Nucleon Sigma Terms from lattice QCD

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2. Light- and strange sigma terms
3. Charm sigma terms
4. Heavy quark relations and the bottom and top sigma terms
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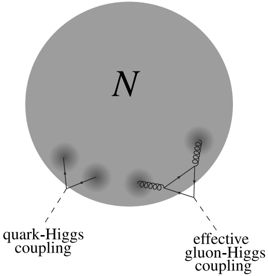
Nucleon sigma terms?



Nucleon couples to the external Higgs field. That coupling is mediated by the couplings g_q of the quark flavors q to the Higgs field.

Elementary fermions have a mass, which is proportional to the fermion-Higgs coupling g_f. As a consequence $m_f = g_f \frac{\partial m_f}{\partial g_f}$.

We can define this logarithmic derivative also for non-elementary particles, like the nucleon:

$$M_N f_{qN} = \sigma_{qN} := g_q \frac{\partial M_N}{\partial g_q} = m_q \frac{\partial M_N}{\partial m_q}$$

The Feynman-Hellmann theorem states

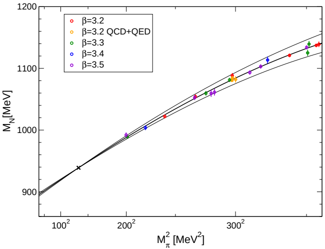
$$\sigma_{qN} = m_q \frac{\partial M_N}{\partial m_q} = m_q \langle N \mid \bar{q}q \mid N \rangle \quad , \quad \langle N \mid N \rangle = 1$$

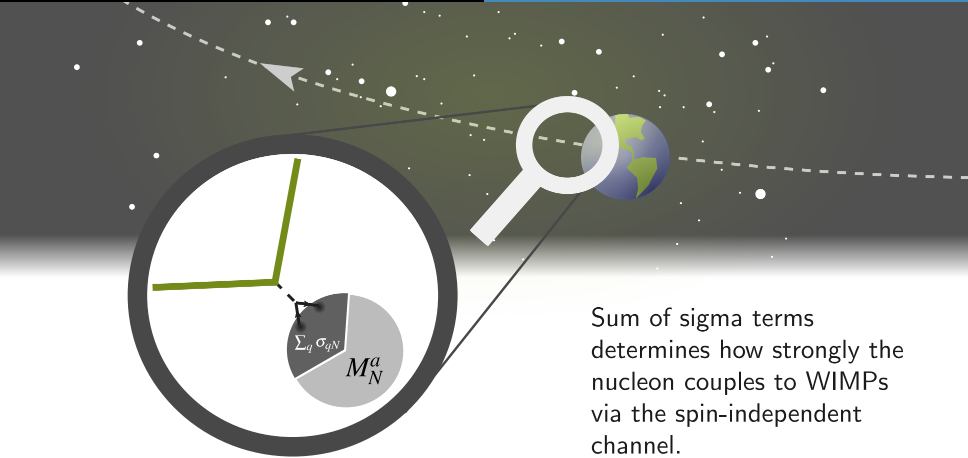
$$M_N = \sum_q \underbrace{m_q \langle N \mid \bar{q}q \mid N \rangle}_{\sigma_{qN}} + \langle N \mid O_{\text{rest}} \mid N \rangle$$

However, the nucleon states do also depend on the quark mass. Therefore

$$M_N = \langle N(m_q) \mid H(m_q) \mid N(m_q) \rangle,$$

$$M_N^{(\phi)} - M_N^{(\chi)} = \sigma_{qN} + \mathcal{O}(m_q^{(\phi)2}).$$





Sum of sigma terms determines how strongly the nucleon couples to WIMPs via the spin-independent channel.

