Gravitational form factor of the nucleon in chiral EFT

Jambul Gegelia

RUB, Bochum, Germany; TSU, Tbilisi, Georgia

in collaboration with
H. Alharazin, D. Djukanovic and M. V. Polyakov

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Outline

- Effective action of chiral EFT in curved spacetime;
- Energy-momentum-tensor;
- Gravitational form factors of the nucleon;
- Summary;

Effective action of chiral EFT in curved spacetame

EChL for pions and nucleons in Minkowski metric can be found in
J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984).
N. Fettes, U.-G. Meißner, M. Mojžiš, and S. Steininger, Ann. Phys. (N.Y.) **283**, 273 (2000); **288**, 249 (2001).

For the purpose of obtaining the EMT one needs to consider the coupling to the gravitational field.
J. F. Donoghue and H. Leutwyler, Z. Phys. C **52**, 343 (1991).

The LO action of pseudoscalar mesons in curved spacetime:

$$S_\pi = \int d^4x \sqrt{-g} \left\{ \frac{F^2}{4} g^{\mu\nu} \mathrm{Tr}(D_\mu U (D_\nu U)^\dagger) + \frac{F^2}{4} \mathrm{Tr}(\chi U^\dagger + U \chi^\dagger) \right\},$$

where $\chi = 2B_0(s + ip)$, $D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$, $U = u^2$ represents the pion fields, B_0 is related to the vacuum condensate of quark fields and s, p, l_μ and r_μ are external sources.

H. Alharazin, D. Djukanovic, J. Gegelia and M. V. Polyakov, Phys. Rev. D **102**, no.7, 076023 (2020).

LO + NLO action of nucleons interacting with pions in curved spacetime:

$$
\begin{aligned}
S_{\pi\mathrm{N}} &= \int d^4x\sqrt{-g}\,\Big\{\frac{1}{2}\,\bar\Psi\, ie_a^\mu\gamma^a\nabla_\mu\Psi - \frac{1}{2}\,\nabla_\mu\bar\Psi\, ie_a^\mu\gamma^a\Psi - m\bar\Psi\Psi \\
&+ \frac{g_A}{2}\,\bar\Psi e_a^\mu\gamma^a\gamma_5 u_\mu\Psi + c_1\langle\chi_+\rangle\bar\Psi\Psi \\
&- \frac{c_2}{8m^2} g^{\mu\alpha}g^{\nu\beta}\langle u_\mu u_\nu\rangle\left(\bar\Psi\left\{\nabla_\alpha,\nabla_\beta\right\}\Psi + \left\{\nabla_\alpha,\nabla_\beta\right\}\bar\Psi\Psi\right) \\
&+ \frac{c_3}{2}\, g^{\mu\nu}\langle u_\mu u_\nu\rangle\bar\Psi\Psi + \frac{ic_4}{4}\,\bar\Psi\, e_a^\mu e_b^\nu\sigma^{ab}\,[u_\mu,u_\nu]\,\Psi + c_5\bar\Psi\hat\chi_+\Psi \\
&+ \frac{c_6}{8m}\bar\Psi\, e_a^\mu e_b^\nu\sigma^{ab}F^+_{\mu\nu}\Psi + \frac{c_7}{8m}\bar\Psi\, e_a^\mu e_b^\nu\sigma^{ab}\langle F^+_{\mu\nu}\rangle\Psi \\
&+ \frac{c_8}{8}\,R\bar\Psi\Psi + \frac{ic_9}{m}\,R^{\mu\nu}\left(\bar\Psi e^a_\mu\gamma_a\nabla_\nu\Psi - \nabla_\nu\bar\Psi e^a_\mu\gamma_a\Psi\right)\Big\}, \qquad (1)
\end{aligned}
$$

This action has been reduced to the above (minimal) form by using field redefinitions.

