

Double parton scattering: collinear and TMD factorization

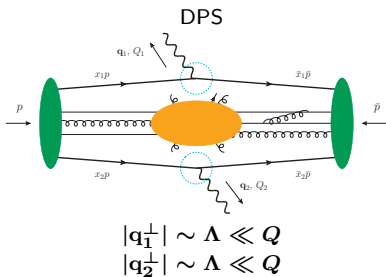
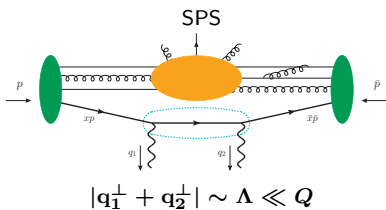
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5th Workshop on the QCD Structure of the Nucleon
Alcalá de Henares, Spain, 3–8 October 2021



What is double parton scattering?



Definition

Double parton scattering (DPS) is a proton-proton scattering process in which two partons from each proton undergo **two separate hard interactions**.

First appearance in theory studies:

Politzer *Nucl. Phys.* B172 (1980) 349

Paver, Treleani *Nuovo Cim.* A70 (1982) 215

Mekhfi *Phys. Rev.* D32 (1985) 2371

Other ground-setting works:

Gaunt, Stirling *JHEP* 03 (2010) 005

Blok et al. *Eur. Phys. J.* C72 (2012) 1963

Diehl et al. *JHEP* 03 (2012) 089

Manohar, Waalewijn *Phys. Rev.* D85 (2012) 114009

Ryskin, Snigirev *Phys. Rev.* D86 (2012) 014018

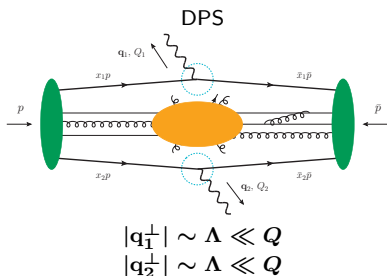
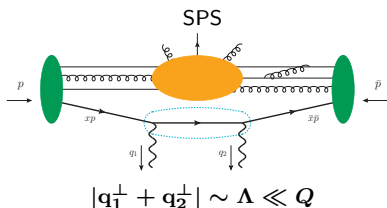
...

hard scale is $Q \sim \min(Q_1, Q_2)$

transverse-momenta scale is Λ

with $\Lambda_{\text{QCD}} \ll \Lambda \ll Q$

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Size comparison to SPS

- ▶ integrated XS: $\frac{\sigma_{\text{DPS}}}{\sigma_{\text{SPS}}} \sim \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$
 \implies phase-space suppressed
- ▶ differential XS: $\frac{d^2\sigma_{\text{SPS}}}{d^2q_1 d^2q_2} \sim \frac{d^2\sigma_{\text{DPS}}}{d^2q_1 d^2q_2}$
 \implies same power counting!

hard scale is $Q \sim \min(Q_1, Q_2)$

transverse-momenta scale is Λ

with $\Lambda_{\text{QCD}} \ll \Lambda \ll Q$

When is DPS important?

Where DPS is enhanced

- ▶ generally, DPS relevance increases with **collision energy**
- ▶ competitive with SPS in regions of **small $|q_1^\perp|, |q_2^\perp|$**
→ e.g. two pairs of back-to-back jets
- ▶ enhanced by parton luminosities at **small- x** , e.g. $F_{gg} \propto (f_g)^2$
- ▶ DPS dominant contribution for **coupling-suppressed processes** in SPS
→ same-sign **WW** production at $\mathcal{O}(\alpha_s^2)$ in SPS, but $\mathcal{O}(1)$ in DPS

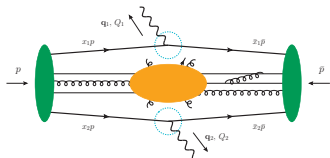
Peculiarities of DPS

- ▶ **polarization** gains importance, since an unpolarized proton can contain two polarized partons
- ▶ **color non-singlet** distributions enter the cross section even for color singlet final states
- ▶ need to account for the **overlap of single and double parton scattering**

DPS cross section

For colorless final states, an analogous factorized form to the SPS case can be derived

