The Regge Limit and Infrared Factorisation of 2 to 3 scattering amplitudes in QCD

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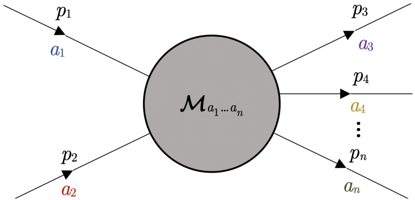
Outline

Scattering Amplitude in Color Space

Consider QCD amplitude for n massless partons (quarks and gluons).

- $p_i^2 = 0$
- Kinematic variables: s_{ij}
- $D = 4 \quad 2'' \quad)$
- Colour index: a_i



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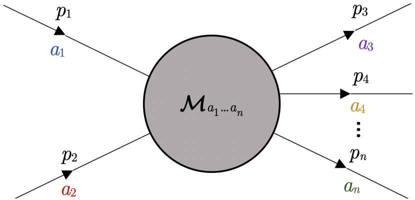
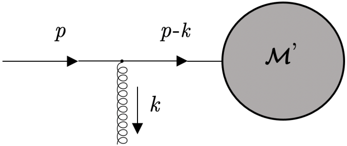


Amplitude color space vector:

$$M_{a_1:::a_n} \quad \underline{p_i}; \quad {}_s(\quad {}^2); '' \quad = \underset{J}{X} \; M_J \quad \underline{p_i}; \quad {}_s(\quad {}^2); '' \quad (c^J)_{a_1:::a_n}$$

Infrared Divergences in QCD

Consider a particular case:



$$\Rightarrow \quad \frac{1}{(p-k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2p_0k_0(1-\cos\theta)}$$

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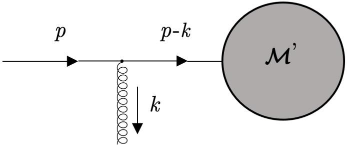

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Types of singularities:

- Vanishing energy ($k_0 \to 0$): Soft divergence
- Parallel momentum ($\theta \to 0$): Collinear divergence
- Both $k_0 \to 0$ and $\theta \to 0$: Soft-Collinear divergence

Infrared Factorisation

QCD amplitudes present exponentiation of divergences [Einan Gardi and Lorenzo Magnea, arXiv:0901.1091]

$$M = P \exp \left(\frac{1}{2} \int_0^{\ ^2} \frac{d\ ^2}{\ ^2} \ \left(\frac{p_i}{\ }; \ _s \right) \right) H$$

- **Soft anomalous dimension ( )**: Universal, captures all infrared singularities, known to $O(\ _s^3)$
- **Hard part of amplitude**, $H = M^{(0;0)} + O(\ _s)$: Finite and process dependant
- **Dipole formula** is ansatz for soft anomalous dimension valid to $O(\ _s^2)$

Regge Limit

For $2 \to 2$, high-energy regime: $|s/t| \to \infty$

Regge limit (RL)

$$s + t + u = 0 \Rightarrow s \simeq -u \gg -t$$

Logarithmic enhancements, $L \equiv \ln\left(\frac{s}{-t}\right)$, at all orders.

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Expansion in loops (α_s) and in L:

$$\mathcal{M} = \sum_{l,n} \alpha_s^l L^n \mathcal{M}^{(l,n)} = \mathcal{M}^{(0,0)} + \alpha_s L\, \mathcal{M}^{(1,1)} + \alpha_s \mathcal{M}^{(1,0)}$$
$$+ \underbrace{\alpha_s^2 L^2 \mathcal{M}^{(2,2)} + \vdots}_{LL} + \underbrace{\alpha_s^2 L\, \mathcal{M}^{(2,1)} + \vdots}_{NLL} + \underbrace{\alpha_s^2 \mathcal{M}^{(2,0)} + \vdots}_{NNLL} \cdots$$

Amplitudes dominated by t-channel exchanges, at LL factorises [Kuraev et al. Sov.Phys.JETP 44 (1976)]

$$M|_{LL} = M^{(0;0)} \frac{s}{t}^{(t)} = M^{(0;0)} e^{(t)L}$$

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- All orders of $_s$
- Same expression for every process in QCD (Universality)
- Regge trajectory (t) of exchanged particle
- Gluon Reggeizes at NLL) Generalisation of Regge factorisation with new features (Impact Factors)