Here $g^{\mu\nu}$ and e_a^μ are the metric and vielbein gravitational fields,

$$
\begin{aligned}
u_\mu &= i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger - i(u^\dagger v_\mu u - u v_\mu u^\dagger)\right] \,, \\
F^+_{\mu\nu} &= u^\dagger F_{R\mu\nu} u + u F_{L\mu\nu} u^\dagger \,, \\
F_{R\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu] \,, \\
F_{L\mu\nu} &= \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu] \,, \\
\chi_+ &= u^\dagger \chi u^\dagger + u \chi^\dagger u \,, \\
\hat{\chi}_+ &= \chi_+ - \frac{1}{2}\langle \chi_+ \rangle \,,
\end{aligned}
\tag{2}
$$

and the covariant derivative acting on the nucleon field has the form

$$
\begin{aligned}
\nabla_\mu \Psi &= \partial_\mu \Psi + \frac{i}{2}\omega_\mu^{ab}\sigma_{ab}\Psi + \left(\Gamma_\mu - i v_\mu^{(s)}\right)\Psi, \\
\nabla_\mu \bar{\Psi} &= \partial_\mu \bar{\Psi} - \frac{i}{2}\bar{\Psi}\,\sigma_{ab}\,\omega_\mu^{ab} - \bar{\Psi}\left(\Gamma_\mu - i v_\mu^{(s)}\right) \,,
\end{aligned}
\tag{3}
$$

$v_\mu^{(s)}$ is an iso-scalar external vector source, $\sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b]$ and

$$
\begin{aligned}
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger\partial_\mu u + u\partial_\mu u^\dagger - i(u^\dagger v_\mu u + u v_\mu u^\dagger)\right] \;, \\
\omega_\mu^{ab} &= -g^{\nu\lambda} e_\lambda^a \left(\partial_\mu e_\nu^b - e_\sigma^b \Gamma^\sigma_{\mu\nu}\right), \\
\Gamma^\lambda_{\alpha\beta} &= \frac{1}{2} g^{\lambda\sigma}\left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right) \;, \\
R^\rho{}_{\sigma\mu\nu} &= \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \,, \\
R &= g^{\mu\nu} R^\lambda{}_{\mu\lambda\nu} \,.
\end{aligned} \tag{4}
$$

NLO action contains two new LECs, c_8 and c_9 which are not present in the theory formulated in flat metric.

Energy-momentum-tensor

Using the definition of the EMT for bosonic matter fields

$$T_{\mu\nu}(g,\psi) = \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{m}}}{\delta g^{\mu\nu}}\,, \tag{5}$$

we obtain in flat spacetime

$$\begin{aligned} T^{(\pi)}_{\mu\nu} &= \frac{F^2}{4}\,\mathrm{Tr}(D_\mu U (D_\nu U)^\dagger + D_\nu U (D_\mu U)^\dagger) \\ &- \eta_{\mu\nu}\left\{\frac{F^2}{4}\,\mathrm{Tr}(D^\alpha U (D_\alpha U)^\dagger) + \frac{F^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\right\}, \end{aligned} \tag{6}$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor.

For the fermion fields we use

$$T_{\mu\nu}(g,\psi) = \frac{1}{2e}\left[\frac{\delta S}{\delta e^{a\mu}}\, e^a_\nu + \frac{\delta S}{\delta e^{a\nu}}\, e^a_\mu\right], \tag{7}$$

where e is the determinant of e^a_μ, and obtain in flat spacetime:

$$\begin{aligned}
T^{(\pi N)}_{\mu\nu} &= \frac{i}{4}\left(\bar\Psi\,\gamma_\mu D_\nu \Psi + \bar\Psi\,\gamma_\nu D_\mu\Psi - D_\mu\bar\Psi\,\gamma_\nu\Psi - D_\nu\bar\Psi\,\gamma_\mu\Psi\right)\\
&+ \frac{g_A}{4}\left(\bar\Psi\,\gamma_\mu\gamma_5 u_\nu\Psi + \bar\Psi\,\gamma_\nu\gamma_5 u_\mu\Psi\right)\\
\cdots&\\
&+ \frac{c_8}{4}(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\bar\Psi\Psi\\
&+ \frac{ic_9}{2m}\left(\eta_{\mu\alpha}\eta_{\nu\beta}\partial^2 + \eta_{\mu\nu}\partial_\alpha\partial_\beta - \eta_{\mu\alpha}\partial_\nu\partial_\beta - \eta_{\nu\alpha}\partial_\mu\partial_\beta\right)\\
&\times \left(\bar\Psi\gamma^\alpha D^\beta\Psi - D^\beta\bar\Psi\gamma^\alpha\Psi + \bar\Psi\gamma^\beta D^\alpha\Psi - D^\alpha\bar\Psi\gamma^\beta\Psi\right),
\end{aligned}$$