- $\hat{\sigma}^{(i)}$ are regular partonic cross sections
- F_{ab} are double parton distributions (DPDs)
- y [GeV⁻¹] is inter-parton transverse separation



here neglecting color indices and x_i, \bar{x}_i dependence in the functions

C is a symmetry factor

Transverse-momentum dependent (TMD) factorization:

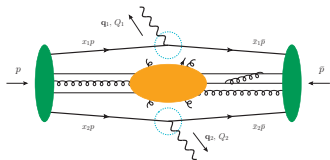
$$\frac{d\sigma_{\text{DPS}}}{dq_1^\perp dq_2^\perp} = \frac{1}{C} \sum_{a_1 a_2 b_1 b_2} \hat{\sigma}_{a_1 b_1}^{(1)} \hat{\sigma}_{a_2 b_2}^{(2)} \\ \times \int d^2 y \frac{d^2 z_1}{2\pi^2} \frac{d^2 z_2}{2\pi^2} e^{-iq_1^\perp z_1 - iq_2^\perp z_2} F_{a_1 a_2}(z_1, z_2, y) F_{b_1 b_2}(z_1, z_2, y)$$

In TMD factorization, $F_{ab}(z_1, z_2, y)$ are the TMDDPDs in position space.

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C is a symmetry factor

Collinear factorization:

$$d\sigma_{\text{DPS}} = \frac{1}{C} \sum_{a_1 a_2 b_1 b_2} \hat{\sigma}_{a_1 b_1}^{(1)} \otimes \hat{\sigma}_{a_2 b_2}^{(2)} \otimes \int d^2 \mathbf{y} F_{a_1 a_2}(\mathbf{y}) \otimes F_{b_1 b_2}(\mathbf{y})$$

In collinear factorization, $F_{ab}(\mathbf{y})$ are the collinear DPDs in position space.

Assuming no inter-partonic correlations whatsoever, obtain convenient XS formula (the **DPS pocket formula**)

$$\sigma_{\text{DPS}} = \frac{1}{C} \frac{\sigma_1^{\text{SPS}} \sigma_2^{\text{SPS}}}{\sigma_{\text{eff}}}$$

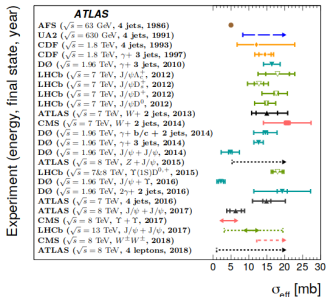
σ_{eff} used as a “measure” of DPS in exp’s

Experimental searches

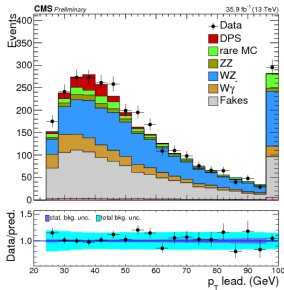
DPS observed since the '80s (4 jets, γ +3 jets, etc)

typical observables: WW , WJ/Ψ , $J/\Psi J/\Psi$, W +jets, ZZ , ...

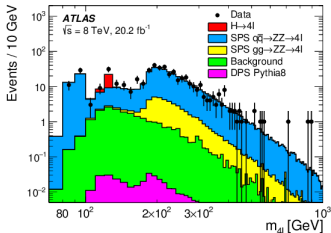
σ_{eff} measurements [CERN-EP-2018-274]



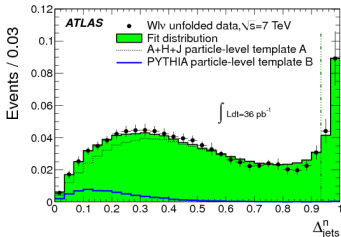
like-sign WW [CMS-PAS-FSQ-16-009]



4ℓ final state [CERN-EP-2018-274]



W + 2 jets [CERN-PH-EP-2012-355]



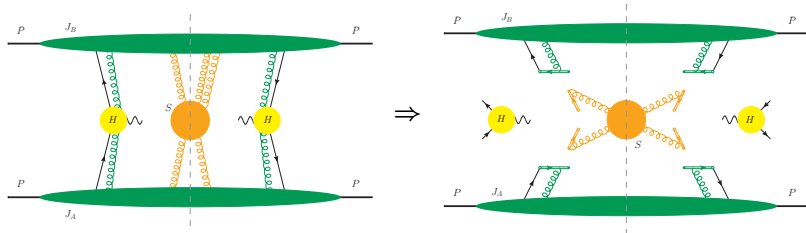
Status of factorization

A **formal all-order proof** of the factorization formulae in perturbative QCD **has been achieved for DPS** in the case of a **colorless final state**, both for the TMD and the collinear case. Current status is at the **same level as for the SPS** counterpart.