In the Regge limit (RL) [V. Del Duca et al. arXiv:1109.3581]

$$ {}^{RL} = \frac{b_K(\ \ s)}{2} \; L\,T_t^2 + i\ \ T_s^2 \; + \;\tilde{}\,1 $$

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- T_s^2 and T_t^2 describe color ow in channels
- Up to two loops, valid to all logarithmic orders!
- Factorisations are compatible

$$\exp\left\{ \frac{1}{2} \int_0^{\ 2} \frac{d\ ^2}{^2} \quad ^{RL} \right\} \quad H \;() \qquad \exp f \quad (t)\,L\,g M^{(0;0)}$$

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$$ \exp\left(\frac{1}{2} \int_0^{\;2} \frac{d\;\;^2}{\;^2}\;\;{}^{RL} \right) H\,(\,) \qquad \exp f\;\;(t) L\, g M^{(0;0)} $$

- First term gives LL reproducing Regge factorisation: $T_t^2 M^{(0;0)} / M^{(0;0)}$
- Second term gives NLL contributions. Breaks Regge factorisation: $T_s^2 M^{(0;0)} \not/ M^{(0;0)}$

We have considered $qq \to qq$ and $gg \to gg$

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- Explicit expressions for T_t^2 and T_s^2 in t-channel color bases
- Infrared structure of $qq \to qq$ using dipole formula up to $O(\alpha_s^2)$
- Compare infrared factorisation and Regge factorisation for $gg \to gg$ to extract features of Regge factorisation at NLL

Consider ve partons with momenta $p = (p_0; p_x; p_y; p_z)$:

- Light-cone coordinates: $p = p_0 \quad p_z$
- Transverse momenta: $p^? = p_x + ip_y$

$) \quad p = \; p^+; p \; ; p_?$

Incoming partons: $p_1 = (p_1^+; 0; 0)$ and $p_2 = (0; p_2; 0)$

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Multi-Regge Limit (MR)

1. $p_1^+ \; ' \;\; p_3^+ \;\;\; p_4^+ \;\;\; p_5^+$ and $p_3 \;\;\; p_4 \;\;\; p_5 \; ' \;\; p_2$
2. $jp_3^? j \; ' \; j \;\; p_4^? j \; ' \; j \;\; p_5^? j$

De ne rapidities: $y_i = \ln \quad \frac{p_i^+}{p_i}$. Condition 1 becomes:

$$y_3 \quad y_4 \quad y_5 \;\;) \qquad y_k = y_k \quad y_{k+1} = \ln \quad \frac{p_k^+ \, p_{k+1}}{p_k \, p_{k+1}^+} \;\; ! \qquad 0$$

Amplitude factorises at leading logarithm accuracy [Lipatov, Sov. J. Nucl. Phys. 23 (1976)]

$$M|_{LL} = M^{(0;0)} \frac{s_{34}}{t_1}^{(t_1)} \frac{s_{45}}{t_2}^{(t_2)}$$

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- $t_1 = q_1^2$ and $t_2 = q_2^2$
- In MR limit: $s_{34} \simeq p_3^+ p_4$ and $s_{45} \simeq p_4^+ p_5$
- $\ln(s_{34})$ and $\ln(s_{45})$ generate LL structure at all orders of $_s$
- More general Regge factorisation expression at NLL for gluon exchanges (Impact factors and Lipatov Vertex)

The dipole formula in the MR limit [V. Del Duca et al. arXiv:1109.3581]

$$\mathrm{MR} = \frac{b_K(\quad s)}{2} \quad y_3 \mathbf{T}^2_{t_1} + \quad y_4 \mathbf{T}^2_{t_2} + i \quad \mathbf{T}^2_{s_{12}} \quad + \tilde{\ }_5\, 1$$

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$$^{MR} = \frac{b_K(\quad_s)}{2} \quad y_3 T^2_{t_1} + \quad y_4 T^2_{t_2} + i \quad T^2_{s_{12}} \quad + \tilde{\ }_5 1$$