where

$$\begin{aligned}
D_\mu\Psi &= \partial_\mu\Psi + \left(\Gamma_\mu - iv^{(s)}_\mu\right)\Psi,\\
D_\mu\bar\Psi &= \partial_\mu\bar\Psi - \bar\Psi\left(\Gamma_\mu - iv^{(s)}_\mu\right).
\end{aligned} \tag{8}$$

Gravitational form factors of the nucleon

At chiral order four there are tree and one-loop contributions to the nucleon matrix element of the EMT.

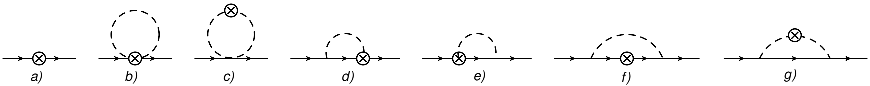

Figure: Diagrams contributing to gravitational form factors of the nucleon. Solid and dashed lines correspond to nucleons and pions, respectively. Crosses stand for EMT insertions.

Standard power counting rules apply to these diagrams:

- The pion lines count as of chiral order minus two;
- The nucleon lines have order minus one;
- Interaction vertices originating from the effective Lagrangian of order N count also as of chiral order N;
- The vertices generated by the EMT have the orders corresponding to the number of quark mass factors and derivatives acting on the pion fields, derivatives acting on the nucleon fields count as of chiral order zero;
- The momentum transfer between the initial and final nucleons counts as of chiral order one.
- Integration over loop momenta is counted as of chiral order four.

This power counting is realized in the results only after performing an appropriate renormalization.

M. V. Polyakov, P. Schweitzer, Int. J. Mod. Phys. A **33**, 26, 1830025 (2018).

The one-nucleon matrix element of the EMT is parameterised as

$$\langle p_f, s_f | T_{\mu\nu} | p_i, s_i \rangle = \bar{u}(p_f, s_f) \left[A(t) \frac{P_\mu P_\nu}{m_N} + iJ(t) \frac{P_\mu \sigma_{\nu\alpha} \Delta^\alpha + P_\nu \sigma_{\mu\alpha} \Delta^\alpha}{2m_N} + D(t) \frac{\Delta_\mu \Delta_\nu - \eta_{\mu\nu} \Delta^2}{4m_N} \right] u(p_i, s_i)\,,$$

where (p_i, s_i) and (p_f, s_f) are the momentum and polarization of the incoming and outgoing nucleons, respectively, and $P = (p_i + p_f)/2$, $\Delta = p_f - p_i$, $t = \Delta^2$.

The tree-order diagrams up to chiral order four lead to

$$\begin{aligned} A_{\rm tree}(t) &= 1 - \frac{2c_9}{m_N}\, t + x_1 M_\pi^2 t + x_2 t^2\,, \\ J_{\rm tree}(t) &= \frac{1}{2} - \frac{c_9}{m_N}\, t\,, \\ D_{\rm tree}(t) &= c_8 m_N + y_1 t + y_2 M_\pi^2\,. \end{aligned} \tag{9}$$

y_i and x_i parameterize tree-order contributions of the third and fourth chiral orders.

We renormalize loop diagrams by applying the EOMS scheme
J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999),
T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D **68**, 056005 (2003).

There is a power counting violating contribution to $A(t)$ which is absorbed into the renormalization of c_9.