Diehl et al. [JHEP 03 \(2012\) 089](#), [JHEP 01 \(2016\) 076](#)
Vladimirov [JHEP 04 \(2018\) 045](#)
Buffing et al. [JHEP 01 \(2018\) 044](#)
Diehl, RN [JHEP 04 \(2019\) 124](#)

The factorization procedure can be understood visually using cut diagrams:

SPS TMD factorization



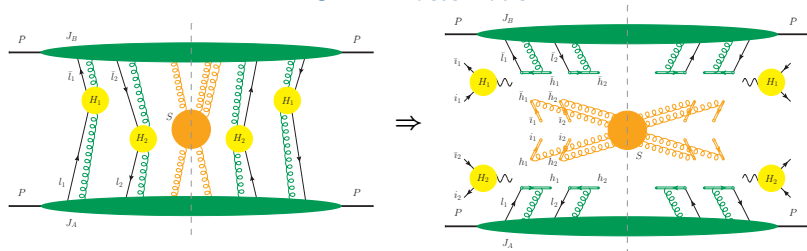
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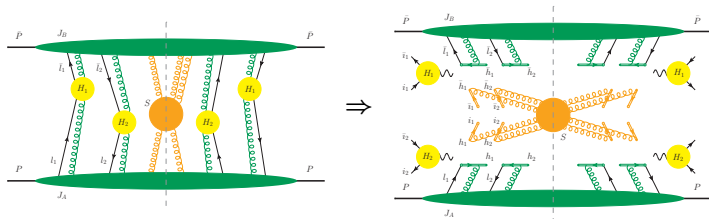


Structure of the proof

The all-order **factorization proof for DPS** generalizes the proofs by **Collins, Soper, Sterman** (CSS). It suffices to prove the TMD factorization; collinear factorization can be derived by integration of the TMD formula.

Sketch:

1. define a **power counting**, identify **leading regions** & introduce **kinematical approximations** Diehl, Ostermeier, Schäfer [JHEP 03 \(2012\) 089](#)
2. loops have to be cut: establish a **subtraction** mechanism Collins "Foundations of pQCD" (2011)
3. decouple **collinear gluons** from the hard interactions CSS [Nucl.Phys.B261 \(1985\) 104](#)
4. show factorization of **Glauber gluons** Diehl et al. [JHEP 01 \(2016\) 076](#)
5. factorize **soft gluons** from collinear graphs Diehl, RN [JHEP 04 \(2019\) 124](#)
6. obtain renormalized **operator definitions** of soft & collinear factors Diehl, Ostermeier, Schäfer [JHEP 03 \(2012\) 089](#)



Soft & collinear approximations

using light-cone coordinates $a \equiv (a^+, a^-, \vec{a})$
with $a^\pm = (a^0 \pm a^3)/\sqrt{2}$

Define scaling of momenta $\ell = (\ell^+, \ell^-, |\ell|)$

- ▶ **hard:** (Q, Q, Q)
- ▶ **right-collinear:** $(Q, \frac{\Lambda^2}{Q}, \Lambda)$; **left-collinear:** $(\frac{\Lambda^2}{Q}, Q, \Lambda)$
- ▶ **central or ultra soft:** $(\Lambda, \Lambda, \Lambda)$ or $(\frac{\Lambda^2}{Q}, \frac{\Lambda^2}{Q}, \frac{\Lambda^2}{Q})$
- ▶ **Glauber soft (bad):** $(\Lambda, \frac{\Lambda^2}{Q}, \Lambda)$ or $(\frac{\Lambda^2}{Q}, \Lambda, \Lambda)$

Approximation example

At leading power:

- ▶ **collinear gluons:**
 $\ell_c^R = (\ell^+, 0, 0)$
 $\ell_c^L = (0, \ell^-, 0)$
- ▶ **soft gluons:**
 $\ell_s^R = (0, \ell^-, \vec{\ell})$
 $\ell_s^L = (\ell^+, 0, \vec{\ell})$

NOTE: in DPS must keep soft trans. momenta!