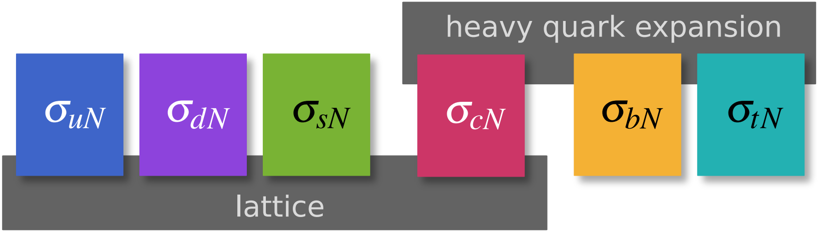
Sigma terms and dark matter detection

The Higgs field and WIMP dark matter interacts with the nucleon via the scalar quark density.

Sigma terms are required for the interpretation of direct dark matter detection experiments. [1]

[1] J. Ellis, N. Nagata and K. A. Olive, Eur. Phys. J. C **78** (2018) no.7, 569 [arXiv:1805.09795].

How to determine the sigma terms?

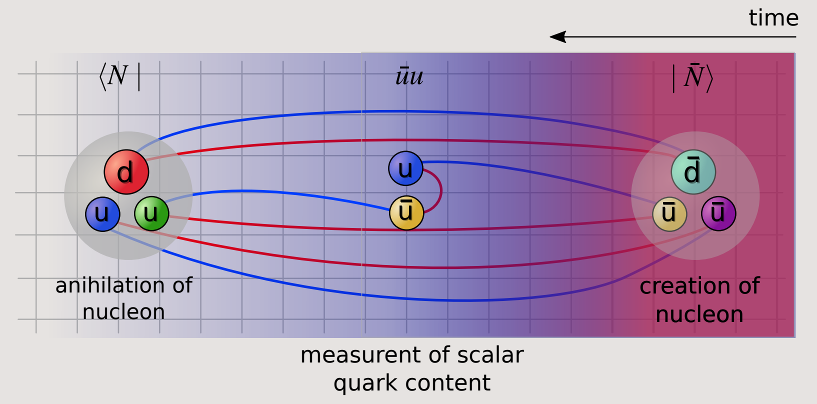


Up, down, strange, and charm sigma terms can be determined on the lattice.

For the top, bottom, and charm sigma terms an expansion in $1/m_q$ can be employed. The validity can be checked in case of σ_{cN}, where the $\mathcal{O}(m_q^{-2})$ effects are biggest.

σ_{uN} σ_{dN} σ_{sN} σ_{cN}

Direct evaluation of matrix elements $\langle N \mid m_q \bar{q}q \mid N \rangle$ requires disconnected contributions:



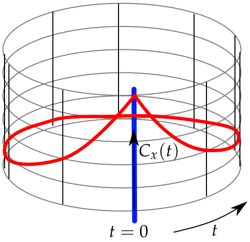
We use the Feynman-Hellmann method: $m_q \frac{\partial M_N}{\partial m_q} = m_q \langle N \mid \bar{q}q \mid N \rangle$

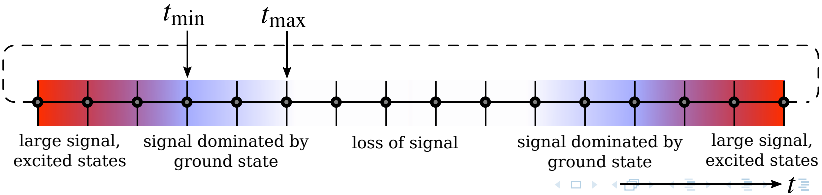
σ_{uN} σ_{dN} σ_{sN}

Extraction of hadron masses

Hadron correlation functions asymptotically behave as

$$C(t) = \begin{cases} A\sinh(-m(t - {}^{T_t}/_2)) & \text{for baryons} \\ A\cosh(-m(t - {}^{T_t}/_2)) & \text{for mesons} \end{cases}$$