The coupling c_8 has to cancel the divergent part and the power counting violating piece of the one-loop contribution to $D(t)$.

$D(0)$ expanded in powers of the pion mass:

$$\frac{D(0)}{m_N} = c_8 + \frac{g_A^2}{16\pi F^2} M_\pi + \frac{2\,(c_2 + 2c_3 - 4c_1) - \frac{3g_A^2}{m_N}}{8\pi^2 F^2} M_\pi^2 \ln\left(\frac{M_\pi}{m_N}\right) + \frac{(8c_3 - 16c_1) - g_A^2\left(3c_8 + \frac{14}{m_N}\right)}{32\pi^2 F^2} M_\pi^2 + \frac{y_2}{m_N} M_\pi^2 + \mathcal{O}(M_\pi^3)\,.$$

Next we define the slopes of GFFs by writing the form factors as:

$$\begin{aligned} A(t) &= 1 + s_A t + \mathcal{O}(t^2)\,, \\ J(t) &= \frac{1}{2} + s_J t + \mathcal{O}(t^2)\,, \\ D(t) &= D(0) + s_D t + \mathcal{O}(t^2)\,. \end{aligned} \tag{10}$$

Chiral expansion of the loop contributions to the slopes:

$$
\begin{aligned}
s_A &= -\frac{7 g_A^2}{128\pi F^2 m_N}\, M_\pi + \frac{c_2 m_N - 4 g_A^2}{16\pi^2 F^2 m_N^2}\, M_\pi^2 \ln\left(\frac{M_\pi}{m_N}\right) \\
&- \frac{3 g_A^2 (2 c_9 m_N + 1)}{32\pi^2 F^2 m_N^2}\, M_\pi^2 + \mathcal{O}(M_\pi^3)\,, \\
s_J &= -\frac{g_A^2}{32\pi^2 F^2} \ln\left(\frac{M_\pi}{m_N}\right) + \frac{g_A^2 (4 c_9 m_N - 5)}{64\pi^2 F^2} \\
&+ \frac{7 g_A^2}{128\pi F^2 m_N}\, M_\pi + \mathcal{O}(M_\pi^2)\,, \\
s_D &= -\frac{g_A^2 m_N}{40\pi F^2}\, \frac{1}{M_\pi} - \frac{(5 g_A^2 + 4 (c_2 + 5 c_3)\, m_N)}{80\pi^2 F^2} \ln\left(\frac{M_\pi}{m_N}\right) \\
&+ \frac{g_A^2 (24 + (15 c_8 + 40 c_9) m_N)}{480\pi^2 F^2} \\
&+ \frac{(4 c_1 - c_2 - 7 c_3)\, m_N}{40\pi^2 F^2} + \mathcal{O}(M_\pi)\,. \qquad (11)
\end{aligned}
$$

These expressions can be used for the analysis of lattice data.

M. V. Polyakov, Phys. Lett. B **555**, 57 (2003),
M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A **33**, no. 26, 1830025 (2018),

The GFFs of the nucleon $A(t), J(t)$ and $D(t)$ can be related to the energy and spin densities as

$$\rho_E(r) = m_N \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-ir\Delta} \Big[A(-\Delta^2) + \frac{\Delta^2}{4m_N^2}\left[A(-\Delta^2) - 2J(-\Delta^2) + D(-\Delta^2)\right]\Big], \quad (12)$$

$$\rho_J(r) = \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-ir\Delta} \left[J(-\Delta^2) + \frac{2}{3}\Delta^2 \frac{dJ(-\Delta^2)}{d\Delta^2}\right]. \quad (13)$$

The distribution of the pressure $p(r)$ and shear force $s(r)$ are obtained through

$$s(r) = -\frac{1}{4m_N}\, r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{6m_N}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r), \tag{14}$$

$$\widetilde{D}(r) = \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\Delta r}\, D(-\Delta^2). \tag{15}$$

Large distance behavior in the region $1/\Lambda_{\text{strong}} \ll r \ll 1/M_\pi$ can be obtained from FFs for small t in chiral limit.