Grammer-Yennie approximation

Grammer, Yennie [Phys. Rev. D8 \(1973\) 4332](#)

In soft-to-collinear gluon propagators:

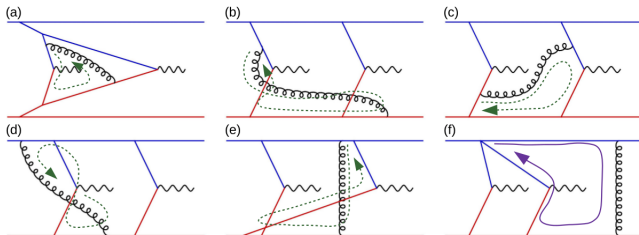
$$g^{\mu\nu} \rightarrow \frac{v^\mu \ell_s^\nu}{\ell \cdot v + i\epsilon}, \quad (v \text{ aux. vector})$$

example: $S_\mu(\ell) J_A^\mu(\ell_s) \simeq S_\mu(\ell) \frac{v^\mu \ell_s^\nu}{\ell \cdot v + i\epsilon} J_{A\nu}(\ell_s)$

Combinations of **Grammer-Yennie factors** usually result in the appearance of **Wilson lines**.

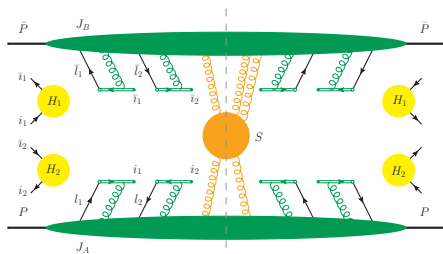
Glauber gluons cancellation

- ▶ The Grammer–Yennie approximation is not valid in the **Glauber region**, hence we must find a way to avoid this region.
- ▶ Glauber cancellation does not depend on the color structure, but only on the **kinematics** and **topology**.
- ▶ After checking all possible cases, **only non-trivial case** connects spectators before final state cut (see figure).
- ▶ Cancellation analogous to the one in SPS, i.e. due to **unitarity** (sum over final states at the cut).



from Diehl et al. [proc. MPI@LHC 2015, 91-95](#)

Factorization of collinear gluons from hard subgraphs



Collinear gluons factorize exactly like in SPS case

In amplitudes, substitute:

$$\blacktriangleright q_i(x) \rightarrow W_{ij}(x, v) q_j(x)$$

$$\blacktriangleright \bar{q}_i(x) \rightarrow \bar{q}_j(x) W_{ji}^\dagger(x, v)$$

X and Y are final-state cut of J_A and J_B

Z is final-state cut of S

$$H_1 H_2 \quad \mathcal{L}_{XYZ} \quad | \quad \mathcal{L}_{XYZ}^* \quad H_1^* H_2^*$$

After collinear gluons are factorized from the hard subgraphs:

$$\begin{aligned} \mathcal{L}_{XYZ}^{i_1 i_2 \bar{i}_1 \bar{i}_2}(n, \bar{n})(q_1, q_2) &= \frac{1}{n! \bar{n}!} \int [\text{hard momenta } l_m, \bar{l}_m] \int [\text{soft momenta } \ell_k, \bar{\ell}_k] \\ &\times (2\pi)^4 \delta^{(4)}(q_1 - l_1 - \bar{l}_1) (2\pi)^4 \delta^{(4)}(q_2 - l_2 - \bar{l}_2) \left[J_{AX}^{i_1 i_2}(l_1, l_2; \{\tilde{\ell}_n\}) \right]_{\mu_1 \dots \mu_n}^{a_1 \dots a_n} \\ &\times \left[S_Z(\{\ell_n\}, \{\bar{\ell}_n\}) \right]_{a_1 \dots a_n, b_1 \dots b_{\bar{n}}}^{\mu_1 \dots \mu_n, \nu_1 \dots \nu_{\bar{n}}} \left[J_{BY}^{\bar{i}_1 \bar{i}_2}(\bar{l}_1, \bar{l}_2; \{\tilde{\bar{\ell}}_n\}) \right]_{\nu_1 \dots \nu_{\bar{n}}}^{b_1 \dots b_{\bar{n}}} , \end{aligned}$$