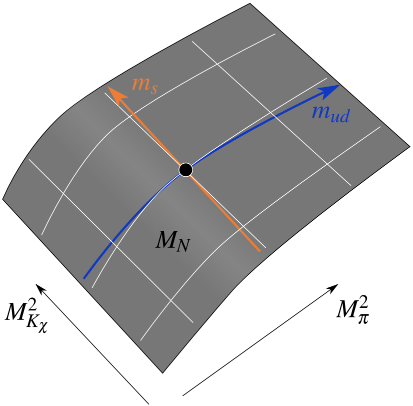


σ_{uN} σ_{dN} σ_{sN}

Strategy for the light and strange sigma terms:

1. Determine $M_N(M_\pi^2, M_{K_\chi}^2)$ from fit to lattice data. Use Wilson fermions.
2. Calculated $J = \partial \log M_{\text{meson}}^2 / \partial \log m_{\text{quarks}}$ to get light and strange sigma term. Use staggered fermions.
3. Use isospin splitting relationships to disentangle up and down sigma terms.

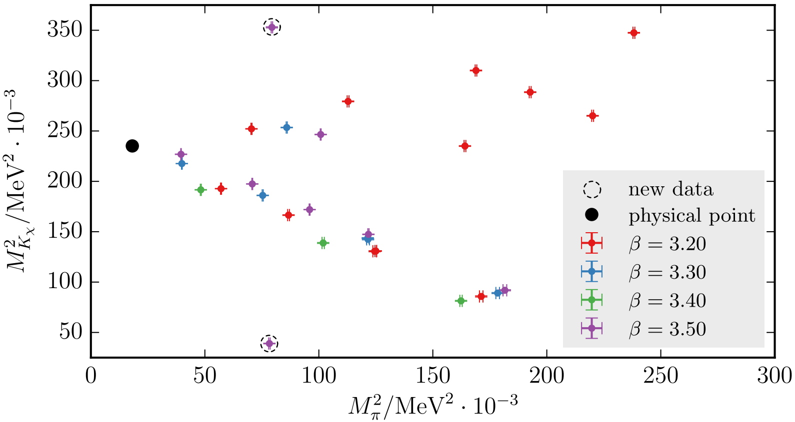
Note that we, because of the logarithmic derivative, do not need to determine the renormalization factors for the quark masses.



$$M_{K_\chi} = 2M_K^2 - M_\pi^2$$

σ_{uN} σ_{dN} σ_{sN}

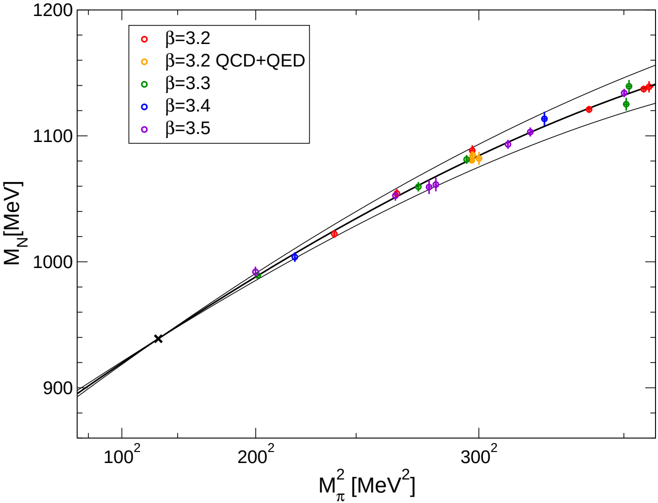
Wilson configurations



Datapoints at four different lattice spacings $a = (0.06 - 0.10)$ fm.