Small t behavior of GFFs in the chiral limit:

$$
\begin{aligned}
A(t) &= 1 - \frac{2c_9}{m_N}\, t + \frac{3g_A^2}{512F^2 m_N}(-t)^{\frac{3}{2}} - \frac{(c_2 m_N - 10g_A^2)}{320\pi^2 F^2 m_N^2}\, t^2 \ln\left(\frac{-t}{m_N^2}\right) \\
&- \frac{(25g_A^2(12c_9 m_N - 7) - 62c_2 m_N)}{9600\pi^2 F^2 m_N^2} t^2 + O(t^{\frac{5}{2}}) , \\
J(t) &= \frac{1}{2} - \frac{c_9}{m_N}\, t - \frac{g_A^2}{64\pi^2 F^2}\, t \ln\left(\frac{-t}{m_N^2}\right) + \frac{g_A^2(12c_9 m_N - 7)}{192\pi^2 F^2}\, t \\
&- \frac{3g_A^2}{512F^2 m_N}(-t)^{\frac{3}{2}} + O(t^2) , \\
D(t) &= m_N c_8 + \frac{3g_A^2 m_N}{128F^2}\sqrt{-t} - \frac{(5g_A^2 + 4(c_2 + 5c_3)m_N)}{160\pi^2 F^2}\, t \ln\left(\frac{-t}{m_N^2}\right) \\
&+ \frac{(5g_A^2(40c_9 m_N + 15c_8 m_N + 28) + 94c_2 m_N + 200c_3 m_N)}{2400\pi^2 F^2}\, t \\
&+ O(t^{\frac{3}{2}}) .
\end{aligned}
$$

Performing Fourier transformation we obtain the large distance behavior in the region $1/\Lambda_{\rm strong} \ll r \ll 1/M_\pi$:

$$
\begin{aligned}
\rho_E(r) &= \frac{9g_A^2}{64\pi^2F^2}\frac{1}{r^6} - \frac{3\left(10g_A^2/m_N + (c_2+10c_3)\right)}{16\pi^3F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right), \\
\rho_J(r) &= \frac{5g_A^2}{64\pi^3F^2}\frac{1}{r^5} - \frac{9g_A^2}{64\pi^2F^2m_N}\frac{1}{r^6} + O\left(\frac{1}{r^7}\right), \\
\widetilde{D}(r) &= -\frac{3g_A^2m_N}{128\pi^2F^2}\frac{1}{r^4} + \frac{3\left(5g_A^2 + 4\left(c_2+5c_3\right)m_N\right)}{160\pi^3F^2}\frac{1}{r^5} + O\left(\frac{1}{r^6}\right), \\
p(r) &= -\frac{3g_A^2}{64\pi^2F^2}\frac{1}{r^6} + \frac{\left(5g_A^2/m_N + 4\left(c_2+5c_3\right)\right)}{16\pi^3F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right), \\
s(r) &= \frac{9g_A^2}{64\pi^2F^2}\frac{1}{r^6} - \frac{21\left(5g_A^2/m_N + 4\left(c_2+5c_3\right)\right)}{128\pi^3F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right).
\end{aligned}
$$

These asymptotics can be used for the analysis of lattice data.

The above expressions satisfy the general stability conditions - $\rho_E(r) > 0$ and $\frac{2}{3}s(r) + p(r) > 0$.

Summary

- Presented the effective chiral Lagrangian of pions and nucleons up to the second chiral order in curved spacetime.
- Derived the corresponding EMT of pions and nucleons in flat spacetime.
- Calculated the one-loop contributions to the one-nucleon matrix element of the EMT at fourth chiral order and extracted the corresponding gravitational form factors of the nucleon.
- Obtained the chiral expansion of the $D(0)$ and slope parameters for all GFFs to the fourth order of the chiral expansion.
- Calculated large distance asymptotic behaviour of the energy, spin, pressure and shear force distributions.