Ward identities

Many Grammer-Yennie factors produce Lorentz-contracted expressions:

$$\ell_1^{\mu_1} \dots \ell_n^{\mu_n} \left[J_{AX}^{i_1 i_2} (l_1, l_2; \{\ell_n\}) \right]_{\mu_1 \dots \mu_n}^{a_1 \dots a_n}$$

Apply **Ward identity** for an amplitude with n external gauge bosons $A_{\mu_i}(\ell_i)$

$$\ell_1^{\mu_1} \dots \ell_n^{\mu_n} \langle 0 | \dots A_{\mu_1}(\ell_1) \dots A_{\mu_n}(\ell_n) \dots | 0 \rangle = 0$$

with all momenta contracted, and **order-by-order** ('t Hooft, Veltman).

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$$\mathcal{L}_{XYZ}^{i_1 i_2 \bar{i}_1 \bar{i}_2 (n, \bar{n})} = \int [\text{F.T. on } \vec{\xi}_m] \int [\text{F.T. on } \vec{l}_m, l_m^-]$$

$$\times S_Z^{h_1 h_2 \bar{h}_1 \bar{h}_2}(\vec{\xi}_m)$$

$$\times J_{AX}^{h_1 h_2}(l_1, l_2) \Big|_{l_1^+ = q_1^+, l_2^+ = q_2^+}$$

$$\times J_{BY}^{\bar{h}_1 \bar{h}_2}(\bar{l}_1, \bar{l}_2) \Big|_{\bar{l}_1^- = q_1^-, \bar{l}_2^- = q_2^-}$$

- ▶ one soft factor S is decoupled
- ▶ two collinear factors $J_{A,B}$
→ DPDs

Ward identities

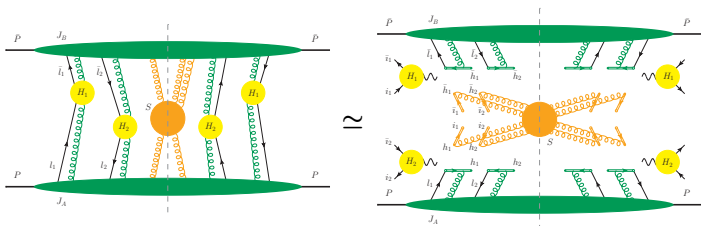
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with all momenta contracted, and **order-by-order** ('t Hooft, Veltman).



Soft factor

Buffing et al. JHEP 01 (2018) 044

Vladimirov JHEP 12 (2016) 038

The soft factor has a few properties:

- ▶ **hermitian in color space:** $S_{a_1 a_2} = S_{a_1 a_2}^\dagger$
- ▶ **hermitian in rep. space:** ${}^{RR'} S_{a_1 a_2} = ({}^{R'R} S_{a_1 a_2})^*$



TMD factorization

$$J_B^c S^{cd} J_A^d$$

In SPS and DPS, the soft factor depends on the color structure and on the Wilson-lines rapidity $y = \log \frac{v^+}{v^-}$

↓

Collins-Soper equation:

$$\frac{\partial}{\partial y} S(y) = K S(y)$$

collinear factorization

$$J_B^c S^{cd,ef} J_A^d H_1^e H_2^f$$

In SPS, integration over transverse momenta implies $S_q = 1$.

In DPS, ${}^{RR'} S_{a_1 a_2} \propto \delta_{RR'}$, color singlet ${}^{11} S_{qq} = 1$, but color non-singlets like ${}^{88} S_{qq}$ depend on rapidity (however Sudakov suppressed).

At NNLO, S can be expressed completely in terms of SPS TMD soft factor

“Bare” TMD DPDs

From factorization formula, definition of “bare” DPDs is similar to PDFs

$$F_{a_1 a_2}^{(0)}(x_1, x_2, z_1, z_2, y) \propto \langle p | \mathcal{O}_{a_1}(y, z_1) \mathcal{O}_{a_2}(0, z_2) | p \rangle \Big|_{z_i^+ = y_i^+ = 0}$$

in terms of operators $\mathcal{O}(y, z) \sim \bar{\psi}(y - \frac{1}{2}z) \Gamma \psi(y + \frac{1}{2}z)$.