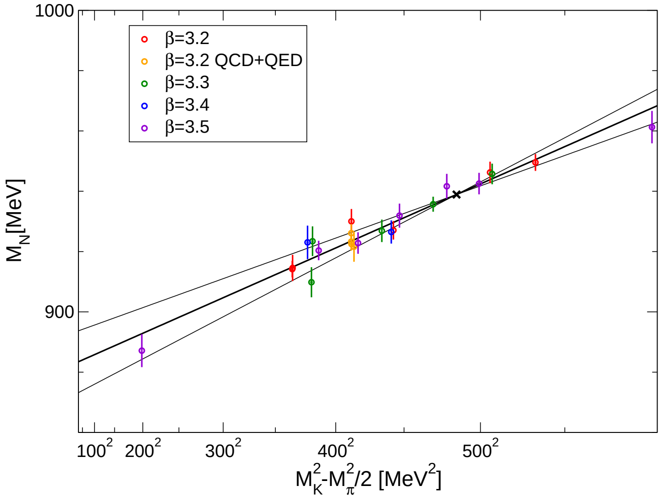
σ_{uN} σ_{dN} σ_{sN}

Nucleon fits:



σ_{uN} σ_{dN} σ_{sN}

Nucleon fits:



σ_{uN} σ_{dN} σ_{sN}

Mixing matrix: We can use a simple quadratic expansion around the physical point

$$M^2_{\text{meson}} = c_0 + (c'_{1,ud} + d_{1,ud}a^2)\Delta_{ud} + (c'_{1,s} + d_{1,s}a^2)\Delta_s + c_{2,ud,s}\Delta_{ud}\Delta_s + c_{2,ud}\Delta^2_{ud} + c_{2,s}\Delta^2_s + c_{c/s}\Delta_{c/s}$$

Normally one would like to use

$$\Delta_{ud} \propto m_{ud} - m^{(\phi)}_{ud},$$
$$\Delta_s \propto m_s - m^{(\phi)}_s,$$
$$\Delta_{c/s} = \frac{m_c}{m_s} - \left(\frac{m_c}{m_s}\right)^{(\phi)}$$

But: Quark masses require renormalization.

σ_{uN} σ_{dN} σ_{sN}

Mixing matrix: We can use a simple quadratic expansion around the physical point

$$M^2_{\text{meson}} = c_0 + (c'_{1,ud} + d_{1,ud}a^2)\Delta_{ud} + (c'_{1,s} + d_{1,s}a^2)\Delta_s + c_{2,ud,s}\Delta_{ud}\Delta_s + c_{2,ud}\Delta^2_{ud} + c_{2,s}\Delta^2_s + c_{c/s}\Delta_{c/s}$$

where we used

$$\Delta_{ud} = \frac{m_{ud}(r_0 + r_1 a^2)}{m_s^{(\phi)}[\beta]} - 1,$$

$$\Delta_s = \frac{m_s}{m_s^{(\phi)}[\beta]} - 1$$

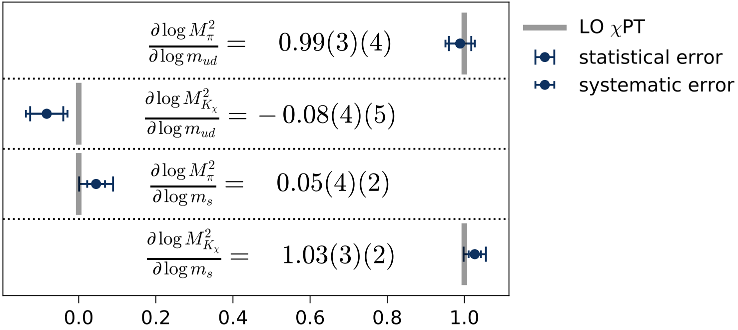
$$\Delta_{c/s} = \frac{m_c}{m_s} - \left(\frac{m_c}{m_s}\right)^{(\phi)}$$

Note that $m_x^{(\phi)}[\beta]$ is an additional fit parameter per β.

