpha_P(0, Q^2) - 1$$

The effective intercept increases by a factor 2 from $Q^2 = 2$ to $Q^2 = 150$ GeV^2

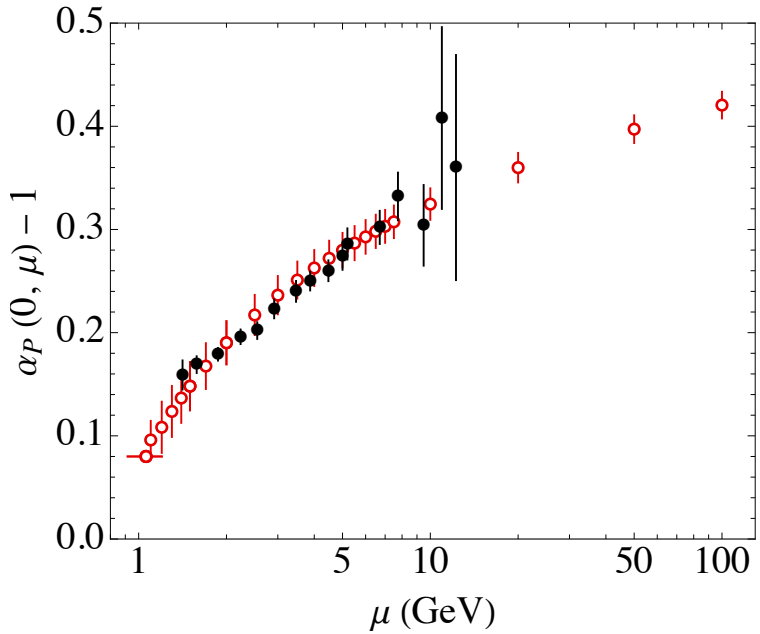
In PRD 105, 034029 (2022) we have examined the possibility that the HLFQCD expression for the gluon distribution is not only valid at the hadronic scale but for any virtuality scale μ

$$x\,g(x,\mu) = \sum_{\tau} \frac{1}{N_\tau(\mu)} c_\tau(\mu) w'(x,\mu)[1-w(x,\mu)]^{\tau(\mu)-2}\, w(x,\mu)^{1-\alpha_P(0,\mu)}$$

leading to a single, scale dependent Pomeron

The small-x behavior is determined uniquely by the the scale-dependent intercept and can be extracted unambiguously

DGLAP evolved results for the effective power $\lambda(\mu)$ are extracted from the expansion of $x\,g(x,\mu)$ in the range $0.0001 \leq x \leq 0.00016$



The effective power $\lambda(\mu) = \alpha_P(0,\mu) - 1$ from the DGLAP evolution of the gluon distribution for the pion (red) compared with the HERA values (black)

Conclusion and outlook

The relation between DIS entropy, gluon density and Pomeron intercept

$$dS_{DIS} = d \ln \left(x\, g(x, \mu) \right) \right) \sim 2 \left(\alpha_P(0) - 1 \right) \frac{dW}{W}$$

shows that the hypercritical intercept $\alpha_P(0)$ is related to the increase of the DIS entropy and the gluon density $x\, g(x, \mu)$ with decreasing x, thus for increasing W

For very small values of x the structure function $F_2(x, Q^2)$ is dominated by the gluon distribution and therefore the x dependence obtained by HERA yields a model independent determination of the Pomeron intercept which is consistent with the DGLAP evolution of the HLFQCD model

It was further suggested by Kharzeev and E. M. Levin (2017) that

$$S_{partons} = \ln \left(x\, g(x, Q^2) \right) \equiv S_{hadrons}.$$

namely a relation between the entropy in the final state of the collision and the entropy of partons

If the final state entropy is also scale dependent, also the hadron multiplicity depends on the scale set by the average momentum of the hadrons in the final state , as we have shown in [arXiv: 2304.14207 [hep-ph]

Parton entanglement entropy, within the context of a scale-dependent Pomeron, leads to a rather consistent picture which goes beyond the usual parton model described in terms of free partons in deep inelastic scattering experiments