- Valid to all rapidities orders up to two loops
- Two t-channel operators $T^2_{t_1}$ and $T^2_{t_2}$ give LL structure reproducing Regge factorisation
- $i \quad T^2_{s_{12}}$ gives NLL structure and breaks Regge factorisation
- Generalisation to $2 ! \quad n$

We have studied the process

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Color t-channel basis c^J and explicit expressions for $T_{t_1}^2$, $T_{t_2}^2$ and $T_{s_{12}}^2$

Extraction of M_{qgq} in the MR limit, tree level and one loop

Infrared structure using dipole formula up to $O(\alpha_s)$

Comparison with Regge Factorisation (Lipatov Vertex)

QCD amplitudes present two types of factorisation

Regge factorisation in high energy limit, gives all-order constraints on amplitude, e.g. Mj $_{LL}$ / M $^{(0;0)}$

Infrared factorisation for long-distance divergences, in high energy limit shows structure beyond LL accuracy

Analyse amplitude results and verify factorisations and compatibility

Obtain universal features of factorisations

Future work: obtain the two-loop qq! qggqamplitude in MR limit and extract features of Regge factorisation at NLL or study the next-to-multi-Regge (NMR) regime.

V. Del Duca, C. Duhr, E. Gardi, L. Magnea and C. D. White , The infrared structure of gauge theory amplitudes in the high-energy limit, (2011), https://arxiv.org/pdf/1109.3581.pdf

V. Del Duca, G. Falcioni, L. Magnea and L. Vernazza , Analyzing high-energy factorization beyond next-to-leading logarithmic accuracy, (2015), https://arxiv.org/abs/1409.8330

S. Caron-Huot, E. Gardi and L. Vernazza , Two-parton scattering in the high-energy limit, (2017), https://arxiv.org/pdf/1701.05241.pdf

M. Canay, V. Del Duca , One-loop impact factor for the emission of two gluons, (2021), https://arxiv.org/abs/2103.16593

Amplitude for exchange of Reggeized gluon

$$M\ (s;t) = 2\,g^2\,\frac{s}{t}\ (T^b)_{a_1 a_3} C_{\ 1\ \ 3}(p_1;p_3)\qquad \frac{s}{t}^{\quad\ g\,(t)}\ [(T_b)_{a_2 a_4} C_{\ 2\ \ 4}(p_2;p_4)]$$

- $g^2 = 4\qquad_s$
- T^b is color generator in corresponding representation of scattered particle
- $C_{\ i\ \ j}(p_i;p_j)$ are the impact factors, depend on helicities (Universal)

Amplitude for exchange of two Reggeized gluons

$$M = 2g^3 s\,(T^b)^{a_5}{}_{a_1} C_{1\,5}\,\frac{1}{t_1}\left(\frac{s_{45}}{t_1}\right)^{g(t_1)} [(T^{a_4})_{bc} V_{4}]\,\frac{1}{t_2}\left(\frac{s_{34}}{t_2}\right)^{g(t_2)} (T^c)^{a_3}{}_{a_2} C_{2\,3}$$

$t_i = q_i^2 \simeq |q_i^?|^2$ for $i = 1, 2$

$s_{ij} = (p_i + p_j)^2 \simeq p_i^+ p_j$
for $i \neq j \in \{3, 4, 5\}$ and $y_i > y_j$

$V_4(q_1, q_2)$
is the central-emission
vertex (Universal)

Channel colour operators:

$$\left.\begin{array}{l} \mathbf{T}_s = \mathbf{T}_1 + \mathbf{T}_2 = \quad (\mathbf{T}_3 + \mathbf{T}_4) \\ \mathbf{T}_t = \mathbf{T}_1 + \mathbf{T}_3 = \quad (\mathbf{T}_2 + \mathbf{T}_4) \\ \mathbf{T}_u = \mathbf{T}_1 + \mathbf{T}_4 = \quad (\mathbf{T}_2 + \mathbf{T}_3) \end{array}\right\} \Longrightarrow \mathbf{T}_s^2 + \mathbf{T}_t^2 + \mathbf{T}_u^2 = \sum_{i=1}^{4} C_i \quad C_{tot}$$

Graphically