Mixing matrix:

σ_{uN} σ_{dN} σ_{sN}

The results for the mixing matrix are:

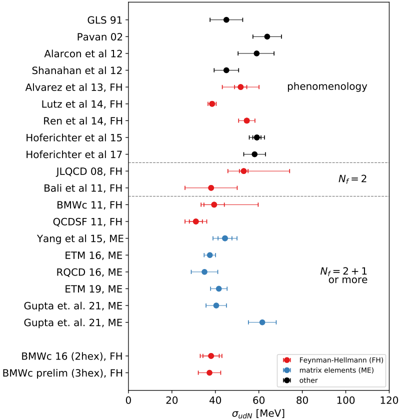


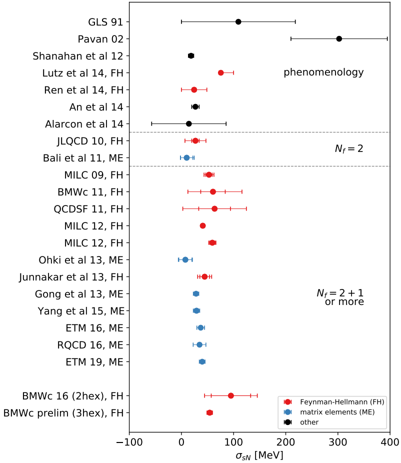
The result for the strange to light quark mass ratio are

$$\frac{m_s}{m_{ud}} = 27.293(33)(08)$$

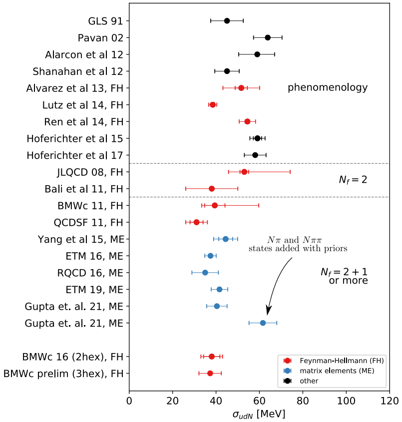
(FLAG result: $\frac{m_s}{m_{ud}} = 27.30(34)$).

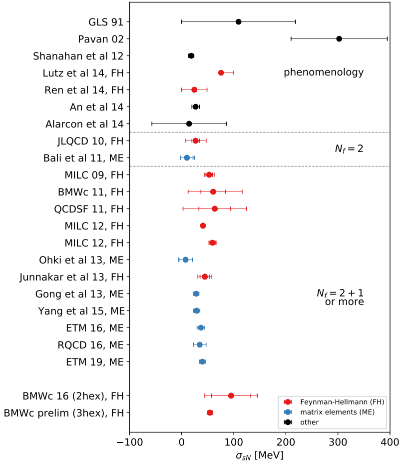
Other determinations





Other determinations

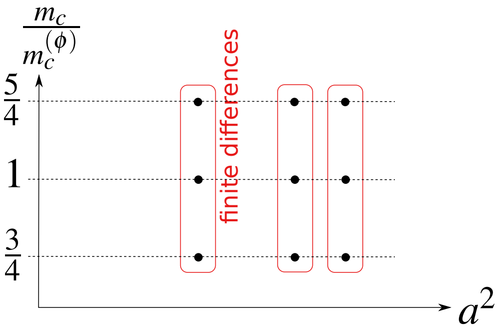




Strategy for the charm sigma term:

σ_{cN}

We measured the nucleon mass on 9 staggered, Symmanzik improved, 4 times stout smeared ensembles:



We cover

- Three lattice spacings: $\beta = 3.75$, $\beta = 3.7753$, and $\beta = 3.84$.
- Three charm masses: $\frac{3}{4} m_c^{(\phi)}$, $m_c^{(\phi)}$, $\frac{5}{4} m_c^{(\phi)}$.

We use finite differences to determine the charm sigma term.