“Bare” collinear DPDs

Collinear DPDs are obtained from TMD DPDs:

- ▶ position-space DPDs $F_{a_1 a_2}^{(0)}(x_1, x_2, y)$ by letting $z_1, z_2 \rightarrow 0$
↪ appear in cross section
- ▶ momentum-space DPDs $F_{a_1 a_2}^{(0)}(x_1, x_2, \Delta)$ by Fourier transform
↪ appear in DPD sum rules

Renormalized collinear DPDs

- ▶ position-space DPDs $F_{a_1 a_2}^1(x_1, x_2, y, \mu_1, \mu_2)$ obey **double DGLAP equations**
↪ evolution and flavor matching up to NNLO available with **ChiliPDF**
- ▶ momentum-space DPDs $F_{a_1 a_2}^1(x_1, x_2, \Delta, \mu_1, \mu_2)$ obey **inhomogeneous generalized DGLAP equations**
↪ numerically solved by Gaunt, Stirling [JHEP 03 \(2010\) 005](#)

DPDs from perturbative splitting

At present, DPDs cannot be extracted from exp's \rightarrow Ansatz necessary

A class of DPD Ansätze at small y

From OPE, at small y DPDs are sum of “intrinsic” and “splitting” piece

$$F(y) = F_{\text{int}}(y) + F_{\text{spl}}(y)$$

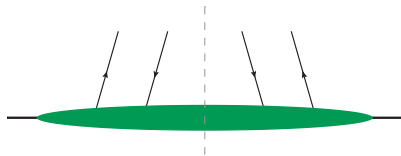
At large y DPDs can be modeled so that $\lim_{y \rightarrow \infty} F(y) = 0$.

Perturbative splitting

- ▶ from MEs: $F_{\text{spl}}(y) \propto \frac{1}{y^2}$
- ▶ UV divergence in cross-section

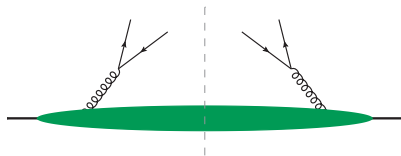
$$\int d^2y F_1 F_2 \sim \int \frac{d^2y}{y^4}$$

- ▶ reason: region of overlap between SPS and DPS



intrinsic (F_{int} or “2”)

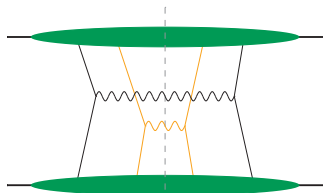
twist-4 distribution at small y , nonperturbative



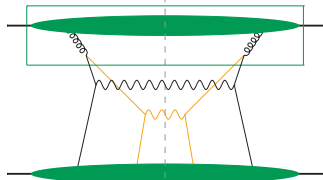
perturbative splitting (F_{spl} or “1”)

- ▶ LO: $F_{ab} \propto P_{a_0 \rightarrow ab} \cdot f_{a_0}$
Diehl et al. [JHEP 03 \(2012\) 089](#)
- ▶ NLO: computed, also color non-singlet
Diehl et al. [SciPost Phys. 7 \(2019\) 017](#)
Diehl et al. [JHEP 08 \(2021\) 040](#)

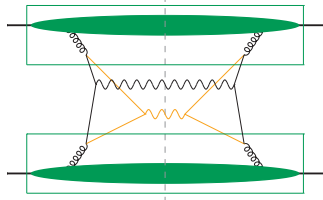
Interplay of splitting and intrinsic contributions



$2v2 \rightarrow$ not divergent



$2v1 \rightarrow$ divergence is $\frac{d^2 y}{y^2} \rightarrow \log y$ terms

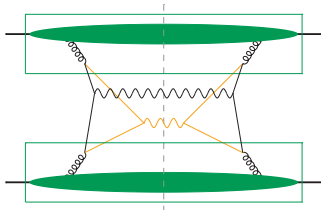


$1v1 \rightarrow$ divergence is $\frac{d^2 y}{y^4}$, must be subtracted

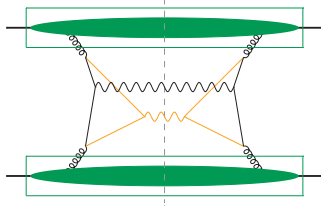
Double-counting between SPS and DPS

The UV divergence in y is associated to the **double counting of SPS and DPS** contributions in the region where $y \rightarrow 0$:

DPS interpretation (1v1)



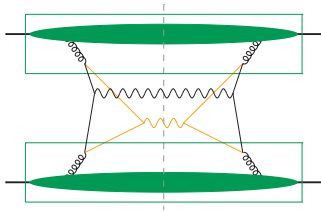
SPS interpretation



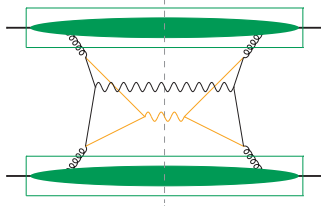
Double-counting between SPS and DPS

The UV divergence in \mathbf{y} is associated to the **double counting of SPS and DPS** contributions in the region where $\mathbf{y} \rightarrow 0$:

DPS interpretation (1v1)



SPS interpretation



Solution: DGS scheme

The DGS subtraction scheme cancels the UV divergence at all orders:

$$\sigma = \sigma_{\text{SPS}} + \sigma_{\text{DPS}} - \sigma_{\text{sub}}, \quad \sigma_{\text{sub}} = \sigma_{\text{DPS}} \text{ with } F_{1,2} = F_{\text{spl}}$$

where the DPS cross section is regularized introducing a cutoff $\nu \sim Q$

$$\sigma_{\text{DPS}} \propto \int d^2\mathbf{y} F_1(\mathbf{y}) F_2(\mathbf{y}) \rightarrow \int d^2\mathbf{y} \Phi^2(\mathbf{y}\nu) F_1(\mathbf{y}) F_2(\mathbf{y})$$

Simple cutoff regulator $\Phi(\mathbf{y}\nu) = \Theta(\mathbf{y}\nu - 2e^{-\gamma_E})$.

State of the art

Recent theory developments

- ▶ dShower: a **parton shower** combining SPS and DPS, implementing the “1 \rightarrow 2” splitting and treating the SPS-DPS double counting
Cabouat, Gaunt [JHEP 10 \(2020\) 012](#)
- ▶ **lattice QCD**: extracted moments of the pion DPD and of the proton DPD
Bali et al. [JHEP 02 \(2021\) 067](#), [JHEP 09 \(2021\) 106](#)
- ▶ a lot **more insight on DPDs**: evolution, sum rules, NLO splitting, color non-singlet distributions
Diehl et al. [SciPost Phys. 7 \(2019\) 2, 017](#), [Eur.Phys.J.C 80 \(2020\) 5, 468](#),
[JHEP 08 \(2021\) 040](#), [arXiv:2109.14304](#)

DPS phenomenology

- ▶ DPD models
 \hookrightarrow constituent quark models ([Rinaldi, Scopetta, Ceccopieri](#)), “bag” model ([Manohar, Waalewijn](#))
 valence quark models ([Broniowski, Ruiz Arriola](#)), KMR approach ([Golec-Biernat, Staśto](#)), . . .
- ▶ multitude of phenomenological studies that include DPS
[Blok, Dokshitzer, Frankfurt, Strikman, Maciuła, Szczurek, Kutak, van Hameren, Gaunt, Kom, Kulesza, Stirling, Fedkyevich,](#)
[Kasemets, Myska, Cotogno, Lansberg, Yamanaka, Zhang, Shao, Ceccopieri, Rinaldi, Scopetta,](#)
- ▶ DPS in pA collisions and TPS (triple parton scattering) [D’Enterria, Snigirev](#)

Summary

- ▶ **DPS** contributions can be **comparable or even dominant** w.r.t. SPS in some cases
- ▶ status of DPS **factorization proofs** is at the same level as for SPS
- ▶ **double-counting** of SPS and DPS in small- y region is understood
- ▶ double **DGLAP evolution** and flavor matching are under control thanks to developments of new tools
- ▶ perturbative **splitting** form of DPDs known **up to NLO**, also for color non-singlet distributions
- ▶ a **DPS parton shower** is being developed

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Thank you!

The latest extractions of σ_{eff}

latest measurements (CMS, 4-jets at 13 TeV) [\[CMS-PAS-SMP-20-007\]](#)