σ_{cN}

Finite difference approximation: For each lattice spacing we used finite differences to estimate the charm sigma term. We used two differences:

$$\Delta^+ M_N = M_N(m_c = 5/4 m_c^{\text{central}}) - M_N(m_c = m_c^{\text{central}})$$
$$\Delta^- M_N = M_N(m_c = m_c^{\text{central}}) - M_N(m_c = 3/4 m_c^{\text{central}})$$

We combined them in two ways:

- The standard finite difference formula (error: $\mathcal{O}((\delta m_c/m_c)^2) = \mathcal{O}(1/16))$

$$\sigma_{cN} = m_c \frac{\partial M_N}{\partial m_c} = 2\frac{\Delta^+ M_N + \Delta^- M_N}{M_N^{(\phi)}}.$$

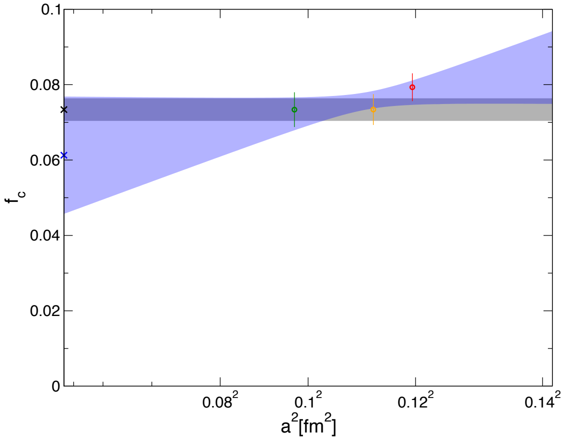
- Based on the HQ behaviour of sigma terms (error: $\mathcal{O}((\sigma_{cN}/M_N^{(\phi)})^3) = \mathcal{O}(3 \times 10^{-4}))$

$$\sigma_{cN} = \frac{1}{\log\frac{5}{4}\log\frac{4}{3}\log\frac{5}{3}}\left(\log^2\frac{4}{3}\Delta^+ M_N + \log^2\frac{5}{4}\Delta^- M_N\right)$$

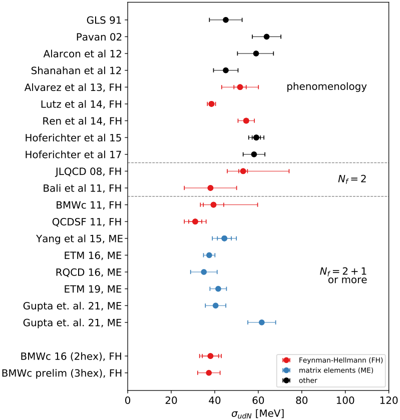
σ_{cN}

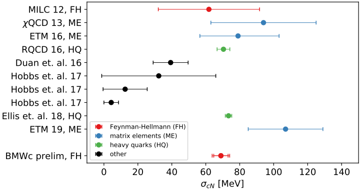
For the continuum extrapolation we used:

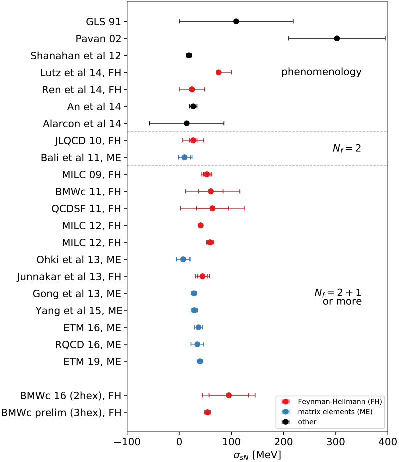
1. A constant fit in a^2 with only the two finest lattice spacings included.
2. A constant fit in a^2 with all lattice spacings included.
3. A linear fit in a^2 with all lattice spacings included.



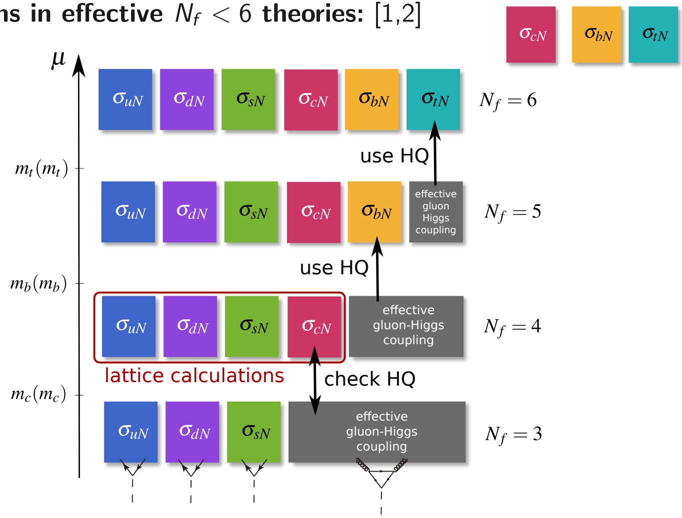
Other determinations







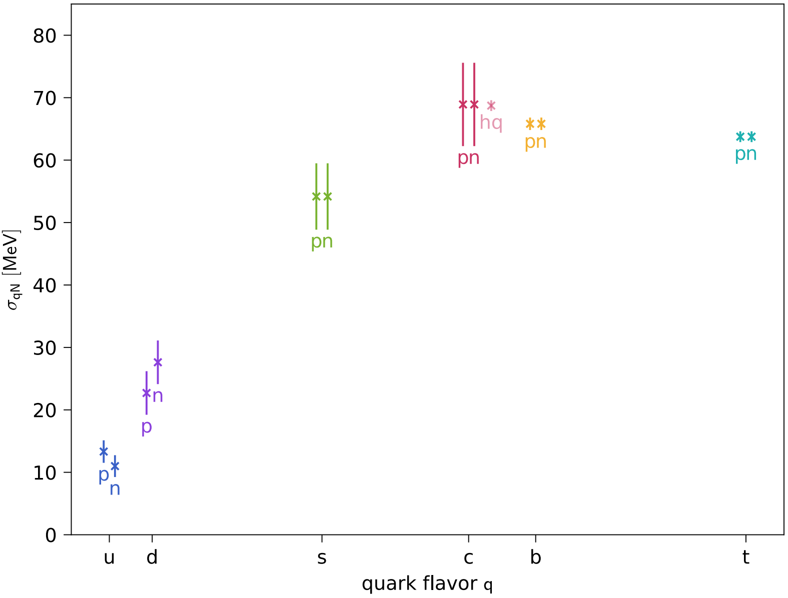
Sigma terms in effective $N_f < 6$ theories: [1,2]



[1] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. **78B** (1978) 443.

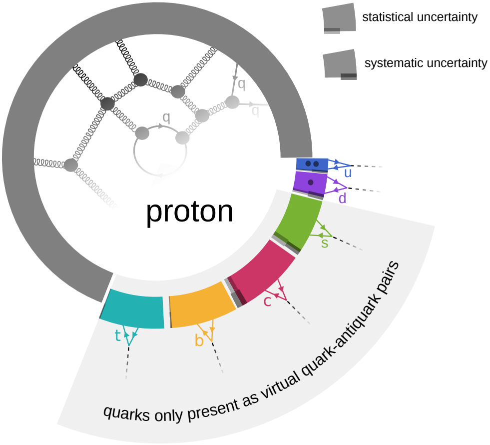
[2] R. J. Hill and M. P. Solon, Phys. Rev. D **91** (2015) 043505 [arXiv:1409.8290].

Results



80
70
60
50
40
30
20
10
0
σ_{qN} [MeV]
p
n
p
n
pn
pn
hq
pn
pn
u
d
s
c
b
t
quark flavor q

Higgs coupling of the nucleon



The nucleons couple only $\sim \frac{1}{3}$ as strongly to the Higgs field as fundamental fermions with the same mass would.